higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N3

(16030143)

29 July 2021 (X-paper)
09:00–12:00

Programmable calculators may NOT be used.

This question paper consists of 6 pages and a formula sheet of 2 pages.

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N3
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but sub-questions must remain together.
5. Show all calculations and intermediary steps.
6. Do all graph work in the ANSWER BOOK, NOT on graph paper.
7. Round all final answers off to THREE decimal places.
8. Diagrams are not drawn to scale.
9. Use only a black or blue pen.
10. Write neatly and legibly.

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QUESTION 1

1.1 Factorise the following as far as possible in prime factors:

$$2(a-1)^2+11(a-1)-6$$ (4)

1.2 Determine the value of a if $f(x)$ gives a remainder of $\frac{23}{8}$ when divided by $2x+1$.

$$f(x)=x^3-ax^2+3x+5$$ (5)

1.3 Simplify the following expression:

$$\frac{\frac{3}{x}+\frac{3}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}\times\frac{x-y}{2xy}$$ (6)

[15]

QUESTION 2

2.1 Solve for x in the following equations:

2.1.1 $5.3^x+3^{x+1}=24$ (3)

2.1.2 $x^{\log x}=1000x^2$ (6)

2.2 Simplify the following:

$$\frac{m-n}{m^{\frac{1}{2}}-n^{\frac{1}{2}}}$$ (2)

[11]

QUESTION 3

3.1 The lengths of two adjacent sides of a rectangle are in the ratio $3:1$ and the diagonal measures 8 metres.

Calculate the area of the rectangle. (5)

3.2 The number of cubic metres of gas which can be delivered by a pipe is given by $N=\frac{1}{100}\sqrt{\frac{d^4}{5,4L}}$

Make the diameter d the subject of the formula. (3)

3.3 Solve for a: $5+\frac{3a}{2-a}=\frac{4a+5}{a-2}$ (4)

[12]

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QUESTION 4

Study FIGURE A below:

A (a; b), B (-2; -3) and C (9; -2) are the vertices of triangle ABC. AB is parallel to DF, while D$(x; y)$, $E\left(\frac{3}{2};\frac{1}{2}\right)$ and F(7;1) are the midpoints of BC, AB and AC respectively.

Determine the following:

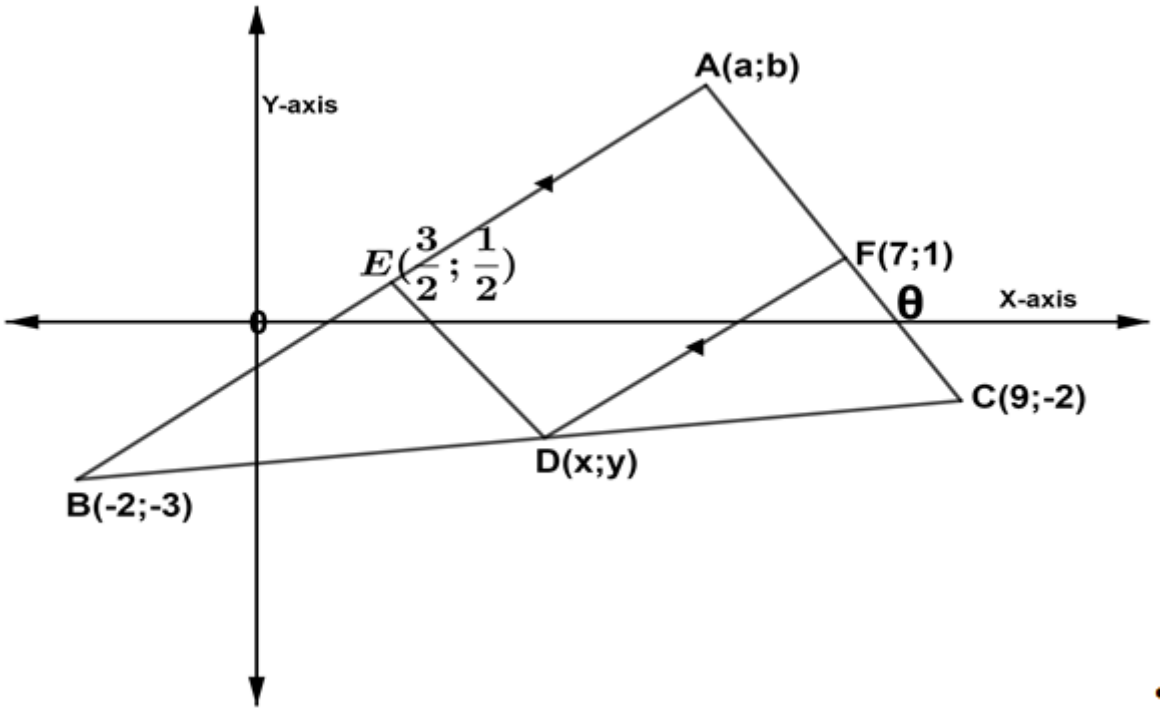

FIGURE A

4.1 The gradient of line AB. (2)

4.2 The co-ordinates of point A. (3)

4.3 The equation of line segment DF. Leave your answer in the form of $y = mx + c$. (3)

4.4 The co-ordinates of D, the midpoint of line BC. (2)

4.5 The length of line segment DF. (2)

4.6 The equation of a line that passes through point C, which is perpendicular to line AC in the form of $y = mx + c$. (3)

4.7 The size of angle of incline θ, which is between line AC and the x axis. (3)

[18]

QUESTION 5

5.1 Given the function defined by $y = 4x^2 + 3x$.

5.1.1 Use the technique of completion of square to find the roots of the given function. (5)

5.1.2 Determine the co-ordinates of the turning point of the given function. (2)

5.1.3 Draw the graph of the given function in your answer book. Show ALL the values at the points of intersections with the system of axes and the co-ordinates of the turning point. (3)

5.2 Determine $\frac{dy}{dx}$ of the function $y = \frac{-2}{x} - 2\sqrt{x}$ by making use of the rules of differentiation. The final answer must be with a positive exponent and in surd form where applicable. (4)

5.3 In FIGURE B, A$(-2;16)$ is a point on the graph $f(x) = ax^3 + bx^2$. The gradient of the tangent at A is -18.

5.3.1 Prove that $a = -\frac{1}{2}$ and $b = 3$ (6)

5.3.2 Determine the coordinates of the turning point of the graph $f(x)$ at the point B (5)

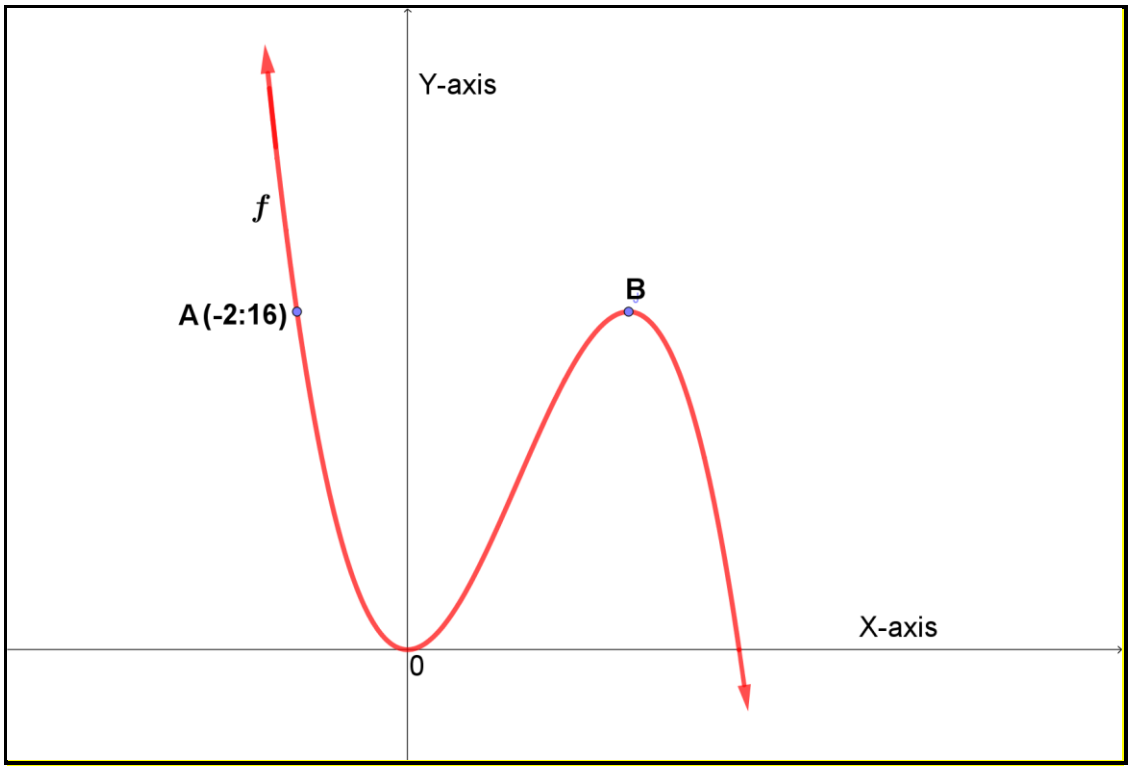

FIGURE B

[25]

QUESTION 6

6.1 Make use of trigonometric identities to prove that:

$$\sec^2 x \times \cot^2 x - 1 = \cot^2 x$$ (4)

6.2 ABC is a triangle with $10\sin B = 6$ and $\hat{A} + \hat{B} = 90^o$. Draw a diagram and calculate the value of $\dfrac{1-2\sin^2 A}{\tan A + \cos B}$.

(4)

6.3 Consider FIGURE C: P,Q,R and S are vertices of a quadrilateral which represents a plot of a land for cultivating different types of vegetables. The plot is the divided onto two portions according to the grouping of the vegetable's favourable seasons and weather. This divisions make portion triangle PQS and triangle QRS.

Determine the area of triangle PQS if $PQ = 138m$, $QR = 179m$, $\hat{R} = 67^o$, $P\hat{Q}R = 90^o$ and $S\hat{Q}R = 53^o$

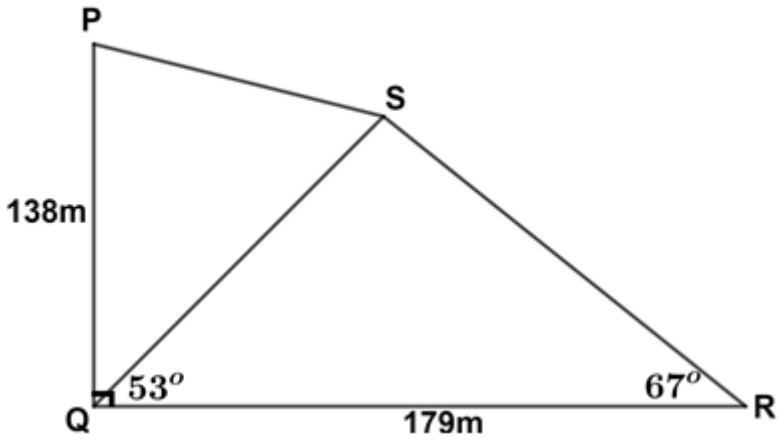

FIGURE C (5)

6.4 Consider the functions $f(x) = \sin x - 1$ and $g(x) = 2\cos x - 1$.

6.4.1 Draw the graph represented by $f(x)$ and $g(x)$ given on the same set of axes for $0^o \le x \le 180^o$.

Show the co-ordinates of the turning points and the intercepts with the axes. (5)

6.4.2 From the graphs find the value of x satisfying the following:

$$\sin x - 1 = 2\cos x - 1$$ (1)

[19]

TOTAL: 100

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MATHEMATICS N3

FORMULA SHEET

Any applicable formula may also be used.

1. **Factors**

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

2. **Logarithms**

$$\log ab = \log a + \log b$$

$$\log \frac{a}{b} = \log a - \log b$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\log a^m = m \log a$$

$$\log_b a = \frac{1}{\log_a b}$$

$$\log_a a = 1 \therefore 1n\ e = 1$$

$$a^{\log_a t} = t \therefore e^{1n\ m} = m$$

3. **Quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. **Parabola**

$$y = ax^2 + bx + c$$

$$y = \frac{4ac - b^2}{4a}$$

$$x = \frac{-b}{2a}$$

5. **Circle**

$$x^2 + y^2 = r^2$$

$$D = \frac{x^2}{4h} + h$$

$$x = \sqrt{4Dh - 4h^2}$$

6. **Straight line**

$$y - y_1 = m(x - x_1)$$

Perpendicular: $m_1 \cdot m_2 = -1$

Parallel lines: $m_1 = m_2$

Distance: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint: $P = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

Angle of inclination: $\theta = tan^{-1} m$

7. **Differentiation**

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

Max/Min

For turning points: $f'(x) = 0$

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8. **Trigonometry**

$$\sin\theta = \frac{y}{r} = \frac{1}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

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