higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N3

23 JULY 2018

This marking guideline consists of 10 pages.

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QUESTION 1

1.1
$$\frac{x^2-4}{3x^2+4x-4}\div\frac{x^2-4x+4}{2x^2-11x+14}\times\frac{3x-2}{6x-21}$$
$$=\frac{x^2-4}{3x^2+4x-4}\times\frac{2x^2-11x+14}{x^2-4x+4}\times\frac{3x-2}{6x-21}\quad\checkmark$$
$$=\frac{(x-2)(x+2)}{(3x-2)(x+2)}\times\frac{(2x-7)(x-2)}{(x-2)(x-2)}\times\frac{3x-2}{3(2x-7)}\quad\checkmark\quad\checkmark\quad\checkmark$$
$$=\frac{1}{3}\quad\checkmark$$
(5)

1.2
$$f(x)=2x^3-px^2+6x-5$$
$$R=f(\tfrac{3}{2})=2(\tfrac{3}{2})^3-p(\tfrac{3}{2})^2+6(\tfrac{3}{2})-5\quad\checkmark$$
$$=2(\tfrac{27}{8})-p(\tfrac{9}{4})+9-5$$
$$\therefore 4=\frac{27}{4}-\frac{9p}{4}+4\quad\checkmark$$
$$\therefore 0=27-9p$$
$$\therefore 9p=27$$
$$\therefore p=3\quad\checkmark$$
(4)

1.3
$$ax+9by-3ay-3bx$$
$$=(ax-3ay)+(9by-3bx)$$
$$=a(x-3y)+3b(3y-x)\ \checkmark$$
$$=a(x-3y)-3b(x-3y)\ \checkmark$$
$$=(x-3y)(a-3b)\checkmark$$

OR

$$=(ax-3ay)-(3bx-9by)\ \checkmark$$
$$=a(x-3y)-3b(x-3y)\checkmark$$
$$=(x-3y)(a-3b)\ \checkmark$$
(3)

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1.4
$$\frac{x+9x^{\frac{1}{2}}}{x+7x^{\frac{1}{2}}-18}\div\left(1-\frac{2}{x^{\frac{1}{2}}}\right)^{-1}$$

✓

$$=\frac{x^{\frac{1}{2}}(x^{\frac{1}{2}}+9)}{(x^{\frac{1}{2}}+9)(x^{\frac{1}{2}}-2)}\div\left(\frac{x^{\frac{1}{2}}-2}{x^{\frac{1}{2}}}\right)^{-1}$$ ✓

✓

$$=\frac{x^{\frac{1}{2}}(x^{\frac{1}{2}}+9)}{(x^{\frac{1}{2}}+9)(x^{\frac{1}{2}}-2)}\times\left(\frac{x^{\frac{1}{2}}-2}{x^{\frac{1}{2}}}\right)^{1}$$ ✓

$=1$ ✓ (5)
[17]

QUESTION 2

2.1
$$2^0+2^{1-2x}=9.2^{-1-x}$$
$$1+2.2^{-2x}=9.2^{-1}.2^{-x}$$
$$\therefore 2.2^{-2x}-\frac{9}{2}.2^{-x}+1=0$$ ✓
$$\therefore 4.2^{-2x}-9.2^{-x}+2=0$$
$$\therefore (4.2^{-x}-1)(2^{-x}-2)=0$$ ✓
$$\therefore 4.2^{-x}=1 \ \ or\ 2^{-x}=2$$
$$\therefore 2^{-x}=\frac{1}{2^2}$$ ✓ *or* $2^{-x}=2$ ✓
$$\therefore 2^{-x}=2^{-2} \quad or \quad 2^{-x}=2^1$$
$$\therefore -x=-2 \quad or \quad -x=1$$
$\therefore x=2$ ✓ or $x=-1$ ✓ (6)

2.2
$$\log x+\frac{1}{\log_x 10}=2$$
$$\therefore \log x+\frac{1}{\frac{\log 10}{\log x}}=2$$ ✓
$$\therefore \log x+\log x=2$$ ✓
$$\therefore 2\log x=2$$ ✓
$$\therefore \log x=1$$
$\therefore x=10$ ✓ (4)

2.3 $2(\log x)^2 + \log x^3 - 2 = 0$

$\therefore 2(\log x)^2 + 3\log x - 2 = 0$ ✓

$\therefore (2\log x - 1)(\log x + 2) = 0$ ✓

$2\log x - 1 = 0 \quad \text{or} \quad \log x + 2 = 0$

$1\log x = \frac{1}{2} \quad \text{or} \quad \log x = -2$ ✓ ✓

$10^{\frac{1}{2}} = x \qquad \text{or} \qquad 10^{-2} = x$

$x = \sqrt{10} = 3{,}162$ ✓ or $x = \frac{1}{100} = 0{,}01$ ✓

(6)

[16]

QUESTION 3

3.1 $M = \frac{K(H^2 + h^2)}{B}$

$\therefore BM = KH^2 + Kh^2$ ✓

$\therefore BM - KH^2 = Kh^2$ ✓

$\therefore \frac{BM - KH^2}{K} = h^2$ ✓

$\therefore \quad h = \pm\sqrt{\frac{BM - KH^2}{K}}$ ✓

(4)

3.2 $\frac{x-2}{x-1} - \frac{5}{x^2 - 4x + 3} - \frac{2}{3-x} = 1$

$\therefore \frac{x-2}{x-1} - \frac{5}{(x-1)(x-3)} + \frac{2}{(x-3)} = 1$ ✓

$\therefore (x-2)(x-3) - 5 + 2(x-1) = (x-1)(x-3)$ ✓ ✓ ✓ ✓

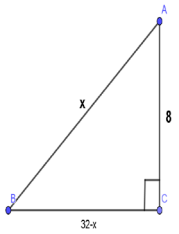
$\therefore x^2 - 5x + 6 - 5 + 2x - 2 = x^2 - 4x + 3$

$\therefore -5x + 2x + 4x = 3 - 6 + 5 + 2$ ✓

$\therefore \quad x = 4$ ✓

(7)

3.3 Let the hypotenuse be x meter



The other side is $=(40-8-x)$ meter $=(32-x)$ meter ✓

$\therefore x^2=(32-x)^2+64$ ✓

$=1024-64x+x^2+64$

$64x=1088$

$\therefore x=17\text{m}$ ✓

The other side =15m ✓

The area of the triangle=$(\frac{1}{2}\times 8\times 15)\text{m}^2$ =60m^2

✓ ✓ (6)

[17]

QUESTION 4

4.1 $(x;y)=(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2})$

$\therefore x=\frac{x_1+x_2}{2}$ and $y=\frac{y_1+y_2}{2}$

$\frac{0+x_2}{2}=4$ ✓ $\frac{10+y_2}{2}=8$ ✓

$x_2=8$ $y_2=6$

$\therefore \text{B}(8;6)$ ✓ (3)

4.2 $m_{\text{AB}}=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{8-10}{4-0}$ ✓

$=\frac{-2}{4}=\frac{-1}{2}$ ✓ (2)

4.3 Equation of the line through A and B

$\therefore m_{\text{AB}}=-\frac{1}{2}$

$\therefore y-y_{\text{A}}=m_{\text{AB}}(x-x_{\text{A}})$

$\therefore y-10=-\frac{1}{2}(x-0)$ ✓

$\therefore y=-\frac{1}{2}x+10$ ✓

Alternative answer:

$y=mx+c$

$\therefore m_{\text{AB}}=-\frac{1}{2}$

$\therefore c=10$

$\therefore y=-\frac{1}{2}x+10$ ✓ ✓

(2)

4.4

$$m_{OP} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{8-0}{4-0}$$
$$= 2 \quad \checkmark$$
$$m_{OP} \times m_{AB} = 2 \times -\frac{1}{2} = -1$$
$$\therefore \text{OP} \perp \text{AB} \quad \checkmark$$

(2)

4.5

The gradient of the line parallel to $y = 2x$ ✓ is also 2.

$y - y_A = m(x - x_A)$ ✓

$\therefore y - 10 = 2(x - 0)$

$\therefore y = 2x + 10$ ✓

Alternative answer:

$y = mx + c$

$\therefore y = 2x + 10$ ✓ ✓ ✓

(3)

4.6

$\theta = \tan^{-1} m$

$\theta = \tan^{-1}\left(\frac{-1}{2}\right)$ ✓

$= -26,565° ref$

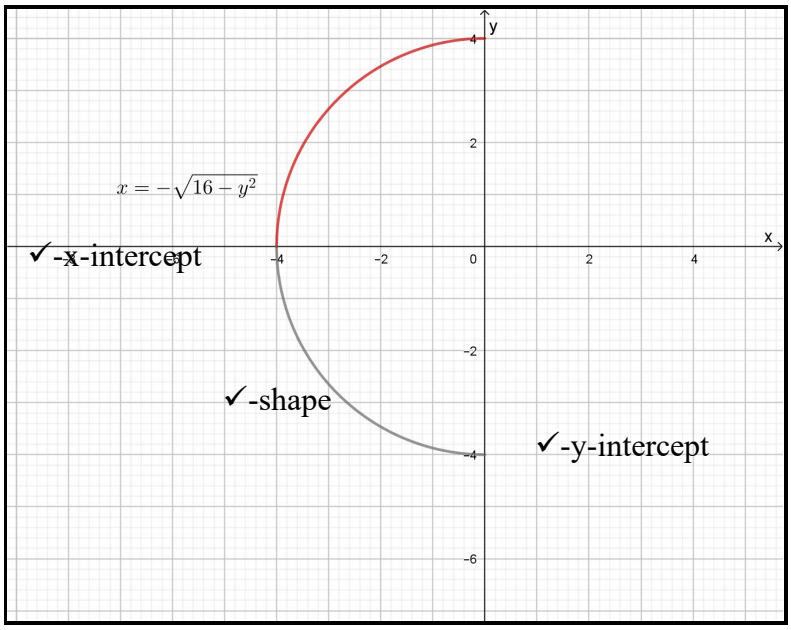
$\theta = 153,435°$ ✓ ✓

(3)

[15]

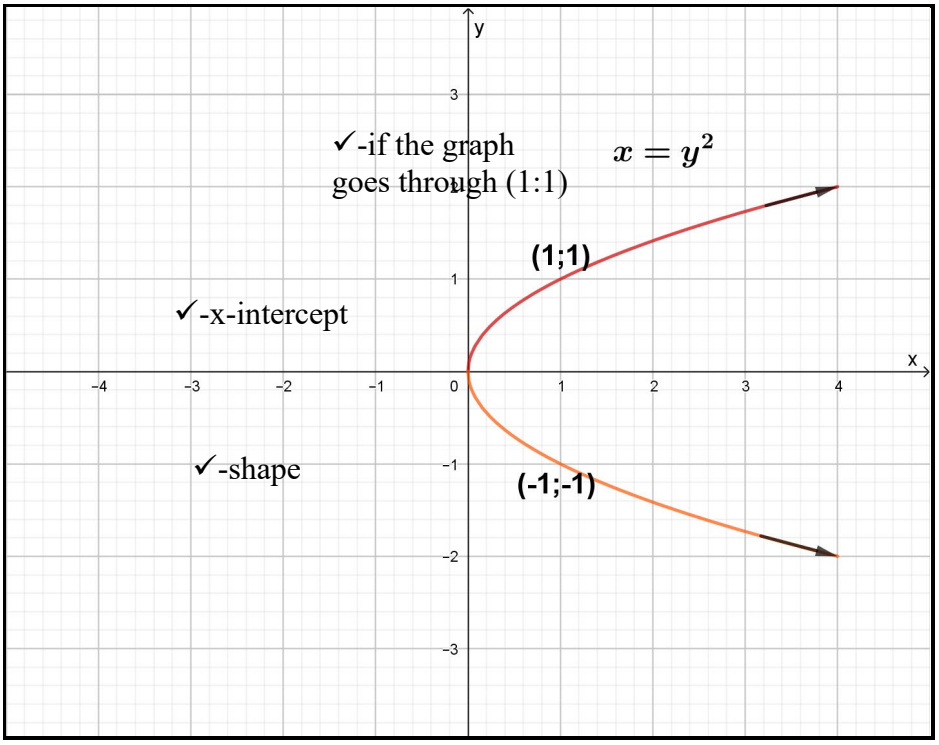
QUESTION 5

5.1 5.1.1



Marks may only be awarded for the x-intercept and y-intercept under the condition that the shape of the semi-circle is correct. (3)

5.1.2



Marks may only be awarded for the x-intercept and y-intercept under the condition that the shape of the parabola is correct. (3)

5.2 $f(x) = x(x^2 - 6x + 9)$ ✓

$\therefore f(x) = x^3 - 6x^2 + 9x$

$\therefore f'(x) = 3x^2 - 12x + 9$ ✓

For maximum and minimum turning points,let $f'(x) = 0$ ✓

$\therefore\ 3x^2 - 12x + 9 = 0$

$\therefore x^2 - 4x + 3 = 0$

$\therefore (x-3)(x-1) = 0$ ✓

$\therefore x = 3 \quad \text{or} \quad x = 1$

If $x = 3$ then $f(x) = (3)^3 - 6(3)^2 + 9(3)$

$= 27 - 54 + 27$

$= 0$

If $x = 1$, then $f(x) = (1)^3 - 6(1)^2 + 9(1)$

$= 4$ ✓ ✓

$\therefore$ *T*urning points are A = $(1;4)$ and B= $(3;0)$ (6)

5.3 $f(x) = -4\sqrt{x} + \dfrac{1}{2x^2}$

$= -4x^{\frac{1}{2}} + \dfrac{x^{-2}}{2}$ ✓ ✓

$f'(x) = \dfrac{dy}{dx} = -4 \times \dfrac{1}{2} x^{-\frac{1}{2}} - \dfrac{2x^{-3}}{2}$ ✓ ✓

$= \dfrac{-2}{x^{\frac{1}{2}}} - \dfrac{1}{x^3}$

$= \dfrac{-2}{\sqrt{x}} - \dfrac{1}{x^3}$ ✓ (5)

[17]

QUESTION 6

6.1 $\text{cosec}\,\alpha = \cot 20^\circ \; for \alpha \in [90°;360°]$

$\therefore \text{cosec}\;\alpha = 2,747$ ✓

$\therefore \sin\alpha = 0,364$

$\alpha = 180^\circ - 21,346^\circ$ ✓

$= 158,654^\circ$ ✓

Alternative answer:

$\text{cosec}\,\alpha = \cot 20^\circ$

$\therefore \sin\;\alpha = \tan 20^\circ$ ✓

$\therefore \sin\alpha = 0,364$ ✓

$\alpha = 180^\circ - 21,346^\circ$

$= 158,654^\circ$ ✓ (3)

6.2 $5\sin B = 3$

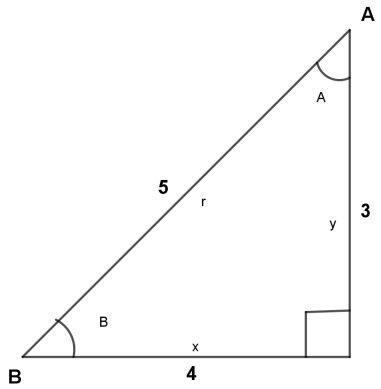
$\therefore \sin B = \frac{3}{5}$

$\therefore x^2 = 5^2 - 3^2 \,(Pythagoras)$

$\therefore x^2 = 25 - 9$

$\therefore x^2 = 16$ ✓

$\therefore x = 4$



$\therefore \frac{1-2\cot^2 A}{\tan A + \text{cosec}B}$

$= \frac{1-2\left(\frac{3}{4}\right)^2}{\left(\frac{4}{3}\right)+\left(\frac{5}{3}\right)}$ ✓ $= \frac{1-\frac{9}{8}}{3}$ ✓ $= -\frac{1}{8}\times\frac{1}{3}$ ✓

$= -\frac{1}{24} = -0,042$ ✓ (5)

6.3 $LHS = \cos^2(180^\circ + x)\left[2 + \tan^2(360^\circ - x)\right]$ RHS=$2 - \sin^2 x$

$= \cos^2 x\left[2 + \tan^2 x\right]$ ✓ ✓

$= 2\cos^2 x + \cos^2 x \tan^2 x$

$= 2\cos^2 x + \cos^2 x\left(\frac{\sin^2 x}{\cos^2 x}\right)$

$= 2\cos^2 x + \sin^2 x$ ✓

$= 2(1 - \sin^2 x) + \sin^2 x$

$= 2 - 2\sin^2 x + \sin^2 x$ ✓

$= 2 - \sin^2 x$ ✓

$= RHS$ (5)

6.4 $AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos B$

$AC^2 = (50)^2 + (100)^2 - 2(50)(100)\cos 45°$✓

$= 12\,500 - 7\,071{,}068$

$= 5\,428{,}932$✓

$AC = 73{,}681$ m✓ (3)

6.5 $a = 1$ ✓

$b = 3$ ✓ (2)

[18]

TOTAL: 100

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