higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First paper)
NQF LEVEL 4

(10501064)

16 November 2020 (X-paper)
09:00–12:00

Nonprogrammable scientific calculators may be used.

This question paper consists of 11 pages, 2 addenda and a formula sheet of 2 pages.

009Q1N2016

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.

2. Read all the questions carefully.

3. Number the answers according to the numbering system used in this question paper.

4. Leave at least THREE lines after each question.

5. Start each section on a new page.

6. Diagrams are not drawn to scale.

7. Where necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

8. Work neatly.

QUESTION 1

1.1 Various options are given as possible answers to the following questions. Choose the correct answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5) in the ANSWER BOOK.

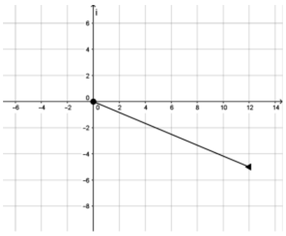
1.1.1 If $z = a + bi$ where a and b are real numbers and $i = \sqrt{-1}$, then the product $z \times \bar{\bar{z}}$ will always be ...

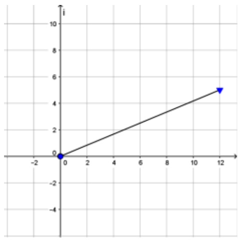
A undefined.
B an irrational number.
C a real number.
D an imaginary number.

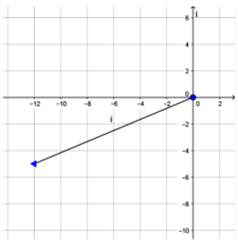
1.1.2 Given $i = \sqrt{-1}$. Then $\frac{1}{i} = \cdots$

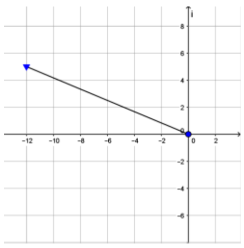
A 1
B i
C $-i$
D $1-i$

1.1.3 $12-5i$ represented on an Argand diagram corresponds with …

A


B


C


D


1.1.4 The complex number $z = -3-4i$ in polar form is …

A $5cis126,870°$
B $5cis233,130°$
C $5cis53,130°$
D $5cis306,870°$

1.1.5 Solve for x and y if $12+6i=-24x+12yi$.

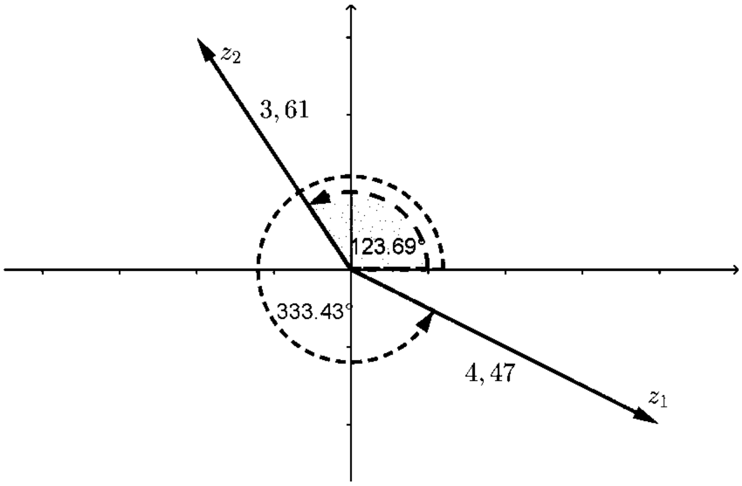
A $x=-2$ and y=2
B $x=-12$ and $y=6$
C $x=-2$ and $y=2i$
D $x=-\frac{1}{2}$ and $y=\frac{1}{2}$

(5 × 1) (5)

1.2 The Argand diagram represents two complex numbers in the form $r\text{cis }\theta$.

Use the information on the diagram below to answer questions 1.2.1 to 1.2.3.

Various options are given as possible answers. Choose the correct answer and write only the letter (A–D) next to the question number (1.2.1–1.2.3) in the ANSWER BOOK.



GRAPH 1

1.2.1 The product of z_1 and z_2 in polar form, with positive argument and $0° \leq \theta \leq 360°$, is equal to …

A $16{,}14cis\ 209{,}74$
B $16{,}14cis\ 97{,}12^0$
C $8{,}08cis\ 97{,}12^0$
D $16{,}14cis\ 457{,}12^0$

1.2.2 Apply De Moivre's theorem and simplify $(z_1)^4$ to polar form with positive argument where $0° \leq \theta \leq 360°$. The answer is equal to …

A $399{,}24cis\ 253{,}72^0$

B $17{,}88cis\ 253{,}72^0$

C $17{,}88cis\ 333{,}43^0$

D $399{,}24cis\ 1333{,}72^0$

1.2.3 $\frac{z_1}{z_2}$ simplified and in polar form with positive argument where $0° \leq \theta \leq 360°$ is equal to …

A $-0{,}86cis - 209{,}74^0$

B $-0{,}86cis150{,}26^0$

C $1{,}24cis209{,}74^0$

D $-0{,}8cis97{,}12^0$

(3 × 2) (6)

1.3 Simplify the following complex expression and write the answer in the form $a + bi$.

$$\frac{1+i}{i} - \frac{3}{3-i}$$

(3)

1.4 Solve for x and y in the following identity and express any non-real solutions in the form $a + bi$.

$$yi^3 + x^2 + 2i^5 = yxi^4 - 20i^8 - 6i$$

(5)

[19]

QUESTION 2

2.1 Given $g(x) = -2x^3 - 3x^2 + kx + 20$

When $-2x^3 - 3x^2 + kx + 20$ is divided by $x + 4$ the remainder is -56.

Determine the value of k by applying the remainder theorem. (2)

2.2 Given: $2x^3 - x^2 - 16x + 15$

2.2.1 Use the factor theorem to show that $x + 3$ is a factor of $2x^3 - x^2 - 16x + 15$ (1)

2.2.2 Hence, factorise $2x^3 - x^2 - 16x + 15$ completely. (4)

2.3 $f(x) = 2x^3 - 5x^2 + ax + 13b$ and are third-degree polynomials which intersect at the point $(2;60)$.

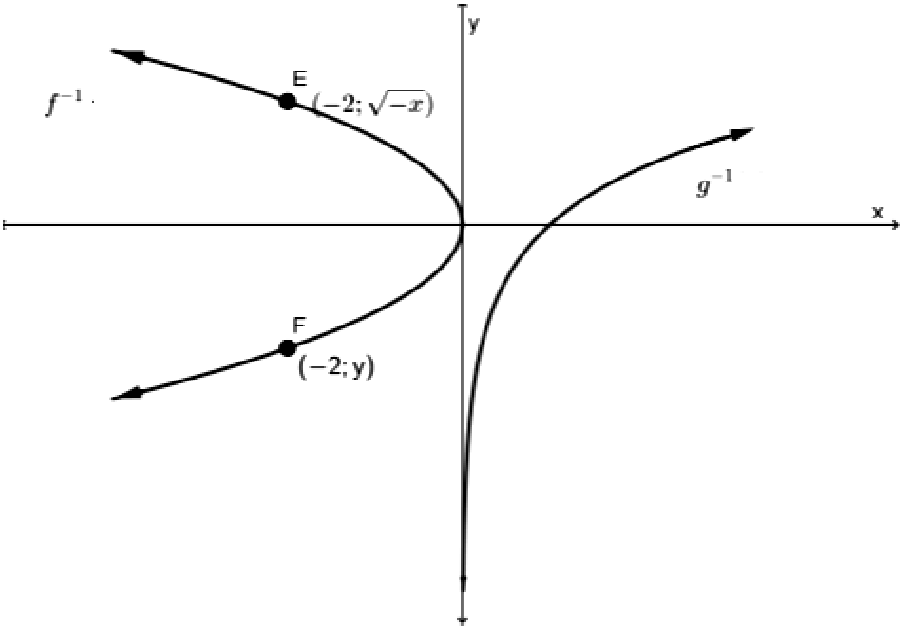
Determine the values of a and b. (5)

2.4 Given $f(x) = \frac{3}{2}x + 3$ where $x \epsilon [-4; 2)$ and $x \in \mathbb{R}$

Use ADDENDUM A and draw a neat graph of $f(x)$ and $f^{-1}(x)$ on the same system of axes, clearly showing the line about which f^{-1} reflects $f(x)$ and label it. Show the x intercepts and y intercepts clearly and indicate the nature of endpoints that correspond with the given domain of $f(x)$. (6)

2.5 Study the graph below and answer the questions. The graphs of the INVERSES of f and g are drawn where

$f(x) = -x^2$ and
$g(x) = 3^x$.



2.5.1 Determine the value of y at the point $\text{F}(-2; y)$ on the graph f^{-1} (1)

2.5.2 Write down the range of f. (1)

2.5.3 Determine the equation of the inverse of f. (1)

2.5.4 The following statements are FALSE. Give ONE reason to explain why.

(a) Both f^{-1} and g^{-1} have asymptotes at $x = 0$. (1)

(b) The inverse of f is a function. (1)

(c) g^{-1} is decreasing in the interval $x \in \{0 < x < \infty\}$ where $x \in \mathbb{R}$. (1)

(d) The range of g is $y \in \{-\infty < y < \infty\}$ where $y \in \mathbb{R}$. (1)

[25]

QUESTION 3

3.1 Given the function $f(x)=-\frac{3}{x}$, determine the derivative of f by applying first principles. (3)

3.2 Use differentiation rules to determine the derivatives of each of the following, leaving your answer with positive exponents and in surd form where applicable:

3.2.1 $f(x)=3x^2+2\ln x+4x+5$ (2)

3.2.2 $f(x)=\sqrt[3]{x^2}+2e^{3x}-\tan 2x-\operatorname{cosec} 3x$ (4)

3.2.3 $y=\frac{x^3+3}{x^2-3x}$ (3)

3.2.4 $y=\left(x^3-5x\right)^5$ (2)

3.3 Given $f(x)=2x^3-x^2-16x+15$.

3.3.1 Determine the y-intercept of $f(x)$. (1)

3.3.2 $f(x)=2x^3-x^2-16x+15$ has turning points at $x=1{,}81$ and $x=-1{,}47$.

Determine at which x values the minimum and maximum turning points occur by applying the second derivative rule. (3)

3.3.3 Will the function f increase or decrease between $x=1{,}81$ and $x=-1{,}47$? (1)

3.3.4 Verify your answer to QUESTION 3.3.3 by determining the gradient of any $x-$value within the interval $-1{,}47<x<1{,}81$ where $x\in R$. (2)

3.4 Given: $f(x)=x^3+mx^2+nx-4$ and $f(x)$ has a turning point at $x=3$.

If $f(x)$ is divided by $x-2$, the remainder is -2 .

Determine the values of m and n in $f(x)$. (4)

[25]

QUESTION 4

4.1 Determine the following integrals and simplify the answers. Leave the answer with positive exponents and in surd form where applicable.

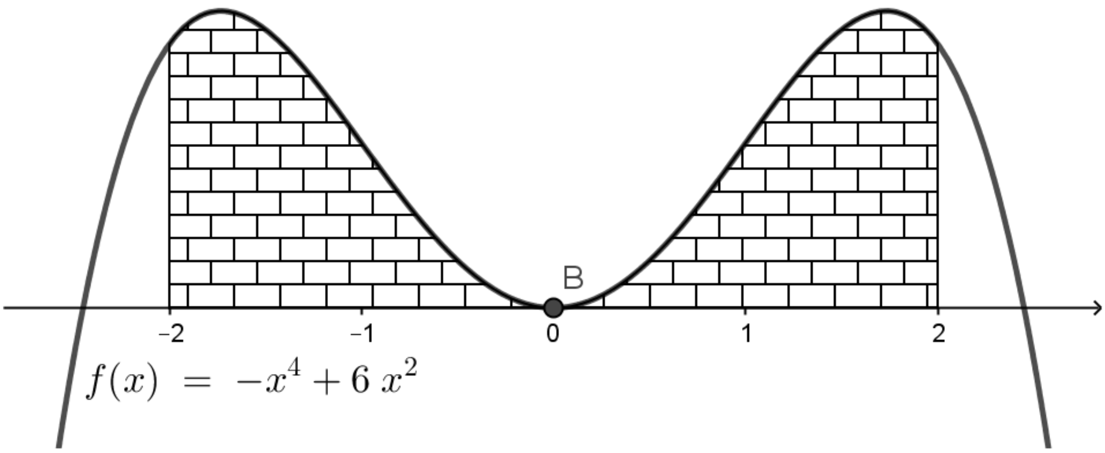
4.1.1 $\int\left(\frac{x}{2}+\frac{2}{3}x^{-3}+5\cos 5x+4x\right)dx$ (3)

4.1.2 $\int\left(\frac{1}{2x}+\frac{2}{\sqrt[3]{x^2}}-e^{3x}+8\sec^2(4x)+5\right)dx$ (5)

4.2 The velocity equation of an object is given as $v=-3t^2+5t+8$ metres per second, where t is time in seconds.

Determine the distance travelled between $t=0$ and $t=2$. Apply integration to solve the problem. Hint: The area below the velocity-time graph gives distance. (3)

4.3 A skateboard ramp has to be built using the function $f(x)=-x^4+6x^2$ for the design.



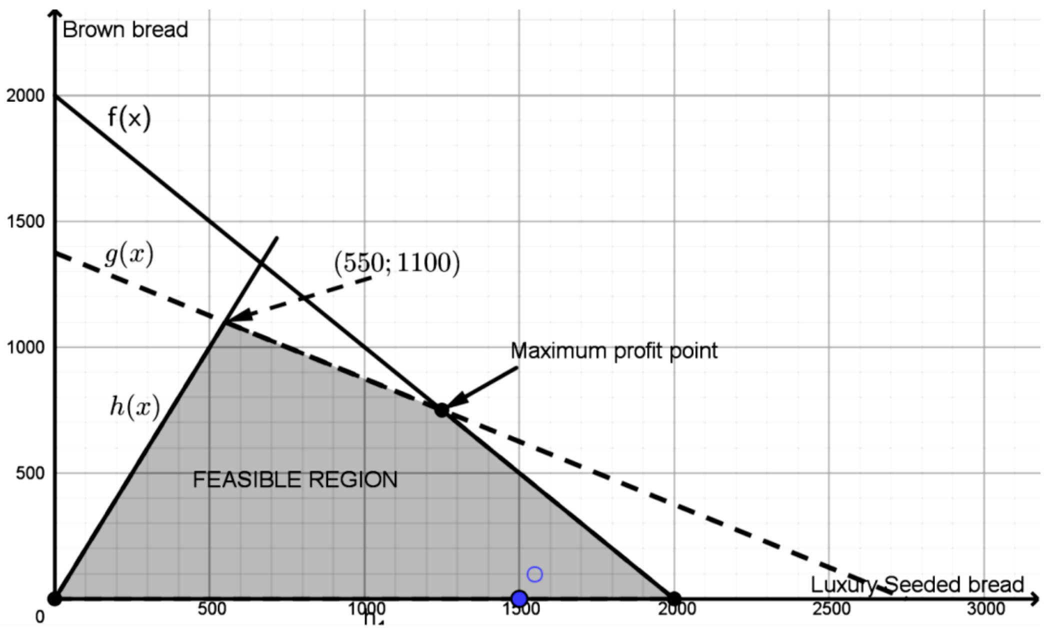
4.3.1 Determine the area of the bricked section enclosed by the curve of $f(x)$, the lines $x=-2$ and $x=2$, and the x axis. (3)

4.3.2 Use calculus methods to determine the maximum gradient of the ramp. (2)

[16]

QUESTION 5

5.1 A shop sells two types of bread loaves, normal brown and luxury seeded bread. The graph below shows the feasible region which applies to the given constraints. The luxury seeded bread loaves are at the most twice as popular as the normal brown bread. The function $h(x)$ shows the preferred ratio of luxury seeded bread to normal brown bread. Let x be the number of normal brown bread loaves and y the number of luxury seeded bread loaves. x and y cannot be negative, hence $x \geq 0$ and $y \geq 0$.



5.1.1 The function $h(x)$ shows the preferred ratio of luxury seeded bread to normal brown bread.

Determine the equation of the line $h(x)$. (2)

5.1.2 The equations of the other two lines are given as $f(x) = 2000 - x$, $g(x) = 1375 - \frac{1}{2}x$. Write down the three constraints that fully describe the feasible region. (3)

5.2 A snack shop sells two different platters, A and B. Each platter contains biltong and nuts. The following are the conditions that apply to each type of platter.

- Platter A contains 200 g of biltong and platter B contains 400 g of biltong.
- The shop stocks at most 8000g of biltong.
- Platter A contains 400 g of nuts and Platter B contains 250 g of nuts.
- The shop always has less than 7 750g of nuts in stock.
- The minimum number of platters of type B must be 5.

Let the number of A type platters be *x* and the number of B type platters be *y*.

5.2.1 Two of the constraints are $x \geq 0$ and $y \geq 0$.

Model the other constraints with respect to the above information in terms of x and y. (3)

5.2.2 a) Using scale 1 unit:5 platters, represent the inequalities graphically on the attached ADDENDUM B. (3)

b) Shade in the feasible region on the graph. (1)

c) Platter A yields a profit of R140 and platter B yields a profit of R160.

Write down the equation that represents the total profit P. (1)

d) Draw a search line on the graph to find the maximum profit. (1)

e) Determine the maximum profit. (1)

[15]

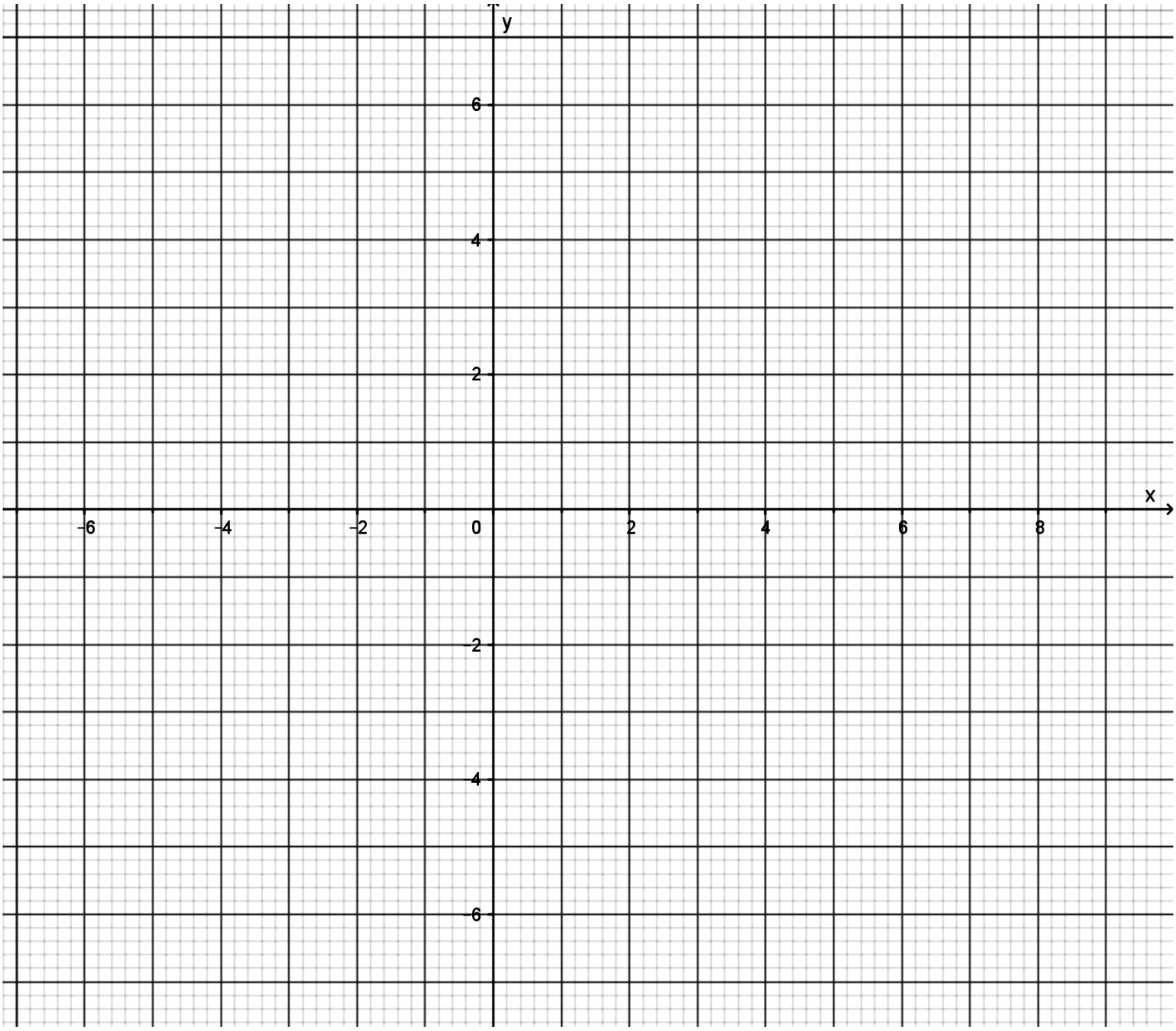
TOTAL: 100

MATHEMATICS L4 P1

ADDENDUM A

EXAMINATION NUMBER: | | | | | | | | | | | | | |

QUESTION 2.4



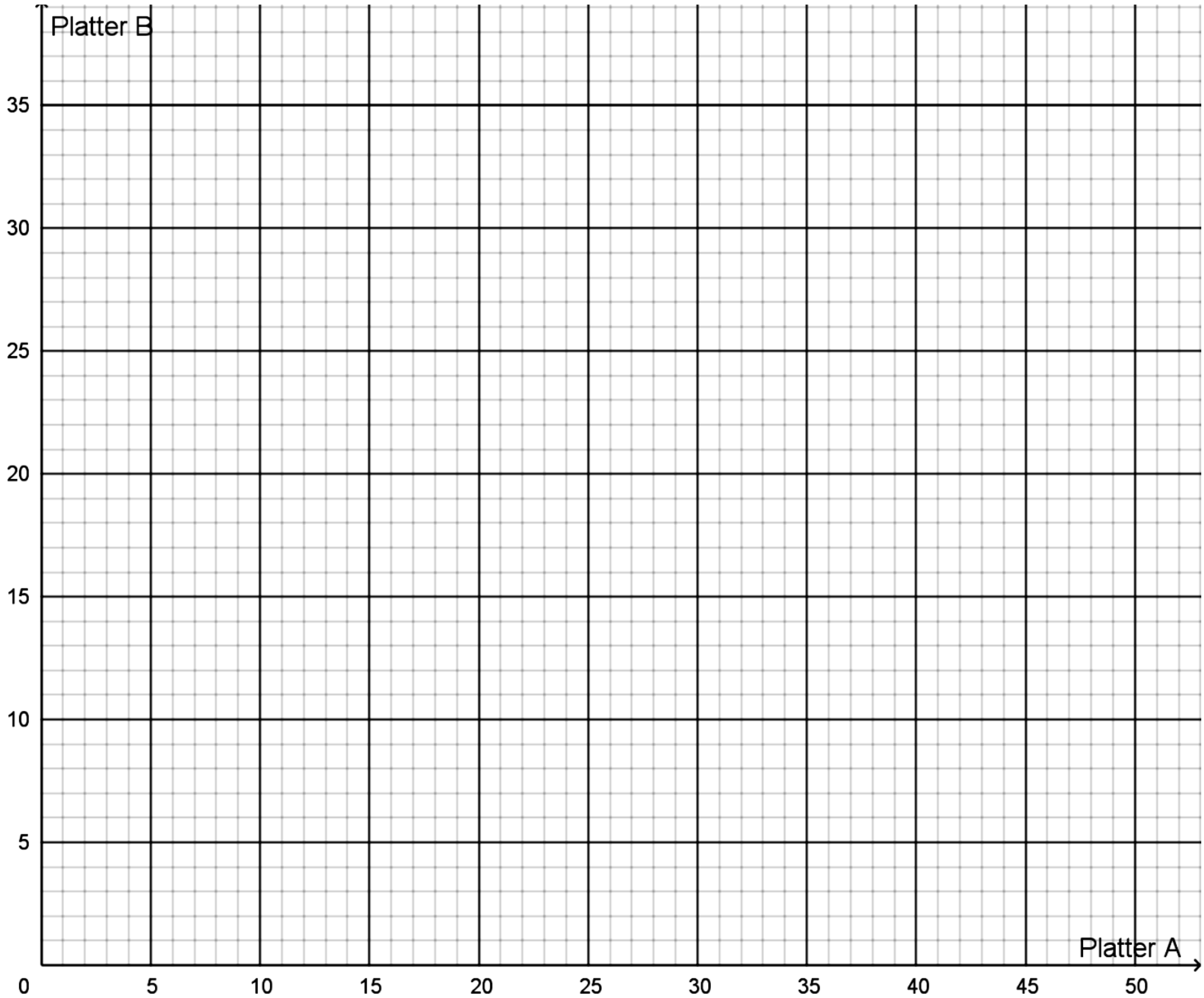
(6)

MATHEMATICS L4 P1

ADDENDUM B

EXAMINATION NUMBER: | | | | | | | | | | | | | |

QUESTION 5.2.2 (a), (b) and (d)



(3+1+1)

FORMULA SHEET

1. $Z = r\cos\theta + r\, j\sin\theta$

2. $Z = a \pm bj$ or $Z = a \pm bi$ *where* $i = j = \sqrt{-1}$

3. $r \lfloor \theta = r\ cis\ \theta$

4. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

5. $\dfrac{d}{dx} x^n = nx^{n-1}$

6. $y = ka^x = ke^{x\ln a}$

7. $\dfrac{d}{dx} k = 0$

8. $\dfrac{dy}{dx} = u.\dfrac{dv}{dx} + v.\dfrac{du}{dx}$ or $\dfrac{d}{dx}\left[f(x).g(x)\right] = f(x)\ g'(x) + f'(x)\ g(x)$

9. $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ or $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\ f'(x) - f(x)\ g'(x)}{[g(x)]^2}$

10. $\dfrac{dy}{dx} = \dfrac{du}{dx} \times \dfrac{dy}{du}$ or $\dfrac{d}{dx} f\left(g(x)\right) = f'\left(g(x)\right) g'(x)$

11. If $y = \ln kx$ then $\dfrac{dy}{dx} = \dfrac{k}{kx}$ or if $f(x) = \ln kx$ then $f'(x) = \dfrac{k}{kx}$

12. If $y = e^x$ then $\dfrac{dy}{dx} = e^x$ or if $f(x) = e^x$ then $f'(x) = e^x$

13. If $y = e^{kx}$ then $\dfrac{dy}{dx} = ke^{kx}$ or if $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$

14. If $y = \sin x$ then $\dfrac{dy}{dx} = \cos x$ or if $f(x) = \sin x$ then $f'(x) = \cos x$

15. If $y = \cos x$ then $\dfrac{dy}{dx} = -\sin x$ or if $f(x) = \cos x$ then $f'(x) = -\sin x$

16 If $y = \tan x$ then $\dfrac{dy}{dx} = \sec^2 x$ or if $f(x) = \tan x$ then $f'(x) = \sec^2 x$

17. If $y = \cot x$ then $\frac{dy}{dx} = -\cos ec^2 x$ or if $f(x) = \cot x$ then $f'(x) = -\cos ec^2 x$

18. If $y = \sec x$ then $\frac{dy}{dx} = \sec x \tan x$ or if $f(x) = \sec x$ then $f'(x) = \sec x \tan x$

19. If $y = \text{cosec } x$ then $\frac{dy}{dx} = -\text{cosec } x \cot x$ or if $f(x) = \text{cosec } x$ then $f'(x) = -\text{cosec } x \cot x$

20. If $y = \ln \sec x$ then $\frac{dy}{dx} = \tan x$ or if $f(x) = \ln \sec x$ then $f'(x) = \tan x$

21. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

22. $\int k\, x^n \, dx = k \int x^n \, dx$ where k is a constant.

23. $\int [\, f(x) \pm g(x)\,] dx = \int f(x)\, dx \pm \int g(x)\, dx$

24. $\int \frac{k}{x} \, dx = k \ln x + c$

25. $\int k a^x dx = \frac{k a^x}{\ln a} + c$

26. $\int e^{kx} \, dx = \frac{e^{kx}}{k} + c$

27. $\int \sin x \, dx = -\cos x + c$

28. $\int \cos x \, dx = \sin x + c$

29. $\int \tan x \, dx = \ln \sec x + c$

30. $\left[\, f(x) \,\right]_a^b = f(b) - f(a)$