higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1040(**E**)(J25)T

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186

25 July 2018 (X-Paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. Round off calculations to THREE decimals.
7. Write down ALL the formulae used.
8. Questions must be answered in BLUE or BLACK ink.
9. Work neatly.

QUESTION 1

1.1 Given $z = \cos^2\left(x^2 y^2\right)$

Determine $\frac{\partial z}{\partial x}$ (2)

1.2 Given $z = \sin^{-1}\frac{y}{x}$

Determine $\frac{\partial z}{\partial y}$ (1)

1.3 The sides of a right-angled triangle are x and y while the hypotenuse is r.

Calculate the approximate change in area if x increases from 3 to 3,2 and y decreases from 4 to 3,8. (3)

[6]

QUESTION 2

Determine $\int$ y dx if :

2.1 $y = x^2 \ln x^2$ (3)

2.2 $y = x\tan^2 x$ (3)

2.3 $y = \frac{1}{x(x-1)+1}$ (4)

2.4 $y = \frac{1}{\tan^4 \frac{x}{4}}$ (4)

2.5 $y = \sin^5 3x \cos^3 3x$ (4)

[18]

QUESTION 3

Use partial fractions to integrate the following:

3.1 $\int \frac{3x^2 - 3x + 7}{(3x+1)(x^2 - 2x + 2)} dx$ (6)

3.2 $\int \frac{x^2 - 7x + 15}{(x^3 - 6x^2 + 9x)} dx$ (6)

[12]

QUESTION 4

4.1 Calculate the particular solution of $\ln x \frac{dy}{dx} + \frac{y}{x} - \cos x = 0$ when $x = \frac{\pi}{3}$ and $y = 2$. (6)

4.2 Find the general solution of $3\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x^2 - x + 5$. (6)

[12]

QUESTION 5

5.1 5.1.1 Sketch the graphs of $y = 2\ln\frac{x}{2}$. Show the area bounded by the graph, the lines $x = 0$, $y = 0$ and $y = 3$. Show the representative strip/element that you use to calculate the volume when the bounded area rotates about the *x*-axis. (3)

5.1.2 Calculate the volume of the solid of revolution when the area described in QUESTION 5.1.1 rotates about the *x*-axis. (4)

5.1.3 Calculate the volume moment about the y-axis of the solid generated when the area described in QUESTION 5.1.1 rotates about the *x*-axis. Also calculate the distance of the centre of gravity from the *y*-axis of the solid. (5)

5.2 5.2.1 Determine the points of intersection of $y = \frac{1}{4}x^2$ and $y = \frac{1}{4}(x+2)$.

Sketch the two graphs and show the area enclosed between the graphs. Also show the representative strip/element perpendicular to the *x*-axis that you use to calculate the enclosed area. (3)

5.2.2 Calculate the area described in QUESTION 5.2.1. (4)

5.2.3 Calculate the y-ordinate of the centroid of the area described in QUESTION 5.2.1. (5)

5.3 5.3.1 Sketch the graph of $y = e^{-x}$. Show the area bounded by the graph, the lines $x = 1$, $x = 0$ and $y = 0$. Show the representative strip/element that you use for calculations. (2)

5.3.2 Calculate the moment of inertia about the x-axis of the solid obtained when the area described in QUESTION 5.3.1 rotates about the x-axis. (4)

5.4 5.4.1 A sluice gate in the shape of an isosceles triangle is vertically placed with its vertex down in a water canal of the same shape. The height of the gate is 4 m and the depth of the water in the canal is 5 m. The gate is 4 m wide. Make a neat sketch of the cross-section of the canal and show the representative strip that you use for calculations. (2)

5.4.2 Calculate the area moment of the sluice gate about the water level by means of integration. (4)

5.4.3 Calculate the second moment of area of the sluice gate about the water level by integration as well as the depth of the centre of pressure on the gate. (4)

[40]

QUESTION 6

6.1 Calculate the length of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ $y = \frac{x^3}{6} + \frac{1}{2x}$ from the point $x = 1$ to $x = 2$. (6)

6.2 Calculate the surface area generated when the curve $y = \cos x$ $0 \leq x \leq \frac{\pi}{2}$ rotates about the x-axis. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any other applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec } x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \quad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}.\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}.\frac{d}{dx}f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}.\frac{d}{dx}f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$-a\,\mathrm{cosec}^2\,ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln[\sec ax+\tan ax]+C$
$\mathrm{cosec}\,ax$	$-a\,\mathrm{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\text{cosec}\, f(x)$	$-\text{cosec}\, f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$$

$$\int [f(x)]^n\, f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\,dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx \; ; \; A_x = \int_a^b (y_1 - y_2)\,dx$$

$$A_y = \int_a^b x\,dy \; ; \; A_y = \int_a^b (x_1 - x_2)\,dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2\pi \int_a^b xydy$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2\pi \int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho \pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho \pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3 (cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore ye^{\int P dx} = \int Qe^{\int P dx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$