

# higher education \& training 

Department:
Higher Education and Training REPUBLIC OF SOUTH AFRICA

## NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS (Second Paper)
NQF LEVEL 4
(10501064)

6 November 2018 (Y-Paper)
13:00-16:00

Nonprogrammable calculators may be used except where indicated otherwise.

This question paper consists of $\mathbf{1 0}$ pages, $\mathbf{1}$ addendum and a formula sheet of $\mathbf{2}$ pages.

## TIME: 3 HOURS

MARKS: 100

## INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. used to determine the answers.
5. Round off answers to THREE decimal places, unless stated otherwise.
6. Diagrams are NOT drawn to scale.
7. Use the ADDENDUM (attached) to answer QUESTION 3.2.1 and QUESTION 3.2.2.
8. Write neatly and legibly.

## QUESTION 1

1.1 In the diagram below, O is the centre of the circle and chord DG is parallel to chord EF. Chords DE, EG and FG are drawn. DÔE $=100^{\circ}$ and $\widehat{F}=110^{\circ}$.


Calculate the size of each of the following angles, giving reasons.

### 1.1.1 $\quad \widehat{\mathrm{G}}_{1}$

1.1.2 $\widehat{\mathrm{E}}_{3}$
1.1.3 GDE
1.1.4 $\widehat{\mathrm{E}}_{2}$
1.2 In the diagram below, CP is a diameter of the circle and TP is a tangent at P . The line TC cuts the circle at B . Chord BQ is parallel to CP . CQ produced meets TP at R. BP is drawn. Let $\widehat{\mathrm{P}}_{1}=y$ and $\widehat{\mathrm{P}}_{2}=x$.

1.2.1 $\quad$ Name two other angles that are equal to $x$.
1.2.2 Name, with reasons, 4 other angles equal to $y$.
1.2.3 Prove that BQRT is a cyclic quadrilateral.
1.3 C is the centre of the circle below. The line AB with equation $x+y+2=0$ passes through the centre of the circle. The equation of the diameter PQ is $y-1=0$.

1.3.1 Determine the coordinates of C .
1.3.2 If P has coordinates $(-8 ; 1)$, determine the equation of the circle.
1.3.3 What is the equation of the tangent to the circle at Q ?

## QUESTION 2

2.1 Consider the trigonometric expression below:

$$
\frac{\cos 2 x}{\cos x+\sin x}
$$

2.1.1 Prove that

$$
\begin{equation*}
\frac{\cos 2 x}{\cos x+\sin x}=\cos x-\sin x \tag{3}
\end{equation*}
$$

2.1.2 Show that the equation

$$
\cos x\left(\frac{\cos 2 x}{\cos x+\sin x}\right)=\frac{1}{2}
$$

$$
\begin{equation*}
\text { can be reduced to } \cos 2 x=\sin 2 x \tag{3}
\end{equation*}
$$

2.1.3 Hence or otherwise, solve for $\theta$ in the following equation if $0^{0} \leq \theta \leq 360^{\circ}$ :

$$
\begin{equation*}
\cos \theta\left(\frac{\cos 2 \theta}{\cos \theta+\sin \theta}\right)=\frac{1}{2} \tag{3}
\end{equation*}
$$

2.2 If $\sin 76^{\circ}=x$ and $\cos 76^{\circ}=y$ show that $x^{2}-y^{2}=\sin 62^{\circ}$.
2.3 If $\sin \theta=-\frac{3}{5}$, calculate, without the use of a calculator, the numerical value of

$$
\begin{equation*}
\frac{\sin 2 \theta}{\cos \theta} \tag{2}
\end{equation*}
$$

2.4 Without the use of a calculator, determine the value of:

$$
\begin{equation*}
\sin 85^{\circ} \times \cos 35^{\circ}+\cos 85^{\circ} \times \sin 35^{\circ} \tag{3}
\end{equation*}
$$

2.5 In the figure below, a crane ABC carries a load $\mathrm{D} . \mathrm{AB}$ and CD are perpendicular to the ground XY . The height of the crane $(\mathrm{AB})$ is $8,3 \mathrm{~m}$. At a given time the angle between the mast (tower) of the crane ( AB ) and its jib (represented by BC) is $120,3^{\circ}$. The angle of elevation of the end of the jib (C) from the bottom (A) of the crane is $75,8^{\circ}$.


Mast (tower)

## Calculate:

2.5.1 The length BC, correct to one decimal place
2.5.2 The height ( $h$ ) of the load (D) above the ground if the length of CD is 5,2m.

## QUESTION 3

3.1 The marks of eight learners in a test for which the maximum mark is 10 were as follows:

| 7 | 4 | 9 | 4 | 9 | 5 | 4 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Calculate the standard deviation of the given data.
3.2 A government medical clinic carried out a survey to study the effect of a drug on the reaction time of patients of different ages.

The results are shown in the table below:

| Ages in years $(x)$ | 15 | 17 | 18 | 20 | 21 | 24 | 25 | 26 | 29 | 31 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Reaction time in seconds $(y)$ | 21 | 29 | 33 | 32 | 40 | 45 | 65 | 52 | 62 | 66 |

3.2.1 Draw a scatter plot to represent the given information. (Use the ADDENDUM.)
3.2.2 Draw the line of best fit on the scatter plot. (Use the ADDENDUM.)
3.2.3 From the scatter plot write down the age of a patient who is an outlier.
3.2.4 Calculate the equation of the least-square regression line for the given data. (Round off to 1 decimal place)
3.2.5 Predict the reaction time of a 28 year old patient, using the least-squares regression equation.

## QUESTION 4

4.1 Complete the following (write the answers in your answer book):
4.1.1 If $A$ and $B$ are mutually exclusive events, then $P(A$ and $B)=\cdots$
4.1.2 If $A$ and $B$ are independents events, then $P(A$ and $B)=\cdots$
4.2 Three fair coins are tossed.
4.2.1 Represent all possible outcomes using a tree diagram. Show clearly the probabilities on each branch.
4.2.2 Calculate the probability of obtaining precisely two heads and a tail in any order.
4.2.3 What is the probability of obtaining at least one tail.
4.3 It is estimated that $60 \%$ of all households in South Africa have post-paid contracts with internet service providers, while $80 \%$ of all households use prepaid data bundles. Also, $50 \%$ of households have both post-paid contracts and prepaid data bundles.
4.3.1 Use a Venn diagram to represent the given information.
4.3.2 What is the probability that a randomly selected household in South Africa does not use prepaid data bundles?
4.3.3 What is the probability that a randomly chosen household in South Africa will have a post-paid contract or will use prepaid data bundles?

## QUESTION 5

5.1 Hlengiwe invests R25 000 in a savings account that offers interest at $6,25 \%$ p.a. compounded quarterly.
How much will Hlengiwe have after 8,5 years?
5.2 Harry wants to borrow money from a bank to purchase a second-hand car that costs R150 000. He plans to repay the full amount over a period of four years in equal monthly instalments. The bank charges simple interest at the rate of $10,5 \%$ p.a. How much will Harry pay the bank in equal monthly instalments over four years?
5.3 Given below is the individual rate table obtained from the South African Revenue Services (SARS).

Individuals and trusts

| Income tax rates for natural persons and special trusts |  |
| :--- | :--- |
| Year of assessment ending 28 February $\mathbf{2 0 1 8}$ |  |
| Taxable income (R) | Taxable rates |
|  |  |
| $0-189880$ | $18 \%$ of each R1 |
| $189881-296540$ | $34178+26 \%$ of the amount above 189880 |
| $296541-410460$ | $61910+31 \%$ of the amount above 296540 |
| $410461-555600$ | $97225+36 \%$ of the amount above 410460 |
| $555601-708310$ | $149475+39 \%$ of the amount above 555600 |
| $708311-1500000$ | $209032+41 \%$ of the amount above 708310 |
| 1500001 and above | $533625+45 \%$ of the amount above 1500000 |

Natural persons

| Tax thresholds | $\mathbf{2 0 1 7 / 1 8}$ |
| :--- | :--- |
|  | R |
| Below 65 years of age | 75750 |
| Aged 65 and below 75 | 117300 |
| Aged 75 and over | 131150 |


| Tax rebates | $\mathbf{2 0 1 7 / 1 8}$ |
| :--- | :--- |
|  | $\mathbf{R}$ |
|  | 13635 |
| Primary - all natural persons | 7479 |
| Secondary - persons aged 65 and below 75 | 2493 |
| Tertiary - persons aged 75 and above |  |


| Medical aid tax credits |  |
| :--- | :--- |
| Monthly amounts deductible from tax payable | $\mathbf{2 0 1 7 / 1 8}$ |
| Main member | R303 |
| Main member with one dependent | R606 |
| Main member with two dependents | R810 |
| Each additional dependant qualifies for a further credit of R204 per month. |  |

[Source: South African Revenue Service (SARS) 2017/2018]

Given below are the monthly earnings and deductions of Patrick Khumalo, a 55-year-old male with two dependents, for the 2017/2018 tax year.

| Gross monthly salary: | R48 000 |
| :--- | :--- |
| 13th cheque: | R56 000 |
| Performance bonus: | R80 000 |

Mr Khumalo also has a business from which he receives a monthly income of R20 000.

His pension fund contribution amounts to R4 300 per month (which is not taxable).
Mr Khumalo also makes monthly contributions towards medical aid for himself and his two dependents.

His total annual tax payment to SARS was R250 000.
5.3.1 Calculate Mr Khumalo's total taxable income for the tax year.
5.3.2 Determine his total tax payable after taking into consideration applicable rebates and tax credits
5.3.3 Calculate the amount due by or due to Mr Khumalo
5.4 Ms Lee's car was bought five years ago for R240000. It is now worth R160650.

If the car depreciated at $x \%$ p.a. compounded monthly, calculate $x$.

## ADDENDUM



QUESTION 3.2.1 and 3.2.2


## FORMULAE SHEET

NOTE: Some of the formulae in this formulae sheet may not be generally true, but may hold under certain conditions. The candidate is required to select the appropriate formulae for a given situation.

1. $y=m x+c$
2. $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
3. $y-y_{1}=m\left(x-x_{1}\right)$
4. $(x-a)^{2}+(y-b)^{2}=r^{2}$
5. $M\left(\frac{x_{1}+x_{2}}{2} ; \frac{y_{1}+y_{2}}{2}\right)$
6. $m=\tan \theta$
7. $\quad$ Distance $=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
8. $\frac{\sin \theta}{\cos \theta}=\tan \theta$
9. $\sin ^{2} \theta+\cos ^{2} \theta=1$
10. $\sin (\alpha \pm \beta)=\sin \alpha \cos \beta \pm \sin \beta \cos \alpha$
11. $\cos (\alpha \pm \beta)=\cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
12. $\sin 2 \theta=2 \sin \theta \cos \theta$
13. $\cos 2 \theta=\cos ^{2} \theta-\sin ^{2} \theta$
14. $\cos 2 \theta=1-2 \sin ^{2} \theta$
15. $\cos 2 \theta=2 \cos ^{2} \theta-1$
16. $\frac{\sin \hat{A}}{a}=\frac{\sin \hat{B}}{b}=\frac{\sin \hat{C}}{c}$
17. $a^{2}=b^{2}+c^{2}-2 b c \cos \hat{A}$
18. $\bar{x}=\frac{\sum_{i=1}^{n} x_{i}}{n}$
19. $\quad$ Variance $=s^{2}=\frac{\sum\left(x_{i}-\bar{x}\right)^{2}}{n}$
20. $\quad$ Standard deviation $=\sqrt{\frac{\sum\left(x_{i}-\bar{x}\right)^{2}}{n}}$
21. $\hat{y}=a+b x$
22. $b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^{2}} \quad$ or $\quad b=\frac{n \sum x y-\sum x \sum y}{n \sum x^{2}-\left(\sum x\right)^{2}}$
23. $a=\bar{y}-b \bar{x}$
24. $I=A_{0} \times \frac{r}{100} \times t$
or $\quad I=\frac{P r t}{100}$
or $\quad A=P(1+$ in $)$
25. $A_{t}=A_{0}\left(1+\frac{r}{100 \times m}\right)^{t \times m}$
or
$A_{t}=P\left(1+\frac{r}{100}\right)^{n} \quad$ or $\quad A=P(1+i)^{n}$
26. $\quad A_{t}=A_{o}\left(1-\frac{r}{100}\right)^{t}$
or $\quad A=P(1-$ in $)$
or
$A=P(1-i)^{n}$
27. $i=\frac{r}{100}$
28. $\quad \mathrm{P}(\mathrm{A})=\frac{n(\mathrm{~A})}{n(\mathrm{~S})}$
29. $\quad \mathrm{P}(\mathrm{A}$ and B$)=\mathrm{P}(\mathrm{A}) \times \mathrm{P}(\mathrm{B})$
30. $\quad \mathrm{P}(\mathrm{A}$ or B$)=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})$
31. $P(A$ or $B)=P(A)+P(B)-P(A$ and $B)$
32. $\quad \mathrm{P}(\mathrm{A} / \mathrm{B})=\frac{\mathrm{P}(\mathrm{A} \text { and } \mathrm{B})}{\mathrm{P}(\mathrm{B})}$
