higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

2 April 2020

This marking guideline consists of 11 pages.

QUESTION 1

1.1 1.1.1 $\lim\limits_{x\to\frac{1}{9}} \dfrac{9x-1}{3\sqrt{x}-1} \quad \left(\dfrac{0}{0}\right)$

$= \lim\limits_{x\to\frac{1}{9}} \dfrac{9}{\frac{3}{2\sqrt{x}}}$ ✓

$= \lim\limits_{x\to\frac{1}{9}} 6\sqrt{x}$ ✓

$= 2$ ✓ (3)

1.1.2 $\lim\limits_{x\to 0}(\operatorname{cosec} x - \cot x) \quad (\infty - \infty)$

$= \lim\limits_{x\to 0}\left(\dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}\right)$ ✓

$= \lim\limits_{x\to 0}\left(\dfrac{1-\cos x}{\sin x}\right)$ ✓ $\left(\dfrac{0}{0}\right)$

$= \lim\limits_{x\to 0}\left(\dfrac{\sin x}{\cos x}\right)$ ✓

$= 0$ ✓ (4)

1.2 $f(x) = \dfrac{x^2+4x-12}{x^2-2x}$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0$ ✓ $\quad or \quad x = 2$ ✓ (2)

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QUESTION 2

2.1 $y = arc\cot x$

$\cot y = x$

$-\operatorname{cosec}^2 y \dfrac{dy}{dx} = 1$ ✓

$= -\dfrac{1}{1+\cot^2 y}$ ✓

$= -\dfrac{1}{1+x^2}$ (2)

2.2 2.2.1 $y = \ln(\cos^5(3x^4))$

$$\frac{dy}{dx} = \frac{1}{\cos^5(3x^4)}\checkmark \times 5\cos^4(3x^4)\checkmark \times\ (-\sin 3x^4)\checkmark \times 12x^3\ \checkmark \quad (4)$$

2.2.2 $$y = \left(\frac{8x - x^6}{x^3}\right)^{-\frac{4}{5}}$$

$$\frac{dy}{dx} = -\frac{4}{5}\left(\frac{8x - x^6}{x^3}\right)^{-\frac{9}{5}}\checkmark \times \frac{x^3(8 - 6x^5)\checkmark - 3x^2(8x - x^6)\checkmark}{(x^3)^2\checkmark} \quad (4)$$

2.2.3 $$y = \sqrt{1 + \sqrt{1 + x^2}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(1 + \sqrt{1 + x^2}\right)^{-\frac{1}{2}}\checkmark \times \frac{1}{2}(1 + x^2)^{-\frac{1}{2}}\checkmark \times 2x\checkmark \quad (3)$$

2.3 $$y = \left(1 + \frac{1}{x}\right)^x$$

$$\ln y = x\ln\left(1 + \frac{1}{x}\right)$$

$$\frac{1}{y}\frac{dy}{dx}\checkmark = \ln\left(1 + \frac{1}{x}\right)\checkmark + x \times \frac{1}{\left(1 + \frac{1}{x}\right)} \times\ -x^{-2}\checkmark$$

$$\frac{dy}{dx} = y\left[\ln\left(1 + \frac{1}{x}\right) - \frac{x}{x^2\left(1 + \frac{1}{x}\right)}\right]$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{x}\right)\left[\ln\left(1 + \frac{1}{x}\right) - \frac{1}{x\left(1 + \frac{1}{x}\right)}\right]\checkmark \quad (4)$$

2.4 2.4.1 $25x^2 + 9y^2 - 70x - 30y + 49 = 0$

$$50x + 18y\frac{dy}{dx} - 70 - 30\frac{dy}{dx} + 0 = 0\checkmark$$

$$18y\frac{dy}{dx} - 30\frac{dy}{dx} = 70 - 50x\checkmark$$

$$\frac{dy}{dx}(18y - 30) = 70 - 50x$$

$$\frac{dy}{dx} = \frac{30 - 25x}{9y - 15}\checkmark \quad (3)$$

2.4.2 $\frac{dy}{dx} = -\frac{5}{4}$ ✓

$y = mx + c$

$y = -\frac{5}{4}x + c$

$3 = -\frac{5}{4}(2) + c$

$c = \frac{11}{2}$ ✓

$y = -\frac{5}{4}x + \frac{11}{2}$ ✓ (3)

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QUESTION 3

3.1 3.1.1 $f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$x = -1 \quad or \quad x = 1$
$y = 3 \quad or \quad y = -1$

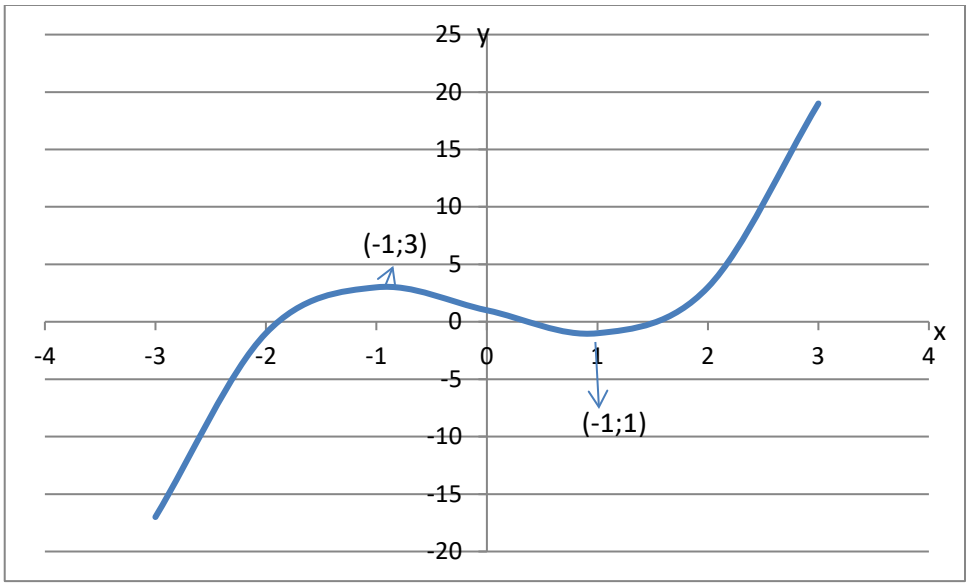
Thus, the coordinates of the turning points are $(-1; 3)$✓ and $(1; -1)$✓ (2)

3.1.2

x	-2	-1	0	1	2
y	-1	3	1	-1	3

✓= 1 mark each for any 3 correct answers (3)

3.1.3



✓ = 1 mark for shape
✓= 1 mark for indication of turning point on graph (2)

3.1.4 $Let\ x_0 = 1.5$

$f(1.5) = -0.125$

$f'(1.5) = 3.75$

$x_1 = 1.5 - \frac{(-0.125)}{3.75}$✓

$= 1.533$✓

$Let\ x_1 = 1.533$

$f(1.53) = 0.005$

$f'(1.533) = 4.053$

$x_1 = 1.533 - \frac{0.005}{4.053}$✓

$= 1.532$✓ (4)

3.2 $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$= \frac{\pi h^3}{12}$ ✓

$\frac{dV}{dt} = \frac{\pi}{12}.3h^2\frac{dh}{dt}$ ✓

$\pi = \frac{\pi}{4}(6)^2.\frac{dh}{dt}$ ✓

$\frac{dh}{dt} = \frac{1}{9}m/s$ ✓ (4)

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QUESTION 4

4.1 4.1.1 $\int \frac{4}{\sec x e^{\sin x}}dx$

$let\ u = \sin x$
$du = \cos x\, dx$ ✓

$= \int \frac{1}{e^u}du$ ✓

$= \int e^{-u}\, du$

$= -e^{-u} + C$

$= e^{-\sin x}$ ✓ $+ C$ (3)

4.1.2 $\int \frac{(\cos^{-1} 2x)^4}{\sqrt{1-4x^2}}dx$

$let\ u = \cos^{-1} 2x$
$-\frac{du}{2} = \frac{1}{\sqrt{1-4x^2}}dx$ ✓

$= -\frac{1}{2}\int u^4\, du$ ✓

$= -\frac{1}{10}u^5 + c$

$= -\frac{1}{10}(\cos^{-1} 2x)^5$ ✓ $+ C$ (3)

4.1.3 $$\int \frac{x^3 + 2x^2 + 9x - 17}{x + 4}\,dx$$

$$= \int \left(x^2 - 2x + 17 + \frac{51}{x + 4}\right) dx\checkmark$$

$$= \frac{1}{3}x^3\checkmark - x^2\checkmark + 17x\checkmark + 51\ln(x + 4)\checkmark + C$$ (5)

4.1.4 $$\int \cos 3\pi x . \cos \pi x\,dx$$

$$= \frac{1}{2}\int (\cos 4\pi x + \cos 2\pi x)\,dx\checkmark$$

$$= \frac{1}{8\pi}\sin 4\pi x\checkmark + \frac{1}{4\pi}\sin 2\pi x\checkmark + C$$ (3)

4.1.5 $$\int x\sqrt{1 + 2x}\,dx$$

$$let\ u = 1 + 2x$$

$$\frac{du}{2} = dx\ \checkmark$$

$$x = \frac{u - 1}{2}$$

$$= \int \frac{u - 1}{2}\sqrt{u}\,du\checkmark$$

$$= \frac{1}{4}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$$

$$= \frac{1}{10}u^{\frac{5}{2}} - \frac{1}{6}u^{\frac{3}{2}} + C$$

$$= \frac{1}{10}(1 + 2x)^{\frac{5}{2}}\checkmark - \frac{1}{6}(1 + 2x)^{\frac{3}{2}}\checkmark + C$$ (4)

4.2 $$\int \frac{3x}{(2x+1)(x+4)}dx$$

$$\frac{3x}{(2x+1)(x+4)} = \frac{A}{2x+1} + \frac{B}{x+4}$$

$$3x = A(x+4) + B(2x+1)\checkmark$$

$let\ x = -4, \qquad B = \frac{12}{7}\checkmark$
$or\ \ x = -\frac{1}{2},\ \ A = -\frac{3}{7}\checkmark$

$$= -\frac{3}{7}\int \frac{1}{2x+1}dx + \frac{12}{7}\int \frac{1}{x+4}dx$$

$$= -\frac{3}{14}\ln(2x+1)\checkmark + \frac{12}{7}\ln(x+4)\checkmark + C$$

(5)
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QUESTION 5

5.1 $$\int_0^{\infty} e^{-st}.f(t)\,dt$$

$$= \frac{1}{\pi}\int_0^{\infty} te^{-st}\,dt\checkmark$$

$let\ u = t \quad dv = e^{-st}\,dt$
$du = dt\ \ v = -\frac{1}{s}e^{-st}\,dt\ \checkmark$

$$= \frac{1}{\pi}\left[-\frac{t}{s}e^{-st}\right]_0^{\infty} + \frac{1}{\pi s}\int_0^{\infty} te^{-st}\,dt\ \checkmark$$

$$= -\frac{1}{s\pi}[te^{-st}]_0^{\infty} - \frac{1}{s^2\pi}[e^{-st}]_0^{\infty}\checkmark$$

$$= -\frac{1}{s\pi}(0-0) - \frac{1}{s^2\pi}(0-1)$$

$$= \frac{1}{s^2\pi}\checkmark$$

(5)

5.2 5.2.1

$$2x^2 = x + 1$$

$$2x^2 - x - 1 = 0$$

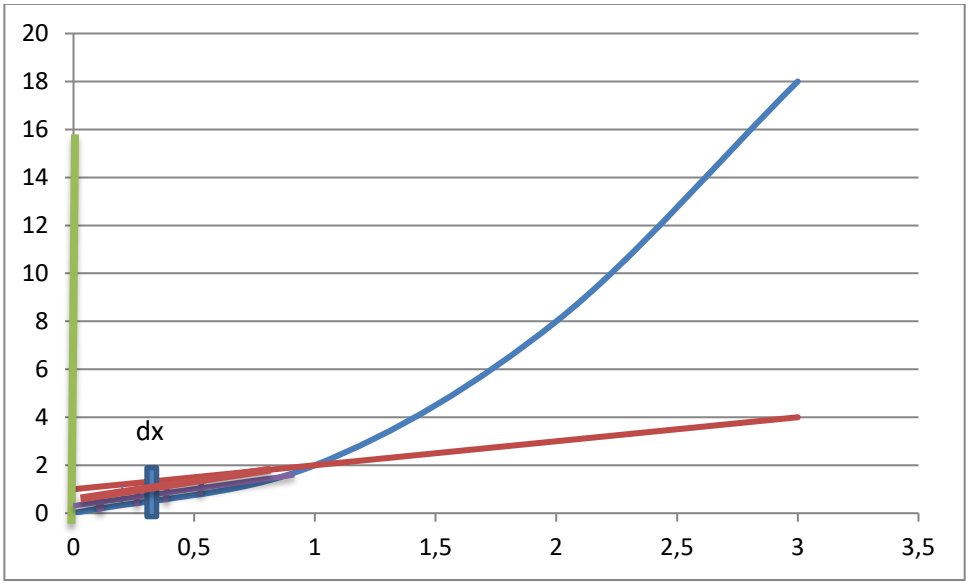
$$(2x + 1)(x - 1) = 0$$

$$x = -\frac{1}{2} \ or \ x = 1$$

$$y = \frac{1}{2} \ or \ y = 2$$

Thus, the coordinates of the points of intersection are $\left(-\frac{1}{2};\frac{1}{2}\right)$✓ and $(1;2)$✓ (2)

5.2.2



✓= 1 mark for indication of enclosed area
✓= 1 mark for indication of vertical or horizontal strip (2)

5.2.3

$$A = \int_a^b (y_1 - y_2)\,dx$$

$$= \int_0^1 (x + 1 - 2x^2)\,dx$$✓

$$= \left[\frac{1}{2}x^2 + x - \frac{2}{3}x^3\right]_0^1$$ ✓

$$= \frac{5}{6} \ or \ 0.833 \ units^2$$✓ (3)

5.2.4

$$V = \pi \int_a^b (y_1^2 - y_2^2)\, dx$$

$$V = \pi \int_0^1 [(x+1)^2 - (2x^2)^2]\, dx$$

$$= \pi \int_0^1 (x^2 + 2x + 1 - 4x^4)\, dx \checkmark\checkmark$$

$$= \pi \left[\frac{1}{3}x^3 + x^2 + x - \frac{4}{5}x^5\right]_0^1 \checkmark$$

$$= \frac{23\pi}{15} \ or \ 4.817\ units^3 \checkmark$$

(4)

5.3 $)_y = r^2\, dA$

$= x^2.3\, dx \checkmark$

$= 3\int_0^6 x^2\, dx \checkmark$

$= 3\left[\frac{x^3}{3}\right]_0^6 \checkmark$

$= 216 units^4 \checkmark$

(4)
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QUESTION 6

6.1 $\frac{dy}{dx} = \frac{x\left(e^{x^2} + 2\right)}{6y^2}$

$6y^2\,dy = x\left(e^{x^2} + 2\right)dx$

$6y^2\,dy = \left(xe^{x^2} + 2x\right)dx$

$2y^3 ✓ = \frac{1}{2}e^{x^2} + x^2 + C ✓$

$2(1)^3 = \frac{1}{2}e^{(0)^2} + (0)^2 + c$

$c = \frac{3}{2} ✓$

$2y^3 = \frac{1}{2}e^{x^2} + x^2 + \frac{3}{2} ✓$ (4)

6.2 $x^2.\frac{d^2y}{dx^2} = x^6 + x^3 - 2x^2$

$\frac{d^2y}{dx^2} = x^4 + x - 2$

$\frac{dy}{dx} = \frac{1}{5}x^5 + \frac{1}{2}x^2 - 2x + A ✓$

$\frac{3}{5} = \frac{1}{5}(1)^5 + \frac{1}{2}(1)^2 - 2(1) + A$

$A = 2 ✓$

$\frac{dy}{dx} = \frac{1}{5}x^5 + \frac{1}{2}x^2 - 2x + 2 ✓$

$y = \frac{1}{30}x^6 + \frac{1}{6}x^3 - x^2 + 2x + B ✓$

$\frac{1}{5} = \frac{1}{30}(1)^6 + \frac{1}{6}(1)^3 - (1)^2 + 2(1) + B$

$B = -1 ✓$

$y = \frac{1}{30}x^6 + \frac{1}{6}x^3 - x^2 + 2x - 1 ✓$ (6)

[10]

TOTAL: 100