higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First paper)
NQF LEVEL 4

16 NOVEMBER 2020

This marking guideline consists of 11 pages.

INSTRUCTIONS TO MARKERS

1. Mark all mathematically correct answers.

2. Adhere consistently to the mark allocation.

3. Errors in the first step of an answer does not imply that the candidate does not know what to do. Follow up.

4. In the in the explanation column, (A) denotes an accuracy mark, while (CA) indicates accuracy consistent with an error in the previous step.

5. Marks are allocated per step, but if a student omitted a step and there is evidence that he/she can derive the next step without calculation, then the mark must still be given.

QUESTION 1

1.1 1.1.1 C
1.1.2 C
1.1.3 A
1.1.4 B
1.1.5 D

(5 × 1) (5)

1.2 1.2.1 B
1.2.2 A
1.2.3 C

(3 × 2) (6)

1.3

✓ 1 for using the conjugate (A)

✓ ½ for simplifying each fraction (A)

$$\frac{1+i}{i}-\frac{3}{3-i}$$

$$=\left(\frac{1+i}{i}\times\frac{-i}{-i}\right)-\left(\frac{3}{3-i}\times\frac{3+i}{3+i}\right)$$

$$=\frac{-i-i^2}{-i^2}\surd-\frac{9+3i\surd}{9-i^2}$$

$$=\frac{-i+1}{1}-\frac{9+3i}{10}$$

$$=\frac{10(-i+1)-(9+3i)}{10}$$

$$=\frac{-10i+10-9-3i\surd}{10}$$

$$=\frac{1-13i\surd}{10}=\frac{1}{10}-\frac{13i}{10}\checkmark$$

$$=0{,}1-1{,}3i$$

½ for ‘a’ value and ½ for “b” value (CA)

Alternate

½ for simplifying fraction (A)
½ for using conjugate (CA)

½ for simplifying numerator (CA)
½ for simplifying denominator (CA)
½ for ‘a’ value and ½ for “b” value (CA)

(3)

$$\frac{1+i}{i}-\frac{3}{3-i}$$
$$=\left(\frac{(1+i)(3-i)-3i}{(3-i)i}\right)$$
$$=\frac{3+2i-i^2-3i}{3i-i^2}$$
$$=\frac{4-i\surd}{3i+1}\times\frac{3i-1}{3i-1}\surd$$
$$=\frac{12i-3i^2-4+i}{9i^2-1}$$
$$=\frac{-1+13i\surd}{-10\surd}=\frac{1}{10}-\frac{13i}{10}\checkmark$$
$$=0,1-1,3i$$

1.4

$$yi^3+x^2+2i^5=yxi^4-20i^8-6i$$
$$\therefore -yi\surd+x^2+2i=yx-20\surd-6i$$ Simplify i^n half mark each (A)
$$\therefore -yi+x^2+2i=yx-20-6i$$
$$\therefore -y=-8\ \checkmark\ldots\ldots\ldots\ldots\ldots 1$$ Equation 1- 1 mark (A)
$$and\ \ x^2-yx+20=0\ \checkmark\ \ldots\ldots 2$$ Equation 2- 1 mark (A)
$$from\ 1:\ y=8$$
$$Substitute\ in\ ..2$$
$$If\ \ y=8,\ x^2-8x+20=0$$
$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$\therefore x=\frac{8\pm\sqrt{64-4(20)}}{2}$$ ½ for substitution in formula (CA)
$$x=\frac{8\pm\sqrt{-16}}{2}=\frac{8\pm 4i}{2}\checkmark$$ ½ for solving
$$\therefore x=4+2i\surd\ \ or\ \ x=4-2i\surd$$ ½ for ‘a’ value and ½ for “b” value (CA)

(5)

[19]

QUESTION 2

2.1

$$-2x^3-3x^2+kx+20$$
$$-2(-4)^3-3(4)^2-4k+20\surd=-56\surd$$ ½ for substituting and ½ for equating (A)
$$\therefore 128-48-4k+20=-56$$
$$\therefore -4k=-156$$
$$\therefore k=\frac{-156}{-4}=39\checkmark$$ 1 for answer (A)

(2)

2.2.1 $x = -3$

$f(-3) = 2(-3)^3 - (-3)^2 - 16(-3) + 15 = 0$ ½ for substituting

½ for arriving at 0

$\therefore x + 3$ is a factor (1)

2.2.2 By inspection:

$2x^3 - x^2 - 16x + 15 = (x+3)(ax^2 + bx + c)$

$a = 2$ √ and $c = 5$ √ ½ each for 2 and 5 by inspect (A)

From $3bx + cx = -16x$

$3bx + 5x = -16x$

$(3b + 5)x = -16x$

$3b + 5 = -16$ 1 for b value in the quadratic (CA)

$b = -7$ ✓

$2x^3 - x^2 - 16x + 15 = (x+3)(2x^2 - 7x + 5)$ 1 each for the two new factors (A)

$= (x+3)(x-1)$ ✓$(2x-5)$✓ (4)

Alternatives

Long/synthetic division to obtain quadratic

factor $2x^2 - 7x + 5$ ✓ ✓ 2 marks

Factorise quadratic factor into

$(x-1)$✓$(2x-5)$✓ 1 each for new linear factors

2.3 The point $(2; 60)$ satisfies both equations:

Substitute $(2; 60)$ in

$f(x) = 2x^3 - 5x^2 + ax + 13b$

$2(2)^3 - 5(2)^2 + 2a + 13b = 60$ ✓ 1 for substitution (A)

$2a + 13b = 64$ ... (i) √ ½ for equation (CA)

Substitute $(2; 60)$in

$g(x) = x^3 + 9x^2 + bx + 7x - a = 60$

$(2)^3 + 9(2)^2 + 2b + 7(2) - a = 60$ ✓ 1 for substitution (A)

$-a + 2b = 2$ ... (ii) √ ½ for equation (CA)

Equation (i) + 2(ii):

$17b = 68$

$b = 4$ ✓ 1 for answer (CA)

Substituting $b = 4$ in (ii): $-a + 2(4) = 2$

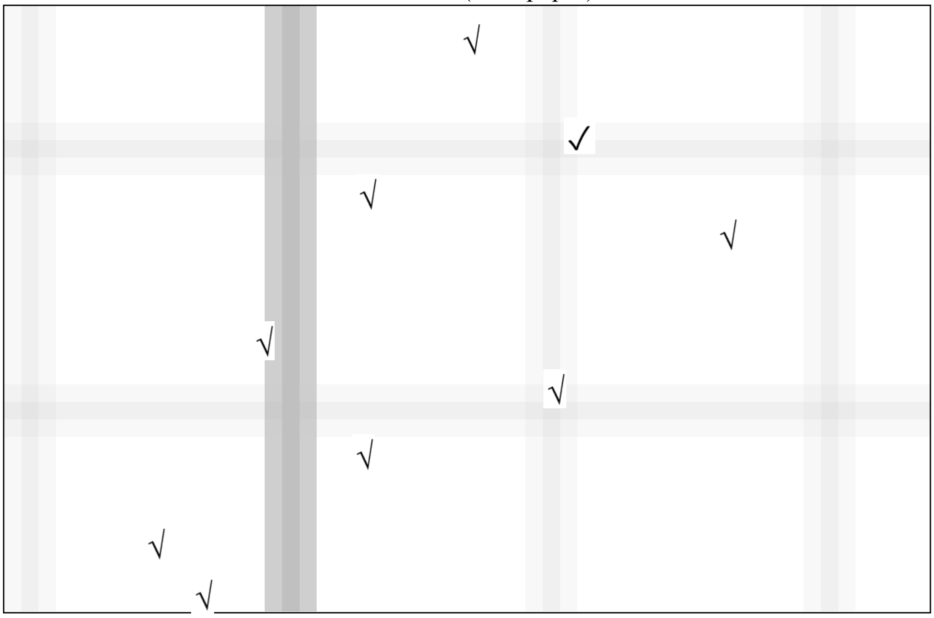
$a = 6$ ✓ 1 for answer (CA)

(5)

2.4 $f(x) = \frac{3}{2}x + 3$ for $x \in [-4; 2)$

Therefore $f^{-1}(x) = \frac{2}{3}x - 2$ ✓ for $x \in [-3; 6)$ 1 for correct inverse function (A)

(6)



Graph of f
½ for each correct endpoint (A)
½ each for x and y intercepts (A)

Graph of f^{-1}
½ for each correct endpoint (A)
½ each for x and y intercepts (CA)

$\boldsymbol{y = x}$
1 for draw & label

2.5.1 Point F is

$\left(-2; -\sqrt{-(-2)}\right)$

$\therefore y = -\sqrt{2}$ ✓ 1 for answer (A) (1)

2.5.2 $x \in (-\infty; 0]$ $\quad$ *where* $x \in R$ ✓ 1 for answer (A)

or $x \in \{-\infty < x \leq 0\}$ $\quad$ *where* $x \in R$

or $x \leq 0$ $\quad$ *where* $x \in R$ (1)

2.5.3 $f^{-1} : y = \sqrt{-x}$

$\therefore f : x = \sqrt{-y}$

$\therefore \quad -y = x^2$ √ ½ for working (A)

$\therefore \quad f(x) = -x^2$ √ ½ for answer (A) (1)

2.5.4 (a) Only g^{-1} has an asymptote 1 for answer (A)
Or f^{-1} touches the y axis

(b) It is a 2-1 function 1 for answer (A)
Or There exist vertical lines which cut f^{-1} twice.

1 for answer (A)

(c) g^{-1} is increasing in this interval
Or y gets larger as x gets larger

1 for answer (A)

(d) The range of g is $y > 0$. (4)

[25]

QUESTION 3

3.1

$$f(x) = -\frac{3}{x}$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{-\frac{3}{x+h}+\frac{3}{x}}{h}\surd$$ ½ for substitution in formula (A)

$$= \lim_{h\to 0}\frac{\frac{-3x+3x+3h}{x(x+h)}\checkmark}{h}$$ 1 for adding (A)

$$= \lim_{h\to 0}\frac{3h}{x(x+h)}\times\frac{1}{h}$$

$$= \lim_{h\to 0}\frac{3}{x(x+h)}\checkmark$$ 1 for simplifying (A)

$$= \frac{3}{x^2}\surd$$ ½ for answer (A)

Note: Zero for answer only (3)

3.2.1

$$f(x) = 3x^2 + 2\ln x + 4x + 5$$

$$f'(x) = 6x\surd + \frac{2}{x}\surd + 4\surd + 0\surd$$ ½ for each term (A)

(2)

3.2.2

$$f(x) = \sqrt[3]{x^2} + 2e^{3x} - \tan 2x - \cos ec3x$$

$$f(x) = x^{\frac{2}{3}} + 2e^{3x} - \tan 2x - \cos ec3x$$ ½ for first term convert (A)

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}\surd + 6e^{3x} - 2\sec^2 2x + 3co\sec 3x\cot 3x$$ Ignore this half tick

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}\surd + 6e^{3x} - 2\sec^2 2x + 3co\sec 3x\cot 3x$$ ½ for derivative (A)

$$f'(x) = \frac{2}{3\sqrt[3]{x}}\surd + 6e^{3x}\surd - 2\surd\sec^2 2x\surd + 3\surd co\sec 3x\cot 3x\surd$$ ½ for each term (A)

(4)

3.2.3

$$y = \frac{x^3+3}{x^2-3x}$$

$$\frac{dy}{dx} = \frac{3x^2\surd(x^2-3x)\surd-(2x-3)\surd(x^3+3)\surd}{(x^2-3x)^2\checkmark}$$

½ for each correct factor in the numerator (A)
1 for correct denominator (A)

ALTERNATIVE
Allocate on the same basis for alternatives

(3)

3.2.4 FOF

$y = \left(x^3 - 5x\right)^5$

$Let\ u = x^3 - 5x \ \therefore \frac{du}{dx} = 3x^2 - 5$

$\frac{dy}{du} = 5\left(u\right)^4$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\therefore \frac{dy}{dx} = 5\surd\left(x^3 - 5x\right)^4 \surd(3x^2 - 5)\surd$ — ½ each for the three factors and ½ for the exponent "4" (A)

$\therefore \frac{dy}{dx} = (15x^2 - 25)\left(x^3 - 5x\right)^4 \surd$

Alternate

Chain rule — ½ each for the three factors and ½ for the exponent "4" (A)

$$\frac{dy}{dx} = 5\surd(x^3 - 5x)^{4\surd}\surd(3x^2 - 5)\surd$$

(2)

3.3.1 $f(x) = 2x^3 - x^2 - 16x + 15$

$Let\ x = 0 \therefore f(0) = 15$

$(0;15)$✓ — 1 for the answer "15" (A) (1)

3.3.2 $f(x) = 2x^3 - x^2 - 16x + 15$

$f'(x) = 6x^2 - 2x - 16\surd$

$f''(x) = 12x - 2\surd$ — ½ for 1st derivative (A); ½ for 2nd derivative (CA)

$x = 1{,}81$ Substitute in $f''(x)$

$f''(1{,}81) = 12(1{,}81) - 2 = \text{positive}\surd$

$\therefore$ At $x = 1{,}81$ the turning point is a minimum $TP\surd$ — ½ for 2nd derivative at 1,81 (CA); ½ for conclusion (CA)

$x = -1{,}47$ Substitute in $f''(x)$

$f''(-1{,}47) = 12(-1{,}47) - 2 = \text{negative}\surd$

$\therefore$ At $x = -1{,}47$ the turning point is a maximum $TP\surd$ — ½ for 2nd derivative at -1,47(CA); ½ for conclusion (CA) (3)

3.3.3 Decrease ✓ — 1 for answer (A) (1)

3.3.4 $f(x) = 2x^3 - x^2 - 16x + 15$

$f'(x) = 6x^2 - 2x - 16$ *gradient*

$Use\ x = 1$

$f'(1) = 6(1)^2 - 2(1) - 16 = -12$✓

Gradient negative

Graph is decreasing✓

NOTE: Students can use any point. Marks are allocated if the outcome is the same. Negative gradient

(2)

3.4 $f(x)=x^3+mx^2+nx-4 \quad x-2$ leaves remainder of -2

$f(2)=(2)^3+m(2)^2+n(2)-4=-2$ √ — ½ Subst.

$\therefore 4m+2n=-6 \quad eq\ 1$√ — ½ equation

$f(x)=x^3+mx^2+nx-4$

$f'(x)=3x^2+2mx+n=0$ √at TP $x=3$ *given* — ½ $f'=0$

$\therefore 3(3)^2+2m(3)+n=0$

$6m+n=-27 \quad$ eq 2√ — ½ equation

$\therefore (6m+n=-27 \quad)\times 2$

$12m+2n=-54 \quad eq\ 3$

$4m+2n=-6 \quad eq\ 1$

$eq3-eq1 \qquad 8m=-48$

$\therefore m=-6$✓ — 1 answer

Substiute in eq2

$\therefore 6(-6)+n=-27$

$n=9$✓ — 1 answer (4)

[25]

QUESTION 4

4.1.1 $\int\left(\frac{x}{2}+\frac{2}{3}x^{-3}+5\cos 5x+4x\right)dx$

$=\frac{x^2}{4}-\frac{2}{6}x^{-2}+\sin 5x+2x^2+c$

$=\frac{x^2}{4}$√$-\frac{1}{3x^2}$✓$+\sin 5x$√$+2x^2$√$+c$√

1 for 2nd term and ½ each for the rest (A) (3)

4.1.2 $\int\left(\frac{1}{2x}+\frac{2}{\sqrt[3]{x^2}}-e^{3x}+8\sec^2 4x+5\right)dx$

$\int\left(\frac{1}{2x}+2x^{\frac{-2}{3}}-e^{3x}+8\sec^2 4x+5\right)dx$

$=\frac{1}{2}\ln x+\left(\frac{2x^{\frac{1}{3}}}{\frac{1}{3}}\right)-\frac{e^{3x}}{3}+\frac{8\tan 4x}{4}+5x+c$

$=\frac{1}{2}\ln x$√$+6\sqrt[3]{x}$✓$-\frac{e^{3x}}{3}$✓$+2\tan 4x$✓$+5x$✓$+c$

1 per term (A)
Ignore the constant c. It was credited in 4.1.1 (5)

4.2 Distance $= \int_0^2 (-3t^2 + 5t + 8)\,dt$ — 1 for setting up correct integral (A)

$= \left(-t^3 + \frac{5}{2}t^2 + 8t\right)\Big|_0^2$ — 1 for integral (CA)

$= (-8 + 10 + 16) - (0)$ — 1 for answer (CA)

$= 18\,m$ (3)

4.3.1 $A = \int_a^b y\,dx$

$A = \int_{-2}^{2} (-x^4 + 6x^2)\,dx\,\surd$ — ½ setting up integral (A)

$A = \frac{-x^5}{5}\surd + \frac{6x^3}{3}\surd\Big|_{-2}^{2}$ — ½ each for integrals (CA)

$A = \left|\frac{-(2)^5}{5} + \frac{6(2)^3}{3}\right| - \left|\frac{-(-2)^5}{5} + \frac{6(-2)^3}{3}\right|\surd$ — ½ for calculation (CA)

$A = 9,6 + 9,6 = 19,2$ ✓ — 1 for answer (CA) (3)

4.3.2 $f'(x) = -4x^3 + 12x\,\surd$ — ½ for gradient function f' (A)
½ for derivative of gradient function

$f''(x) = -12x^2 + 12 = 0\,\surd$

$\therefore x = \pm 1\,\surd$ — ½ for stationary points of f'

$\therefore$ at $x = 1$ $f'(x) = -4(1)^3 + 12(1) = 8\,\surd$ — ½ for gradient at $x = 1$

Maximum gradient is 8 (2)

[16]

QUESTION 5

5.1.1 Gradient: $m = \frac{1100}{550}$
$= 2$ — 1 for gradient (A)

$h(x) = 2x$ — 1 for answer (CA)
Full marks for answer only (2)

5.1.2 $h(x) \leq 2x$ ✓ — I for 1st constraint (CA)

$f(x) \leq$ ✓ $2000 - x$ — 1 each for remaining two (A)

$g(x) <$ ✓ $1375 - \frac{1}{2}x$ (3)

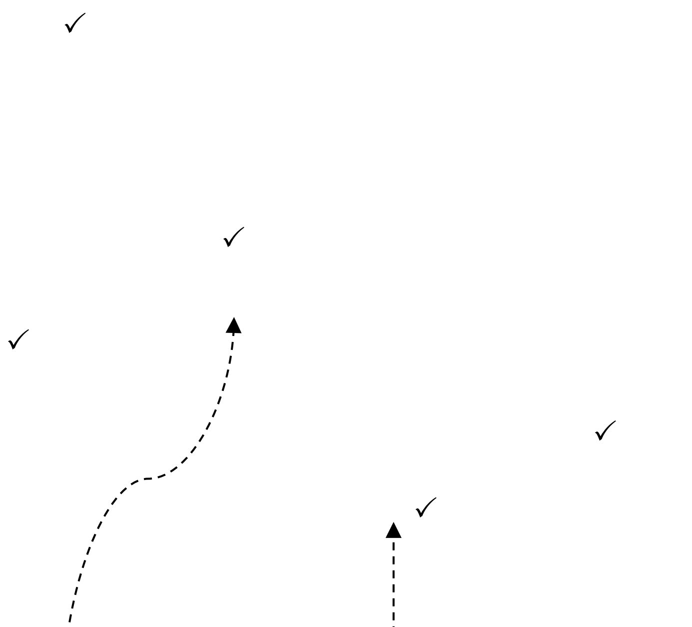
5.2.1 $200x + 400y \leq 8000$ ✓ (or $x + 2y \leq 40$) 1 for each constraint (A)

$400x + 250y \leq 7750$ √✓(or $8x + 5y \leq 155$)

$y \geq 5$ √✓ (3)

5.2.2

a)



(3)

1 mark for graphs =3 marks
1mark feasible region = 3 marks
1 mark search line

b) Indicate the feasible region on the graph. SEE GRAPH (1)

c) $P = 140x + 160y$✓ (1)

d) Draw a search line on the graph. SEE GRAPH ✓ (1)

e) Max at $(10; 15)$ √
Max P $= 140(10) + 160(15) =$ R3800 √ (1)

[15]

TOTAL: 100