higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N4

1 APRIL 2020

Hierdie nasienriglyn bestaan uit 10 bladsye.

Kopiereg voorbehou

VRAAG 1

1.1
$$\frac{\cos x.\cos ecx}{\tan x.\cos x.-\sec x}\checkmark \qquad \frac{\cos x.\sec x}{\tan x.\sin x-\csc x}\checkmark$$
$$=\frac{\cos x.\frac{1}{\sin x}}{\tan x.\cos x.\frac{-1}{\cos x}}\checkmark \qquad =\frac{\frac{\cos x}{1}.\frac{1}{\cos x}}{-\frac{\sin x}{\cos x}.\frac{\sin x}{1}.\frac{1}{\sin x}}\checkmark$$
$$=\frac{\tan x}{\tan x.-\cot x} \qquad =-\frac{1}{\tan x}\checkmark$$
$$=-\tan x\ \checkmark \qquad =-\cot x$$ (3)

1.2
$$2\cos^2\beta+\cos\beta-1=0\ ;-180^0\le\beta\le 180^0$$
$$(2\cos 2\beta-1)(\cos 2\beta+1)=0\checkmark$$
$$\cos 2\beta=\frac{1}{2}\text{ or }\cos 2\beta=-1$$
$$2\beta=60^0\text{ or }2\beta=270^0$$
$$\beta=30^0\text{ or }\beta=135^0\checkmark\checkmark$$
$$GS=\left[30^0;45^0\right]\checkmark\checkmark$$ (5)

1.3 1.3.1
$$1-\sin x=2\cos^2\frac{x}{2}$$
$$2\cos^2\frac{x}{2}=1-\sin x$$
$$\cos^2\frac{x}{2}=\frac{1-\sin x}{2}\checkmark$$
$$\cos\frac{x}{2}=\pm\sqrt{\frac{1-\sin x}{2}}\checkmark$$ (2)

1.3.2
$$\cos 15^0\rightarrow\cos\frac{30^0}{2}=\sqrt{\frac{1-\sin 30^0}{2}}\checkmark$$
$$=\sqrt{\frac{1-\frac{1}{2}}{2}}\checkmark$$
$$=\sqrt{\frac{1}{4}}$$
$$=\frac{1}{2}\checkmark$$ (3)

1.4 $$\frac{\sin x+\sin 2x}{1+\cos x+\cos 2x}=\tan x$$

$$\text{LHS}=\frac{\sin x+\sin 2x}{1+\cos x+\cos 2x}$$

$$=\frac{\sin x+2\sin x\cos x}{1+\cos x+\cos^2 x-\sin^2 x}\ \checkmark\checkmark$$

$$=\frac{\sin x(1+2\cos x)}{\cos x(1+2\cos x)}\ \checkmark$$

$$=\frac{\sin x}{\cos x}$$

$$=\tan x\ \checkmark$$

$$=\text{RK}$$ (4)

1.5 $$\cos 75^0=\cos(45^0+30^0)$$

$$=\cos 45^0.\cos 30^0-\sin 45^0.\sin 30^0\ \checkmark$$

$$=\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}.\frac{1}{2}$$

$$=\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}=\checkmark$$

$$=\frac{\sqrt{3-1}}{2\sqrt{2}}\ \checkmark$$

(3)
[20]

VRAAG 2

2.1 2.1.1

$$Area=\int_0^{\frac{\pi}{4}}2\cos x dx$$

$$=2\int_0^{\frac{\pi}{4}}\cos x dx$$

$$=2\left[\sin x+c\right]_0^{\frac{\pi}{4}}\ \checkmark$$

$$=2\left[\sin\left(\frac{\pi}{4}\right)+c-(\sin 0^0+c)\right]\checkmark$$

$$=2\left[\frac{1}{\sqrt{2}}+c-c\right]\checkmark$$

$$=1{,}414\ units^2\ \checkmark$$ (4)

2.1.2 $y = 2\cos x$

(i) Vir DP, laat $x = 0$

$-2\sin x = 0$

$x = \sin^{-1} 0$

$x = 0^0$ or $x = 180^0$ ✓✓

$y - values,\ if\ x = 0^0$

$y = 2\cos 0^0$

$y = 2$ ✓

$if\ x = 180^0$

$y = 2\cos 180^0$

$y = -2$ ✓

$T.P_1 = (0^0;2)\ and\ T.P_2 = (180^0;-2)$ ✓✓ (6)

2.1.3 $\frac{d^2y}{dx^2} = -2\cos x$

$= -2\cos 0^0$

$= -2$ maximum

$\frac{d^2y}{dx^2} = -2\cos x$ ✓

$= -2\cos 180^0$

$= 2$ minimum

$T.P_1 = (0^0;2)$maximum ✓ $T.P_2 = (180^0;-2)$minimum ✓ (3)

2.2 $\frac{-1}{2x^2} + \frac{1}{2}\cos 3x + 2x^{\frac{3}{2}} - \frac{10.10^x}{\ln 10} + \frac{1}{2}\sin 2x - 2x + c$

✓ ✓ ✓ ✓ ✓ ✓ ✓ (7)

[20]

VRAAG 3

3.1 $y = \sqrt{\tan x}$ $\quad$ $\ell et\ \mathrm{u} = \tan x$

$y = \sqrt{u} = u^{\frac{1}{2}}$ $\quad$ $\frac{dy}{du} = \sec^2 x$ ✓

$\frac{dy}{du} = \frac{2u^{\frac{3}{2}}}{3}$

$\therefore \quad \frac{dy}{du} = \frac{dy}{du} \text{ x } \frac{dy}{du}$ ✓

$= \frac{2u^{\frac{3}{2}}}{3} \text{ x } \sec^2 x$

$= \frac{2}{3}\sqrt{\mathrm{u}^3}.\sec^2 x$ ✓

$= \frac{2}{3}\sqrt{\tan^3 x}.\sec^2 x$ ✓ (4)

3.2 $y = 2^x + \frac{1}{2}\ln x + \frac{1}{2}e^x + 6t$

$\frac{dy}{dx} = 2^x.\ln 2 + \frac{1}{2x} + \frac{1}{2}e^x + 0$

✓ ✓ ✓ ✓ (4)

3.3 $\lim\limits_{x\to 2}\left(\frac{x^2-4}{x-2}\right)$

$= \frac{(x-2)(x+2)}{(x-2)}$ ✓

$= (x+2)$ ✓

$= 2+2$

$= 4$ ✓ (3)

3.4 $y = -x^3 + 5x$

$\lim\limits_{x\to 0}\frac{(-x+h)^3 + 5(x+h) - (-x^3+5x)}{h}$ ✓

$= \frac{-x^3 - 3x^2.h - 3x.h^2 + 5x + 5h + x^3 - 5x}{h}$ ✓

$= \frac{h(-3x^2 - 3xh + 5)}{h}$ ✓

$= -3x^2 + 5$ ✓ (4)

3.5
$$S = \int_1^4 \left(\sqrt{t} - 5\right)^2 dt$$
$$= \int_1^4 \left(\sqrt{t} - 5\right)\left(\sqrt{t} - 5\right)dt$$
$$= \int_1^4 \left(t - 5\sqrt{t} - 5\sqrt{t} + 25\right)dt \checkmark$$
$$= \int_1^4 \left(t - 10\sqrt{t} + 25\right)dt$$
$$= \left[\frac{t^2}{2} - \frac{20t^{3/2}}{3} + 25t\right]_1^4$$
$$= \frac{4^2}{2} - \frac{20(4)^{3/2}}{3} + 25(4) - \left(\frac{(1)^2}{2} - \frac{20(1)^{3/2}}{3} + 25(1)\right) \checkmark$$
$$= 8 - 53{,}333 + 100 - \frac{1}{2} + 6{,}667 - 25$$
$$= 35{,}834 \checkmark$$ (3)

3.6
$$\int \left(\frac{4\sec^4\theta - 9}{2\sec^2\theta + 3}\right)d\theta$$
$$= \int \frac{\left(2\sec^2\theta + 3\right)\left(2\sec^2\theta - 3\right)}{\left(2\sec^2\theta + 3\right)} \checkmark$$
$$= \int \left(2\sec^2\theta - 3\right)d\theta$$
$$= 2\tan\theta - 3\theta + c \checkmark$$ (2)

[20]

VRAAG 4

4.1

$$(3-4j)^2 = \frac{-x-yj}{j^2}$$

$$j^2(3-4)(3-4j) = -x-yj$$

$$j^2(9-12j-12j+16j^2) = -x-yj \checkmark$$

$$j^2(9-24j+16j^2) = -x-yj$$

$$-(9-24j-16) = -x-yj \checkmark$$
$$-9+24j+16 = -x-yj$$
$$24j+7 = -x-yj$$
$$x = -7 \text{ en } y = -24 \checkmark\checkmark$$ (4)

4.2 $\left(\sqrt{3}-j\right)^5$

$$|r| = \sqrt{x^2+y^2}$$
$$= \sqrt{\left|(3)^2+\left(-1^2\right)\right|}$$
$$= \sqrt{4}$$
$$= 2 \quad \checkmark$$

$$\tan\theta = \frac{y}{x}$$
$$= \frac{-1}{\sqrt{3}}$$
$$\tan\theta = 0{,}578$$
$$\theta = 30{,}023$$
$$\therefore \quad \theta = 360^0 - 30{,}023$$
$$\theta = 329{,}977 \checkmark$$

$\therefore = (r\angle\theta)^2$	$(r\angle\Theta)^5$
$= (2\angle 329{,}977)^2$	$(2\angle 329{,}977)^5$
$= 4\angle 659{,}954 \checkmark$	$32\angle 1649{,}855$
$\therefore r(\cos\theta + i\sin\theta)$	$\therefore r(\cos\theta + i\sin\theta)$
$= 4(\cos 659{,}954 + i\sin 659{,}954)$	$32(\cos 1649{,}855 + \text{i sin } 1649{,}855)$
$= 1{,}997 - i3{,}466 \checkmark$	

(4)

4.3
$$\frac{2cis(-60^0).5cis45^0}{4cis33^0.3cis(-21^0)}$$
$$=\frac{2\angle -60^\circ.5\angle 45^\circ}{4\angle 33^\circ.3\angle 21^\circ}$$
$$=\frac{10\angle 60^\circ+45^\circ}{12\angle 33^\circ-21^\circ}\checkmark$$
$$=\frac{10\angle -15}{12\angle 12}\checkmark$$
$$=0{,}833\angle -27\checkmark$$
(3)

4.4
$$7a+6b=27$$
$$3a+4b=5$$
$$D=\begin{vmatrix}-7 & 6\\ 3 & 4\end{vmatrix}$$
$$D=-28-18$$
$$D=-46\checkmark$$
$$D_a=\begin{vmatrix}27 & 6\\ 5 & 4\end{vmatrix}\quad \therefore a=\frac{Da}{D}=\frac{78}{-46}=-1{,}696\checkmark$$
$$=108-30\checkmark$$
$$=78$$
$$D_b=\begin{vmatrix}-7 & 27\\ 3 & 5\end{vmatrix}\quad \therefore b=\frac{Db}{D}=\frac{-116}{-46}=2{,}522\checkmark$$
$$=-35-81$$
$$=-116\checkmark$$
(5)

4.5 4.5.1
$$\begin{vmatrix}\frac{2}{3} & -1 & -\frac{1}{2}\\ -\frac{1}{4} & 2 & 0\\ 5 & -8 & \frac{-2}{3}\end{vmatrix}$$
$$D=\begin{vmatrix}\frac{-1}{4} & 0\\ 5 & \frac{-2}{3}\end{vmatrix}+2\begin{vmatrix}\frac{2}{3} & \frac{-1}{2}\\ 5 & \frac{-2}{3}\end{vmatrix}-8\begin{vmatrix}\frac{2}{3} & \frac{-1}{2}\\ \frac{-1}{4} & 0\end{vmatrix}\checkmark$$
$$=0{,}167+2(2{,}945)-8(0-0{,}125)\checkmark$$
$$=0{,}167+5{,}89+1$$
$$=7{,}059\checkmark$$
(3)

4.5.2 $= (-1)^3 \begin{vmatrix} \frac{-1}{4} & 0 \\ 5 & \frac{-2}{3} \end{vmatrix}$

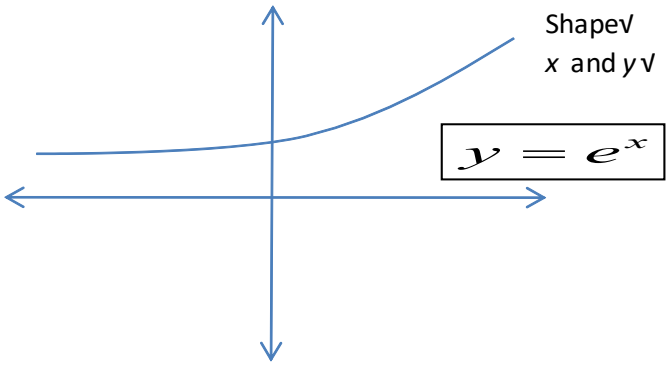
$= -(0{,}167 - 0)$

$= -0{,}167$ ✓ (1)

[20]

VRAAG 5

5.1 5.1.1

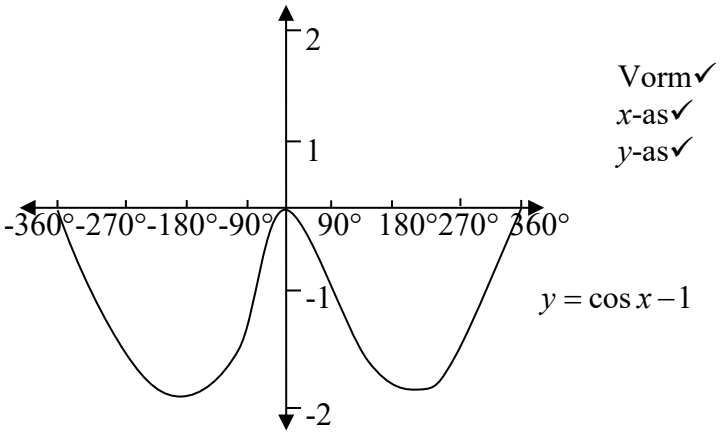


(2)

5.1.2 $= \{0 < y < \infty\}$ ✓ (1)

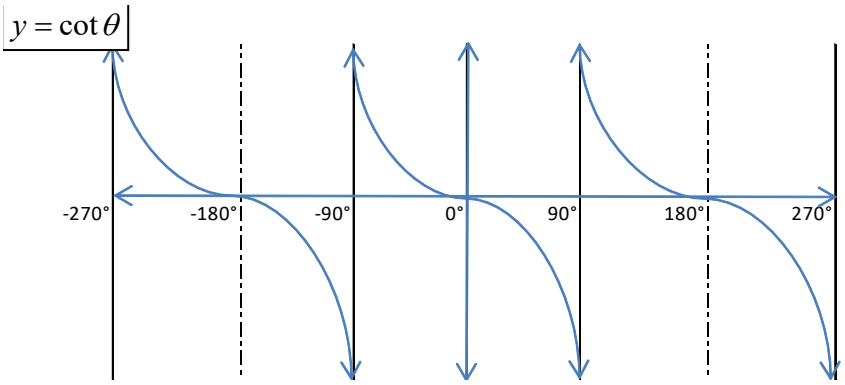
5.1.3 $x = e^y$ ✓ (1)

5.2



(3)

5.3



Shape √
x-axis √
y-axis √

(3)

5.4

$$\ln\sqrt{x-1}+\ln\sqrt{x+1}=a$$
$$\ln\sqrt{(x-1)(x+1)}=a \checkmark$$
$$\frac{1}{2}\ln(x^2-1)=a$$
$$\ln(x^2-1)=2a \checkmark$$
$$x^2-1=e^{2a}$$
$$x^2=e^{2a}+1 \checkmark$$
$$x=\pm\sqrt{e^{2a}+1} \checkmark$$

(4)

5.5

$$27(k-1)^3-8(k+1)^3$$
$$[3(k-1)-2(k+1)][(3(k-1))^2+6(k-1)(k+1)+(2(k+1))^2]$$
$$\checkmark \qquad \checkmark\checkmark$$

(3)

5.6

$$3^{2x-1}=\ln 5$$
$$\ln 3(2x-1)=\ln 5$$
$$2{,}197x-1{,}099=1{,}609 \checkmark$$
$$x=\frac{2{,}708}{2{,}197} \checkmark$$
$$=1{,}233 \checkmark$$

(3)
[20]

TOTAAL: 100

Kopiereg voorbehou