higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

2 April 2020 (X-paper)
09:00–12:00

Scientific calculators may be used.

This question paper consists of 5 pages and a formula sheet of 4 pages.

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
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MATHEMATICS N5

TIME: 3 HOURS

MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show all intermediate steps and simplify where possible.
5. All final answers must be rounded off to THREE decimals.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Sketches must be large, neat and fully labelled
8. Write neatly and legibly.

QUESTION 1

1.1 Determine the following limits:

1.1.1 $\lim\limits_{x \to \frac{1}{9}} \dfrac{9x - 1}{3\sqrt{x} - 1}$ (3)

1.1.2 $\lim\limits_{x \to 0} (\operatorname{cosec} x - \cot x)$ (4)

1.2 Determine the value(s) of x for which $f(x)$ is discontinuous:

$$f(x) = \frac{x^2 + 4x - 12}{x^2 - 2x}$$

(2)

[9]

QUESTION 2

2.1 Derive a formula to determine $\dfrac{dy}{dx}$; if $y = arc \cot x$. (2)

2.2 Determine $\dfrac{dy}{dx}$ in each of the following cases (simplification NOT required):

2.2.1 $y = \ln[\cos^5(3x^4)]$ (4)

2.2.2 $y = \left(\dfrac{8x - x^6}{x^3}\right)^{-\frac{4}{5}}$ (4)

2.2.3 $y = \sqrt{1 + \sqrt{1 + x^2}}$ (3)

2.3 Calculate $\dfrac{dy}{dx}$ if $y = \left(1 + \dfrac{1}{x}\right)^x$ with aid of logarithmic differentiation. (4)

2.4 Given the implicit function $25x^2 + 9y^2 - 70x - 30y + 49 = 0$

2.4.1 Determine $\dfrac{dy}{dx}$ (3)

2.4.2 Determine the equation of the tangent to the graph at the point (2;3). (3)

[23]

QUESTION 3

3.1 Given: $f(x) = x^3 - 3x + 1$

3.1.1 Determine the coordinates of the turning points of $f(x)$ (2)

3.1.2 Draw up a table of x and $f(x)$, where x is ranging from $x = -2$ to $x = 2$. (3)

3.1.3 Draw a neat graph of $f(x)$ between these values showing the turning points on it. (2)

3.1.4 Use the table and the graph to estimate a value for the best root between $x = 1$ and $x = 2$ of the equation $x^3 - 3x + 1 = 0$ and then use Taylor's/Newton's method TWICE to determine a better approximation of this root. (Root correct to THREE decimal figures) (4)

3.2 Water is being poured into a conical reservoir at a rate of $\pi m^3/s$. The reservoir has a radius of 6 metres across the top and a height of 12 metres.

At what rate is the depth of water increasing when the depth is 6 metres?

HINT: $V = \frac{1}{3}\pi r^2 h$

(4)

[15]

QUESTION 4

4.1 Determine $\int y\, dx$ in each of the following cases:

4.1.1 $y = \dfrac{4}{\sec x e^{\sin x}}$ (3)

4.1.2 $y = \dfrac{(\cos^{-1} 2x)^4}{\sqrt{1 - 4x^2}}$ (3)

4.1.3 $y = \dfrac{x^3 + 2x^2 + 9x - 17}{x + 4}$ (5)

4.1.4 $y = \cos 3\pi x . \cos \pi x$ (3)

4.1.5 $y = x\sqrt{1 + 2x}$ (4)

4.2 Determine $\int y\,dx$ by resolving the integral into partial fractions:

$$y = \frac{3x}{(2x+1)(x+4)}$$

(5)
[23]

QUESTION 5

5.1 Determine $\int_0^\infty e^{-st}.f(t)dt$ if $f(t) = \frac{t}{\pi}$ (5)

5.2 Given : $y = x + 1,\ x = 0$ and $y = 2x^2$

5.2.1 Calculate the coordinates of the points of intersection. (2)

5.2.2 Make a neat sketch to show the enclosed area, the representative strip and the point of intersection. (2)

5.2.3 Calculate the magnitude of the area in QUESTION 5.2.2. (3)

5.2.4 Calculate the volume of the solid of revolution formed when the area in QUESTION 5.2.2 is rotated about the x-axis. (4)

5.3 Calculate the second moment of area of a rectangular lamina with sides 6 cm × 3 cm and about a 3 cm side. (4)
[20]

QUESTION 6

6.1 Determine the particular solution of $\frac{dy}{dx} = \frac{x\left(e^{x^2}+2\right)}{6y^2}, y(0) = 1$ (4)

6.2 Determine the particular solution of $x^2.\frac{d^2y}{dx^2} = x^6 + x^3 - 2x^2$ if $\frac{dy}{dx} = \frac{3}{5}, x = 1$ and $y = \frac{1}{5}$

(6)
[10]

TOTAL: 100

MATHEMATICS N5

FORMULA SHEET

Any applicable formula may also be used

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$1 + \cot^2 x = \text{cosec}^2 x$
$\sin 2A = 2\sin A \cos A$
$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$
$\sin^2 A = {}^1/_2 - {}^1/_2 \cos 2A$
$\cos^2 A = {}^1/_2 + {}^1/_2 \cos 2A$

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$
$\cos(A \pm B) = \cos A \cos B \pm \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \pm \tan A \tan B}$

$\sin A \cos B = {}^1/_2 [\sin(A + B) + \sin(A - B)]$
$\cos A \sin B = {}^1/_2 [\sin(A + B) - \sin(A - B)]$
$\cos A \cos B = {}^1/_2 [\cos(A + B) + \cos(A - B)]$
$\sin A \sin B = {}^1/_2 [\cos(A - B) - \cos(A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}; \sin x = \dfrac{1}{cosec\ x}; \cos x = \dfrac{1}{\sec x}$

BINOMIAL THEOREM

$(x + h)^n = x^n + \text{n}x^{n-1}\, h + \dfrac{n(n-1)}{2!} x^{n-2}\, h^2 + \cdots$

DIFFERENTIATION

$e = -\dfrac{f(a)}{f'(a)}$
$r = a + e$

PRODUCT RULE

$\text{y} = \text{u}(x) \,.\, v(x)$
$\dfrac{dy}{dx} = u \,.\dfrac{dv}{dx} + v \,.\dfrac{du}{dx}$
$= u \,.\, v' + v \,.\, u'$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v \,.\, \frac{du}{dx} - u \,.\, \frac{dv}{dx}}{v^2}$$

$$\frac{v \,.\, u' - u \,.\, v'}{v^2}$$

CHAIN RULE

$$y = f\big(u(x)\big)$$

$$\frac{dy}{dx} = \frac{dy}{du} \,.\, \frac{du}{dx}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
ax^n	nax^{n-1}	$\frac{ax^{n+1}}{n+1} + c$
a	0	$ax + c$
e^x	e^x	$e^x + c$
a^x	$a^x . lna$	$\frac{a^x}{lna} + c$
$\log_e x$	$\frac{1}{x}$	——
$\log_a x$	$\frac{1}{xlna}$	——
$\sin x$	$\cos x$	$-\cos x + c$
$\cos x$	$-\sin x$	$\sin x + c$
$\tan x$	$\sec^2 x$	$ln(\sec x) + c$
$\cot x$	$-\operatorname{cosec}^2 x$	$ln(\sin x) + c$
$\sec x$	$\sec x \,.\tan x$	$ln[\sec x + \tan x] + c$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \,.\cos x$	$ln(\operatorname{cosec} x + \cot x) + c$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	——
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	——
$\tan^{-1} x$	$\frac{1}{1+x^2}$	——
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	——
$\sec^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$	——
$\operatorname{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$	——

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$

$\frac{1}{\sqrt{a^2-x^2}}$	—	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	—	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+ c$
$\frac{1}{x\sqrt{x^2+a^2}}$	—	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+c$
$\sqrt{a^2-x^2}$	—	$\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+$ $\frac{x}{2}\sqrt{a^2-x^2}\ +c$
$\frac{1}{x^2-a^2}$	—	$\frac{1}{2a}ln\left(\frac{x-a}{x+a}\right)+c$
$\frac{1}{a^2-x^2}$	—	$\frac{1}{2a}ln\left(\frac{a+x}{a-x}\right)+c$

INTEGRATION

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)g\,(x)dx$$

$$\int \frac{f'(x)}{f(x)}dx = ln\,f(x)\ + c$$

$$\int [f(x)]^n\,f'(x)dx = \frac{[f(x)]^{n+1}}{n\ +1} + c$$

$$\frac{f(x)}{(ax+b)(cx\ +d)} = \frac{A}{ax\ +b} + \frac{B}{cx\ +d}$$

$$\frac{f(x)}{(x\ +a\ )^n} = \frac{A}{(x\ +a)} + \frac{B}{(x\ +a)^2} + \frac{C}{(x+a)^3} + \cdots \frac{Z}{(x\ +a)^n}$$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y dx\ ;\ A_x = \int_a^b (y_1 - y_1)dx$$

$$A_x = \int_a^b xdy\ ; A_{Y=} \int_a^b (x_1 - x_2)dy$$

VOLUMES

$$V_x = \pi\int_a^b y^2\,dx\ ;\ V_{X=\pi}\int_a^b (y_1^2 - y_2^2)dx$$

$$V_y = \pi\int_a^b x^2\,dy\ ;\ V_{y=\pi}\int_a^b (x_1^2 - x_2^2)dy$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2\, dA\ ;\ I_{y=} \int_a^b r^2\, dA$$

MOMENT OF INERTIA

$Mass\ = density\ \times volume$

$M\ = pv$

$DEFINITION : I\ = m\, r^2$

$$GENERAL : I\ =\ \int_a^b r^2 dm\ = p \int_a^b r^2 dV$$