

higher education & training

Department: Higher Education and Training REPUBLIC OF SOUTH AFRICA

T1020(E)(J25)

NATIONAL CERTIFICATE

MATHEMATICS N4

(16030164)

25 July 2018 (X-Paper) 09:00–12:00

Scientific calculators may be used.

This question paper consists of 6 pages and 1 formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING REPUBLIC OF SOUTH AFRICA

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NATIONAL CERTIFICATE MATHEMATICS N4 TIME: 3 HOURS MARKS: 100

INSTRUCTIONS AND INFORMATION

- 1. Answer ALL the questions.
- 2. Read ALL the questions carefully.
- 3. Number the answers according to the numbering system used in this question paper.
- 4. Show ALL intermediate steps and simplify where possible.
- 5. ALL final answers must be rounded off to THREE decimals unless indicated otherwise.
- 6. Questions may be answered in any order, but subsections of questions must be kept together.
- 7. Use only BLUE or BLACK ink.
- 8. Work neatly.

QUESTION 1

1.1 Simplify:

$$\frac{\sin 210^{\circ} . \tan 225^{\circ} . \cos 315^{\circ}}{\sin(-45^{\circ}) . \cos 240^{\circ}}$$
(3)

1.2 Solve for c if
$$2\cos c + 3\sin c - 1 = 0; 0^{\circ} \le c \le 360^{\circ}$$
 (5)

1.3 1.3.1 Derive the identity for
$$\sin \frac{x}{2}$$
 if $1 - \cos x = 2\sin^2 \frac{x}{2}$. (2)

1.3.2 Determine the value of sin15° without the use of a calculator if

$$1 - \cos x = 2\sin^2 \frac{x}{2} \tag{3}$$

1.4 Prove that
$$2 - \sec^2 y = \cos 2y(\tan^2 y + 1)$$
 (4)

1.5 Determine $(\cos 15^{\circ})^2$ without the use of a calculator. Simplify as far as possible. (3) [20]

QUESTION 2

2.1

| $y = 3 \sin x$ | |
|-----------------|---|
| $\frac{\pi}{2}$ | X |

- 2.1.1 Determine, by using integration, the value of the shaded area indicated in the graph. (4)
- 2.1.2 Using differentiation and calculate the minimum and the maximum turning points of the graph.
- 2.1.3 Use the second derivative and distinguish between the minimum and the maximum turning points of the graph. (3)

(6)

-4-

2.2 Integrate the following in terms of *x*:

$$\int \left(-4e^{3x} + \frac{\sin 2x + \tan x}{\cos x} - \sqrt{5x} + 5 - 2^{-3x} \right) dx$$
[20]

QUESTION 3

3.2

3.1 Use the function of a function (chain rule) to differentiate the following:

$$y = \tan(x^5 - 7x)$$
Differentiate the following in terms of y:
$$(4)$$

$$x = \frac{3}{\sqrt[5]{y}} - \cos 3\pi y - e^{-22y} - \ln(y^6)$$
(4)

- 3.3 Determine the limit of $\lim_{x \to \infty} \left(\frac{2x^2 8}{x^2 + 2} \right)$ (3)
- 3.4 Differentiate $y = -x^3$ by using first principles. (4)
- 3.5 The displacement of an object is represented by $S = \int_{0}^{4} (t^{3} + t) dt$

Determine the value of *S*.

3.6 Simplify $\int \frac{1}{2}\sqrt{1 + \tan^2 x} \cdot 2\tan x dx$

(2) [**20**]

(3)

QUESTION 4

4.1 Solve for x and y if
$$\frac{7+j5}{-j+2} = 1,8x+j3,4y$$
 (4)

4.2 Simplify
$$(-4-j)^5$$

Leave answer in rectangular form.

(4)

(5)

[20]

4.3 Determine the argument and the modulus of

$$z = \frac{2cis - 60^{\circ}.5cis45^{\circ}}{4cis33^{\circ}.3cis - 21^{\circ}}$$
(3)

Solve for a and b only using Cramer's rule. 4.4

$$-6b - 27 = -7a$$
$$3a + 4b - 5 = 0$$

Determine the value of the following determinant by only expanding on 4.5 4.5.1 column 2:

4.5.2

(3) l 3 Determine the value of the co-factor of -1 in QUESTION 4.5.1. (1) (16030164)

QUESTION 5

| 5.1 | 5.1.1 | Sketch the graph of $y = -\sqrt{144 - x^2}$ | | (2) | |
|-----|--|---|-----------|----------------------|--|
| | 5.1.2 | State the range of the graph in QUESTION 5.1.1. | | (1) | |
| | 5.1.3 | Write down the equation of the inverse of the graph in QUESTIC | ON 5.1.1. | (1) | |
| 5.2 | Sketch the | Sketch the graph of $y = \sin x + 1; 0^{\circ} \le x \le 360^{\circ}$ (| | | |
| 5.3 | Sketch the | e graph of $y = \sec \theta$; for $\frac{-3\pi}{2} \le 0 \le \frac{3\pi}{2}$ | | (3) | |
| 5.4 | Given: | | | | |
| | $R_1 = R_5 \left[\frac{1}{2} \right]$ | $\frac{1-y}{1-y^n} \end{bmatrix}$ | | | |
| | Make n | the subject of the formula. | | (4) | |
| 5.5 | Factorise, | but do NOT simplify. | | | |
| | $64(a-1)^3$ | $-(a+1)^3$ | | (3) | |
| 5.6 | Solve for | the unknown: | | | |
| | $15^{3y+2} = 2$ | 2 ^{8+y} | | (3) [20] | |
| | | | TOTAL: | 100 | |
| | | | | | |

MATHEMATICS N4

FORMULA SHEET

| $a^x = b \Leftrightarrow \log a^x = \log b$ | | $\ln x = \log_e x$ |
|---|--|---|
| $(r \underline{\theta})^n = r^n \underline{n}\underline{\theta}$ $a+b$ | $j = c + dj \Leftrightarrow a = candb = d$ | |
| $\sin(a\pm b) = \sin a \cos b \pm b$ | $\sin b \cos a$ | $\sin^2 x + \cos^2 x = 1$ |
| $\cos(a\pm b) = \cos a \cos \mp \sin a \sin b$ | | $1 + \cot^2 x = \cos ec^2 x$ |
| | | $1 + \tan^2 x = \sec^2 x$ |
| $\tan(a \pm b) = \tan a \pm \tan a$ | | |
| $\tan(a \pm b) = \frac{1}{1 \mp \tan a \tan b}$ | | |
| V | dv | $y = u(x) \cdot v(x)$ |
| 2 | $\frac{dy}{dx}$ | $y = u(x) \cdot v(x)$ |
| an ⁿ | nax^{n-1} | $\Rightarrow \frac{dy}{dx} = u(x)v^{1}(x) + u^{1}(x)v(x)$ |
| ka^{x} | $ka^{x} \ln a$ | u(x) |
| $k \ln x$ | k | $y - \frac{1}{v(x)}$ |
| | \overline{x} | $\Rightarrow \frac{dy}{dx} = \frac{v(x)u^{1}(x) - u(x)v^{1}(x)}{u^{1}(x) - u(x)v^{1}(x)}$ |
| sin <i>x</i> | $\cos x$ | $\rightarrow dx - [v(x)]^2$ |
| $\cos x$ | $-\sin x$ | $\frac{dy}{dt} = \frac{dy}{dt} \frac{du}{dt}$ |
| tan x | $\sec^2 x$ | dx du dx |
| $\cot x$ | $-\cos ec^2 x$ | |
| sec x | sec x tan x | |
| cos ecx | $-\cos ecx \cot x$ | |
| ax^{n+1} | | $\int \sin x dx = \cos x + a$ |
| $\int ax^n dx = \frac{ax}{n+1} + C$ | | $\int \sin x dx = -\cos x + c$ |
| | | $\int \cos x dx = \sin x + c$ |
| $\int_{x}^{-ax} = a \ell nx + c$ | | $\int \tan x dx = \ln \sec x + c$ |
| $\int ka^x dx = \frac{ka^x}{\ln a} + c$ | | $\int \sec x dx = \ln(\sec x + \tan x) + c$ |
| - ena | | |
| $A_{ox} = \int_{a} yax$ | | |