higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1030(**E**)(M29)T

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

29 March 2018 (X-Paper)
09:00–12:00

This question paper consists of 6 pages and a formula sheet of 5 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. ALL final answers must be rounded off to THREE decimal places.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Questions must be answered in blue or black ink.
8. Work neatly.

QUESTION 1

1.1 Determine the following limit:

$$\lim_{x\to 0}\left(\frac{arc\sin 4x}{arc\tan 5x}\right)$$ (3)

1.2 Given: $\ln y = \lim_{x\to 0}\frac{\int_0^x e^t\,dt}{x}$, calculate the numerical value of:

HINT: $\int_0^x e^t\,dt = e^x - 1$

1.2.1 $\ln y$ (3)

1.2.2 y (1)

1.3 Determine the value(s) of x for which $f(x)$ is discontinuous if:

$$f(x) = \frac{\sin 3x}{x^3 - 4x}$$ (3)

[10]

QUESTION 2

2.1 Determine the derivative of $f(x) = \frac{-3}{x^{-7}}$ from first principles. (5)

2.2 Make a neat sketch of the graph $y = arc\tan x$ for the range $\left[\frac{\pi}{2}; \frac{3\pi}{2}\right]$. (2)

2.3 Calculate $\frac{dy}{dx}$ of the following, by making use of the derivatives of $\sin x$ and $\cos x$, as well as the rules of differentiation:

$$y = 10^{\log(\cot x)}$$

HINT: $a^{\log_a x} = x$ (4)

2.4 Determine $\frac{dy}{dx}$ in each of the following cases:

(Simplification NOT required)

2.4.1 $y = \sin(arc\sin x) - \pi^t$ (2)

2.4.2 $y = \ln^3(\cos^{-1} x)^{(\cos x)^0}$ (4)

2.5 Calculate $\frac{dy}{dx}$ with the aid of logarithmic differentiation if $y = x^{\ln x}$. (4)

2.6 Determine $\frac{dy}{dx}$ of the implicit function $y\sin(x^2) = x\sin(y^2)$. (4)

[25]

QUESTION 3

3.1 Given:

$f(x) = x^3 - 7x^2 + 8x - 3$

3.1.1 Determine the coordinates of the turning points of $f(x)$. (2)

3.1.2 Draw up a table of x and $f(x)$, where x is ranging from $x = -2$ to $x = 7$. (2)

3.1.3 Draw a neat graph of $f(x)$ between these values and show the turning points on it. (2)

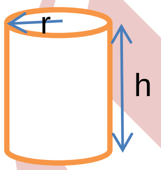
3.1.4 One root of the equation $f(x) = x^3 - 7x^2 + 8x - 3$ is close to 5.

Use this value and one approximation of Taylor's/Newton's method to determine a better approximation of this root (correct to THREE decimal figures). (4)

3.2 A thin sheet of ice is in the form of a circle. If the ice is melting in such a way that the area of the sheet is decreasing at a rate of $0{,}5\ m^2/s$ at what rate is the radius decreasing when the area of the sheet is $12\ \text{m}^2$? (5)

3.3 A cylindrical can with bottom but no top with a volume of $30\ \text{cm}^3$ must be constructed.

Determine the dimensions of the can that will minimise the amount of material needed to construct the can.



HINT: $V = \pi r^2 h$ and $A = 2\pi rh + \pi r^2$ (5)

[20]

QUESTION 4

4.1 Determine $\int y\,dx$ in each of the following cases:

4.1.1 $y = \dfrac{\sec^2 \pi x}{1 + \tan \pi x}$ (3)

4.1.2 $y = \dfrac{x}{x + 4}$ (3)

4.1.3 $y = \dfrac{1}{5 + 25x^2}$ (2)

4.1.4 $y = \cot^3 x$ (4)

4.1.5 $y = \sqrt{x} \ln x$ (3)

4.2 Determine $\int y\,dx$ by resolving the integral into partial fractions:

$\dfrac{3 - x}{x^2 - 5x}$ (5)

[20]

QUESTION 5

5.1 Evaluate the definite integral:

$\int_0^1 \frac{x^2}{x^3+1} dx$ (4)

5.2 Given:

$y = x - 1$ and $y = (x - 1)^2$

5.2.1 Calculate the coordinates of the points of intersection. (2)

5.2.2 Make a neat sketch to show the enclosed area, the representative strip and the point of intersection. (2)

5.2.3 Calculate the magnitude of the area in QUESTION 5.2.2. (3)

5.2.4 Calculate the volume of the solid of revolution formed when the area in QUESTION 5.2.2 is rotated about the x-axis. (4)

5.3 Calculate the second moment of area of a rectangular lamina with sides 8 cm × 4 cm and about a 4 cm side. (4)

[19]

QUESTION 6

6.1 Determine the particular solution of $x\,dy = y\ln y\,dx$, given $x = 2$ when $y = e$. (4)

6.2 Determine the general solution of $\frac{d^2y}{dx^2} = x^4 - \sin x$. (2)
[6]

TOTAL: 100

MATHEMATICS N5

FORMULA SHEET

Any other applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cos}ec^2 x$

$\sin 2A = 2\sin A.\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\sin^2 A = \dfrac{1}{2} - \dfrac{1}{2}\cos 2A$

$\cos^2 A = \dfrac{1}{2} + \dfrac{1}{2}\cos 2A$

$\sin(A \pm B) = \sin A.\cos B \pm \sin B.\cos A$

$\cos(A \pm B) = \cos A.\cos B \mp \sin A.\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 - \tan A.\tan B}$

$\sin A.\cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\cos A.\sin B = \tfrac{1}{2}[\sin(A+B) - \sin(A-B)]$

$\cos A.\cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\sin A.\sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$

$\tan x = \dfrac{\sin x}{\cos x};\sin x = \dfrac{1}{\operatorname{cos}ecx};\cos x = \dfrac{1}{\sec x}$

BINOMIAL THEOREM

$$(x+h)^n = x^n + n.x^{n-1}h + \frac{n(n-1)}{2!}x^{n-2}h^2 + \ldots$$

DIFFERENTIATION

$$e = -\frac{f(a)}{f'(a)}$$

$$r = a + e$$

PRODUCT RULE

$$y = u(x).v(x)$$

$$\frac{dy}{dx} = u.\frac{dv}{dx} + v.\frac{du}{dx}$$

$$= u.v' + v.u'$$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$$

$$= \frac{v.u' - u.v'}{v^2}$$

CHAIN RULE

$$y = f(u(x))$$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$

ax^n	nax^{n-1}	$\dfrac{ax^{n+1}}{n+1}+c$
a	0	$ax+c$
e^x	e^x	e^x+c
a^x	$a^x.\ln a$	$\dfrac{a^x}{\ln a}+c$
$\log_e x$	$\dfrac{1}{x}$	—
$\log_a x$	$\dfrac{1}{x\ln a}$	—
$\sin x$	$\cos x$	$-\cos x+c$
$\cos x$	$-\sin x$	$\sin x+c$
$\tan x$	$\sec^2 x$	$\ln(\sec x)+c$
$\cot x$	$-\cos ec^2 x$	$\ln(\sin x)+c$
$\sec x$	$\sec x.\tan.$	$\ln[\sec x+\tan x]+c$
$\cos ecx$	$-\cos ecx.\cot x$	$\ln[\cos ecx-\cot x\}+c$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$	—
$\cos^{-1} x$	$\dfrac{-1}{\sqrt{1-x^2}}$	—
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$	—
$\cot^{-1} x$	$\dfrac{-1}{1+x^2}$	—
$\sec^{-1} x$	$\dfrac{1}{x\sqrt{x^2-1}}$	—
$\cos ec^{-1}x$	$\dfrac{-1}{x\sqrt{x^2-1}}$	—
$f(x)$	$\dfrac{d}{dx}f(x)$	$\int f(x)dx$

$\frac{1}{\sqrt{a^2-x^2}}$ — $\sin^{-1}(\frac{x}{a})+c$

$\frac{1}{a^2+x^2}$ — $\frac{1}{a}\tan^{-1}(\frac{x}{a})+c$

$\frac{1}{x\sqrt{x^2-a^2}}$ — $\frac{1}{a}\sec^{-1}(\frac{x}{a})+c$

$\sqrt{a^2-x^2}$ — $\frac{a^2}{2}\sin^{-1}(\frac{x}{a})+\frac{x}{2}\sqrt{a^2-x^2}+c$

$\frac{1}{x^2-a^2}$ — $\frac{1}{2a}\ln(\frac{x-a}{x+a})+c$

$\frac{1}{a^2-x^2}$ — $\frac{1}{2a}\ln(\frac{a+x}{a-x})+c$

INTEGRATION

$$\int f(x).g'(x) = f(x).g(x)-\int f'(x).g(x)dx$$

$$\int \frac{f'(x)}{f(x)}dx = \ln f(x)+c$$

$$\int [f(x)]^n.f'(x)dx = \frac{f(x)^{n+1}}{n+1}+c$$

$$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b}+\frac{B}{cx+d}$$

$$\frac{f(x)}{(x+a)^n} = \frac{A}{(x+a)}+\frac{B}{(x+a)^2}+\frac{C}{(x+a)^3}+...+\frac{Z}{(x+a)^n}$$

Applications of integration

AREAS

$$A_x = \int_a^b ydx; \quad A_x = \int_a^b (y_2-y_1)dx$$

$$A_y = \int_a^b xdy; \quad A_y = \int_a^b (x_2-x_1)dy$$

VOLUMES

$V_x = \pi \int_a^b y^2 dx; \ \ V_x = \pi \int_a^b (y_2{}^2 - y_1{}^2) dx$

$V_y = \pi \int_a^b x^2 dy; \ \ V_y = \pi \int_a^b (x_2{}^2 - x_1{}^2) dy$

SECOND MOMENT OFAREA

$I_x = \int_a^b r^2 dA; \ \ I_y = \int_a^b r^2 dA$

MOMENTS OF INERTIA

Mass = density x volume/Massa = digtheid x volume

$M = pV$

Definition: $I = mr^2$

General: $I = \int_a^b r^2 dm = p \int_a^b r^2 dV$