higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N4

(16030164)

27 July 2021 (X-paper)
09:00–12:00

Scientific calculators may be used.

This question paper consists of 5 pages and 1 formula sheet.

050Q1G2127

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N4
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show all intermediate steps and simplify where possible.
5. All final answers must be rounded off to THREE decimal places (unless indicated otherwise).
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Only use a black or blue pen.
8. Write neatly and legibly.

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QUESTION 1

1.1 Integrate:

$$\int \left(\frac{2}{\sin x \,.\, \tan x} - \frac{7}{\sin^2 x} + 4\pi^{3x} - \frac{4.e^{-4x}}{3} + \frac{1}{\cos x \,.\, \cot x} + \frac{7\pi}{8} \right) dx \quad (7)$$

1.2 Simplify and integrate the following:

$$\int \left(\frac{7\cos x.\sin x + 9}{\cos^2 x} \right) dx \quad (5)$$

1.3 1.3.1 Sketch and clearly indicate the area enclosed by the graph $y = 2e^x$, the *x-axis* and the ordinates $x = -1$ and $x = 1$.

Also show the representative strip to calculate the area. (4)

1.3.2 Calculate using INTEGRATION the magnitude of the area indicated in QUESTION 1.3.1. (4)

[20]

QUESTION 2

2.1 Determine the general solution of:

$$\sin 2x = \cos\left(\frac{x}{2}\right) \quad (5)$$

2.2 Prove the following identity:

$$\frac{\tan x}{\sec x - 1} + \frac{\tan x}{\sec x + 1} = \frac{2}{\cos x} \quad (5)$$

2.3 Simplify as far as possible:

$$\frac{\sin^2(180^0 + \theta) + \cos(180^0 - \theta).\cos ec(90^0 - \theta)}{\tan(180^0 - \theta).\tan(90^0 - \theta)} \quad (5)$$

2.4 If $\tan\alpha = \frac{12}{5}$, where $\alpha \,\varepsilon\, [0^0, 90^0]$, determine, without using a calculator, the value of: $5\sin x + 12\cos x$. (5)

[20]

QUESTION 3

3.1 Sketch the graph of $xy = 10$. (3)

3.1.1 Is the graph $xy = 10$ asymmetrical or symmetrical about the *x-axis*?

3.1.2 Is the graph $xy = 10$ continuous or discontinuous?

(2 × 1) (2)

3.2 Sketch the graph of $x = 2^{-y}$. (3)

3.2.1 Is the graph $x = 2^{-y}$ continuous or discontinuous? (1)

3.3 Sketch the graph of $y = \cot x, -360^0 \leq x \leq 360^0$. (4)

3.4 Solve for x if $: 3^{2x+1} = \ln(\ln 3)$. (3)

3.5 Expand $\left(x - \frac{3}{2}\right)^{\frac{3}{4}}$ to FOUR terms. (4)

[20]

QUESTION 4

4.1 Given:

$$a + 2b + 3c = 1$$
$$2a + 3b - 2c = 3$$
$$-2a - 3b - c = 5$$

4.1.1 Determine the determinant of coefficients expanding along the first column. (4)

4.1.2 Write down and evaluate the minor of (-1). (2)

4.1.3 Write down and evaluate the co-factor of (-3). (2)

4.2 Given: $Z = -2 - j\sqrt{6}$

4.2.1 Write down the conjugate of Z. (1)

4.2.2 Determine the modulus of Z. (2)

4.2.3 Determine the argument of Z and convert it to polar form. (4)

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4.2.4 Present Z in an Argand diagram/complex plane and indicate ALL the values on it. (3)

4.3 Simplify j^{19}. (2)

[20]

QUESTION 5

5.1 Differentiate with respect to x from first principles if given:

$$y = ax^2 + bx + c$$ (5)

5.2 Differentiate with respect to x, using the product rule:

$$y = \sec x \text{ x } \tan x$$ (3)

5.3 Determine $\frac{dy}{dx}$ if:

$$y = \frac{-2}{\cot x} + \frac{2}{\sqrt{x^3}} - \frac{5}{\sin x} + 4^{-3x} + \frac{1}{7e^{3x}}$$ (5)

5.4 Determine, with the aid of DIFFERENTIATION, the coordinates of the minimum and maximum turning points given: $y = x^3 - 6x^2 + 9x - 18$, and distinguish between the MAXIMUM and MINIMUM turning points by using the second derivatives. (7)

[20]

TOTAL: 100

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MATHEMATICS N4

FORMULA SHEET

NEW SYLLABUS

$a^x = b \Leftrightarrow \log a^x = \log b$

$\ln x = \log_e x$

$\left(r\left|\underline{\theta}\right.\right)^n = r^n\left|\underline{n\theta}\right. \qquad a + bj = c + dj \Leftrightarrow a = c and b = d$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$

$\cos(a \pm b) = \cos a \cos \mp \sin a \sin b$

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \cos ec^2 x$

$1 + \tan^2 x = \sec^2 x$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

y	$\frac{dy}{dx}$
an^n	nax^{n-1}
ka^x	$ka^x \ln a$
$k \ln x$	$\frac{k}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\cos ec^2 x$
$\sec x$	$\sec x \tan x$
$\cos ecx$	$-\cos ecx \cot x$

$$y = u(x) \cdot v(x)$$

$$\Rightarrow \frac{dy}{dx} = u(x)v^1(x) + u^1(x)v(x)$$

$$y = \frac{u(x)}{v(x)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{v(x)u^1(x) - u(x)v^1(x)}{[v(x)]^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} x \frac{du}{dx}$$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$$

$$\int \frac{a}{x} dx = a\ell nx + c$$

$$\int ka^x dx = \frac{ka^x}{\ell na} + c$$

$$A_{ox} = \int_a^b y dx$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \tan x dx = \ell n \sec x + c$$

$$\int \sec x dx = \ell n(\sec x + \tan x) + c$$

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