



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N4

1 APRIL 2020

This marking guideline consists of 10 pages.

QUESTION 1

$$\begin{aligned}
 1.1 \quad & \frac{\cos x \cdot \operatorname{csc} x}{\tan x \cdot \cos x - \sec x} \checkmark & \frac{\cos x \cdot \sec x}{\tan x \cdot \sin x - \csc x} \checkmark \\
 & = \frac{\cos x \cdot \frac{1}{\sin x}}{\tan x \cdot \cos x \cdot \frac{-1}{\cos x}} \checkmark & = \frac{\frac{\cos x}{1} \cdot \frac{1}{\cos x}}{\frac{\sin x}{\cos x} \cdot \frac{1}{1} \cdot \frac{1}{\sin x}} \checkmark \\
 & = \frac{\tan x}{\tan x - \cot x} & = -\frac{1}{\tan x} \checkmark \\
 & = -\tan x \checkmark & = -\cot x \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 1.2 \quad & 2 \cos^2 \beta + \cos \beta - 1 = 0 ; -180^\circ \leq \beta \leq 180^\circ \\
 & (2 \cos 2\beta - 1)(\cos 2\beta + 1) = 0 \checkmark \\
 & \cos 2\beta = \frac{1}{2} \text{ or } \cos 2\beta = -1 \\
 & 2\beta = 60^\circ \text{ or } 2\beta = 270^\circ \\
 & \beta = 30^\circ \text{ or } \beta = 135^\circ \checkmark \checkmark \\
 & GS = [30^\circ; 45^\circ] \checkmark \checkmark
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 1.3 \quad 1.3.1 \quad & 1 - \sin x = 2 \cos^2 \frac{x}{2} \\
 & 2 \cos^2 \frac{x}{2} = 1 - \sin x \\
 & \cos^2 \frac{x}{2} = \frac{1 - \sin x}{2} \checkmark \\
 & \cos \frac{x}{2} = \pm \sqrt{\frac{1 - \sin x}{2}} \checkmark
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 1.3.2 \quad & \cos 15^\circ \rightarrow \cos \frac{30^\circ}{2} = \sqrt{\frac{1 - \sin 30^\circ}{2}} \checkmark \\
 & = \sqrt{\frac{1 - \frac{1}{2}}{2}} \checkmark \\
 & = \sqrt{\frac{1}{4}} \\
 & = \frac{1}{2} \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 1.4 \quad \frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} &= \tan x \\
 \text{LHS} &= \frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} \\
 &= \frac{\sin x + 2 \sin x \cos x}{1 + \cos x + \cos^2 x - \sin^2 x} \checkmark \checkmark \\
 &= \frac{\sin x(1 + 2 \cos x)}{\cos x(1 + 2 \cos x)} \checkmark \\
 &= \frac{\sin x}{\cos x} \\
 &= \tan x \checkmark \\
 &= \text{RHS}
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 1.5 \quad \cos 75^\circ &= \cos(45^\circ + 30^\circ) \\
 &= \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \cdot \sin 30^\circ \checkmark \\
 &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\
 &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \checkmark \\
 &= \frac{\sqrt{3}-1}{2\sqrt{2}} \checkmark
 \end{aligned} \tag{3}$$

[20]

QUESTION 2

$$\begin{aligned}
 2.1 \quad 2.1.1 \quad \text{Area} &= \int_0^{\frac{\pi}{4}} 2 \cos x dx \\
 &= 2 \int_0^{\frac{\pi}{4}} \cos x dx \\
 &= 2 \left[\sin x + c \right]_0^{\frac{\pi}{4}} \checkmark \\
 &= 2 \left[\sin\left(\frac{\pi}{4}\right) + c - (\sin 0^\circ + c) \right] \checkmark \\
 &= 2 \left[\frac{1}{\sqrt{2}} + c - c \right] \checkmark \\
 &= 1,414 \text{ units}^2 \checkmark
 \end{aligned} \tag{4}$$

2.1.2 $y = 2 \cos x$
 (i) For T.P let $x = 0$
 $-2 \sin x = 0$
 $x = \sin^{-1} 0$
 $x = 0^\circ$ or $x = 180^\circ$ ✓✓
 y - values, if $x = 0^\circ$
 $y = 2 \cos 0^\circ$
 $y = 2$ ✓
 if $x = 180^\circ$
 $y = 2 \cos 180^\circ$
 $y = -2$ ✓
 $T.P_1 = (0^\circ; 2)$ and $T.P_2 = (180^\circ; -2)$ ✓✓ (6)

2.1.3 $\frac{d^2 y}{dx^2} = -2 \cos x$
 $= -2 \cos 0^\circ$
 $= -2$ maximum

$\frac{d^2 y}{dx^2} = -2 \cos x$ ✓
 $= -2 \cos 180^\circ$
 $= 2$ minimum

$T.P_1 = (0^\circ; 2)$ maximum ✓
 $T.P_2 = (180^\circ; -2)$ minimum ✓ (3)

2.2 $\frac{-1}{2x^2} + \frac{1}{2} \cos 3x + 2x^{\frac{3}{2}} - \frac{10 \cdot 10^x}{\ln 10} + \frac{1}{2} \sin 2x - 2x + c$

✓ ✓ ✓ ✓ ✓ ✓ ✓ (7)

[20]

QUESTION 3

$$\begin{aligned}
 3.1 \quad y &= \sqrt{\tan x} && \text{let } u = \tan x \\
 y &= \sqrt{u} = u^{\frac{1}{2}} && \frac{dy}{du} = \sec^2 x \checkmark \\
 \frac{dy}{du} &= \frac{2u^{\frac{3}{2}}}{3} \\
 \therefore \frac{dy}{du} &= \frac{dy}{du} \times \frac{du}{dx} \checkmark \\
 &= \frac{2u^{\frac{3}{2}}}{3} \times \sec^2 x \\
 &= \frac{2}{3} \sqrt{u^3} \cdot \sec^2 x \checkmark \\
 &= \frac{2}{3} \sqrt{\tan^3 x} \cdot \sec^2 x \checkmark
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 3.2 \quad y &= 2^x + \frac{1}{2} \ln x + \frac{1}{2} e^x + 6t \\
 \frac{dy}{dx} &= 2^x \cdot \ln 2 + \frac{1}{2x} + \frac{1}{2} e^x + 0 \\
 &\quad \checkmark \quad \quad \checkmark \quad \quad \checkmark \quad \quad \checkmark
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 3.3 \quad &\lim_{x \rightarrow 2} \left(\frac{x^2 - 4}{x - 2} \right) \\
 &= \frac{(x-2)(x+2)}{(x-2)} \checkmark \\
 &= (x+2) \quad \checkmark \\
 &= 2+2 \\
 &= 4 \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 3.4 \quad y &= -x^3 + 5x \\
 \lim_{x \rightarrow 0} &\frac{(-x+h)^3 + 5(x+h) - (-x^3 + 5x)}{h} \checkmark \\
 &= \frac{-x^3 - 3x^2 \cdot h - 3x \cdot h^2 + 5x + 5h + x^3 - 5x}{h} \checkmark \\
 &= \frac{h(-3x^2 - 3xh + 5)}{h} \checkmark \\
 &= -3x^2 + 5 \checkmark
 \end{aligned} \tag{4}$$

$$\begin{aligned}
3.5 \quad S &= \int_1^4 (\sqrt{t} - 5)^2 dt \\
&= \int_1^4 (\sqrt{t} - 5)(\sqrt{t} - 5) dt \\
&= \int_1^4 (t - 5\sqrt{t} - 5\sqrt{t} + 25) dt \checkmark \\
&= \int_1^4 (t - 10\sqrt{t} + 25) dt \\
&= \left[\frac{t^2}{2} - \frac{20t^{3/2}}{3} + 25t \right]_1^4 \\
&= \frac{4^2}{2} - \frac{20(4)^{3/2}}{3} + 25(4) - \left(\frac{(1)^2}{2} - \frac{20(1)^{3/2}}{3} + 25(1) \right) \checkmark \\
&= 8 - 53,333 + 100 - \frac{1}{2} + 6,667 - 25 \\
&= 35,834 \checkmark
\end{aligned}
\tag{3}$$

$$\begin{aligned}
3.6 \quad &\int \left(\frac{4 \sec^4 \theta - 9}{2 \sec^2 \theta + 3} \right) d\theta \\
&= \int \frac{(2 \sec^2 \theta + 3)(2 \sec^2 \theta - 3)}{(2 \sec^2 \theta + 3)} \checkmark \\
&= \int (2 \sec^2 \theta - 3) d\theta \\
&= 2 \tan \theta - 3\theta + c \checkmark
\end{aligned}
\tag{2}$$

[20]

QUESTION 4

4.1

$$(3 - 4j)^2 = \frac{-x - yj}{j^2}$$

$$j^2(3 - 4)(3 - 4j) = -x - yj$$

$$j^2(9 - 12j - 12j + 16j^2) = -x - yj \checkmark$$

$$j^2(9 - 24j + 16j^2) = -x - yj$$

$$-(9 - 24j - 16) = -x - yj \checkmark$$

$$-9 + 24j + 16 = -x - yj$$

$$24j + 7 = -x - yj$$

$$x = -7 \text{ and } y = -24 \checkmark \checkmark$$

(4)

4.2

$$(\sqrt{3} - j)^5$$

$$|r| = \sqrt{x^2 + y^2}$$

$$= \sqrt{(3)^2 + (-1)^2}$$

$$= \sqrt{4}$$

$$= 2 \quad \checkmark$$

$$\tan \theta = \frac{y}{x}$$

$$= \frac{-1}{\sqrt{3}}$$

$$\tan \theta = 0,578$$

$$\theta = 30,023$$

$$\therefore \theta = 360^\circ - 30,023$$

$$\theta = 329,977 \quad \checkmark$$

$$\therefore = (r \angle \theta)^2$$

$$= (2 \angle 329,977)^2$$

$$= 4 \angle 659,954 \quad \checkmark$$

$$\therefore r(\cos \theta + i \sin \theta)$$

$$= 4(\cos 659,954 + i \sin 659,954)$$

$$= 1,997 - i3,466 \checkmark$$

$$(r \angle \theta)^5$$

$$(2 \angle 329,977)^5$$

$$32 \angle 1649,855$$

$$\therefore r(\cos \theta + i \sin \theta)$$

$$32(\cos 1649,855 + i \sin 1649,855)$$

(4)

$$\begin{aligned}
 4.3 \quad & \frac{2\text{cis}(-60^\circ) \cdot 5\text{cis}45^\circ}{4\text{cis}33^\circ \cdot 3\text{cis}(-21^\circ)} \\
 &= \frac{2|-60^\circ \cdot 5|45^\circ}{4|33^\circ \cdot 3|21^\circ} \\
 &= \frac{10|60^\circ + 45^\circ}{12|33^\circ - 21^\circ} \quad \checkmark \\
 &= \frac{10|-15}{12|12} \quad \checkmark \\
 &= 0,833|-27 \quad \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 4.4 \quad & 7a + 6b = 27 \\
 & 3a + 4b = 5 \\
 & D = \begin{vmatrix} -7 & 6 \\ 3 & 4 \end{vmatrix} \\
 & D = -28 - 18 \\
 & D = -46 \quad \checkmark \\
 & D_a = \begin{vmatrix} 27 & 6 \\ 5 & 4 \end{vmatrix} \quad \therefore a = \frac{Da}{D} = \frac{78}{-46} = -1,696 \quad \checkmark \\
 & = 108 - 30 \quad \checkmark \\
 & = 78 \\
 & D_b = \begin{vmatrix} -7 & 27 \\ 3 & 5 \end{vmatrix} \quad \therefore b = \frac{Db}{D} = \frac{-116}{-46} = 2,522 \quad \checkmark \\
 & = -35 - 81 \\
 & = -116 \quad \checkmark
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 4.5 \quad 4.5.1 \quad & \begin{vmatrix} \frac{2}{3} & -1 & -\frac{1}{2} \\ -\frac{1}{4} & 2 & 0 \\ 5 & -8 & \frac{-2}{3} \end{vmatrix} \\
 & D = \begin{vmatrix} -1 & 0 \\ 4 & -2 \end{vmatrix} + 2 \begin{vmatrix} \frac{2}{3} & -1 \\ 5 & \frac{-2}{3} \end{vmatrix} - 8 \begin{vmatrix} \frac{2}{3} & -1 \\ \frac{-1}{4} & 0 \end{vmatrix} \quad \checkmark \\
 & = 0,167 + 2(2,945) - 8(0 - 0,125) \quad \checkmark \\
 & = 0,167 + 5,89 + 1 \\
 & = 7,059 \quad \checkmark
 \end{aligned} \tag{3}$$

4.5.2

$$= (-1)^3 \begin{vmatrix} \frac{-1}{4} & 0 \\ 5 & \frac{-2}{3} \end{vmatrix}$$

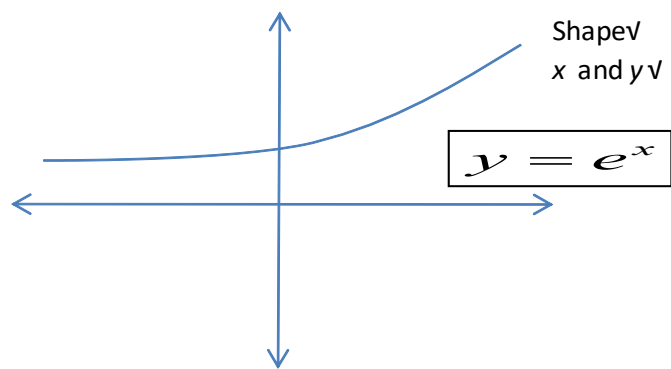
$$= -(0,167 - 0)$$

$$= -0,167 \checkmark$$

(1)
[20]

QUESTION 5

5.1 5.1.1



(2)

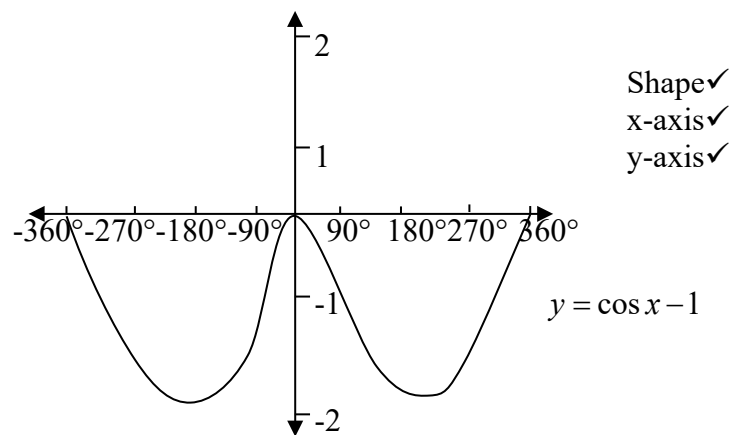
5.1.2 $= \{0 < y < \infty\} \checkmark$

(1)

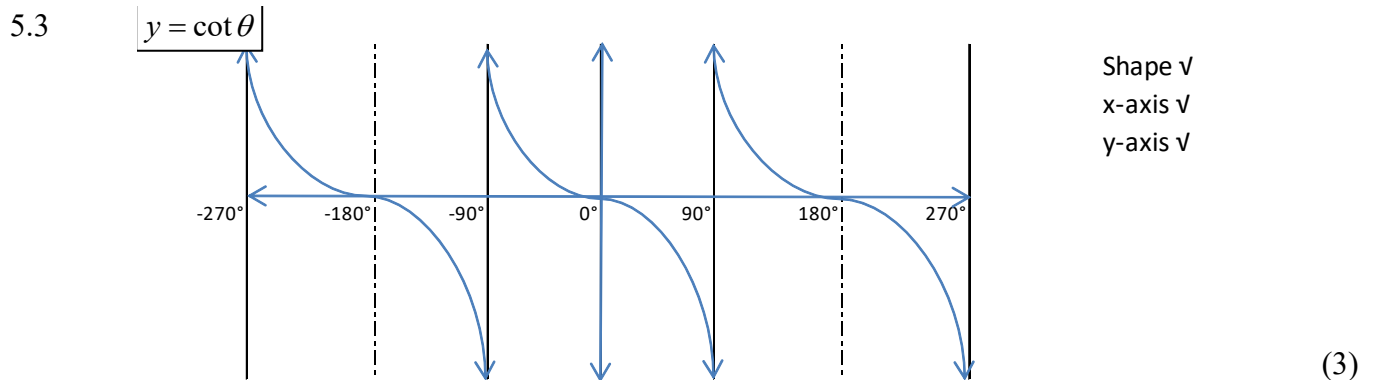
5.1.3 $x = e^y \checkmark$

(1)

5.2



(3)



5.4

$$\ln \sqrt{x-1} + \ln \sqrt{x+1} = a$$

$$\ln \sqrt{(x-1)(x+1)} = a \quad \checkmark$$

$$\frac{1}{2} \ln(x^2 - 1) = a$$

$$\ln(x^2 - 1) = 2a \quad \checkmark$$

$$x^2 - 1 = e^{2a}$$

$$x^2 = e^{2a} + 1 \quad \checkmark$$

$$x = \pm \sqrt{e^{2a} + 1} \quad \checkmark$$

(4)

5.5

$$27(k-1)^3 - 8(k+1)^3$$

$$[3(k-1) - 2(k+1)] [(3(k-1))^2 + 6(k-1)(k+1) + (2(k+1))^2]$$

(3)

5.6

$$3^{2x-1} = \ln 5$$

$$\ln 3(2x-1) = \ln 5$$

$$2,197x - 1,099 = 1,609 \quad \checkmark$$

$$x = \frac{2,708}{2,197} \quad \checkmark$$

$$= 1,233 \quad \checkmark$$

(3)
[20]

TOTAL: 100