higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T960(E)(A7)T
APRIL EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N4

(16030164)

7 April 2016 (X-Paper)
09:00–12:00

Scientific calculators may be used.

This question paper consists of 5 pages and 1 formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
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NATIONAL CERTIFICATE
MATHEMATICS N4
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. ALL final answers must be rounded off to THREE decimal places (unless indicated otherwise).
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Use only BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 1.1.1 Draw the graph of $y = 2\cos ecx, 0^0 \le x \le 2\pi$ (3)

1.1.2 Is the graph of $y = 2\cos ecx$ in QUESTION 1.1.1 above a function or a relation? (1)

1.1.3 Is the graph of $y = 2\cos ecx$ in QUESTION 1.1.1 above symmetrical or asymmetrical about the X-axis? (1)

1.2 1.2.1 Draw the graph of the inverse of $y = 11^x$ (3)

1.2.2 What is the domain of the graph of the inverse of $y = 11^x$ in QUESTION 1.2.1 above? (2)

1.3 Given:

$$a + b = c + 3$$
$$2a + c = 3 - b$$
$$-b - \frac{c}{2} = -a$$

Solve the value of 'a' by using Cramer's rule only. (8)

1.4 Given:

$$\begin{vmatrix} \frac{1}{2} & 0 \\ -1 & \frac{1}{3} \end{vmatrix}$$

Calculate the value of the determinant. (2)
[20]

QUESTION 2

2.1 Calculate the value of $\sin 2A$ if $\cos A = \frac{24}{25}$ and A is in the first quadrant. (3)

2.2 Solve for the value of 'A' if:

$$5\cos 2A + 4 = 3\cos A; 0^0 \le A \le 180^0$$ (5)

2.3 Prove that:

$$\frac{2\sin A + 1}{\cos A + \sin 2A} = \sec A$$ (3)

2.4 Simplify:

$$\frac{\sin(180^0 - a)\tan(180^0 + a)\sin(90^0 + a)}{\tan(180^0 + a)\cos(-a).\sin(-a)}$$ (2)

2.5 If $\cos x = \frac{3}{5}$ and $\sin y = \frac{12}{13}$ and both 'x' and 'y' are acute angles, determine without the use of a calculator the value of $\cos(x - y)$. (4)

2.6 Prove that:

$$\sqrt{2 + 2\cos 2A} = 2\cos A$$ (3)

[20]

QUESTION 3

3.1 The perimeter of a field in the form of a right-angled triangle is 60 m. The length of the hypotenuse is 25 m.

Calculate the lengths of the other two sides. (5)

3.2 Solve for 'x' if:

$$4.4^x = 29$$ (3)

3.3 Make 'c' the subject of the formula if:

$$i = \mathrm{I}_0\left(1 - e^{-\frac{t}{Rc}}\right)$$ (3)

3.4 Solve for 'x' if:

$$x + \frac{7}{x} + 5 = 0$$ (4)

3.5 Rationalise:

$$\frac{1}{\cos\theta - j\sin\theta}$$ (3)

3.6 Given:

$$z = 3 - j6$$

Convert $\bar{z}$ into polar form. The argument may only be positive. (2)

[20]

QUESTION 4

4.1 Determine the following limit:

$$\lim_{x \to \frac{1}{2}} \left(\frac{x^3 - x}{x^3 - 1} \right)$$ (4)

4.2 Given:

$$y = (x-1)(x+5)^2$$

Calculate, using differentiation, the co-ordinates of the maximum and minimum turning points. (7)

4.3 Differentiate with regard to 'x':

$$y = -2\cot x + \ln x^6 - \frac{3}{\cos x} + e^{\pi} - \frac{2}{x}$$ (5)

4.4 Use the quotient rule to differentiate the following:

$$y = \frac{\sqrt{x}}{x+4}$$ (4)

[20]

QUESTION 5

5.1 Simplify:

$$\int (\cos^2 A - \sin^2 A) dA$$ (3)

5.2 Evaluate:

$$\int_2^4 \left(\ln 3 - \frac{1}{x} \right) dx$$ (3)

5.3 5.3.1 Sketch and indicate clearly the area enclosed by the graph of $y = x^2 + 2$, $x = 0$ and $x = 2$. Also indicate the representative strip used to calculate the indicated area. (3)

5.3.2 Calculate, using integration, the magnitude of the area indicated in QUESTION 5.3.1 above. (4)

5.4 Integrate:

$$\int \left(\frac{\sin 2x}{\cos x} - \frac{2}{\sqrt{x}} - 3 + \frac{1}{a} - 3\cos\frac{x}{3} - 3^{-4x} \right) dx$$ (7)

[20]

TOTAL: 100

FORMULA SHEET

MATHEMATICS N4

$a^x = b \Leftrightarrow \log a^x = \log b$

$\ell n\, x = \log_e x$

$(r\,|\,\underline{\theta})^n = r^n\,|\,\underline{n\theta}$ $\quad a + bj = c + dj \Leftrightarrow a = c$ and $b = d$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$
$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$\sin^2 x + \cos^2 x = 1$
$1 + \cot^2 x = \operatorname{cosec}^2 x$
$1 + \tan^2 x = \sec^2 x$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

y	$\frac{dy}{dx}$
ax^n	nax^{n-1}
ka^x	$ka^x \ell na$
$k\ell nx$	$\frac{k}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$y = u(x) \cdot v(x)$

$$\Rightarrow \frac{dy}{dx} u(x)v^1(x) + u^1(x)v(x)$$

$$y = \frac{u(x)}{v(x)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{v(x)u^1(x) - u(x)v^1(x)}{[v(x)]^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$$

$$\int \frac{a}{x} dx = a\ell nx + c$$

$$\int ka^x dx = \frac{ka^x}{\ell na} + c$$

$$A_{ox} = \int_u^b y dx$$

$\int \sin x dx = -\cos x + c$

$\int \cos x dx = \sin x + c$

$\int \tan x dx = \ell n \sec x + c$

$\int \sec x dx = \ell n(\sec x + \tan x) + c$