higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N4

29 JULY 2019

This marking guideline consists of 10 pages.

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QUESTION 1

1.1

$$f(x) = 2x^{-4}$$

$$f(x+h) = 2(x+h)^{-4}$$

$$= 2\left[x^{-4} + (4)x^{-5}.h + (-4)(-5)x^{-6}h^2 + \frac{(-4)(-5)(-6)x^{-7}h^3}{6}\right]$$

$$f(x+h) = 2x^{-4} - 8x^{-5}h + 2x^{-6}h^2 - 40x^{-7}h^3 \quad \checkmark$$

$$f(x+h) - f(x) = 2x^{-4} - 8x^{-5}h + 2x^{-6}h^2 - 40x^{-7}h^3 - 2x^{-4}$$

$$f(x+h) - f(x) = -8x^{-5}h + 2x^{-6}h^2 - 40x^{-7}h^3 \quad \checkmark$$

$$\frac{f(x+h) - f(x)}{h} = \frac{-8x^{-5}h + 2x^{-6}h^2 - 40x^{-7}h^3}{h}$$

$$= -8x^{-5} + 2x^{-6}h - 40x^{-7}h^2 \quad \checkmark$$

$$\lim_{\substack{h \to \theta \\ h \to 0}} \left[\frac{f(x+h) - f(x)}{h}\right] = -8x^{-5} + 2x^{-6}(0) - 40x^{-7}(0)^2$$

$$= -8x^{-5} \quad \checkmark \qquad (4)$$

1.2

$$y = (5x+1)^{\frac{1}{2}}$$

$$let\, u = 5x+1 \qquad y = u^{\frac{1}{2}}$$

$$\frac{du}{dx} = 5 \quad \checkmark \qquad \frac{dy}{du} = \frac{1}{2\sqrt{u}} \quad \checkmark$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$= 5 \times \frac{1}{2\sqrt{5x+1}} \quad \checkmark$$

$$= \frac{5}{2\sqrt{5x+1}} \quad \checkmark \qquad (4)$$

1.3

$$\frac{dx}{dy} = \frac{1}{y} - 0 + 2\pi \sin y - \frac{3}{4y} - \frac{1}{4\sqrt{y^3}} \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \qquad (5)$$

1.4 $\frac{dy}{dx} = 3x^2 - 2$ ✓

$3x^2 = 2$

$x^2 = \pm\frac{2}{3}$

$x_1 = 0{,}816; x_2 = -0{,}816$ ✓

$y_1 = (0{,}816)^3 - 2(-0{,}816) - 4$

$y_1 = -5{,}089$

$y_2 = (-0{,}816)^3 - 2(-0{,}816) - 4$ ✓

$y_2 = -2{,}911$

∴ Co-ordinates of the turning points are: (-0,816; -2,911) and (0,816; -5,089)
✓ ✓

P.O.I :

$\frac{d^2y}{dx^2} = 6x$ ✓

$x = 0$

$y = (0)^3 - 2(0) - 4$

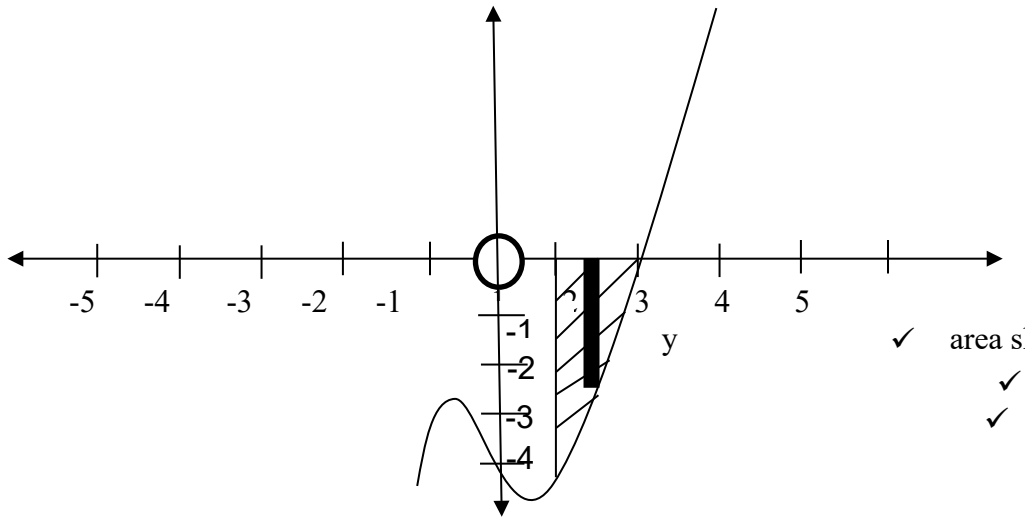
$y = -4$ ✓

$\therefore P.O.I. = (0;-4)$

(7)
[20]

QUESTION 2

2.1 2.1.1



✓ area shaded
✓ strip
✓ shape

(3)

2.1.2
$$Aox = \int_1^2 \left(x^3 - 2x - 4\right)dx \quad \checkmark$$
$$= \left[\frac{x^4}{4} - x^2 - 4x\right]_1^2 \quad \checkmark$$
$$= -8 - (-4{,}75) \quad \checkmark$$
$$Aox = -3{,}25$$
$$= 3{,}25u^2 \quad \checkmark \qquad (4)$$

2.2 2.2.1
$$\int\left(\cos t - \sec^2 t - (2t)^{\frac{1}{2}} + 2e^{-2t} + \frac{2}{t} - 4^{4t} + 4\right)dt$$
$$\sin t - \tan t - \frac{2}{3}(2t)^{\frac{3}{2}} - e^{-2t} + 2\ln^t - \frac{4^{4t}}{4\ln^4} + 4t + c$$
$\checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark$ (7)

2.2.2
$$\left[\ln 3x - \ln x\right]_2^4 \quad \checkmark$$
$$\{[\ln 3(4) - \ln 4] - [\ln 3(2) - \ln 2]\} \quad \checkmark$$
$$= 1{,}098 - 1{,}098$$
$$= 0 \quad \checkmark \qquad (3)$$

2.3
$$\int \frac{(y-1)(y-1)}{y-1}dy$$
$$\int (y-1)dy$$
$$\frac{y^2}{2}\checkmark - y\checkmark + c \checkmark$$
(3)
[20]

QUESTION 3

3.1
$$x + y + 10 = 24 \ldots\ldots\ldots\ldots\ldots\ldots (1)\checkmark$$
$$x^2 + y^2 = 100 \ldots\ldots\ldots\ldots\ldots\ldots (2)\checkmark$$
$$\ln(1)x = 14 - y$$
$$\ln(2)(4-y)^2 + y^2 = 100$$
$$y^2 - 14y + 48 = 0$$
$$(y-8)(y-6) = 6$$
$$y_1 = 8\ cm \quad and \quad y_2 = 6\ cm\checkmark$$
Substituting y_1 *and* y_2 *in* (1)
$$x_1 = 6\ cm$$
$$x_2 = 8\ cm\checkmark \qquad (4)$$

3.2 $(2x-5)\ln 0{,}8 = x\ln 2{,}5$

$x - 0{,}4463x + 1{,}116x = 0{,}916x$ ✓

$-1{,}363x = -1{,}116$ ✓

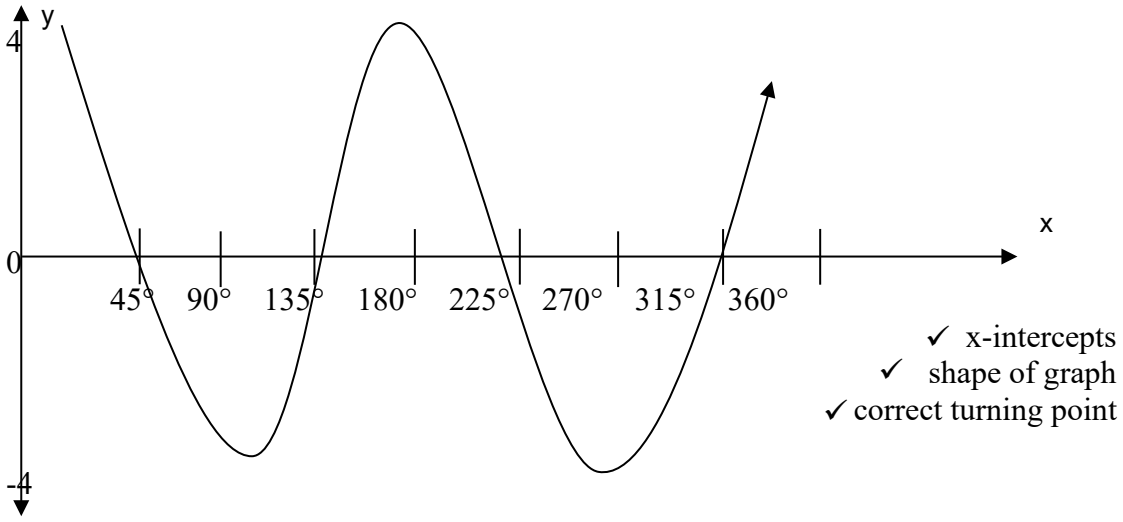
$x = 0{,}819$ ✓ (3)

3.3 $\frac{2RXYZ}{QS} = B^{2P}$ ✓

$2P\log B = \log\left(\frac{2RXYZ}{QS}\right)$ ✓

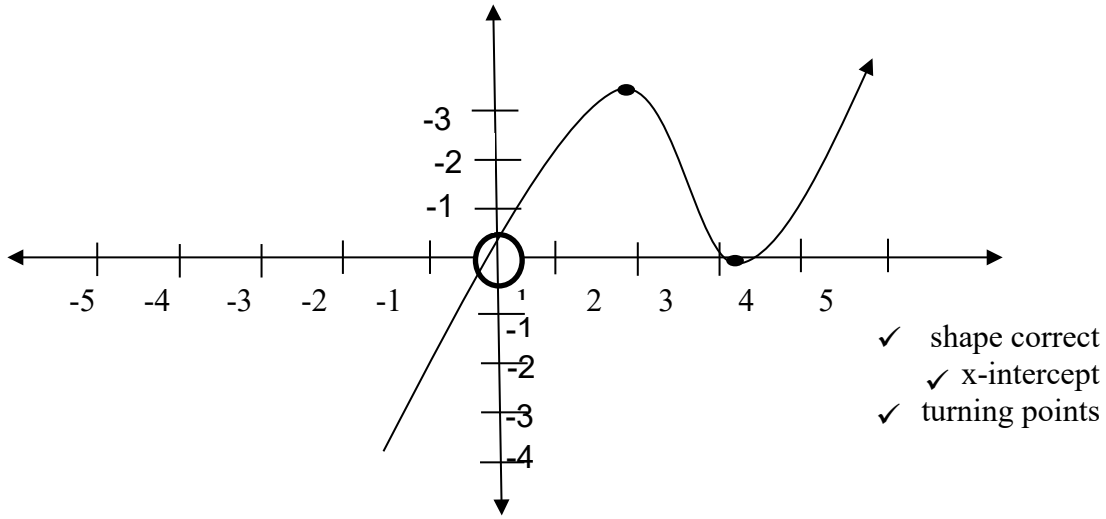
$P = \frac{\log\left(\frac{2RXYZ}{QS}\right)}{2\log B}$ ✓ (3)

3.4



(3)

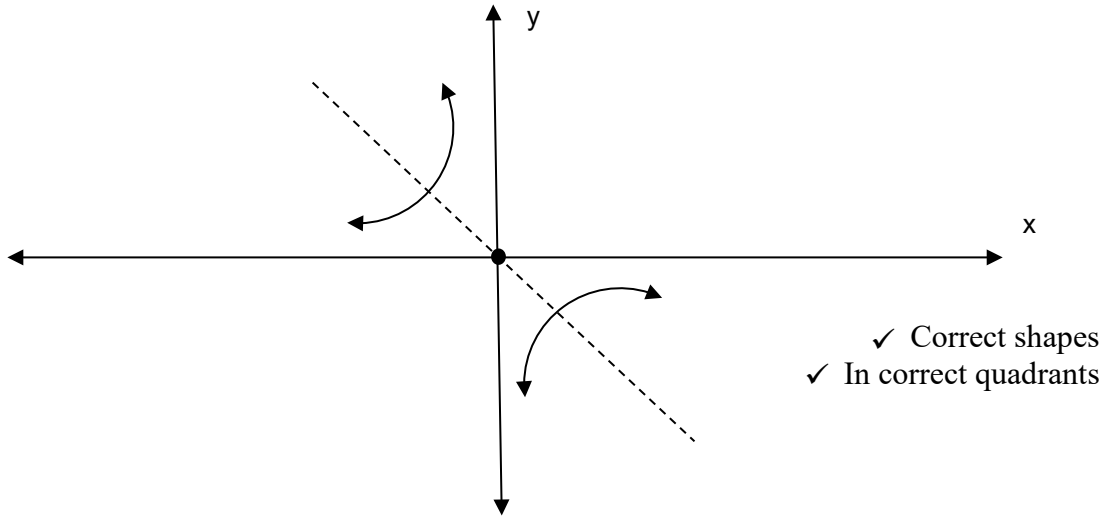
3.5 3.5.1



(3)

3.5.2 $\{x : 0 \leq x \leq 3xER\}$ (1)

3.5.3 Continuous (1)



(2)
[20]

QUESTION 4

4.1

$$x - jy = (j-4) + \frac{2+j}{2-j} \times \frac{2+j}{2+j}$$

$$x - jy = (j-4) + \frac{3-j4}{5} \quad \checkmark$$

$$x - jy = \frac{j5 - 20 + 3 - j4}{5}$$

$$5(x - jy) = -17 + j \quad \checkmark$$

$$5x = -17 \ldots\ldots\ldots\ldots\ldots (1)$$

$$x = \frac{-17}{5} \quad \checkmark$$

$$-5y = 1 \ldots\ldots\ldots\ldots\ldots (2)$$

$$y = \frac{-1}{5} \quad \checkmark$$

$$x - jy = (j-4) + \frac{2+j}{2-j} \times \frac{2+j}{2+j}$$

$$x - jy = (j-4) + \frac{3-j4}{5}$$

$$x - jy = \frac{j5 - 20 + 3 - j4}{5}$$

$$5(x - jy) = -17 + j$$

$$5x = -17 \ldots\ldots\ldots\ldots\ldots (1)$$

$$x = \frac{-17}{5}$$

$$-5y = 1 \ldots\ldots\ldots\ldots\ldots (2)$$

$$y = \frac{-9}{5}$$

(4)

4.2

$$Z_1 = \left(8{,}246 \angle 14{,}036°\right)^{\frac{-1}{2}} \checkmark$$

$$= 0{,}348 \angle 7{,}018° \quad \checkmark$$

$$Z_2 = 3{,}163 \angle 339.517 \quad \checkmark$$

$$Z_1 = 0{,}348 \angle -7{,}018° \quad \checkmark$$

$$= 9{,}089 \angle 346.535 \quad \checkmark$$

(5)

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4.3 1 (1)

4.4 4.4.1

$$|D| = \begin{vmatrix} 2 & 1 & -3 \\ 1 & -2 & 5 \\ 3 & 4 & -1 \end{vmatrix} \quad \checkmark$$

$$|D| = 2\begin{vmatrix} -2 & 5 \\ 4 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 5 \\ 3 & -1 \end{vmatrix} - 3\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} \quad \checkmark$$

$$= 2(-18) - 1(-16) - 3(10)$$

$$= -50 \quad \checkmark$$

$$|Dc| = \begin{vmatrix} 2 & 1 & 4 \\ 1 & -2 & 7 \\ 3 & 4 & 8 \end{vmatrix} \quad \checkmark$$

$$= 2\begin{vmatrix} -2 & 7 \\ 4 & 8 \end{vmatrix} - 1\begin{vmatrix} 1 & 7 \\ 3 & 8 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix}$$

$$= 2(-44) - 1(-13) + 4(10)$$

$$= -35 \quad \checkmark$$

$$c = \frac{|Dc|}{|D|} = \frac{-35}{-50}$$

$$= 0{,}7 \quad \checkmark$$ (6)

4.4.2 Minor of -2: $\begin{vmatrix} 2 & -3 \\ 3 & -1 \end{vmatrix}$ ✓

$= 7$ ✓ (2)

4.4.3 Co-factor of 3: $(+)\begin{vmatrix} 1 & -3 \\ -2 & 5 \end{vmatrix}$ ✓

$= -1$ ✓ (2)

[20]

QUESTION 5

5.1
$$3(1+\tan^2 y)-4\tan y-2=0 \checkmark$$
$$3\tan^2 y-4\tan y+1=0 \checkmark$$
$$(3\tan y-1)(\tan y-1)=0 \checkmark$$
$$\tan y=\frac{1}{3} \quad and \quad \tan y=1 \checkmark$$
$$y_1=18{,}435° \quad y_3=45° \checkmark$$
$$y_2=198{,}434° \quad y_4=225° \checkmark$$
(6)

5.2
$$RHS=\frac{-\sin x.-\cos x+\cos x}{[0-(-1)],\cos x} \checkmark$$
$$=\frac{\sin x\cos x+\cos x}{1.\cos x} \checkmark$$
$$=\frac{\sin x\cos x}{\cos x}+\frac{\cos x}{\cos x}$$
$$=1+\sin x \checkmark$$
$$RHS=LHS \checkmark$$
(4)

5.3
$$=\frac{1-(1-2\sin^2\theta)}{\tan\theta} \checkmark$$
$$=\frac{2\sin\theta}{1}\times\frac{\cos\theta}{\sin\theta}$$
$$=2\sin\theta\cos\theta \checkmark$$
$$=\sin 2\theta \checkmark$$
(3)

5.4
$$\cos\frac{45°}{2}=\pm\sqrt{\frac{\cos 45+1}{2}} \checkmark$$
$$=\pm\sqrt{\frac{\frac{\sqrt{2}}{2}+\frac{2}{2}}{2}} \checkmark$$
$$=\pm\sqrt{\frac{\sqrt{2}+2}{4}} \checkmark$$
$$=\pm\sqrt{\frac{\sqrt{2}+2}{2}} or \frac{\sqrt{2}+2}{4} \checkmark$$
(4)

5.5 $\sin(x-y) = \sin x \cos y - \sin y \cos x$

$= \frac{3}{5}.\frac{8}{9} - \frac{\sqrt{17}}{9}.\frac{4}{5}$ ✓✓

$= \frac{24}{45} - \frac{4\sqrt{17}}{45}$

$= \frac{24-4\sqrt{17}}{45}$ ✓

(3)

[20]

TOTAL: 100

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