

## higher education \& training

Department:
Higher Education and Training REPUBLIC OF SOUTH AFRICA

## NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS (Second Paper) NQF LEVEL 4
(10501064)

23 February 2018 (Y-Paper)
13:00-16:00
Candidates may use nonprogrammable scientific calculators.

This question paper consists of 8 pages, 1 graph paper and ONE formula sheet of $\mathbf{2}$ pages.

## TIME: 3 HOURS

MARKS: 100

## INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
5. Answers should be rounded off to TWO decimals, unless stated otherwise.
6. Diagrams are NOT drawn to scale.
7. Work neatly.

## QUESTION 1

1.1 In the diagram below, ABCD is a quadrilateral with diagonal AC . $\mathrm{Also}, \mathrm{AB}=\mathrm{AC}$ and $\widehat{\mathrm{D}}=50^{\circ}$. Let $\mathrm{B} \widehat{\mathrm{A}} \mathrm{C}=2 \mathrm{y}+20^{\circ}, \mathrm{D} \widehat{\mathrm{A}} \mathrm{C}=2 x-60^{\circ}, \mathrm{A} \widehat{\mathrm{C}}=\mathrm{y}$ and $\widehat{\mathrm{B}}=x-30^{\circ}$

1.1.1 $\quad$ Express AĈB in terms of $x$.
1.1.2 $\quad$ Solve for $x$ and $y$.
1.1.3 Is ABCD a cyclic quadrilateral? Give a reason.
1.2 In the diagram below, $\mathrm{E}, \mathrm{D}$ and P are points on the circle. TPQ is a straight line and $\widehat{\mathrm{E}}=30^{\circ}$. The sizes of the angles marked $a, b$ and $c$ are in the ratio 1:2:3.

1.2.1 $\quad$ Why is $a+b+c=180^{\circ}$ ?
1.2.2 Show that $c=90^{\circ}$.
1.2.3 Prove that TPQ a tangent to the circle at $P$.
1.3 In the diagram below, B is the midpoint of chord AC of the circle with centre O . OE is the radius passing through $\mathrm{B}, \mathrm{AE}$ is produced to meet tangent CD at D such that $\mathrm{AD} \perp \mathrm{CD}$. C and E are joined.

1.3.1 Why is $\mathrm{B}_{3}=90^{\circ}$ ?
1.3.2 Prove that BCDE is a cyclic quadrilateral.
1.3.3 Prove that $\widehat{\mathrm{E}}_{3}=2 \widehat{\mathrm{~A}}$.
1.4 In the figure below, C is the centre of the circle and AB is a diameter. The coordinates of $A$ and $B$ are $(8 ; 5)$ and $(-2 ; 0)$ respectively. DE is the tangent to the circle at A.

1.4.1 Show by calculation that the coordinates of C are $\left(3 ; \frac{5}{2}\right)$.
1.4.2 Determine the equation of the circle.
1.4.3 Determine the equation of the tangent passing through A.

## QUESTION 2

2.1 Without the use of a calculator, determine the value of the following expression:

$$
\begin{equation*}
\sin 70^{\circ} \cos 40^{\circ}-\cos 70^{\circ} \sin 40^{\circ} \tag{2}
\end{equation*}
$$

2.2 Make use of the formulae for the expansion of $\cos (x-y)$, and without using a calculator, prove that:

$$
\begin{equation*}
\cos 15^{\circ}=\frac{\sqrt{3}+1}{2 \sqrt{2}} \tag{5}
\end{equation*}
$$

2.3 If $\tan \theta=-\frac{3}{4}$ and $\theta \in\left[270^{\circ} ; 360^{\circ}\right]$, determine the following, without using a calculator, leaving your answer in surd form where applicable.
2.3.1 $2 \cos ^{2} \theta-1$
2.3.2 $\sin 2 \theta$
2.3.3 $\tan 2 \theta$
2.4 Prove the following trigonometric identity:

$$
\begin{equation*}
\frac{\cos 2 x-\cos x}{\sin 2 x+\sin x}=\frac{\cos x-1}{\sin x} \tag{5}
\end{equation*}
$$

2.5 Solve for $x$ (correct to two decimal places if necessary) in the following equation if $x \in\left[0^{\circ} ; 360^{\circ}\right]$ :

$$
\begin{equation*}
3 \sin x=1+\cos 2 x \tag{4}
\end{equation*}
$$

2.6 In figure below, CD represents a vertical tower at an army camp, with height 55 m . Sipho and Thandi stand apart at points $S$ and $T$ respectively. Points $S$ and $T$ are in the same horizontal plane as C , the foot of the tower. The angle of elevation from S to D is $37^{\circ}$, the angle of elevation from T to D is $36^{\circ}$ and $\mathrm{S} \widehat{\mathrm{C}}=110^{\circ}$.


Calculate ST, the distance between Sipho and Thandi.

## QUESTION 3

3.1 The table below shows the marks obtained by 7 learners in the final chemistry examinations.

| Chemistry <br> results (\%) | 36 | 55 | 60 | 65 | 75 | 70 | 80 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Calculate the standard deviation of these scores.
3.2 The table below shows the marks (\%) scored by a group of NCV level 4 mathematics students in the preparatory examination and subsequently in the final examination.

| Preparatory <br> Examination | 80 | 68 | 94 | 72 | 74 | 83 | 56 | 68 | 65 | 75 | 88 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Final <br> Examination | 72 | 71 | 96 | 77 | 82 | 72 | 58 | 83 | 78 | 80 | 92 |

3.2.1 Draw a scatter plot of the data above on the graph sheet provided.
3.2.2 Using the information above, find the sample regression equation using the method of least squares.
3.2.3 What will the predicted final examination mark be for a learner who scored $75 \%$ in the preparatory examination?

## QUESTION 4

4.1 In a survey at a TVET college 1360 hockey players were asked if they had ever broken a tooth while playing hockey. The results were as follows:

|  | Broken a tooth <br> (B) | Not broken a <br> tooth (not B) | Total |
| :--- | :---: | :---: | :---: |
| Male (M) | 420 | $\mathbf{P}$ | 720 |
| Female (F) | A | $\mathbf{C}$ | D |
| Total | 810 | 550 | 1360 |

4.1.1 Calculate the values of $\mathbf{A}, \mathbf{P}, \mathbf{C}$ and $\mathbf{D}$.
4.1.2 Calculate the probability of randomly choosing a female hockey player from the survey who had NOT broken a tooth.
4.1.3 Are the events, being a female hockey player and not having broken a tooth independent? Show your calculation, correct to ONE decimal place, to motivate the answer.
4.1.4 Calculate the probability that a player chosen at random is male or has a
broken a tooth.
4.2 Three dart board players, Alyss, Busi and Calvin take part in a competition to hit the bull's eye. The probability of Alyss hitting the bull's eye is $\frac{1}{2}$, that is, $\mathrm{P}(\mathrm{A})=\frac{1}{2}$. Also, the probability of Busi and Calvin hitting the bull's eye is given by $P(B)=\frac{1}{3}$ and $P(C)=\frac{1}{4}$ respectively.

4.2.1 What is the probability that all three players hit the bull's eye?
4.2.2 What is the probability that at least one of the players hits the bull's eye?
4.2.3 What is the probability that precisely one of the players hits a bull's eye?

## QUESTION 5

5.1 Zwandile invested a certain sum of money. Her return after 22 years was R36 000.

Calculate the amount that she initially invested if the interest rate was $7 \%$ per annum compounded annually.
5.2 When Jose lost his job on his $55^{\text {th }}$ birthday, he owed the bank R400 000 for a home loan he had taken. He could not afford to make any further payments until he turned 60. The bank charged an interest rate capped at $8 \%$ per annum compounded monthly. How much did Jose owe the bank on his $60^{\text {th }}$ birthday?
5.3 Given below is the individual tax rate table obtained from the South African Revenue Services (SARS). Use the table to answer the questions which follow.

RATES OF TAX FOR INDIVIDUALS
2018 tax year (1 March 2017-28 February 2018)

| Taxable income (R) | Rates of tax (R) |
| :--- | :--- |
| $0-189880$ | $18 \%$ of taxable income |
| $189881-296540$ | $34178+26 \%$ of taxable income above 189880 |
| $296541-410460$ | $61910+31 \%$ of taxable income above 296540 |
| $410461-555600$ | $97225+36 \%$ of taxable income above 410460 |
| $555601-708310$ | $149475+39 \%$ of taxable income above 555600 |
| $708311-1500000$ | $209032+41 \%$ of taxable income above 708310 |
| 1500001 and above | $533625+45 \%$ of taxable income above 1500 000 |
| Tax Rebates for 2018 (1 March 2017 - 28 February 2018) |  |
| Primary R13 635 <br> Secondary (65 and older) R7 479 <br> Tertiary (75 and older) R2 493 |  |

Tax Thresholds for 2018 (1 March 2017-28 February 2018)

| Under 65 | R75 750 |
| :--- | :--- |
| 65 and older | R117 300 |
| 75 and older | R131 150 |

5.3.1 Mrs Khuzwayo is a retired 75 year old. Her only regular income is a pension of R15 300 per month. Determine whether she is liable for income tax. Show calculations to support your answer.
5.3.2 Calculate the total tax rebates that Mrs Khuzwayo is entitled to.
5.3.3 Based on 5.3.1 and 5.3.2, determine the annual tax that Mrs Khuzwayo is expected to pay.
5.3.4 During the year Mrs Khuzwayo cashed in her investment which paid out R150 000. She gifted this to her unemployed daughter. However, SARS charges a tax of $20 \%$ of donations after exempting the first R100 000. Calculate the donations tax that Mrs Khuzwayo will have to pay.
5.3.5 Mrs Khuzwayo pays PAYE tax of R1500 per month. How much will she be owing the SARS at the end of the tax year?

TOTAL:

Question 3.2.1

DEPARTMENT OF HIGHER EDUCATION AND TRAINING

## GRAPH PAPER • GRAFIEKPAPIER

(Return this sheet with the other answers)
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Examination number
Eksamennommer


## FORMULAE SHEET

NOTE: Some of the formulae in this formulae sheet may not be generally true, but may hold under certain conditions. The candidate is required to select the appropriate formulae for a given situation.

1. $y=m x+c$
2. $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
3. $y-y_{1}=m\left(x-x_{1}\right)$
4. $(x-a)^{2}+(y-b)^{2}=r^{2}$
5. $M\left(\frac{x_{1}+x_{2}}{2} ; \frac{y_{1}+y_{2}}{2}\right)$
6. $m=\tan \theta$
7. $\quad$ Distance $=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
8. $\frac{\sin \theta}{\cos \theta}=\tan \theta$
9. $\sin ^{2} \theta+\cos ^{2} \theta=1$
10. $\sin (\alpha \pm \beta)=\sin \alpha \cos \beta \pm \sin \beta \cos \alpha$
11. $\cos (\alpha \pm \beta)=\cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
12. $\sin 2 \theta=2 \sin \theta \cos \theta$
13. $\cos 2 \theta=\cos ^{2} \theta-\sin ^{2} \theta$
14. $\cos 2 \theta=1-2 \sin ^{2} \theta$
15. $\cos 2 \theta=2 \cos ^{2} \theta-1$
16. $\frac{\sin \hat{A}}{a}=\frac{\sin \hat{B}}{b}=\frac{\sin \hat{C}}{c}$
17. $a^{2}=b^{2}+c^{2}-2 b c \cos \hat{A}$
18. $\bar{x}=\frac{\sum_{i=1}^{n} x_{i}}{n}$
19. $\quad$ Variance $=s^{2}=\frac{\sum\left(x_{i}-\bar{x}\right)^{2}}{n}$
20. $\quad$ Standard deviation $=\sqrt{\frac{\sum\left(x_{i}-\bar{x}\right)^{2}}{n}}$
21. $\hat{y}=a+b x$
22. $b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^{2}} \quad$ or $\quad b=\frac{n \sum x y-\sum x \sum y}{n \sum x^{2}-\left(\sum x\right)^{2}}$
23. $a=\bar{y}-b \bar{x}$
24. $I=A_{0} \times \frac{r}{100} \times t$
or $\quad I=\frac{\operatorname{Prt}}{100}$
or $\quad A=P(1+i n)$
25. $A_{t}=A_{0}\left(1+\frac{r}{100 \times m}\right)^{t \times m} \quad$ or
$A_{t}=P\left(1+\frac{r}{100}\right)^{n}$ or $A=P(1+i)^{n}$
26. $\quad A_{t}=A_{o}\left(1-\frac{r}{100}\right)^{t} \quad$ or $\quad A=P(1-$ in $) \quad$ or $\quad A=P(1-i)^{n}$
27. $i=\frac{r}{100}$
28. $\quad \mathrm{P}(\mathrm{A})=\frac{n(\mathrm{~A})}{n(\mathrm{~S})}$
29. $\quad \mathrm{P}(\mathrm{A}$ and B$)=\mathrm{P}(\mathrm{A}) \times \mathrm{P}(\mathrm{B})$
30. $P(A$ or $B)=P(A)+P(B)$
31. $P(A$ or $B)=P(A)+P(B)-P(A$ and $B)$
32. $P(A / B)=\frac{P(A \text { and } B)}{P(B)}$
