higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

# MARKING GUIDELINE

## NATIONAL CERTIFICATE

## APRIL EXAMINATION

## MATHEMATICS N5

## 4 APRIL 2016

**This marking guideline consists of 10 pages.**

## INSTRUCTIONS AND INFORMATION

1. Half marks are not allocated, unless indicated otherwise.

2. Where formula is required, using the wrong formula is a principle error and NO marks are allocated.

3. Students should show ALL formulae and intermediate steps and simplify where possible.

4. ALL final answers must be rounded off to THREE decimal places (unless indicated otherwise).

5. Questions may be answered in any order, but subsections of questions must be kept together. If subsections are separated, the student can be penalised by ONE mark.

6. Where a student copied wrong from the question paper, and the standard of the question is still the same, the student will be penalised by ONE mark.
   If the copying error simplifies the question and makes it easier, the student forfeits the marks.

7. Questions must be answered in blue or black ink. Answers in PENCIL will not be marked as it is regarded as rough work.

## QUESTION 1

1.1 1.1.1 $\lim\limits_{x\to 0}\dfrac{e^x}{xe^x-2x}$

$=\dfrac{1}{0}$ ✓

$=\infty$ ✓ (2)

1.1.2 $\lim\limits_{x\to\frac{\pi}{2}}(\sec x-\tan x)$

$=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{1-\sin x}{\cos x}(\dfrac{0}{0})$ ✓

$=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{-\cos x}{-\sin x}$ ✓

$=\dfrac{0}{-1}$

$=0$ ✓ (3)

1.2 $f(-3)=\dfrac{(-3)^3-27}{x-3}=\dfrac{-54}{-6}=9$ ✓

$f(x)$*is continuous at* $x=$-3 ✓ (2)

**[7]**

## QUESTION 2

2.1 WENK: $\lim\limits_{h\to 0}\dfrac{\cosh-1}{h}=0$ ; $\lim\limits_{h\to 0}\dfrac{\sinh}{h}=1$

$\lim\limits_{h\to 0}\dfrac{\cos(x+h)-\cos x}{h}$ ✓

$=\lim\limits_{h\to 0}\dfrac{\cos x.\cosh-\sin x.\sinh-\cos x}{h}$ ✓

$=\lim\limits_{h\to 0}\cos x.\dfrac{\cosh-1}{h}-\sin x.\dfrac{\sinh}{h}$ ✓

$=\cos x(0)-\sin x(1)$

$=-\sin x$ ✓ (4)

2.2 2.2.1 $y=\cos^4(x^2-4)+\cos(x^2-4)^4$

$=4\cos^3(x^2-4).-\sin(x^2-4).2x \;-\; \sin(x^2-4)^4.4(x^2-4)^3.2x$ ✓ ✓ ✓ ✓ (4)

2.2.2 ✓ ✓ ✓ ✓

$$= \frac{6\tan\sqrt{1-x}.\sec^2\sqrt{1-x}.\frac{-1}{2\sqrt{1-x}}.2\ln\cos ec4x - 3\tan^2\sqrt{1-x}.\frac{-8\cos ec4x.\cot 4x}{\cos ec4x}}{(2\ln\cos ec4x)^2} \quad \checkmark .$$

(5)

2.2.3 $y = arc\sec(2.4^{3x})$

$$\frac{dy}{dx} = \frac{6.4^{3x}\ln 4}{2.4^{3x}\sqrt{(2.4^{3x})^2 - 1}} \quad \checkmark\checkmark$$

(2)

2.3 $y = x^{e^x}$

$\ln y = e^x.\ln x$ ✓

$\frac{1}{y}\frac{dy}{dx} = e^x.\ln x + \frac{e^x}{x}$ ✓✓

$\frac{dy}{dx} = x^{e^x}(e^x \ln x + \frac{e^x}{x})$ ✓

(4)

2.4 2.4.1

$12x^3 - 1y - x\frac{dy}{dx} = 3.2^{3x}\ln 2$ ✓

$\frac{12x^3 - y - 3.2^{3x}\ln 2}{x} = \frac{dy}{dx}$ ✓

: $at(1,-5)$

$\frac{dy}{dx} = \frac{12 - (-5) - 3.8\ln 2}{1}$

$\frac{dy}{dx} = 0{,}364$ ✓

(3)

2.4.2 $y = mx + c$

$-5 = 0{,}364(1) + c$ ✓

$-5{,}364 = c$ ✓

$equation: y = 0{,}364x - 5{,}364$

(2)
**[24]**

## QUESTION 3

3.1 3.1.1 $f(x) = x^3 - 5x - 4$

$3x^2 - 5 = 0$ ✔

$x = 1{,}291$ or $x = -1{,}291$ ✔mark for both x-values

$y = -8{,}303$ $y = 0{,}303$ ✔mark for both y-values (3)

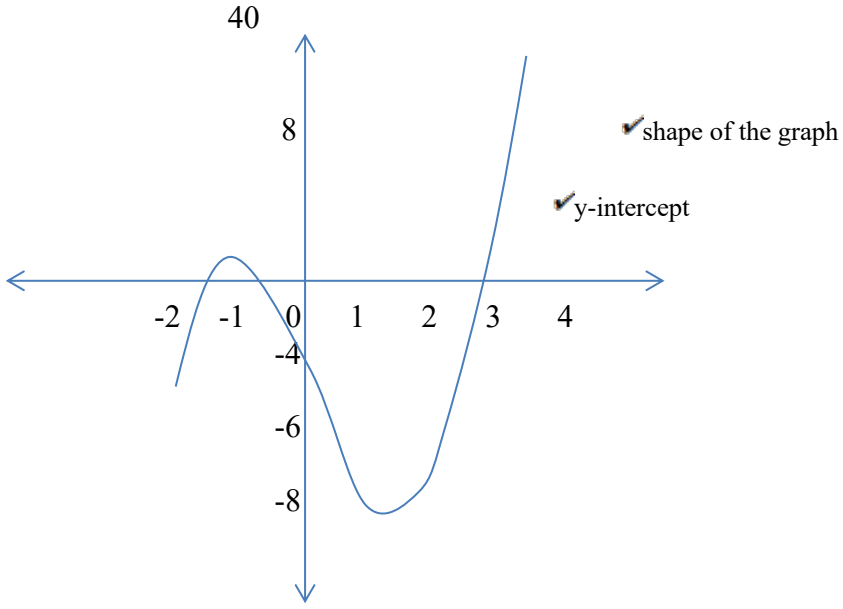
3.1.2 Use values on the table: $0 \le x \le 4$

| X | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Y | -4 | -8 | -6 | 8 | 40 |

✔✔✔ marks for any 4 correct values

y-value changes from negative to positive indicating a root ✔ (4)

3.1.3




✔shape of the graph

✔y-intercept (2)

3.1.4 ✔

$$e = -\frac{f(2{,}7)}{f'(2{,}7)} = -\frac{2{,}183}{16{,}87} = -0{,}129$$ ✔

✔

$a = 2{,}7 - 0{,}129 = 2{,}571$ ✔ (4)

3.2 3.2.1

$A = xy$

$50 = xy$ ✔

$50x^{-1} = y$

$P = 2x + 100x^{-1}$ ✔

$\frac{dP}{dt} = 2\frac{dx}{dt} - \frac{100}{x^2}.\frac{dx}{dt}$ ✔

$\frac{dP}{dt} = 2(3) - \frac{100}{7^2}.3$ ✔

$\frac{dP}{dt} = -0{,}122$ ✔ (5)

3.2.2 $\frac{dP}{dt} = 0$ only if $\frac{dx}{dt} = \frac{dy}{dt}$ ✔

Therefore: $x=y$

Meaning the figure is a square ✔ (2)

3.3 3.3.1

$s(t) = 3t^{\frac{1}{2}} - 3t^{\frac{3}{2}} - t^{\frac{5}{2}}$

$v = \frac{3}{2\sqrt{t}} - \frac{9}{2}t^{\frac{1}{2}} - \frac{5}{2}t^{\frac{3}{2}}$ ✔✔

$v =$ -23,987 ✔✔ (4)

3.3.2

$a = \frac{-3}{4t^{\frac{3}{2}}} - \frac{9}{4t^{\frac{1}{2}}} - \frac{15}{4}t^{\frac{1}{2}}$ ✔✔

$a =$ -7,159 ✔ (3)

**[27]**

## QUESTION 4

4.1

$$\frac{(e^x + e^{-x})^5}{5} + c \quad ✔✔$$ (2)

4.2 4.2.1

$$y = \frac{\sin x}{\sqrt{1+\cos x}}$$

$$u = 1 + \cos x$$

$$dx = \frac{du}{-\sin x} \quad ✔$$

$$\int \frac{\sin x}{\sqrt{u}} \cdot \frac{du}{-\sin x}$$

$$-\int u^{\frac{-1}{2}} du \quad ✔$$

$$-2u^{\frac{1}{2}} + c$$

$$-2\sqrt{1+\cos x} + c \quad ✔$$ (3)

4.2.2

$$y = x.\sec^2 x$$

$$f(x) = x \qquad g'(x) = \sec^2 x$$
$$f'(x) = 1 \qquad g(x) = \tan x \quad ✔$$

$$x.\tan x - \int \tan x dx$$

$$x.\tan x - \ln(\sec x) + c$$

✔ ✔ (3)

4.2.3

$$y = \cos 6x.\cos 2x$$

$$\int \cos(8x) + \cos(4x) dx$$

$$= \frac{\sin 8x}{8} + \frac{\sin 4x}{4} + c$$

✔ ✔ (2)

4.2.4

$$y = \frac{2}{3 + 4x^2}$$

$$\int \frac{2}{3+4x^2} dx \quad ✔$$

$$= \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + c$$ (3)

✔ ✔

4.3 $$y = \frac{x^3 - 2}{x^2 + 2x}$$

$$\frac{x^3 - 1}{(x)(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

$x^3 - 1 = A(x+2) + Bx$ ✔

$x = -2 : \frac{9}{2} = B$ ✔

$x = 0 : -\frac{1}{2} = A$ ✔

$$\int \frac{-\frac{1}{2}}{x} + \frac{\frac{9}{2}}{(x+2)}\,dx$$

$$= -\frac{1}{2}\ln x - \frac{9}{2}\ln(x+2) + c$$

✔ ✔ (5)

4.4

$$= \int x + 5 + \frac{25}{x-5}\,dx$$ ✔✔

$$= \frac{x^2}{2} + 5x + 25\ln(x-5) + c$$

✔ ✔ (4)

**[22]**

## QUESTION 5

5.1 5.1.1

$$\int_0^4 \sqrt{16-x^2}-(4-x)dx \quad ✔$$

$$=[8\sin^{-1}\frac{x}{4}+\frac{x}{2}\sqrt{16-x^2}-4x+\frac{x^2}{2}]_0^4 \quad ✔$$

$$=(\ 4{,}566\text{-}0)$$

$$=4{,}566 \quad ✔$$

(3)

5.1.2

$$\pi\int_0^4 \sqrt{16-x^2}^{\,2}-(4-x)^2\,dx \quad ✔$$

$$=\pi[16x-\frac{x^3}{3}-16x+\frac{8x^2}{2}-\frac{x^3}{3}]_0^4 \quad ✔✔$$

$$=\pi[(21{,}333)-(0)]$$

$$=67{,}02 \quad ✔$$

(4)

5.2 .

$$=-5[\frac{e^{-st}}{-s}]_0^\infty \quad ✔$$

$$=\frac{5}{s}[\frac{1}{e^{st}}]_0^\infty \quad ✔$$

$$=\frac{5}{s}(0-1) \quad ✔$$

$$=\frac{-5}{s} \quad ✔$$

(4)
**[11]**

## QUESTION 6

6.1

$$\frac{dy}{dx} = \frac{\tan^2 y}{\cos ec^2 x}$$

$$\frac{1}{\tan^2 y}dy = \frac{1}{\cos ec^2 x}dx \quad ✔$$

$$\cot^2 y dy = \sin^2 x dx$$

$$✔(\cos ec^2 y - 1)dy = (\frac{1}{2} - \frac{1}{2}✔\cos 2x)dx$$

$$-\cot y - ✔y = \frac{1}{2}x - \frac{\sin 2x}{4} + c \quad ✔$$

(4)

6.2 ✔

$$\frac{dy}{dx} = \frac{-1x^3}{6} + \frac{3x^2}{4} + \pi x + c \quad ✔$$

$$y = \frac{-x^4}{24} + \frac{3x^3}{12} + \frac{\pi x^2}{2} + cx + d$$

$$-3 = 3{,}725 + c \quad ✔$$

$$-6{,}725 = c$$

$$2 = -4{,}946 + d$$

$$6{,}946 = d \quad ✔$$

(5)
**[9]**

**TOTAL: 100**