higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1030**(E)**(A1)T

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

1 April 2019 (X-Paper)
09:00–12:00

Scientific calculators may be used.

This question paper consists of 6 pages and a formula sheet of 5 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. ALL final answers must be rounded off to THREE decimals.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Use only BLUE or BLACK ink.
8. Sketches must be large, neat and fully labelled
9. Work neatly and legibly.

QUESTION 1

1.1 Determine the following limits:

1.1.1 $\lim_{x \to 0} \frac{3^x - 2^x}{x^2 - x}$ (2)

1.1.2 $\lim_{x \to 0} \tan x \, In \, x$ (5)

1.2 Determine the value(s) of x for which $f(x)$ is discontinuous if

$$f(x) = \frac{1}{2 - 4\cos 2x}; \; x \in [0; 2\pi]$$

(2)

[9]

QUESTION 2

2.1 Given:

$$f(x) = \frac{1}{\sqrt{9x}}$$

Determine the simplest form of each of the following:

2.1.1 $f(x+h)$ (2)

2.1.2 $f(x+h) - f(x)$ (1)

2.1.3 $\frac{f(x+h) - f(x)}{h}$ (1)

2.1.4 $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ (1)

2.2 Derive a formula to determine $\frac{dy}{dx}$ if $y = arc \cot x$ (4)

2.3 Determine $\frac{dy}{dx}$ in each of the following cases: (Simplification NOT required)

2.3.1 $y = \tan\left(\sqrt[3]{3x^2} + In\left(5x^4\right)\right)$

2.3.2 $y = In^3(\sin x - \cot x)$

(2 × 3) (6)

2.4 Calculate $\frac{dy}{dx}$ if $y=\left(2x-e^{8x}\right)^{\sin 2x}$ with the aid of logarithmic differentiation. (4)

2.5 Given: the implicit function $2\sin x\cos y=1$

2.5.1 Determine $\frac{dy}{dx}$ (4)

2.5.2 Determine the equation of the tangent to the graph at the point $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ (3)

[26]

QUESTION 3

3.1 Given: $f(x)=3x^3-9x^2+5x+2$

3.1.1 Determine the coordinates of the turning points of $f(x)$ (2)

3.1.2 Draw a table of x and $f(x)$, with x ranging from $x=-1$ to $x=3$ (2)

3.1.3 Draw a neat graph of $f(x)$ between these values, showing the turning points on it. (2)

3.1.4 One root of the equation $f(x)=3x^3-9x^2+5x+2$ is close to 1.

Use this value and an approximation of Taylor's/Newton's method to determine a better approximation of this root (root correct to THREE decimals). (4)

3.2 The position of a particle is given by $s(t)=e^{t^2}+t^2, t\geq 0,$ where s is the displacement in metres and t is time in seconds.

Find the acceleration of the particle when $t=1$. (4)

[14]

QUESTION 4

4.1 Determine $\int y\,dx$ in each of the following cases

4.1.1 $y = e^x \sec^2(e^x)[1 + \tan(e^x)]^{-3}$ (2)

4.1.2 $y = \dfrac{\cos ecx \cot x}{2 - \cos ecx}$ (2)

4.1.3 $y = \dfrac{7}{1 + 5x^2}$ (3)

4.1.4 $y = \tan^3 x$ (5)

4.1.5 $y = \sin(\ln x)$ (5)

4.2 Determine $\int y\,dx$ by resolving the integral into partial fractions:

$y = \dfrac{9 - 9x}{2x^2 + 7x - 4}$ (5)

[22]

QUESTION 5

5.1 Evaluate the definite integral: $\int_{\pi}^{2\pi} (1 - 3x)\sin\left(\frac{1}{2}x\right) dx$ (5)

5.2 Given: $y = \dfrac{8}{x}$; $y = x$ and $y = 4$

5.2.1 Calculate the coordinates of the points of intersection. (2)

5.2.2 Make a neat sketch to show the enclosed area, the representative strip and the point of intersection. (2)

5.2.3 Calculate the magnitude of the area in QUESTION 5.2.2. (3)

5.2.4 Calculate the volume of the solid of revolution formed when the area in QUESTION 5.2.2 is rotated about the y-axis. (4)

5.3 Calculate the moment of inertia of a circular lamina of radius r and mass m about an axis through its centre and perpendicular to the plane of the lamina. (5)

[21]

QUESTION 6

6.1 Determine the general solution of $\frac{dy}{dx}\tan x = \alpha + y$ (3)

6.2 Calculate the particular solution of: $\frac{d^2y}{dx^2} = x^4 - x^2 + 1$ if $\frac{dy}{dx} = 1$, $x = 1$ and $y = 2$ (5)

[8]

TOTAL: 100

MATHEMATICS N5

FORMULA SHEET

Any applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1 \rightarrow \cos^2 x = 1 - \sin^2 x$

$\sin^2 x + \cos^2 x = 1 \rightarrow \sin^2 x = 1 - \cos^2 x$

$1 + \tan^2 x = \sec^2 x \longrightarrow \tan^2 x = \sec^2 x - 1$

$1 + \cot^2 x = \text{cosec}^2 x \longrightarrow \cot^2 x = \text{cosec}^2 x - 1$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = ½ - ½ \cos 2A = ½(1 - \cos 2A)$

$\cos^2 A = ½ + ½ \cos 2A = ½ (1 + \cos 2A)$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x} ; \sin x = \frac{1}{\text{cosec } x} ; \cos x = \frac{1}{\sec x}$$

$$\cot x = \frac{\cos x}{\sin x} ; \cos ecx = \frac{1}{\sin x} ; \sec x = \frac{1}{\cos x}$$

BINOMIAL THEOREM

$$(x+h)^n = x^n + nx^{n-1}\,h + \frac{n(n-1)}{2!}x^{n-2}\,h^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}x^{n-3}h^3 + \ldots\ldots$$

DIFFERENTIATION

$$e = -\frac{f(a)}{f^{'}(a)}$$

$$r = a + e$$

PRODUCT RULE

$y = u(x)\,.\,v(x)$

$$\frac{dy}{dx} = u\,.\,\frac{dv}{dx} + v\,.\,\frac{du}{dx}$$

$$= u \cdot v^{'} + v \cdot u^{'}$$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\,.\,\frac{du}{dx} - u\,.\,\frac{dv}{dx}}{v^2}$$

$$= \frac{v \cdot u' - u \cdot v'}{v^2}$$

CHAIN RULE

$y = f(u(x))$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+c \quad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n dx$
e^{ax+b}	$e^{ax+b} \cdot \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$ln(ax)$	$\frac{1}{ax}.\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}.\frac{d}{dx}f(x)$	—
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}f(x)$	—
$ln\ f(x)$	$\frac{1}{f(x)}.\frac{d}{dx}f(x)$	—
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}.+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}.+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$-a\ \text{cosec}^2\ ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a\sec ax \cdot \tan ax$	$\frac{1}{a}\ln[\sec ax+\tan ax]+C$
$\text{cosec}\ ax$	$-a\ \text{cosec}\ ax \cdot \cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
$\sin f(x)$	$\cos f(x) \bullet f^1(x)$	—
$\cos f(x)$	$-\sin f(x) \bullet f^1(x)$	—
$\tan f(x)$	$\sec^2 f(x) \bullet f^1(x)$	—
$\cot f(x)$	$-\ \mathrm{cosec}^2 f(x) \bullet f^1(x)$	—
$\sec f(x)$	$\sec f(x) \bullet \tan f(x) \bullet f^1(x)$	—
$\mathrm{cosec}\, f(x)$	$-\ \mathrm{cosec}\, f(x) \bullet \cot f(x) \bullet f^1(x)$	—
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	—
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	—
$\tan^{-1} f(x)$	$\frac{f'(x)}{1+[f(x)]^2}$	—
$\cot^{-1} f(x)$	$\frac{-f'(x)}{1+[f(x)]^2}$	—
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	—
$\mathrm{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	—
$\sin^2(ax)$	—	$\frac{x}{2}-\frac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	—	$\frac{x}{2}+\frac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	—	$\frac{1}{a}\tan(ax)-x+C$
$\cot^2(ax)$	—	$-\frac{1}{a}\cot(ax)-x+C$

INTEGRATION

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n \bullet f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y dx \;;\; A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x dy \;;\; A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \;;\; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx$$

$$V_y = \pi \int_a^b x^2 dy \;;\; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \;;\; I_y = \int_a^b r^2 dA$$

MOMENTS OF INERTIA

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$