higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N3

31 JULY 2019

This marking guideline consists of 10 pages.

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QUESTION 1

1.1 $f(x) = 6x^3 - 11x^2 + x + 4$

$R = f(1) = 6(1)^3 - 11(1)^2 + 1 + 4$

$= 6 - 11 + 1 + 4$

$= 0$ ✓

$\therefore x - 1$ is a factor of $f(x)$

$$x-1 \overline{)\, 6x^3 - 11x^2 + x + 4}$$

$6x^2 - 5x - 4$ ✓ ✓

$6x^3 - 6x^2$

$-5x^2 + x$

$-5x^2 + 5x$

$-4x + 4$

$-4x + 4$

0

$f(x) = (x-1)(6x^2 - 5x - 4)$

$= (x-1)(2x+1)(3x-4)$ ✓ ✓

Full marks if candidate proves by using a long method of division only instead of using factor theorem. (5)

1.2 $(a+b)^2 - 2(a+b) + 1$

$= [(a+b) - 1][(a+b) - 1]$ ✓

$= (a+b-1)(a+b-1)$ ✓ (2)

1.3 1.3.1

$$\frac{\frac{1}{\sqrt{a}} + \frac{3}{\sqrt{a}}}{\frac{7}{\sqrt{a}} - \frac{3}{\sqrt{a}}}$$

$$= \frac{\frac{4}{\sqrt{a}}}{\frac{4}{\sqrt{a}}}$$ ✓

$= 1$ ✓

Alternative solution:

$$\frac{\frac{1}{\sqrt{a}} + \frac{3}{\sqrt{a}}}{\frac{7}{\sqrt{a}} - \frac{3}{\sqrt{a}}} = \left(\frac{1}{\sqrt{a}} + \frac{3}{\sqrt{a}}\right) \div \left(\frac{7}{\sqrt{a}} - \frac{3}{\sqrt{a}}\right)$$ ✓

$$= \frac{1+3}{\sqrt{a}} \div \frac{7-3}{\sqrt{a}}$$

$$= \frac{4}{\sqrt{a}} \times \frac{\sqrt{a}}{4}$$

$= 1$ ✓ (2)

1.3.2
$$\frac{-ax-bx+by+ay}{x-y}\div\frac{2a^2-3ab-5b^2}{8a^2-50b^2}$$
$$=\frac{-x(a+b)+y(b+a)}{\checkmark(x-y)}\times\overset{\checkmark}{\frac{8a^2-50b^2}{2a^2-3ab-5b^2}}$$
$$=\frac{(y-x)(a+b)}{(x-y)}\times\frac{2(2a-5b)(2a+5b)}{(2a-5b)(a+b)}\ \checkmark$$
$$=-\frac{(y-x)(a+b)}{(y-x)\ \checkmark}\times\frac{2(2a-5b)(2a+5b)}{(2a-5b)(a+b)}$$
$$=-2(2a+5b)\qquad\checkmark\qquad\checkmark$$

(7)
[16]

QUESTION 2

2.1 Given : $\log 2 = a$

$$\therefore \log 50 = \log\left(\frac{100}{2}\right)$$
$$= \log 100 - \log 2$$
$$= \log 10^2 - \log 2 \quad \checkmark$$
$$= 2 - a \quad \checkmark$$

(2)

2.2 2.2.1
$$\frac{2^{2x-1}-2^{2x}}{4^x}=\sqrt{2^{-3x}}-1$$
$$\therefore\frac{2^{2x}(2^{-1}-1)}{2^{2x}}=2^{\frac{-3x}{2}}-1\quad\checkmark$$
$$\therefore\frac{1}{2}-1=2^{\frac{-3x}{2}}-1$$
$$\therefore\frac{1}{2}=2^{\frac{-3x}{2}}$$
$$\therefore 2^{-1}=2^{\frac{-3x}{2}}\quad\checkmark$$
$$\therefore\frac{3x}{2}=1$$
$$\therefore x=\frac{2}{3}\qquad\checkmark$$

(6)

2.2.2 $\log_2 x - 3\log_x 2 = 2$

$\therefore \dfrac{\log x}{\log 2} - \dfrac{3\log 2}{\log x} = 2$ ✓

$\therefore (\log x)^2 - 3(\log 2)^2 = 2\log x \log 2$ ✓

$\therefore (\log x)^2 - 2\log x \log 2 - 3(\log 2)^2 = 0$

$\therefore (\log x - 3\log 2)(\log x + \log 2) = 0$ ✓

$\therefore \log x - 3\log 2 = 0$ or $\log x + \log 2 = 0$

$\therefore \log x = 3\log 2$ $\log x = -\log 2$

$\therefore \log x = \log 2^3$ ✓ $\log x = \log 2^{-1}$ ✓

$\therefore x = 2^3 = 8$ ✓ $x = 2^{-1} = \dfrac{1}{2}$ ✓

Alternative solution:

$\log_2 x - 3\log_x 2 = 2$

$\therefore \log_2 x - \dfrac{3\log_2 2}{\log_2 x} = 2$ ✓

Let $\log_2 x = a$

$\therefore a - \dfrac{3}{a} = 2$ ✓

$\therefore a^2 - 2a - 3 = 0$

$\therefore (a-3)(a+1) = 0$ ✓

$\therefore a - 3 = 0$ or $a + 1 = 0$

$\therefore a = 3$ or $a = -1$

$\therefore \log_2 x = 3$ ✓ $\log_2 x = -1$ ✓

$\therefore x = 2^3 = 8$ ✓ $x = 2^{-1} = \dfrac{1}{2}$ ✓ (7)

[15]

QUESTION 3

3.1 3.1.1 $(x-5)(y+3) = 0$

$\therefore (x-5)(-2+3) = 0$

$\therefore x - 5 = 0$

$\therefore x = 5$ ✓ (1)

3.1.2 $\therefore (x-5)(-3+3) = 0$

$\therefore (x-5)(0) = 0$ ✓

$\therefore x$ any real number ✓ (2)

3.2 $\dfrac{P}{Q}(R^2 - V^2) = D$

$R^2 - V^2 = \dfrac{Q}{P}(D)$ ✓

$V^2 = R^2 - \dfrac{DQ}{P}$ ✓

$V = \pm\sqrt{R^2 - \dfrac{DQ}{P}}$ ✓ Do NOT penalise for the ± sign. (3)

3.3 Let the class average be x
Then the mark of the student is $x-6$
$\therefore 2(x-6)=x+40$ ✓
$\therefore 2x-12=x+40$ $\therefore x-6$ ✓
$\therefore 2x-x=40+12$ $=52-6$ ✓
$\therefore x=52$ ✓ $=46$

$\therefore$ Class average is 52 and the student's marks are 46.

Alternative solution:
Let the student mark be x ✓
$\therefore$ Class average is $x+6$
$\therefore 2x-40=x+6$ ✓
$\therefore 2x-x=40+6$
$\therefore x=46$ ✓
$\therefore x+6=52$ ✓

(4)

3.4 $-2ab-1=-a^2$
$\therefore a^2-2ab=1$ ✓
$\therefore a^2-2ab+(\frac{2b}{2})^2=1+(\frac{2b}{2})^2$
$\therefore a^2-2ab+b^2=b^2+1$ ✓
$\therefore (a-b)^2=b^2+1$
$\therefore a-b=\pm\sqrt{b^2+1}$ ✓
$\therefore a=b\pm\sqrt{b^2+1}$ ✓

(4)
[14]

QUESTION 4

4.1 $M_{AB}=\frac{y_2-y_1}{x_2-x_1}$
$=\frac{-2-(-6)}{-5-(-1)}$ ✓ $=\frac{-2+6}{-5+1}=\frac{4}{-4}$
$=-1$ ✓

(2)

4.2 $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(-1+5)^2+(-6+2)^2}$ ✓
$=\sqrt{(4)^2+(-4)^2}$
$=\sqrt{32}$
$=4\sqrt{2}$ ✓

(2)

4.3 $M_{BC}=\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$
$=\left(\frac{-1+5}{2};\frac{-6+4}{2}\right)$ ✓
$=\left(\frac{4}{2};\frac{-2}{2}\right)$
$=(2;-1)$ ✓

(2)

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4.4

$$M_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4+6}{5+1} \quad ✓$$

$$= \frac{10}{6}$$

$$= \frac{5}{3} \quad ✓$$

$$\therefore y - y_1 = m(x - x_1)$$

$$\therefore y + 2 = \frac{5}{3}(x+5) \quad ✓✓$$

$$\therefore y + 2 = \frac{5}{3}x + \frac{25}{3}$$

$$\therefore y = \frac{5}{3}x + \frac{19}{3} \quad ✓$$

(5)

4.5

$$\theta = \tan^{-1}\left(\frac{5}{3}\right) \quad ✓$$

$$\theta = 59,036^\circ \quad ✓$$

(2)

4.6

$$y - y_1 = m(x - x_1)$$

$m_{BC} \times m = -1$ (lines are perpendicular)

$$\therefore \frac{5}{3} \times m = -1$$

$$\therefore m = -\frac{3}{5} \quad ✓$$

$$\therefore y + 1 = -\frac{3}{5}(x-2) \quad ✓$$

$$\therefore y + 1 = -\frac{3}{5}x + \frac{6}{5}$$

$$\therefore y = -\frac{3}{5}x + \frac{6}{5} - 1$$

$$\therefore y = -\frac{3}{5}x + \frac{1}{5} \quad ✓$$

(3)
[16]

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QUESTION 5

5.1 5.1.1

$x = 2y^2$.....(1)

$x = y + 1$.....(2)

From (1) and (2): $2y^2 = y + 1$

$\therefore\ 2y^2 - y - 1 = 0$

$\therefore (2y + 1)(y - 1) = 0$

$\therefore 2y = -1$ or $y = 1$

$\therefore y = -\frac{1}{2}$ or $y = 1$ ✓

From (2): If $y = -\frac{1}{2}$, then $x = -\frac{1}{2} + 1$

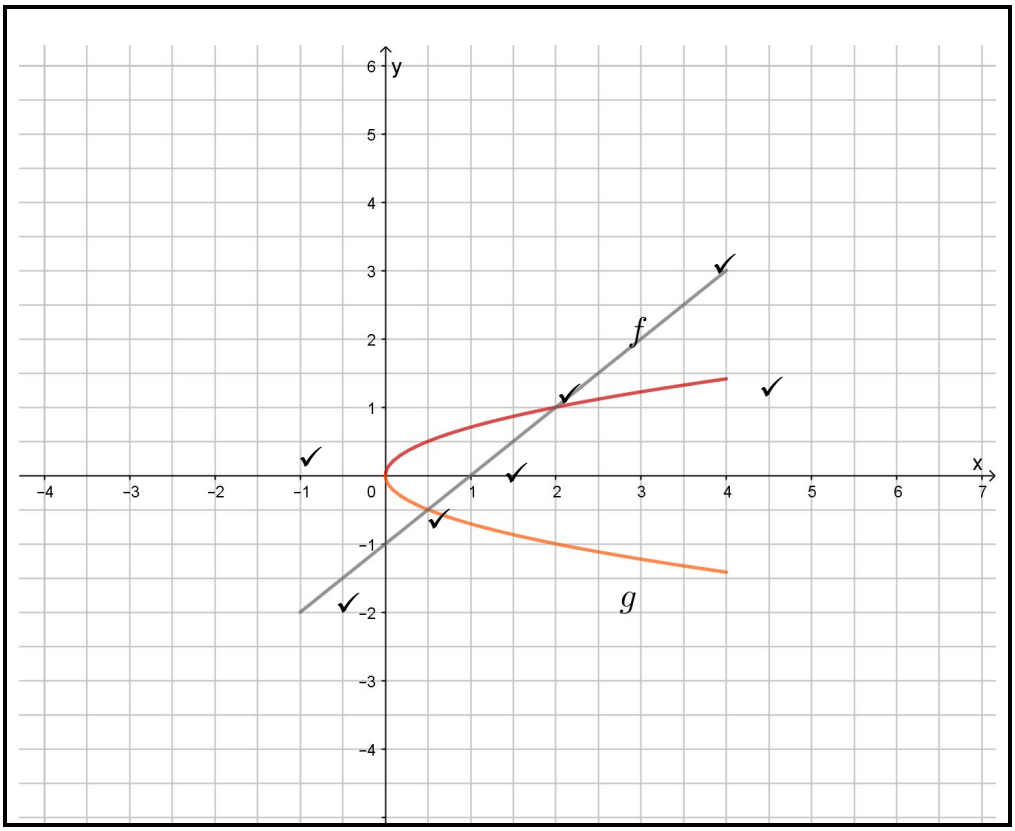
$\therefore x = \frac{1}{2}$ ✓

If $y = 1$, then $x = 2$. ✓

$\therefore$ Points of intersection are $(2;1)$ and $(\frac{1}{2};-\frac{1}{2})$

(4)

5.1.2



intersection of the two graphs = 2
TP of parabola graph = 1
y-intercept of a line graph = 1
x-intercept-t of a line graph = 1
shape = 2 (7)

5.2 $y = 3\sqrt[3]{x^2} + \frac{5}{\sqrt{x}}$

$y = 3 \times x^{\frac{2}{3}} + 5x^{-\frac{1}{2}}$ ✓

$\therefore \frac{dy}{dx} = 3 \times \frac{2}{3}x^{-\frac{1}{3}} + 5 \times -\frac{1}{2}x^{-\frac{3}{2}}$

✓ ✓

$= 2x^{-\frac{1}{3}} - \frac{5}{2}x^{-\frac{3}{2}}$

$= \frac{2}{x^{\frac{1}{3}}} - \frac{5}{2x^{\frac{3}{2}}}$

$= \frac{2}{\sqrt[3]{x}} - \frac{5}{2\sqrt{x^3}}$ ✓✓ (5)

5.3 $f(x) = -x^3 + 6x^2 - 9x$

Gradient is $f'(x) = -3x^2 + 12x - 9$ ✓

If $x = 2$, then $f'(2) = -3(2)^2 + 12(2) - 9$ ✓

$= -12 + 24 - 9$

$= 3$ ✓ (3)

[19]

QUESTION 6

6.1 $\tan(x - 30^\circ) = \frac{1}{\sqrt{3}}$

$x - 30^\circ = \tan^{-1}\frac{1}{\sqrt{3}}$ ✓

$x - 30^\circ = 30^\circ$ or $x - 30^\circ = 180^\circ + 30^\circ$

$x = 60^\circ$ ✓ $x = 240^\circ$ ✓ (3)

6.2

$$\cos(360^\circ - A) = 1 - \frac{\sin^2(180^\circ + A)}{1 - \cos(180^\circ + A)}$$

$$RHS = 1 - \frac{\sin^2(180^\circ + A)}{1 - \cos(180^\circ + A)}$$

$$= 1 - \frac{\sin^2 A}{1 + \cos A} \quad ✓$$

$$= 1 - \frac{1 - \cos^2 A}{1 + \cos A} \quad ✓$$

$$= 1 - \frac{(1 - \cos A)(1 + \cos A)}{1 + \cos A} \quad ✓$$

$$= 1 - (1 - \cos A)$$

$$= 1 - 1 + \cos A$$

$$= \cos A \quad ✓$$

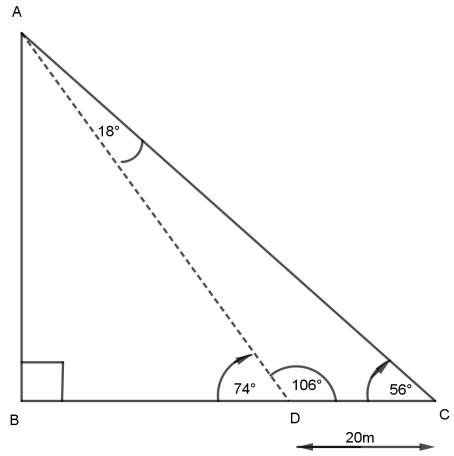
$$LHS = \cos(360^\circ - A)$$

$$= \cos A \quad ✓$$

$$= RHS$$

(6)

6.3



In ΔACD:

$C\hat{A}D = 18^{o}$ ✓

and $A\hat{D}C = 106^{o}$ ✓

$\therefore \dfrac{AC}{\sin 106^{o}} = \dfrac{20}{\sin 18^{o}}$ ✓

$\therefore AC = \dfrac{20\sin 106^{o}}{\sin 18^{o}}$

$\therefore AC = 62,214m$ ✓

In ΔABC

$\dfrac{AB}{AC} = \sin 56^{o}$ ✓

$\therefore AB = AC\sin 56^{o}$

$= 62,214 \sin 56^{o}$ ✓

$= 51,578\text{m}$ ✓

Alternative solution:

$\dfrac{\sin 56^{\circ}}{AD} = \dfrac{\sin 18^{\circ}}{20}$ ✓

$AD\sin 18^{\circ} = 20\sin 56^{\circ}$ ✓

$AD = \dfrac{20\sin 56^{\circ}}{\sin 18^{\circ}}$ ✓

$= 53,656\text{ m}$ ✓

$\sin 74^{\circ} = \dfrac{AB}{AD}$ ✓

$AB = 53,656\sin 74^{\circ}$ ✓

$= 51,577\text{ m}$ ✓

(7)

6.4 6.4.1 $a = 2$ ✓

$b = 1$ ✓

$c = -1$ ✓ (3)

6.4.2 $a\cos x - \sin bx = c$

$\therefore a\cos x = \sin bx + c$

$\therefore x = 90^{o}$ ✓

(1)

[20]

TOTAL: 100

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