higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N4

(16030164)

6 April 2021 (X-Paper)
09:00–12:00

Scientific calculators may be used.

This question paper consists of 5 pages and 1 formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
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NATIONAL CERTIFICATE
MATHEMATICS N4
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show all intermediate steps and simplify where possible.
5. All final answers must be rounded off to three decimal places (unless indicated otherwise).
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Draw all graphs large, clear and neat and they may be done in pencil.
8. Use only a blue or black pen.
9. Write neatly and legibly.

QUESTION 1

1.1 Sketch the graph of $x = \frac{-8}{y}$. (2)

1.2 Sketch the graph of $y = -\sqrt{3-x^2}$ (2)

1.3 1.3.1 Draw the inverse of the graph in QUESTION 1.2 and state whether it is a *function* or a *relation*. (3)

1.3.2 Is the inverse of $y = -\sqrt{3-x^2}$ *continuous* or *discontinuous*? (1)

1.4 Given: $Z = (3-4j)^3$

Convert Z to polar form. θ may always be positive. (6)

1.5 Solve for x and y simultaneously using complex numbers if:

$$x - jy = \frac{7-j^9}{1-j}$$ (5)

1.6 Evaluate: $j^9 - j^2$ (1)

[20]

QUESTION 2

2.1 If $8\sin\theta + 5 = 0$, determine the value of:

$\cos^2\theta + \tan\theta.\cos\theta$

without using a calculator.

2.2 Prove that: $\cot(90^0 + \theta) = -\tan\theta$

2.3 Simplify as far as possible:

$$\frac{\sin x + \tan x}{\cos ecx + \cot x}$$

2.4 Prove the following identity:

$$\frac{\sec\theta + \tan\theta}{\cos\theta} = \frac{1}{1-\sin\theta}$$

2.5 Solve for α: $\sec\left(\frac{\alpha}{2} + 5^0\right) = -\cos ec(20^0 - \alpha)$, for $0^0 \le \alpha \le 90^0$

(5 × 4) **[20]**

QUESTION 3

3.1 Differentiate: $y = \sqrt{\sec x}$ using the chain rule (function of function). (4)

3.2 Differentiate the following expression:

$$y = \frac{\sin 2x + 1}{\cos x} + \frac{e^{7x}}{14} - 2^x + \frac{\pi^{3x}}{4} - 2B + \frac{1}{2}\ln x$$ (7)

3.3 Differentiate from first principle if given:

$y = 5x^5 - 2x$ (5)

3.4 Given: $y = x^3 - 12x^2 + 16x - 12$

Determine the coordinate of the points of inflection using the second derivative. (4)

[20]

QUESTION 4

4.1 Solve for x if : $3^{x+2} = \ln 19$ (3)

4.2 Given: $3x + 4y - 6 = 0$ and $x = 2y - 3$

Solve for x and y using Cramer`s rule. (5)

4.3 Given:

$$\begin{vmatrix} 3 & 7 & 9 \\ -3 & -1 & 6 \\ 4 & 8 & 5 \end{vmatrix}$$

4.3.1 Write down and determine the minor of element 7.

4.3.2 Write down and determine the cofactor of element -1.

(2 × 2) (4)

4.4 A quarter of the sum of two numbers is 4. Four times the difference between the two is –64.

Calculate the two numbers. (5)

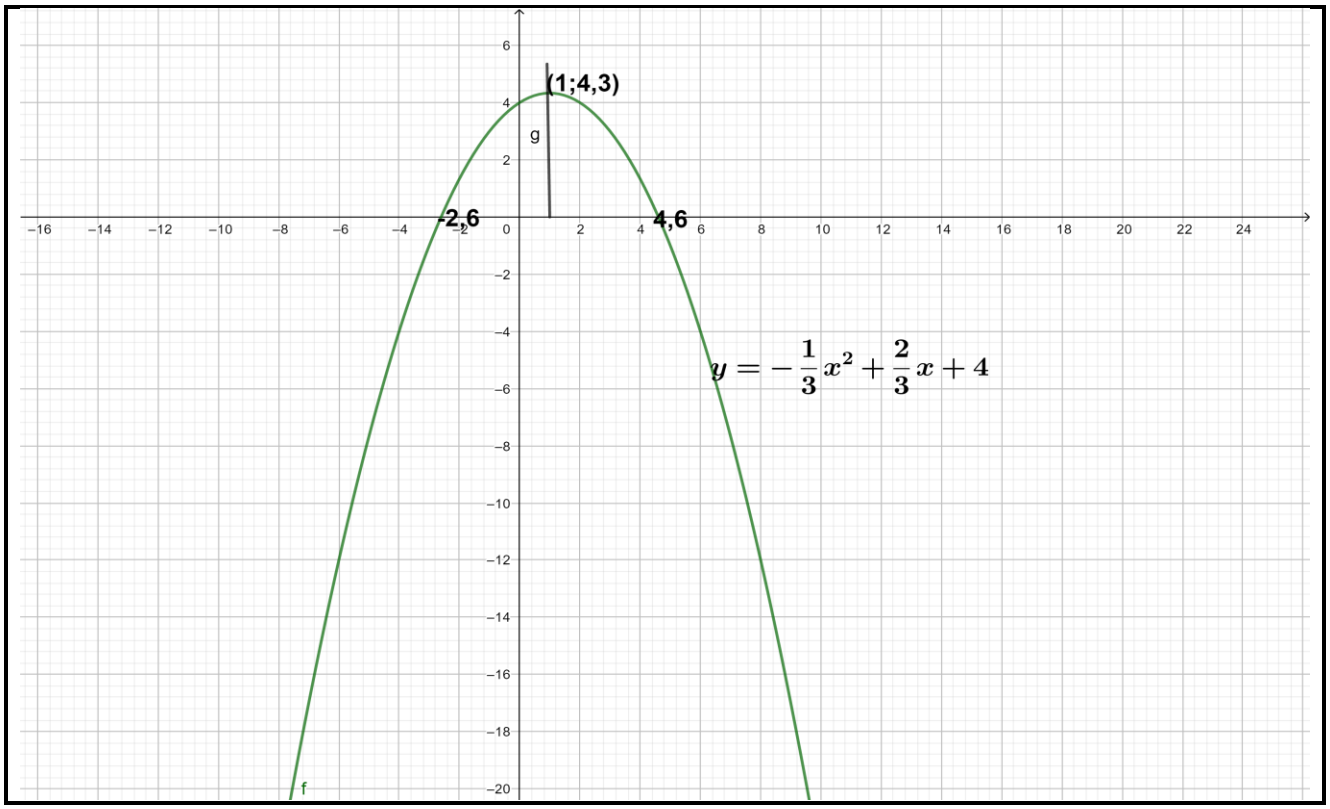
4.5 Given: $P = \frac{DB}{C}\ln\left(\frac{N}{R}\right)$

Make *N* the subject of the formula. (3)

[20]

QUESTION 5

5.1 Given:



5.1.1 Redraw the sketch in the ANSWER BOOK and clearly indicate the area enclosed by the graph: $y = -\frac{1}{3}x^2 + \frac{2}{3}x + 4$, $x = 1$ and $x = 4{,}6$. Also indicate the representative strip used to calculate the indicated area. (2)

5.1.2 Calculate, using integration, the magnitude of the area indicated in QUESTION 5.1.1. (5)

5.2 Integrate the following:

$$\int\left(4\pi^{3x} - \frac{5\pi}{x} + \tan x.\sec x - \cos ec^2 3x + \frac{1}{e^{3x}} + 5^{-7x} + p\right)dx$$ (7)

5.3 Evaluate: $\int_0^{\frac{\pi}{2}} (\sin 3x)dx$ (3)

5.4 Simplify: $\int\left(-\cos ecx.\cot x - \left(-\cos ec^2 x\right)\right)dx$ (3)

[20]

TOTAL: 100

MATHEMATICS N4

FORMULA SHEET

NEW SYLLABUS

$a^x = b \Leftrightarrow \log a^x = \log b$

$\ln x = \log_e x$

$\left(r\left|\underline{\theta}\right.\right)^n = r^n\left|\underline{n\theta}\right.$ $\quad a + bj = c + dj \Leftrightarrow a = c and b = d$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$

$\cos(a \pm b) = \cos a \cos \mp \sin a \sin b$

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \cos ec^2 x$

$1 + \tan^2 x = \sec^2 x$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

y	$\frac{dy}{dx}$
an^n	nax^{n-1}
ka^x	$ka^x \ln a$
$k \ln x$	$\frac{k}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\cos ec^2 x$
$\sec x$	$\sec x \tan x$
$\cos ecx$	$-\cos ecx \cot x$

$$y = u(x) \cdot v(x)$$
$$\Rightarrow \frac{dy}{dx} = u(x)v^1(x) + u^1(x)v(x)$$

$$y = \frac{u(x)}{v(x)}$$
$$\Rightarrow \frac{dy}{dx} = \frac{v(x)u^1(x) - u(x)v^1(x)}{[v(x)]^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} x \frac{du}{dx}$$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$$

$$\int \frac{a}{x} dx = a\ell nx + c$$

$$\int ka^x dx = \frac{ka^x}{\ell na} + c$$

$$A_{ox} = \int_a^b y dx$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \tan x dx = \ell n \sec x + c$$

$$\int \sec x dx = \ell n(\sec x + \tan x) + c$$