higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T940**(E)**(M26)T

NATIONAL CERTIFICATE

MATHEMATICS N3

(16030143)

26 March 2018 (X-Paper)
09:00–12:00

This question paper consists of 6 pages and a formula sheet of 2 pages.

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N3
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL calculations and the intermediary steps.
6. ALL graph work must be done in the ANSWER BOOK. No graph paper needed.
7. ALL final answers must be accurately approximated to THREE decimal places.
8. Diagrams are NOT drawn to scale.
9. Write neatly and legibly.

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QUESTION 1

1.1 Factorise the following as far as possible in prime factors:

1.1.1 $5x^4 - 5y^8$

1.1.2 $15x^2y^2 - 25xy^3 - 10y^4$

(2 × 3) (6)

1.2 Simplify the following algebraic fractions:

1.2.1 $\dfrac{x^2 - y^2}{x^2 - 2xy + y^2}$ (3)

1.2.2 $\dfrac{2}{a-b} \div \dfrac{4}{a^2 - b^2} \times \dfrac{5}{6a + 6b}$ (4)

1.3 If $x - 1$ is a factor of $f(x) = 2x^3 - x^2 - 7x + 6$, determine the other factors. (5)

[18]

QUESTION 2

Solve for *x* in the following equations:

2.1 $\sqrt{10 - 3x} - x = -2$ (4)

2.2 $2^{2x+2} - 2^{2x-1} = 56$ (4)

2.3 $2\log_x 4 + \log_x 54 - \log_x \dfrac{1}{2} = 3$ (4)

[12]

QUESTION 3

3.1 John runs from A to B uphill with an average speed of 8 km/h, and returns from B to A downhill with an average speed of 12km/h. What was his average speed during the total distance?

HINT: Distance = (average speed) × time (4)

3.2 Make *R* the subject of the following formula:

$$B = \frac{A}{\sqrt{R^2 + PQ}}$$ (4)

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3.3 Determine the value of x by means of completing the square:
$x^2 = -6x + 5$ (4)

3.4 Determine the quadratic equation of which the roots are $x = \frac{1}{2}$ and $x = \frac{2}{3}$ (3)
[15]

QUESTION 4

In FIGURE 1 below ABC is a triangle with the vertices A (0 ; 8), B (1 ; 0) and C (4 ; 0). BD, a line with D (p ; 2) on AC, makes an angle of 45° with the x-axis.

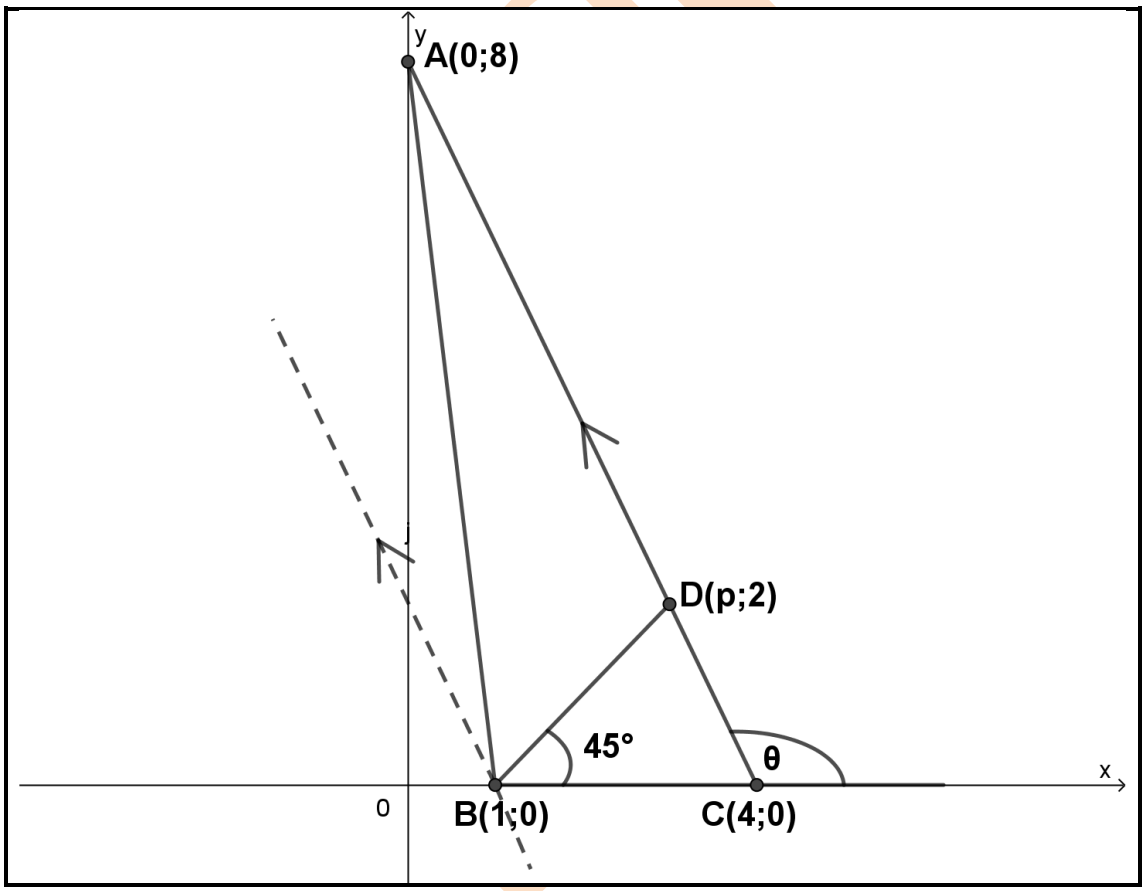

FIGURE 1

Determine the following:

4.1 The gradient of BD; (1)

4.2 The value of p; (2)

4.3 The length of AC. (Leave the answer in its simplest surd form); (2)

4.4 The gradient of AC; (2)

4.5 The size in degrees of an obtuse angle θ between AC and the x-axis; (2)

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4.6 The equation of the line parallel to AC passing through point B. Leave the answer in gradient-intercept form; (3)

4.7 The equation of the line perpendicular to BD passing through point B. Leave the answer in gradient-intercept form. (3)

[15]

QUESTION 5

5.1 Draw the graph defined by $\frac{x^2}{25}+\frac{y^2}{4}=1$. (3)

5.2 Given: $f(x)=x^3-12x^2+36x$

5.2.1 Determine the co-ordinates of the turning points of $f(x)$. (4)

5.2.2 Calculate the x and y-intercepts of $f(x)$. (3)

5.2.3 Sketch the graph of $f(x)$ and show the calculated values in questions 5.2.1 and 5.2.2. on the graph. (3)

5.3 Determine $\frac{dy}{dx}$ by using the rules of differentiation. The answer should be with a positive exponent and in surd form where applicable.

$$f(x)=\frac{1}{x^3}-\frac{2}{3x^{\frac{1}{2}}}$$ (6)

[19]

QUESTION 6

6.1 Calculate the values of A that will satisfy the following equation for $0^\circ \le A \le 360^\circ$:

$$4\tan A-6=0$$ (4)

6.2 Simplify the following:

$$\sin^2 B-\sin^2 B\cos^2 B$$ (3)

6.3 Prove that $\frac{\sin^2\beta}{1+\cos\beta}=1-\cos\beta$ (3)

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6.4 Consider FIGURE 2 below. AB is a lighthouse of 100 metres away from a nearby building CD. The angle of elevation from the top of CD to the top of the lighthouse AB is 30°, while the angle of elevation from the foot of the lighthouse AB to the top of the building is 43°.

Determine the height of the lighthouse AB.

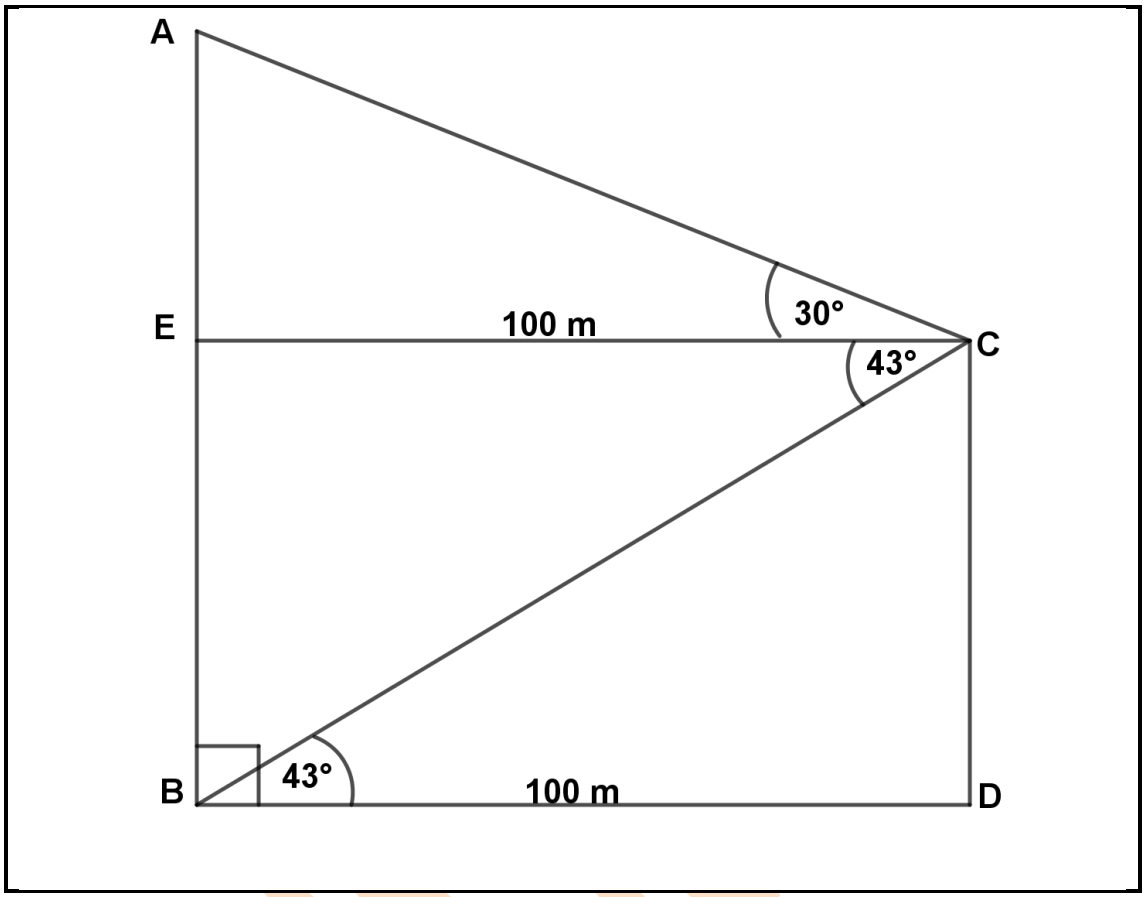

FIGURE 2

(6)

6.5 Draw a graph represented by the following trigonometric equation:

$y = \sin x + 1$ where $0^\circ \leq \theta \leq 360^\circ$

(5)
[21]

TOTAL: 100

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FORMULA SHEET

Any other applicable formula may also be used.

1. **Factors**

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

2. **Logarithms**

$$\log ab = \log a + \log b$$

$$\log \frac{a}{b} = \log a - \log b$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\log a^m = m \log a$$

$$\log_b a = \frac{1}{\log_a b}$$

$$\log_a a = 1 \therefore \ln e = 1$$

$$a^{\log_a t} = t \therefore e^{\ln m} = m$$

3. **Quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. **Parabola/ Parabool**

$$y = ax^2 + bx + c$$

$$y = \frac{4ac - b^2}{4a}$$

$$x = \frac{-b}{2a}$$

5. **Circle**

$$x^2 + y^2 = r^2$$

$$D = \frac{x^2}{4h} + h$$

$$x = \sqrt{4Dh - 4h^2}$$

6. **Straight line**

$$y - y_1 = m(x - x_1)$$

Perpendicular:
Loodreg: $m_1 \cdot m_2 = -1$

Parallel lines:
Ewewydige lyne: $m_1 = m_2$

Distance:
Afstand: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint:
Middelpunt: $P = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$

Angle of inclination:
Hellingshoek: $\theta = \tan^{-1} m$

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7. **Differentiation**

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

Max/Min
Maks/Min

For turning points:
Vir draaipunte: $f'(x) = 0$

8. **Trigonometry**

$$\sin\theta = \frac{y}{r} = \frac{1}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

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