higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1030**(E)**(J30)T

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

30 July 2019 (X-Paper)
09:00–12:00

A scientific calculator may be used.

This question paper consists of 5 pages and a formula sheet of 5 pages.

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. ALL final answers must be rounded off to THREE decimal places.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Questions must be answered in blue or black ink.
8. Sketches must be large, neat and fully labelled.
9. Write neatly and legibly.

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QUESTION 1

1.1 Determine the following limit:

1.1.1 $\lim\limits_{x\to 0} \dfrac{arc\sin 4x}{arc\tan 5x}$ (3)

1.1.2 $\lim\limits_{x\to 0} \sqrt{x}\ln x$ (4)

1.2 Determine the value(s) of x for which $f(x)$ is continuous if $f(x) = \sec x$; $x \in [0; 2\pi]$ (2)

[9]

QUESTION 2

2.1 Determine the derivative of $f(x) = \frac{x+1}{x+2}$ from first principles. (5)

2.2 Prove that if $y = \text{arc cosec}\, x$, then $\frac{dy}{dx} = \frac{-1}{x\sqrt{x^2-1}}$ (3)

2.3 Calculate $\frac{dy}{dx}$ of the following, by making use of the derivatives of $\sin x$ and $\cos x$, as well as the rules of differentiation:

$$y = \cot x$$ (3)

2.4 Determine $\frac{dy}{dx}$ in each of the following cases:
(Simplification is NOT required.)

2.4.1 $y = \sqrt{3x + \sqrt{2x + \sqrt{x}}}$

2.4.2 $y = \left[\left(\frac{1}{x} + 1\right)^{-1} + 1\right]^{-1}$

(2 × 3) (6)

2.5 Calculate $\frac{dy}{dx}$ if $y = (1 - 3x)^{\cos x}$ with the aid of logarithmic differentiation. (4)

2.6 Determine $\frac{dy}{dx}$ of implicit function $e^{2x+3y} = x^2 - \ln(xy^3)$. (4)

[25]

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QUESTION 3

3.1 Given: $f(x) = 2x^3 - x^2 + 5$

3.1.1 Determine the coordinates of the turning points of $f(x)$ (2)

3.1.2 Draw a table of x and $f(x)$, where x is ranging from $x = -2$ to $x = 2$ (2)

3.1.3 Draw a neat graph of $f(x)$ between the values mentioned in QUESTION 3.1.2 and show the turning points on the graph. (2)

3.1.4 Use the table mentioned in QUESTION 3.1.2 and the graph mentioned in QUESTION 3.1.3 to estimate a value for the best root between $x = -2$ and $x = -1$ of the equation $-x^3 - 5 = 0$ and then use Taylor's or Newton's method to determine a better approximation of this root. (The root should be correct to THREE decimal figures.) (4)

3.2 Air is pumped into a spherical balloon at a rate of $5\ cm^3/s$.

Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is $20\ cm$.

HINT: $V = \frac{4}{3}\pi r^3$ (4)

[14]

QUESTION 4

4.1 Determine $\int y\,dx$ for each of the following:

4.1.1 $y = 4\left(\frac{1}{x} - e^{-x}\right)\cos(e^{-x} + \ln x)$ (3)

4.1.2 $y = \frac{1}{\sqrt{4 - 9x^2}}$ (3)

4.1.3 $y = \frac{x^3 + x}{x - 1}$ (5)

4.1.4 $y = \sin 10x \sin 7x$ (3)

4.1.5 $y = \frac{x}{\sec 3x}$ (4)

4.2 Determine $\int y\,dx$ by resolving the integral into partial fractions:

$y = \frac{x^2}{x^2 - 1}$ (6)

[24]

QUESTION 5

5.1 Determine $\int_0^{\infty} e^{-st}.f(t)dt$ if $f(t) = -5$ (4)

5.2 Given: $x = \frac{y}{6}$; $y = 3x^2$

5.2.1 Calculate the coordinates of the points of intersection. (2)

5.2.2 Draw a neat graph to show the enclosed area, the representative strip and the point of intersection. (2)

5.2.3 Calculate the magnitude of the area mentioned in QUESTION 5.2.2. (3)

5.2.4 Calculate the volume of the solid of revolution formed when the area in QUESTION 5.2.2 is rotated about the x-axis. (4)

5.3 Determine from the first principles, the second moment of area of a rectangular lamina with respect to a reference axis parallel to one side of the lamina that bisects the lamina. (4)

[19]

QUESTION 6

6.1 Determine the general solution of $(3y^2 + 2)\cos x dx - 6y \sin x dy$. (3)

6.2 Determine the particular solution of $\frac{d^2y}{dx^2} = 3x^2 - 2x$ if $\frac{dy}{dx} = -1, x = 1$ and $y = 1$ (6)

[9]

TOTAL: 100

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MATHEMATICS N5

FORMULA SHEET

Any applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = {}^1\!/_2 - {}^1\!/_2 \cos 2A$

$\cos^2 A = {}^1\!/_2 + {}^1\!/_2 \cos 2A$

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos(A \pm B) = \cos A \cos B \pm \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \pm \tan A \tan B}$

$\sin A \cos B = {}^1\!/_2 [\sin(A + B) + \sin(A - B)]$

$\cos A \sin B = {}^1\!/_2 [\sin(A + B) - \sin(A - B)]$

$\cos A \cos B = {}^1\!/_2 [\cos(A + B) + \cos(A - B)]$

$\sin A \sin B = {}^1\!/_2 [\cos(A - B) - \cos(A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}; \sin x = \dfrac{1}{cosec\ x}; \cos x = \dfrac{1}{\sec x}$

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BINOMIAL THEOREM

$$(x+h)^n = x^n + \mathrm{n}x^{n-1}\,h + \frac{n(n-1)}{2!}x^{n-2}\,h^2 + \cdots$$

DIFFERENTIATION

$$e = -\frac{f(a)}{f'(a)}$$

$$r = a + e$$

PRODUCT RULE

$$\mathrm{y} = \mathrm{u}(x)\,.\,v(x)$$

$$\frac{dy}{dx} = u\,.\frac{dv}{dx} + v\,.\frac{du}{dx}$$

$$= u\,.\,v' + v\,.\,u'$$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\,.\,\frac{du}{dx} - u\,.\,\frac{dv}{dx}}{v^2}$$

$$\frac{v\,.\,u' - u\,.\,v'}{v^2}$$

CHAIN RULE

$$y = f\big(u(x)\big)$$

$$\frac{dy}{dx} = \frac{dy}{du}\,.\frac{du}{dx}$$

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$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
ax^n	nax^{n-1}	$\frac{ax^{n+1}}{n+1}+c$
a	0	$ax+c$
e^x	e^x	e^x+c
a^x	$a^x.lna$	$\frac{a^x}{lna}+c$
$\log_e x$	$\frac{1}{x}$	—
$\log_a x$	$\frac{1}{xlna}$	—
$\sin x$	$\cos x$	$-\cos x+c$
$\cos x$	$-\sin x$	$\sin x+c$
$\tan x$	$\sec^2 x$	$ln(\sec x)+c$
$\cot x$	$-\operatorname{cosec}^2 x$	$ln(\sin x)+c$
$\sec x$	$\sec x\,.\tan x$	$ln[\sec x+\tan x]+c$
$\operatorname{cosec} x$	$-\operatorname{cosec} x\,.\cos x$	$ln(\operatorname{cosec} x+\cot x)+c$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	—
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	—
$\tan^{-1} x$	$\frac{1}{1+x^2}$	—
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	—
$\sec^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$	—
$\operatorname{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$	—

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$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{a^2-x^2}}$	—	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	—	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+$ c
$\frac{1}{x\sqrt{x^2+a^2}}$	—	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+c$
$\sqrt{a^2-x^2}$	—	$\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+\frac{x}{2}\sqrt{a^2-x^2}\ +c$
$\frac{1}{x^2-a^2}$	—	$\frac{1}{2a}ln\left(\frac{x-a}{x+a}\right)+c$
$\frac{1}{a^2-x^2}$	—	$\frac{1}{2a}ln\left(\frac{a+x}{a-x}\right)+c$

INTEGRATION

$$\int f(x)\ g'(x)\ dx = f(x)\ g(x) - \int f'(x)g\ (x)dx$$

$$\int \frac{f'(x)}{f(x)}dx = ln\ f(x)\ + c$$

$$\int [f(x)]^n\ f'(x)dx = \frac{[f(x)]^{n+1}}{n\ +1} + c$$

$$\frac{f(x)}{(ax+b)(cx\ +d)} = \frac{A}{ax\ +b} + \frac{B}{cx\ +d}$$

$$\frac{f(x)}{(x\ +a\ )^n} = \frac{A}{(x\ +a)} + \frac{B}{(x\ +a)^2} + \frac{C}{(x+a)^3} + \cdots \frac{Z}{(x\ +a)^n}$$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b ydx\ ;\ A_x = \int_a^b (y_1 - y_1)dx$$

$$A_x = \int_a^b xdy\ ; A_{Y=} \int_a^b (x_1 - x_2)dy$$

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VOLUMES

$V_x = \pi \int_a^b y^2\,dx$; $V_{X=\pi} \int_a^b (y_1^2 - y_2^2)dx$

$V_y = \pi \int_a^b x^2\,dy$; $V_{y=\pi} \int_a^b (x_1^2 - x_2^2)dy$

SECOND MOMENT OF AREA

$I_x = \int_a^b r^2\,dA$; $I_{y=} \int_a^b r^2\,dA$

MOMENT OF INERTIA

$\text{Mass} = \text{density} \times \text{volume}$

$\text{M} = \rho\text{V}$

$\text{DEFINITION} : \text{I} = \text{m}\,\text{r}^2$

$\text{GENERAL} : \text{I} = \int_a^b \text{r}^2\text{dm} = \rho \int_a^b \text{r}^2\text{dV}$

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