



**higher education  
& training**

Department:  
Higher Education and Training  
**REPUBLIC OF SOUTH AFRICA**

# **MARKING GUIDELINE**

**NATIONAL CERTIFICATE**

**MATHEMATICS N5**

**7 April 2021**

**This marking guideline consists of 12 pages.**

**QUESTION 1**

1.1 1.1.1

$$\begin{aligned} \lim_{x \rightarrow \infty} \left[ x \ln \left( 1 + \frac{3}{x} \right) \right] \\ \lim_{x \rightarrow \infty} \frac{\ln \left( 1 + \frac{3}{x} \right)}{\frac{1}{x}} \quad \checkmark \quad \left( \frac{0}{0} \right) \\ = \lim_{x \rightarrow \infty} \frac{-3x^{-2} \times \frac{1}{1 + \frac{3}{x}}}{-x^{-2}} \quad \checkmark \\ = \lim_{x \rightarrow \infty} \frac{3}{1 + \frac{3}{x}} \quad \checkmark \\ = 3 \quad \checkmark \end{aligned}$$

(4)

1.1.2

$$\begin{aligned} \lim_{x \rightarrow 7} \frac{\frac{1}{7} - \frac{1}{x}}{x - 7} \quad \left( \frac{0}{0} \right) \\ = \lim_{x \rightarrow 7} \frac{-x^{-2}}{1} \quad \checkmark \\ = -\frac{1}{49} \quad \checkmark \end{aligned}$$

(2)

1.2 1.2.1

$$\begin{aligned} -\log y = \lim_{x \rightarrow 0} \frac{\sin 7x}{x} \quad \left( \frac{0}{0} \right) \\ = \lim_{x \rightarrow 0} \frac{7 \cos 7x}{1} \quad \checkmark \\ = 7 \end{aligned}$$

$$\log y = -7 \quad \checkmark \quad (2)$$

1.2.2

$$y = 10^{-7} \quad \checkmark \quad (1)$$

1.3  $f(x) = \frac{e^{x^2+1}}{e^x - 2e^{1-x}}$

$$e^x - 2e^{1-x} = 0$$

$$e^x - \frac{2e}{e^x} = 0$$

$$e^{2x} - 2e = 0 \checkmark$$

$$e^{2x} = 2e$$

$$2x = 1 + \ln 2$$

$$x = \frac{1 + \ln 2}{2} \checkmark$$

(2)  
[11]

**QUESTION 2**

2.1  $f(x) = \frac{3}{x^5}$

2.1.1  $f(x + h) = 3(x + h)^{-5}$

$$= 3 \left[ \frac{x^{-5}h^0}{0!} + \frac{(-5)x^{-6}h}{1!} + \frac{30x^{-7}h^2}{2!} + \frac{(-210)x^{-8}h^3}{3!} + \dots \dots \dots \right]$$

$$= 3x^{-5} - 15x^{-6}h + 45x^{-7}h^2 - 105x^{-8}h^3 + \dots \dots \dots \checkmark \checkmark \quad (2)$$

2.1.2  $f(x + h) - f(x) = -15x^{-6}h + 45x^{-7}h^2 - 105x^{-8}h^3 + \dots \dots \checkmark \quad (1)$

2.1.3  $\frac{f(x + h) - f(x)}{h} = -15x^{-6} + 45x^{-7}h - 105x^{-8}h^2 + \dots \dots \checkmark \quad (1)$

2.1.4  $\lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} = -15x^{-6} \checkmark \quad (1)$

$$2.2 \quad y = e^{\ln(\tan x)}$$

$$y = \tan x \checkmark$$

$$= \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cos x(\cos x) - \sin x(-\sin x) \checkmark}{\cos^2 x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$$

$$= \frac{1 \checkmark}{\cos^2 x}$$

$$= \sec^2 x$$

(3)

$$2.3 \quad 2.3.1 \quad y = [\ln(x^2 + 1) - \tan^{-1}(6x)]^{10}$$

$$\frac{dy}{dx} = 10[\ln(x^2 + 1) - \tan^{-1}(6x)]^9 \checkmark \times \left[ \frac{2x}{x^2 + 1} \checkmark - \frac{6}{36x^2 + 1} \checkmark \right]$$

$$2.3.2 \quad y = \frac{1 + \sin^{-1} x}{1 - \cos^{-1} x}$$

$$\frac{dy}{dx} = \frac{\frac{1 - \cos^{-1} x}{\sqrt{1-x^2}} \checkmark - \left( -\frac{1 + \sin^{-1} x}{\sqrt{1-x^2}} \right) \checkmark}{(1 - \cos^{-1} x)^2 \checkmark}$$

$$= \frac{\frac{\sin^{-1} x + \cos^{-1} x}{\sqrt{1-x^2}}}{(1 - \cos^{-1} x)^2}$$

(2 × 3) (6)

$$2.4 \quad y = (2x - e^{8x})^{\sin 2x}$$

$$\ln y = \sin 2x \cdot \ln(2x - e^{8x})$$

$$\frac{1}{y} \cdot \frac{dy}{dx} \checkmark = 2 \cos 2x \cdot \ln(2x - e^{8x}) \checkmark + \sin 2x \times \frac{1}{2x - e^{8x}} \times (2 - 8e^{8x}) \checkmark$$

$$\frac{dy}{dx} = y \left[ 2 \cos 2x \cdot \ln(2x - e^{8x}) + \frac{(2 - 8e^{8x}) \sin 2x}{(2x - e^{8x})} \right]$$

$$\frac{dy}{dx} = (2x - e^{8x})^{\sin 2x} \left[ 2 \cos 2x \cdot \ln(2x - e^{8x}) + \frac{(2 - 8e^{8x}) \sin 2x}{(2x - e^{8x})} \right] \checkmark$$

(4)

$$2.5 \quad \tan(x^2y^4) = 3x + y^2$$

$$\sec^2(x^2y^4) \left[ 2xy^4 + 4x^2y^3 \cdot \frac{dy}{dx} \right] \checkmark = 3 + 2y \frac{dy}{dx} \checkmark$$

$$\frac{dy}{dx} [4x^2y^3 \cdot \sec^2(x^2y^4) - 2y] = 3 - 2xy^4 \cdot \sec^2(x^2y^4) \checkmark$$

$$\frac{dy}{dx} = \frac{3 - 2xy^4 \cdot \sec^2(x^2y^4)}{4x^2y^3 \cdot \sec^2(x^2y^4) - 2y} \checkmark$$

(4)  
[22]**QUESTION 3**

3.1 3.1.1

$$f(x) = x^3 - 7x^2 + 8x + 3$$

$$f'(x) = 3x^2 - 14x + 8 = 0$$

$$(3x - 2)(x - 4) = 0$$

$$x = \frac{3}{2} \quad \text{or} \quad x = 4$$

$$y = \frac{21}{8} \quad y = -13$$

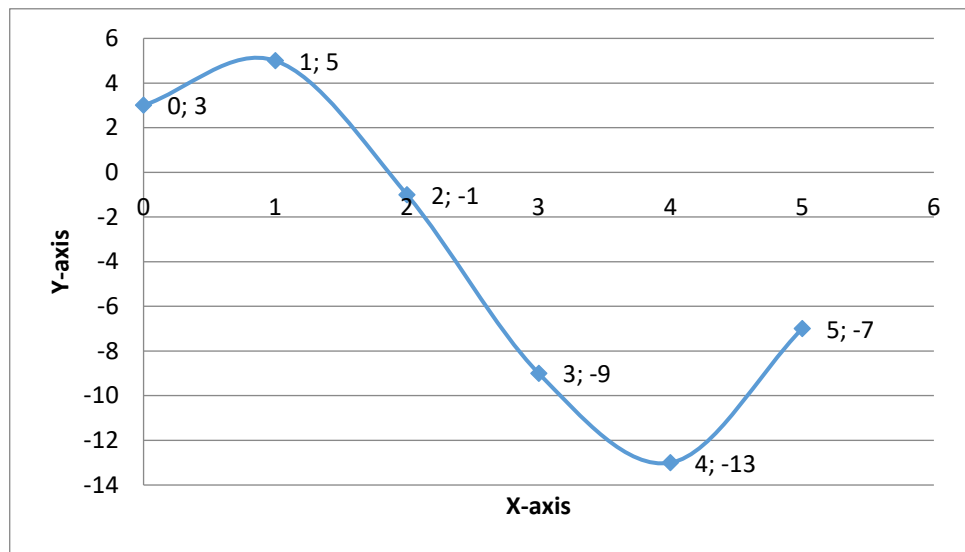
Thus, the coordinates of the turning points are  $\left(\frac{3}{2}; \frac{21}{8}\right) \checkmark$  and  $(4; -13) \checkmark$  (2)

3.1.2

$x$	0	1	2	3	4	5
$y$	3	5	-1	-9	-13	-7

(Any 3 × 1) (3)

3.1.3



(1 × 1 for shape and 1 × 1 for indication of turning points) (2)

3.1.4 Let  $x_0 = 1.9$

$$f(1.9) = -0,211$$

$$f'(1.9) = -7,77 \checkmark$$

$$x_1 = 1.9 - \frac{(-0,211)}{(-7,77)} \checkmark$$

$$= 1.872 \checkmark$$

(3)

3.2  $xy = 750 \dots \dots \dots (1) \checkmark$

$$S = y + 10x \dots \dots \dots (2) \checkmark$$

$$y = \frac{750}{x} \dots \dots \dots (3)$$

$$S = \frac{750}{x} + 10x$$

$$S' = -\frac{750}{x^2} + 10 = 0 \checkmark$$

$$10x^2 = 750$$

$$x = \pm\sqrt{75} = 5\sqrt{3} \checkmark$$

$$y = 50\sqrt{3} \checkmark$$

(5)  
[15]

## QUESTION 4

4.1 4.1.1

$$\int \left[ 4 \left( \frac{1}{x} - e^{-x} \right) \cos(e^{-x} + \ln x) \right] dx$$

$$\text{let } u = e^{-x} + \ln x$$

$$du = \left( \frac{1}{x} - e^{-x} \right) dx \checkmark$$

$$= 4 \int \cos u \, du \checkmark$$

$$= 4 \sin u + C$$

$$= 4 \sin(e^{-x} + \ln x) \checkmark + C \quad (3)$$

4.1.2

$$\int \frac{1}{\sqrt{4 - 9x^2}} dx$$

$$= \frac{1}{2} \checkmark \int \frac{1}{\sqrt{1 - \frac{9}{4}x^2}} dx \checkmark$$

$$= \frac{1}{2} \left( \frac{2}{3} \right) \sin^{-1} \left( \frac{3}{2}x \right) + C$$

$$= \frac{1}{3} \sin^{-1} \left( \frac{3}{2}x \right) + C \checkmark$$

(3)

4.1.3

$$\int \frac{2x^2 - 5x - 1}{x - 3} dx$$

$$= \int \left( 2x + 1 - \frac{2}{x - 3} \right) dx \checkmark$$

$$= x^2 \checkmark + x \checkmark - 1 \ln(x - 3) \checkmark + C \quad (4)$$

4.1.4

$$\int \sin(\pi x) \sin\left(\frac{x}{5}\right) dx$$

$$= \int \frac{1}{2} \left[ \cos\left(\pi - \frac{1}{5}\right)x - \cos\left(\pi + \frac{1}{5}\right)x \right] dx \checkmark \checkmark$$

$$= \frac{\sin\left(\pi - \frac{1}{5}\right)x}{2\left(\pi - \frac{1}{5}\right)} \checkmark - \frac{\sin\left(\pi + \frac{1}{5}\right)x}{2\left(\pi + \frac{1}{5}\right)} \checkmark + C$$

(4)

4.1.5

$$\int \frac{\ln x}{x^2} dx$$

$\begin{aligned} \text{let } u &= \ln x & dv &= \frac{1}{x^2} dx \checkmark \\ du &= \frac{1}{x} dx & v &= -\frac{1}{x} \end{aligned}$
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$$\int \frac{\ln x}{x^2} dx = -\frac{1}{x} \cdot \ln x - \int -\frac{1}{x} \cdot \frac{1}{x} dx \checkmark$$

$$= -\frac{\ln x}{x} \checkmark - \frac{1}{x} \checkmark + C$$

(4)

4.2

$$\int \frac{x+2}{(x-1)^2} dx$$

$$\frac{x+2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$x+2 = A(x-1) + B \checkmark$$

$\text{let } x = 1, \quad B = 3 \checkmark$
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or

$x = 0, \quad A = 1 \checkmark$
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$$= \int \left( \frac{1}{x-1} + \frac{3}{(x-1)^2} \right) dx$$

$$= \ln(x-1) \checkmark - \frac{3}{x-1} \checkmark + C$$

(5)  
[23]**QUESTION 5**

5.1

$$\int_0^2 \left( e^x + \frac{1}{x^2+1} \right) dx$$

$$= [e^x + \tan^{-1} x]_0^2 \checkmark$$

$$= (e^2 + \tan^{-1} 2) - (e^0 + \tan^{-1} 0) \checkmark$$

$$= 69.824 \checkmark$$

(3)



5.2      5.2.1

$$6 + x - x^2 = \frac{2}{3}x^2 - 4x + 6$$

$$18 + 3x - 3x^2 = 2x^2 - 12x + 18$$

$$-18 - 3x + 3x^2 + 2x^2 - 12x + 18 = 0$$

$$5x(x - 3) = 0$$

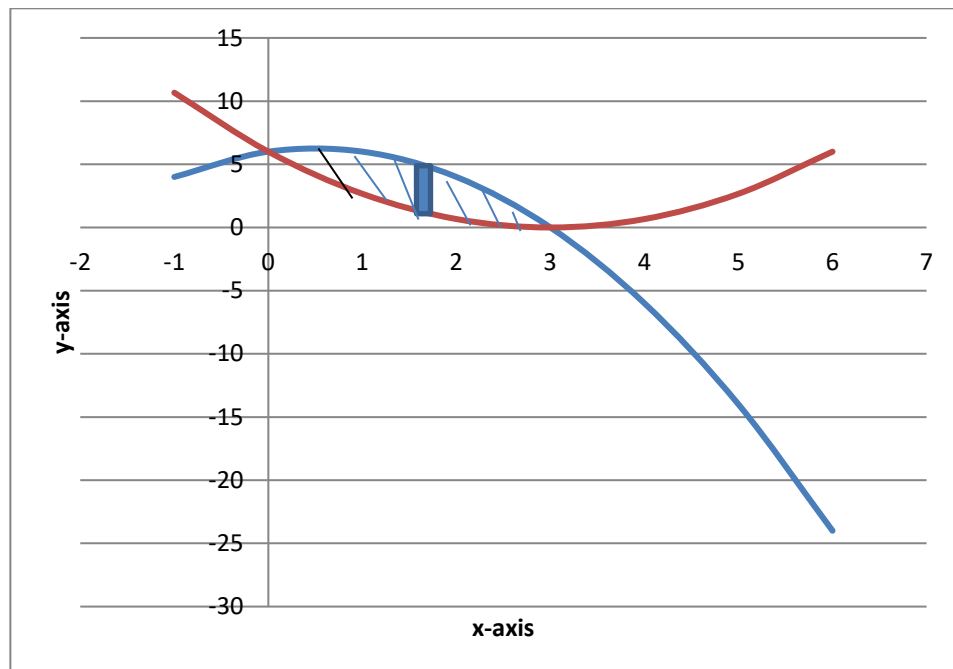
$$x = 0 \text{ or } x = 3$$

$$y = 6 \quad y = 0$$

Thus, the coordinates of the points of intersection are  $(0; 6)$ ✓ and  $(3; 0)$ ✓.

(2)

5.2.2



$(1 \times 1$  for indication of enclosed area and  $1 \times 1$  for indication of the vertical or horizontal strip)

(2)

5.2.3

$$\begin{aligned}
 A &= \int_a^b (y_1 - y_2) dx \\
 A &= \int_0^3 \left[ (6 + x - x^2) - \left( \frac{2}{3}x^2 - 4x + 6 \right) \right] dx \checkmark \\
 &= \int_0^3 \left( 6 + x - x^2 - \frac{2}{3}x^2 + 4x - 6 \right) dx \\
 &= \int_0^3 \left( 5x - \frac{5}{3}x^2 \right) dx \\
 &= \left[ \frac{5}{2}x^2 - \frac{5}{9}x^3 \right]_0^3 \checkmark \\
 &= \frac{15}{2} \text{ or } 7,5 \text{ units}^2 \checkmark
 \end{aligned} \tag{3}$$

5.2.4

$$\begin{aligned}
 V &= \pi \int_0^3 \left[ (6 + x - x^2)^2 - \left( \frac{2}{3}x^2 - 4x + 6 \right)^2 \right] dx \\
 &= \pi \int_0^3 \left( 36 + 12x - 11x^2 - 2x^3 + x^4 - \frac{4}{9}x^4 + \frac{16}{3}x^3 - 24x^2 + 48x \right. \\
 &\quad \left. - 36 \right) dx \checkmark \checkmark \\
 &= \pi \int_0^3 \left( 60x - 35x^2 + \frac{10}{3}x^3 + \frac{5}{9}x^4 \right) dx \checkmark \\
 &= \pi \left[ 30x^2 - \frac{35}{3}x^3 + \frac{5}{6}x^4 + \frac{4}{45}x^5 \right]_0^3 \checkmark \\
 &= \frac{441}{10} \pi \text{ or } 138,544 \text{ units}^3 \checkmark
 \end{aligned} \tag{5}$$

5.3  $m = \sigma A$

$$dm = \sigma dA$$

$$dI_z = r^2$$

$$= x^2 \sigma dm \checkmark$$

$$= x^2 \sigma 2\pi x dx \checkmark$$

$$= 2\pi\sigma \int_0^r x^3 dx \checkmark$$

$$= 2\pi\sigma \left[ \frac{1}{4} x^4 \right]_0^r \checkmark$$

$$= 2\pi\sigma \left[ \frac{r^4}{4} - 0 \right]$$

$$= \frac{1}{2} \pi \sigma r^4 \text{units}^4 \checkmark$$

(5)  
[20]**QUESTION 6**

6.1 
$$\frac{dy}{dx} = \frac{x^2}{1-y^2}$$

$$(1-y^2) dy = x^2 dx \checkmark$$

$$y - \frac{y^3}{3} \checkmark = \frac{x^3}{3} + C \checkmark$$

(3)

$$6.2 \quad \frac{d^2y}{dx^2} = \frac{4}{3}x^3 + x$$

$$\frac{dy}{dx} = \frac{1}{3}x^4 + \frac{1}{2}x^2 + A \checkmark$$

$$\frac{1}{3} = \frac{1}{3}(0)^4 + \frac{1}{2}(0)^2 + A$$

$$A = \frac{1}{3} \checkmark$$

$$\frac{dy}{dx} = \frac{1}{3}x^4 + \frac{1}{2}x^2 + \frac{1}{3} \checkmark$$

$$y = \frac{1}{(3)(5)}x^5 + \frac{1}{(2)(3)}x^3 + \frac{1}{3}x + B \checkmark$$

$$1 = \frac{1}{15}(0)^5 + \frac{1}{6}(0)^3 + \frac{1}{3}(0) + B$$

$$B = 1 \checkmark$$

$$y = \frac{1}{15}x^5 + \frac{1}{6}x^3 + \frac{1}{3}x + 1 \checkmark$$

(6)  
[9]

**TOTAL: 100**