higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT (BEROEPSGERIG)

WISKUNDE
(Tweede vraestel)
NKR VLAK 4

25 Februarie 2020

Hierdie nasienriglyn bestaan uit 10 bladsye.

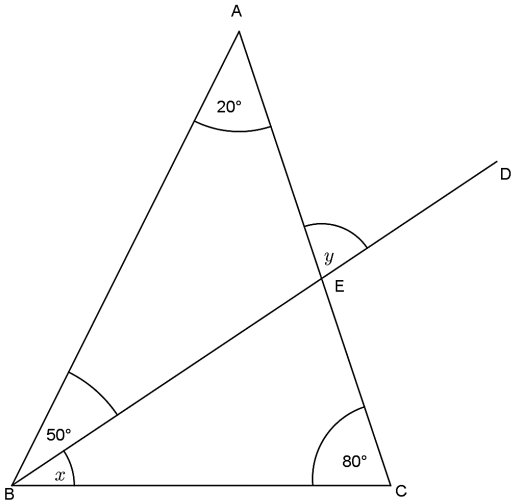
Kopiereg voorbehou

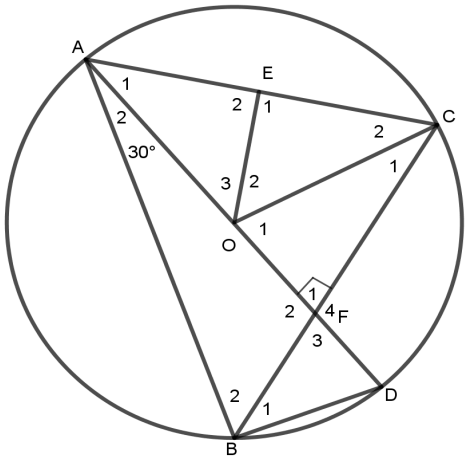
✓ = 1 punt √ = ½ punt

- Aanvaar ALLE wiskundig geldige oplossings.
- Konsekwente akkuraatheid geld as die algemene reël.
- As 'n kandidaat 'n vraag twee keer beantwoord en nie een van die oplossings skrap nie, sien slegs die eerste poging na.
- As 'n kandidaat 'n vraag beantwoord, dit skrap en nie weer doen nie, sien die geskrapte poging na.

NASIENLEGENDE	
SIMBOOL	**VERDUIDELIKING VAN SIMBOOL**
A	Akkuraatheid
M	Metode
KA	Konsekwente akkuraatheid (opvolg)
O/L	Ontleding
S	Stelling
R	Rede
S/R	Stelling en rede

VRAAG 1

1.1	**OPLOSSING**	**VERDUIDELIKING**	
			
	1.1.1 $A\hat{B}C = 180° - 80° - 20°$ $= 80°$ $x = 80° - 50°$ $= 30°$✓	1 vir antwoord (A)	(1)
	1.1.2 $y = 50° + 20°$ ... buite∠ van ΔABE $y = 70°$✓	1 vir antwoord (A)	(1)
	1.1.3 Gelykbenige driehoek✓ ... $A\hat{B}C = 80° = \hat{B}$✓	1 vir antwoord, 1 vir rede (A)	(2)

1.2					
	1.2.1	(a)	$\widehat{B}_1 + \widehat{B}_2 = 90°$✓... onderspan by middellyn $\widehat{D} + 90° + 30° = 180°$ $\widehat{D} = 60°$✓	1 vir 90° (KA) 1 vir antwoord (KA)	
		(b)	$\widehat{D} = \widehat{C}_1 + \widehat{C}_2 = 60°$✓... dieselfde segment $\widehat{A}_1 + 60° + 90° = 180°$... hoek som van ΔAFC $\widehat{A}_1 = 30°$✓	1 vir S (A) 1 vir antwoord (A)	
		(c)	$\widehat{O}_1 = 2\widehat{A}_1 = 60°$✓... hoek by middelpunt - stelling $\widehat{C}_1 + 60° + 90° = 180°$ ... ΔCOF $\widehat{C}_1 = 30°$✓	1 vir S (A) 1 vir antwoord (A)	
				(3 × 2)	(6)
	1.2.2		$\widehat{B}_2 = 60°$... ΔABF $\widehat{B}_2 = \widehat{D}$✓... elk = 60° AB is 'n raaklyn ... conv. them✓	1 vir S (A) 1 vir rede (A)	(2)
	1.2.3		AE = CE ... gegewe $\widehat{E}_1 = 90°$✓... middelpt van koord AC✓ $\widehat{F}_1 = 90°$ ... gegewe $\widehat{F}_1 + \widehat{E}_1 = 180°$✓ ∴ CEOF is ′nsikliese vierhoek omdat die teenoorst. ∠′e van sikl. vierh. supplementêr is	1 vir S, 1 vir R (A) 1 vir S (A) 1 vir rede (A)	(4)

1.3	1.3.1	$r^2 = (-2)^2 + 3^2 = 13\sqrt{}$ $\therefore x^2 + y^2 = 13\sqrt{}$	½ vir S (A) ½ vir antwoord (A)	(1)
	1.3.2	C$(-3; -4)$ $r^2 = (-3 - 0)^2 + (0 - 4)^2 = 25$✓ C lê buite die ⊙ aangesien $25 > 13$✓	1 vir S (A) 1 vir R (A)	(2)

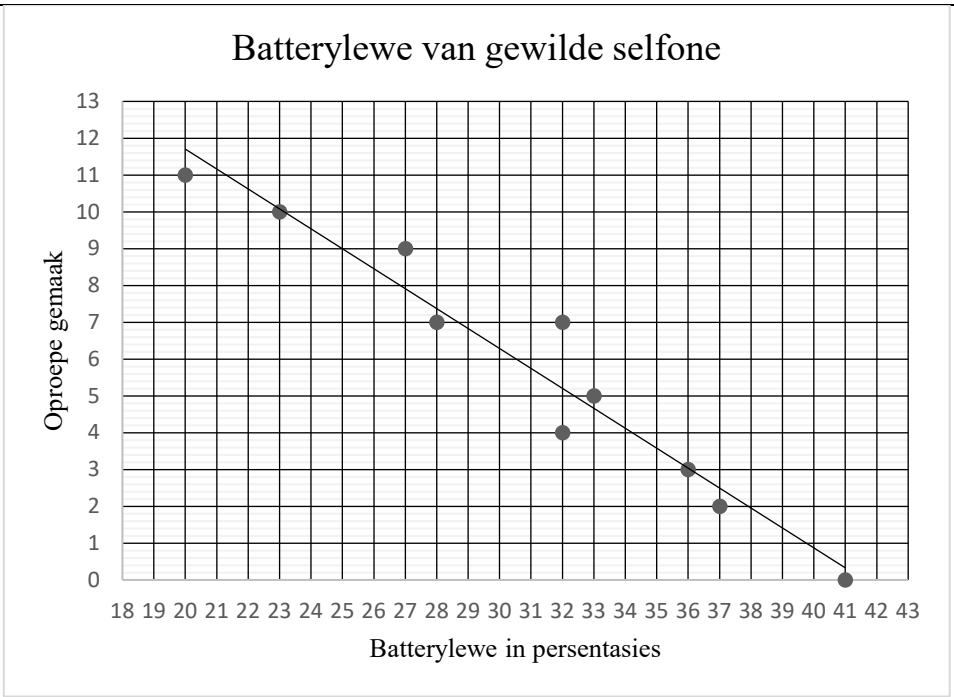
	1.3.3	AB = $2r$... middellyn✓ AB = $2\sqrt{13} = \sqrt{52}$✓	1 vir S/afstandformule (A) 1 vir enige van antwoorde (A)	(2)
	1.3.4	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{-2-0} = -\frac{3}{2}$✓	1 vir antwoord (A)	(1)
	1.3.5	$m_{RAAKLYN} = -1 \times -\frac{2}{3} = \frac{2}{3}$✓ $(-2;3)$: $y - 3 = \frac{2}{3}(x+2)$✓ $y = \frac{2}{3}x + \frac{4}{3} + 3$ $y = \frac{2}{3}x + \frac{13}{3}$✓	1 vir antwoord (KA) 1 vir subst. in formule (KA) 1 vir antwoord (KA)	(3)
				[25]

VRAAG 2			
2.1	$\cos\theta = 0{,}726$ Ref ∠: $\theta = \cos^{-1}0{,}726 = 43{,}4°$✓ $\therefore \theta = 360° - 43{,}4° = 316{,}6°$✓	1 vir verw.hoek (A) 1 vir antwoord (A)	(2)
2.2	$\frac{\sin^2\theta}{\sin(180°-\theta)\times\cos(90°+\theta)+\tan 45°}$ $= \frac{\sin^2\theta}{\sin\theta\sqrt{\times}(-\sin\theta)\sqrt{+1}\sqrt{}}$ $= \frac{\sin^2\theta}{-\sin^2\theta+1\sqrt{}}$ $= \frac{\sin^2\theta}{1-\sin^2\theta}$ $= \frac{\sin^2\theta}{\cos^2\theta}$✓ $= \tan^2\theta$ ✓	½ vir elke term (A) ½ vir vereenvoudiging (A) 1 vir noemer (A) 1 vir antwoord (A)	(4)
2.3	$2\cos^2 2x - 1 = -\frac{1}{2}$ $\cos 4x$✓ $= -0{,}5$ $4x = \pm 120°$✓ $+ 360°k$✓ ; $k \in \mathbb{Z}$ $x = \pm 30° + 90°k$; $k \in \mathbb{Z}$✓ **OF** $x = 30° + 90°k; 60° + 90°k$; $k \in \mathbb{Z}$	1 vir dubbelhoek (A) 1 vir hoek, 1 vir 360k (KA) 1 vir antwoord (KA)	(4)

2.4	$\frac{\sin A}{\sin B}-\frac{\cos A}{\cos B}=\frac{2\sin(A-B)}{\sin 2B}$ $LK=\frac{\sin A}{\sin B}-\frac{\cos A}{\cos B}$ $=\frac{\sin A\cos B-\cos A\sin B}{\sin B\cos B}$ ✓ ✓ $=\frac{\sin(A-B)}{\sin B\cos B}$ ✓ $=\frac{\sin(A-B)}{\sin B\cos B}\times\frac{2}{2}$ $=\frac{2\sin(A-B)}{2\sin B\cos B}$ ✓ $=\frac{2\sin(A-B)}{\sin 2B}$ ✓ $=RK$	1 vir teller (A) 1 vir noemer (A) 1 vir saamgest. hoek (A) 1 vir opstelling (A) 1 vir antwoord (A)	(5)
2.5	$\frac{\cos(45°-\theta)}{\cos 45°\times\cos\theta}-\tan\theta$ $=\frac{\cos 45°\cos\theta+\sin 45°\sin\theta}{\cos 45°\times\cos\theta}$ ✓ $-\frac{\sin\theta}{\cos\theta}$ ✓ $=\frac{\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta-\frac{1}{\sqrt{2}}\sin\theta}{\frac{1}{\sqrt{2}}\times\cos\theta}$ ✓ $=\frac{\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta-\frac{1}{\sqrt{2}}\sin\theta}{\frac{1}{\sqrt{2}}\times\cos\theta}$ ✓ $=\frac{\frac{1}{\sqrt{2}}\cos\theta}{\frac{1}{\sqrt{2}}\times\cos\theta}$ $=1$ ✓	Ken toe soos aangedui Alle akkuraatheids-punte	(5)
2.6	$BC^2=AC^2+AB^2-2AC.AB\cos 20°$ $BC^2=20^2+16^2-2\times 20\times 16\times\cos 20°$ ✓ $BC=\sqrt{54,6}=7,39$m ✓ $D\hat{C}B=70°$ ✓ ... verwisselende hoeke $\frac{DB}{BC}=\tan 70°$ ✓ $DB=7,39\ \tan 70°$ $=20,3$m ✓	1 vir substitusie (A) 1 vir antwoord (A) 1 vir antwoord (A) 1 vir verhouding (A) 1 vir antwoord (KA)	(5)
			[25]

VRAAG 3

3.1 en 3.3



Punte vir stipping (′plotting′):
All 10 punte korrek: 4 punte
7 – 9 punte korrek: 3 punte
4 – 6 punte korrek: 2 punte
2 – 3 punte korrek: 1 punt

(4)

3.3 Lyn van beste passing✓ (1)

3.2

Interne eksamen (x)	Finale eksamen (y)	$x.y$	x^2
32	7	224	1 024
20	11	220	400
27	9	243	729
37	2	74	1 369
32	4	128	1 024
28	7	196	784
41	0	0	1 681
23	10	230	529
33	5	165	1 089
36	3	108	1 296
$\sum x = 309\surd$	$\sum y = 58\surd$	$\sum x.y = 1\,588\surd$	$\sum x^2 = 9\,925\surd$
$\bar{x} = 30{,}9\surd$	$\bar{y} = 5{,}8\surd$		

Ken punte toe soos aangedui is.

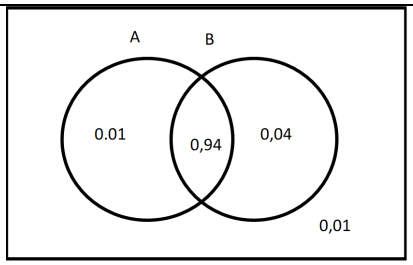
	$b = \frac{n\sum x.y - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$ $= \frac{10 \times 1\,588 - (309)(58)}{10 \times 9\,925 - (309)^2}\surd$ $= \frac{15\,880 - 17\,922}{99\,250 - 95\,481}$ $= \frac{-2\,042}{3\,769}$ $b = -0{,}542\surd$ $a = \bar{y} - b\bar{x}$ $a = 5{,}8 - (-0{,}542)(30{,}9)$ $a = 22{,}54\surd$ $\therefore y = a + bx$ $y = 22{,}486 + (-0{,}542)x\surd$	½ vir substitusie (A) ½ vir b-waarde (A) ½ vir a-waarde (A) ½ vir vergelyking (A)	(5)

Alternatief:

	Interne eksamen (x)	Finale eksamen (y)	$(x - \bar{x})(y - \overline{y})$	$(x - \bar{x})^2$
	32	7	1,32	1,21
	20	11	-56,68	118,81
	27	9	-12,48	15,21
	37	2	-23,18	37,21
	32	4	-1,98	1,21
	28	7	-3,48	8,41
	41	0	-58,58	102,01
	23	10	-33,18	62,41
	33	5	-1,68	4,41
	36	3	-14,28	26,01
	$\sum x = 309$	$\sum y = 58$	$\Sigma(x - \bar{x})(y - \bar{y}) = -204{,}2\checkmark$	$\Sigma(x - \bar{x})^2 = 376{,}9\checkmark$
	$\bar{x} = 30{,}9\surd$	$\bar{y} = 5{,}8\surd$		
	Ken punte toe soos aangedui is.			

	$b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$ $b = \frac{-204,12}{376,9}\surd$ $= 0,54\surd$ $a = \bar{y} + b\bar{x}$ $a = 5,8 - (-0,542)(30,9)$ $a = 22,54\surd$ $y = a + bx$ $y = 22,54 + (-0,542)x\surd$	½ vir substitusie (A) ½ vir b-waarde (A) ½ vir a-waarde (A) ½ vir vergelyking (A)	
3.4	Hoe minder oproepe gemaak word, hoe langer hou die battery.	1 vir antwoord (A)	(2)
3.5	$y = 22,54 + (-0,542)x$✓ $y = 22,54 + (-0,542)(31)$✓ $y = 5,738$✓	Ken punte toe soos aangedui is.	(3)
			[15]

VRAAG 4					
4.1	4.1.1	(a)	$\text{P(man)} = \frac{120}{236}$✓ $= 0,51$	Soos toegeken (A)	(1)
		(b)	$\text{P(vrou en nie gesteriliseer nie)} = \frac{67}{236}$✓ $= 0,28$	Soos toegeken (A)	(1)
	4.1.2		$\text{P(man en gesteriliseer)} = \frac{51}{236}$ $= 0,22$✓ $\neq 0$ Nie wedersyds uitsluitend nie✓	1 vir korrekte berekening (A) 1 vir antwoord (A)	(2)
	4.1.3		$\text{P(man)} \times \text{P(gesteriliseer)}$ $= \frac{120}{236} \times \frac{100}{236}\surd$ $= 0,22$ $\text{P(man en gesteriliseer)} = \frac{51}{236}$✓ $= 0,22$ $\therefore \text{P(man)} \times \text{P(gesteriliseer)}$ $= \text{P(man en gesteriliseer)}\surd$ $\therefore$ onafhanklike gebeurtenisse, korrek tot 2 desimale plekke✓	Alle akkuraatheids-punte, soos aangedui is	(3)

4.2	4.2.1	 ✓✓	2 vir Venndiagram	
		$P(A \cup B) = 0{,}01 + 0{,}94 + 0{,}04$✓ $= 0{,}99$✓ **OF** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$✓ $= 0{,}95 + 0{,}98 - 0{,}94 = 0{,}99$✓	2 akkuraatheidspunte, soos aangedui is	(4)
	4.2.2	$P(A \cup B)'$✓ $= 0{,}01$ of 1%✓ **OF** $P(A \cup B)' = 1 - P(A \cup B)$✓ $= 1 - 0$, $= 0{,}01$✓	2 vir korrekte antwoord op grond van Venndiagram	(2)
	4.2.3	$P(\text{slegs toestel B}) = 0{,}04$✓✓	2 vir korrekte antwoord op grond van Venndiagram	(2)
				[15]

VRAAG 5				
5.1	Prys in 2023 $= P(1+i)^4\sqrt{}$ $= 22{,}50(1 + 0{,}07)^4\sqrt{}$ $=$ R29,493✓			(2)
5.2	Refilwe: $A = P(1 + i \times n)\sqrt{}$ $=$ R6 000$(1 + 0{,}085 \times 5)$✓ $= 6\,000(1{,}425)$ $=$ R8 550✓ Johnny: $A = P(1 + i)^n\sqrt{}$ $A =$ R6 000$(1 + 0{,}02)^{20}$✓ $=$ R8 915,68✓ ∴ Johnny sal die grootste bedrag ontvang.✓		½ vir formule (A) 1 vir substitusie (A) 1 vir antwoord (KA) ½ vir formule (A) 1 vir substitusie (A) 1 vir antwoord (KA) 1 vir antwoord (KA)	(6)
5.3	5.3.1	$A = P(1 - i)^n$✓ $A =$ 120 000$(1 - 0{,}09)^5$✓ $=$ R74 883,86✓	Ken punte toe volgens die regmerkies.	(3)
	5.3.2	$A = P(1 + i)^n$✓ $A =$ 120 000$(1 + 0{,}07)^5$✓ $=$ R168 306,21✓	Ken punte toe volgens die regmerkies.	(3)

5.4	5.4.1	**Bruto inkomste** Inkomste uit salaris $= R36\,000 \times 12$ $= R432\,000$ Jaarlikse bonus $= R\ 45\,000$ Eenmalige aansporingsbonus $= R26\,000$ TOTAAL $= R503\,000$√ **Aftrekkings** Pensioen $= R2\,500 \times 12 = R30\,000$ WVF $= R360 \times 12 = R4\,320$ TOTAAL $= R34\,320$√ ∴ Jaarlikse belasbare inkomste = Bruto inkomste − (Pensioen + WVF) $= R503\,000 - (R30\,000 + R4\,320)$ $= R46\,8680$✓		(2)
	5.4.2	**Normale belasting** $= R100\,263 + \frac{36}{100}(R468\,680 - R423\,300)$ $= R116\,599{,}80$✓ **Minus**: **Kortings en belastingkrediete** Primêr: $R14\,067 + R7\,713 = R\ 21\,780$√ Mediesehulpbelastingkrediet: $R310 \times 12$ $= R\ 3\,720$√ **Totale belasting verskuldig** = Belasbare belasting – (Kortings + Belasting-krediete) $= R116\,599{,}80 - R25\,500$ $= R91\,099{,}80$✓ Totale belasting alreeds betaal = R 95 000 **Aan Barry verskuldig** = Belasting betaal – Belasbare belasting = R95 000 – R91 099,80 $= R3\,900{,}20$✓		(4)
				[20]

TOTAAL:	**100**

Kopiereg voorbehou