higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

29 MARCH 2018

This marking guideline consists of 12 pages.

QUESTION 1

1.1 1.1.1

$$\lim_{x\to 0}\left(\frac{arc\sin 4x}{arc\tan 5x}\right)$$

$$=\lim_{x\to 0}\left(\frac{\frac{1}{\sqrt{1-16x^2}}\checkmark}{\frac{1}{1+25x^2}\checkmark}\right)\left[\frac{0}{0}\right]$$

$$=1\checkmark \quad (3)$$

1.2

$$\ln y=\lim_{x\to 0}\frac{\int_0^x e^t\,dt}{x}$$

1.2.1

$$\ln y=\lim_{x\to 0}\frac{e^x-1}{x}\checkmark\ \left[\frac{0}{0}\right]$$

$$=\lim_{x\to 0}\frac{e^x}{1}\checkmark$$

$$=1\checkmark \quad (3)$$

1.2.2 $y=e^1$

$=,.718\checkmark$ (1)

1.3

$$f(x)=\frac{\sin 3x}{x^3-4x}$$

$x^3-4x=0$

$x(x-2)(x+2)=0$

$x=0\checkmark$ or $x=2\checkmark$ or $x=-2\checkmark$ (3)

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QUESTION 2

2.1 $$f(x) = -\frac{3}{x^{-7}} = -3x^7$$

$$f(x+h) = -3(x+h)^7$$

$$= -3x^7 - 21x^6h - 63x^5h^2 + \cdots .\checkmark\checkmark$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

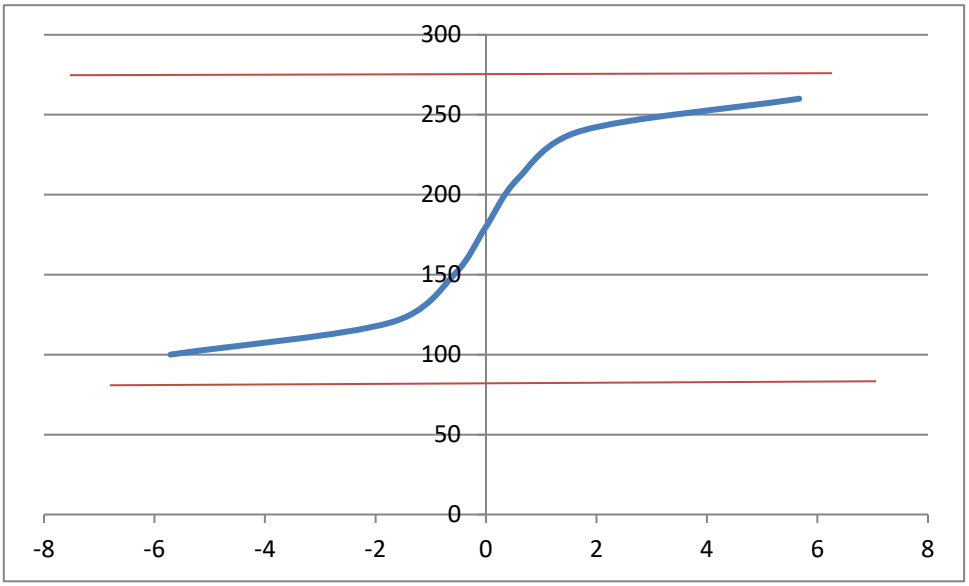
$$= \lim_{h\to 0}\frac{-3x^7 - 21x^6h - 63x^5h^2 + \cdots - (-3x^7)}{h}$$

$$= \lim_{h\to 0}\frac{-21x^6h - 63x^5h^2 + \cdots ..}{h}\checkmark$$

$$= \lim_{h\to 0} -21x^6 - 63x^5h\checkmark$$

$$= -21x^6\checkmark$$ (5)

2.2 $$y = arc\tan x$$



✓ =1 mark for the shape
✓ =1 mark for the range or restriction (2)

2.3 $y = 10^{\log(\cot x)}$

$y = \cot x \surd\checkmark$

$y = \dfrac{\cos x}{\sin x}$

$\dfrac{dy}{dx} = \dfrac{\sin x\,(-\sin x) - \cos x\,(\cos x)\checkmark}{\sin^2 x}$

$= \dfrac{-\sin^2 x - \cos^2 x\,\checkmark}{\sin^2 x}$

$= -\frac{1}{\sin^2 x}\checkmark$

$= -\operatorname{cosec}^2 x$ (4)

2.4 2.4.1 $y = \sin(arc\sin x) - \pi^t$

$\dfrac{dy}{dx} = \cos(arc\sin x)\checkmark \times \dfrac{1}{\sqrt{1-x^2}}\checkmark$ (2)

2.4.2 $y = \ln^3(\cos^{-1} x)^{(\cos x)^0}$

$= \ln^3(\cos^{-1} x)\checkmark$

$\frac{dy}{dx} = 3\ln^2(\cos^{-1} x)\checkmark \times \frac{1}{\cos^{-1} x}\checkmark \times -\frac{1}{\sqrt{1-x^2}}\checkmark\checkmark$ (4)

2.5 $y = x^{\ln x}$

$\ln y = \ln x \ln x$

$\frac{1}{y}\frac{dy}{dx}\surd = \frac{\ln x}{x} + \frac{\ln x}{x}\checkmark\checkmark$

$\frac{dy}{dx} = y\left[\frac{2\ln x}{x}\right]\checkmark$

$= x^{\ln x}\left[\frac{2\ln x}{x}\right]\checkmark$ (4)

2.6 $y\sin(x^2) = x\sin(y^2)$

$y\cos(x^2) \times 2x + \dfrac{dy}{dx}.\sin(x^2)\checkmark = \sin(y^2) + x\cos(y^2) \times 2y.\dfrac{dy}{dx}\checkmark$

$[\sin(x^2) - 2xy\cos(y^2)] \times \dfrac{dy}{dx} = \sin(y^2) - 2xy\cos(x^2)\ \checkmark$

$\dfrac{dy}{dx} = \dfrac{\sin(y^2) - 2xy\cos(x^2)}{\sin(x^2) - 2xy\cos(y^2)}\checkmark$ (4)

[25]

QUESTION 3

3.1 Given:

$f(x) = x^3 - 7x^2 + 8x - 3$

3.1.1 $f'(x) = 3x^2 - 14x + 8$

$(3x - 2)(x - 4) = 0$

$x = \frac{2}{3}$ *or* $x = 4$

$y = -\frac{13}{27}$ *or* $y = -19$

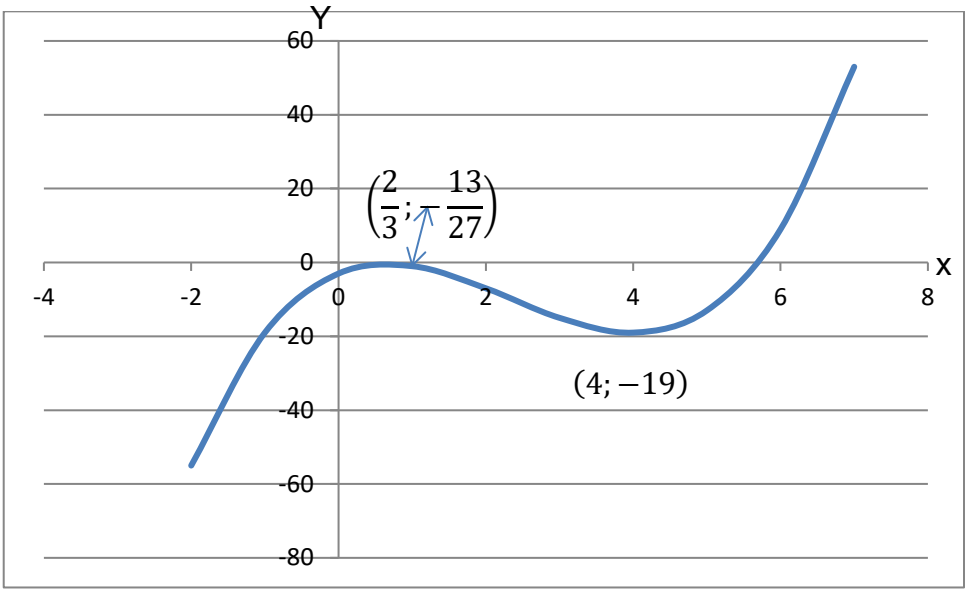
Thus turning points are; $\left(\frac{2}{3}; -\frac{13}{27}\right)$ ✓ *and* $(4; -19)$✓ (2)

3.1.2

x	-2	-1	0	1	2	3	4	5	6
y	-55	-19	-3	-1	-7	-15	-19	-13	9

✓ = 1 mark for any 3 correct answers (2)

3.1.3



✓ =1 mark for the shape
✓ =1 mark for the indication of the turning point on the graph (2)

3.1.4 $Let\ x_0 = 5$
$f(2) = -13$
$f'(2) = 13$

$x_1 = 2 - \frac{(-13)}{13}$✓
$= 6$✓

$f(5) = 9$
$f'(5) = 32$

$x_2 = 6 - \frac{9}{32}$✓
$= 5{,}71875$✓ (4)

3.2 $A = \pi r^2$

$r^2 = \frac{A}{\pi}$

$r = \sqrt{\frac{12}{\pi}}$✓

$= 1{,}954$✓

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$✓

$\frac{dr}{dt} = \frac{-0{,}5}{2\pi(1{,}954)}$✓

$= -0{,}0407$✓ (5)

3.3 $V = \pi r^2 h$

$30 = \pi r^2 h$

$h = \frac{30}{\pi r^2}\surd$

$A = 2\pi rh + \pi r^2$

$A = 2\pi r\left(\frac{30}{\pi r^2}\right) + \pi r^2\checkmark$

$= \frac{60}{r} + \pi r^2$

$\frac{dA}{dr} = -\frac{60}{r^2} + 2\pi r = 0\checkmark$

$r = 2{,}1216\ cm = \ h\checkmark\checkmark$ (5)

[20]

QUESTION 4

4.1 4.1.1 $\int \frac{\sec^2 \pi x}{1 + \tan \pi x}\, dx$

$Let\ u = 1 + tan\ \pi x\checkmark$

$du = \pi \sec^2 \pi x\, dx$

$= \frac{1}{\pi}\int \frac{1}{u} du\ \checkmark$

$= \frac{1}{\pi}\ \ln u + c$

$= \frac{1}{\pi}\ln (1 + tan\ \pi x\ ) + \ c\checkmark$ (3)

4.1.2 $\int \frac{x}{x+4}dx$

Let $u = x + 4$
$du = dx$
$\Rightarrow x = u - 4$✓

$= \int \frac{u-4}{u}du$✓

$= \int \left(1 - \frac{4}{u}\right)du$

$= u - 4\ln u + c$

$= (x+4) - 4\ln(x+4) + c$✓ (3)

4.1.3 $\int \frac{1}{5+25x^2}dx$

$= \frac{1}{25}$✓

$= \frac{\sqrt{5}}{25}tan^{-1}(\sqrt{5}x) + c$✓ (2)

4.1.4 $\int cot^3 x\,dx$

$= \int \cot^2 x \,.\cot x\,dx$ ✓

$= \int (\text{cosec}^2 x - 1)\cot x\,dx$✓

$= \int \text{cosec}^2 x \cot x\,dx - \int \cot x\,dx$

$= -\frac{\cot^2 x}{2}$✓ $- \ln(\sec x + tanx)$✓ $+ c$ (4)

4.1.5 $\int \sqrt{x}\, ln\, x\, dx$

$Let\ u = \ln x \qquad dv = \sqrt{x}\, dx$
$du = \frac{1}{x} dx$ ✓ $\quad v = \frac{2}{3} x^{\frac{3}{2}}$

$= \frac{2}{3} x^{\frac{3}{2}}. \ln x - \frac{2}{3} \int \sqrt{x}\, dx$✓

$= \frac{2}{3} x^{\frac{3}{2}}. \ln x - \frac{4}{9} x^{\frac{3}{2}} + c$✓ (3)

4.2 $\int \frac{3-x}{x^2-5x} dx$

$\frac{3-x}{x(x-5)} = \frac{A}{x} + \frac{B}{x-5}$

$3 - x = A(x-5) + Bx$✓
$Let\ x = 5, B = -\frac{2}{5}$✓

$Let\ x = 0, A = -\frac{3}{5}$✓

$= -\frac{3}{5} \int \frac{1}{x} dx - \frac{2}{5} \int \frac{1}{x-5} dx$
$= -\frac{3}{5} \ln x\sqrt{} - \frac{2}{5} \ln(x-5)$✓ $+ c$ (5)
[20]

QUESTION 5

5.1 $\int_0^1 \frac{x^2}{x^3+1} dx$
$let\ u = x^3 + 1$✓
$\frac{du}{3} = x^2\, dx$
$= \frac{1}{3} \int_{x=0}^{x=1} \frac{1}{u} du$✓
$= \frac{1}{3} [\ln u]_1^{\ 2}$ ✓
$= 0{,}2310$✓ (4)

5.2 Given: $y = x - 1$ *and* $y = (x-1)^2$

5.2.1 $(x-1)^2 = x - 1$

$x^2 - 2x + 1 - x + 1 = 0$

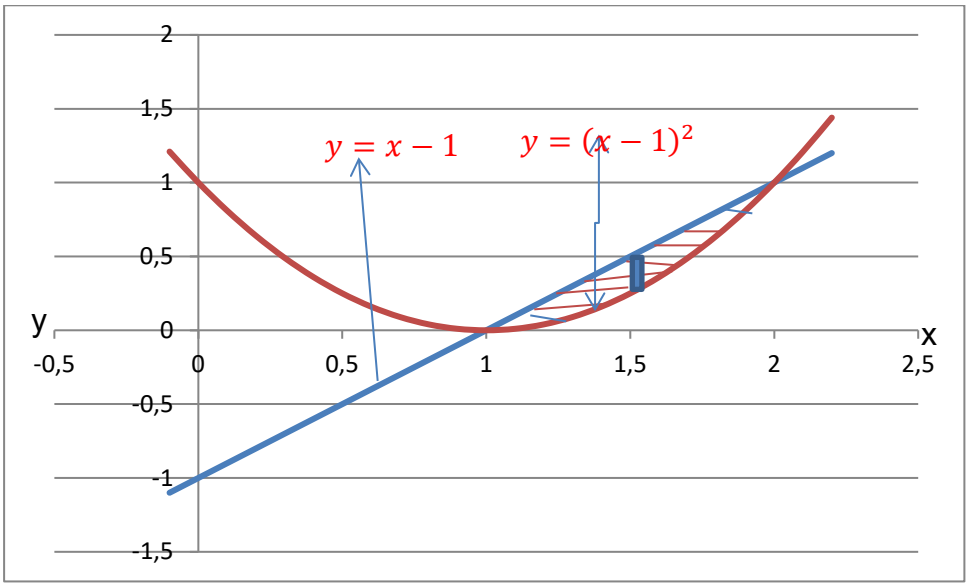
$(x-2)(x-1) = 0$

$x = 1 \; or \; x = 2$

$y = 0 \; or \; y = 1$

Thus, coordinates of point of intersection are: $(1;0)$✓and $(2;1)$✓ (2)

5.2.2



✓= 1 mark for indication of enclosed area
✓= 1 mark for indication of the vertical or horizontal strip (2)

5.2.3 $A = \int_1^2 (y_1 - y_2)\,dx$

$= \int_1^2 [(x-1) - (x-1)^2]\,dx$
$= \int_1^2 (-x^2 + 3x - 2)\,dx$✓

$= \left[-\frac{x^3}{3} + \frac{3}{2}x^2 - 2x\right]_1^2$✓
$= 0{,}167 units^2$✓ (3)

5.2.4 $V = \pi \int_1^2 A(x)\,dx$

$= \pi \int_1^2 [(x-1)^2 - (x-1)^4]\,dx$

$= \pi \int_1^2 [x^2 - 2x + 1 - (-x^4 - 4x^3 + 6x^2 - 4x + 1)]\,dx$✓

$= \pi \int_1^2 (-x^4 + 4x^3 - 5x^2 + 2x)\,dx$✓

$= \pi \left[-\frac{x^5}{5} + x^4 - \frac{5}{3}x^3 + x^2\right]_1^2$✓

$= \frac{2}{15}\pi units^3$✓

$= 0{,}4189 units^3$✓ (4)

5.3 $y = r^2\,dA$

$= x^2.4$✓

$= 4\int_0^8 x^2\,dx$ ✓

$= 4\left[\frac{x^3}{3}\right]_0^8$ ✓

$= 682{,}667$✓ (4)

[19]

QUESTION 6

6.1 $x\,dy = y\ln y\,dx$, given $x = 2$ when $y = e$

$$\frac{dy}{y\ln y} = \frac{dx}{x}\checkmark$$

$$\ln(\ln y) = \ln x + c\checkmark$$

$$\ln(\ln e) = \ln 2 + c$$

$$c = -\ln 2\checkmark$$

$$\ln(\ln y) = \ln x - \ln 2\checkmark \qquad (4)$$

6.2 $$\frac{d^2y}{dx^2} = x^4 - \sin x$$

$$\frac{dy}{dx} = \frac{1}{5}x^5 + \cos x + A\checkmark$$

$$y = \frac{1}{30}x^6 + \sin x + Ax + B\checkmark \qquad (2)$$

[6]

TOTAL: 100