higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1020**(E)**(M28)T

# NATIONAL CERTIFICATE

# MATHEMATICS N4

(16030164)

**28 March 2018 (X-Paper)**
**09:00–12:00**

**Scientific calculators may be used**.

**This question paper consists of 6 pages and 1 formula sheet.**

# DEPARTMENT OF HIGHER EDUCATION AND TRAINING
# REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N4
TIME: 3 HOURS
MARKS: 100

---

**INSTRUCTIONS AND INFORMATION**

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. Round off ALL final answers to THREE decimal places (unless indicated otherwise).
6. Questions may be answered in any order but keep subsections of questions together.
7. Use only blue or black ink.
8. Write neatly and legibly.

---

**QUESTION 1**

1.1 Given:

$$f(x) = x^3 + 5x^2 + 2x - 8$$

Use differentiation to calculate the coordinates of the turning points and the point of inflection of the given cubic function. Distinguish between the maximum and the minimum turning points by using the second derivative. (9)

1.2 1.2.1 Sketch and clearly indicate the area enclosed by the graph of $y = x^2 - 6x + 5$, the $x$-axis, $x = 2$ and $x = 4$. Show the representative strip used to calculate the area. (3)

1.2.2 Use integration to calculate the value of the enclosed area indicated in QUESTION 1.2.1. (4)

1.3 The work done on a particular object is represented by the following formula:

$$W = \int_{0,5}^{1} 100x\,dx$$

Determine $W$. (3)

**[19]**

**QUESTION 2**

2.1 Integrate the following in terms of $x$:

$$\int \left( 2^{-9x} + \frac{1}{2}\cos ec^2 x - \frac{\pi}{x} + x^{\frac{-7}{5}} - 15\sec 15x \tan 15x + \ln y \right) dx$$

(7)

2.2 Simplify:

$$\int \left( \frac{2 - \cos x}{\sin^2 x} \right) dx$$

(3)

2.3 Expand $\sqrt{x + h}$ to only THREE terms using the binomial theorem. (3)

2.4 Differentiate the following by using the quotient rule:

$$y = \tan x$$

(4)

 

2.5 Differentiate the following in terms of $x$:

$$y = \frac{1}{3^{-5x}} + \frac{\cos 2x}{\sin x + \cos x} + \ln \pi$$

(4)
**[21]**

## QUESTION 3

3.1 The breadth of a rectangular field is 7 m less than the length. The area of the rectangular field is 500 m$^2$. Each side of the rectangular field is then increased by 3 m.

Determine the value of the new area. (5)

3.2 Make $g$ the subject of the formula if:

$$I\left(e^{\frac{gv}{ct}} + 1\right) = j$$

(3)

3.3 Solve the unknown if:

$$5^{2x+2} = 4^{x-1}$$ (2)

3.4 The impedance of a circuit is given by the equation $z_T = \dfrac{z_2}{z_3 + z_1}$.

Calculate $z_T$ if $z_1 = 2 + j3$, $z_2 = 3 + j4$ and $z_3 = 5 - j4$. Leave the answer in polar form. Show ALL steps. (5)

3.5 Solve for $t$ if:

$$t^2 - 2t + 4 = 0$$ (4)

3.6 State the conjugate of $j3$. (1)
**[20]**

## QUESTION 4

4.1 In the application of Kirchhoff's law in a circuit, the following equations are obtained:

$$3I_1 + 3I_3 = 7{,}5 + 5I_2$$
$$2I_1 + I_2 = 17{,}5 + 7I_3$$
$$4I_2 + 5I_3 = 16 + 10I_1$$

Determine the value of $I_3$ by only using Cramer's rule. (8)

 

4.2

$$|D| = \begin{vmatrix} 1 & 0 & 5 \\ -2 & 3 & 7 \\ 6 & -1 & 0 \end{vmatrix}$$

4.2.1 Determine the cofactor of -1.

4.2.2 Determine the minor of 5.

(2 × 2) (4)

4.3 4.3.1 Sketch and clearly label the graph of:

$$25y^2 + 16x^2 - 400 = 0$$ (3)

4.3.2 About which axis is the graph of $25y^2 + 16x^2 - 400 = 0$ in QUESTION 4.3.1 symmetrical? (1)

4.4 Sketch and clearly label the graph of $y = \ln x$. (2)

4.5 Sketch and clearly label the graph of:

$$y - 3 = x^2$$ (2)

**[20]**

**QUESTION 5**

5.1 Solve for $\theta$ if:

$$\operatorname{cosec}(2\theta - 70°) = \sec\left(\frac{\theta}{2} + 10°\right), 0° \leq \theta \leq 90°$$ (3)

5.2 Prove that:

$$\cos 2\theta = \frac{1 - \tan^2\theta}{\sqrt{1 + 2\tan^2\theta + \tan^4\theta}}$$ (6)

5.3 Simplify:

$$\frac{1 - \cos 2x}{\tan x}$$ (4)

5.4 Given:

$\tan A = \dfrac{15}{20}$ and A is an acute angle

Determine without the use of a calculator the value of $\cos\dfrac{A}{2}$. (3)

5.5 Simplify:

$$\frac{\tan(-\theta).\sin(270^\circ-\theta).\operatorname{cosec}\theta}{\sin(-\theta).\cot(90^\circ-\theta).\cos(180^\circ-\theta)}$$

(4)

**[20]**

**TOTAL: 100**

**FORMULA SHEET**

$a^x = b \Leftrightarrow \log a^x = \log b$

$\ln x = \log_e x$

$(r|\underline{\theta})^n = r^n|\underline{n\theta}$ $\quad a + bj = c + dj \Leftrightarrow a = c \text{ and } b = d$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$

$\cos(a \pm b) = \cos a \cos \mp \sin a \sin b$

$\tan(a \pm b) = \dfrac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$1 + \tan^2 x = \sec^2 x$

| $y$ | $\dfrac{dy}{dx}$ |
|---|---|
| $an^n$ | $nax^{n-1}$ |
| $ka^x$ | $ka^x \ln a$ |
| $k \ln x$ | $\dfrac{k}{x}$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |

$y = u(x) \cdot v(x)$

$\Rightarrow \dfrac{dy}{dx} = u(x)v^1(x) + u^1(x)v(x)$

$y = \dfrac{u(x)}{v(x)}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{v(x)u^1(x) - u(x)v^1(x)}{[v(x)]^2}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\int ax^n dx = \dfrac{ax^{n+1}}{n+1} + C$

$\int \dfrac{a}{x} dx = a\ell nx + c$

$\int ka^x dx = \dfrac{ka^x}{\ell na} + c$

$A_{ox} = \int_a^b y dx$

$\int \sin x dx = -\cos x + c$

$\int \cos x dx = \sin x + c$

$\int \tan x dx = \ln \sec x + c$

$\int \sec x dx = \ln(\sec x + \tan x) + c$