higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 4

(10501064)

5 November 2019 (X-Paper)
09:00–12:00

Nonprogrammable scientific calculators may be used.

This question paper consists of 12 pages, 2 addenda and formula sheet of 2 pages.
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TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Leave at least THREE lines after each question.
5. Start each section on a NEW page.
6. Diagrams are NOT drawn to scale.
7. Where necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
8. Write neatly and legibly.

QUESTION 1

1.1 1.1.1 Various options are given as possible answers to the following questions. Choose the answer and write only one of the letters (A–D) next to the question number (1.1.1–1.1.5) in the ANSWER BOOK.

1.1.1 $(-6+5i)-(3-2i)$ simplified is …

A $-9+3i$
B $7-9i$
C $-9+7i$
D $3+3i$

1.1.2 $i^3+i^5+i^7$ will be equal to …

A 1
B i
C $3i^{15}$
D $-i$

1.1.3 $(5-3i)(5+3i)=$ …

A 34
B $25-30i+9i^2$
C $25-9i$
D $5-15i-3i^2$

1.1.4 If $z=-2-3i$ is plotted on a complex plane, then it will be in the following quadrant …

A First
B Second
C Third
D Fourth

1.1.5 Solve for x and y if $3x-5yi=-10i+15$ where $i=\sqrt{-1}$

A $x=\frac{-10}{3}$ $y=-3$
B $x=5$ $y=2$
C $x=\frac{1}{5}$ $y=\frac{1}{2}$
D $x=5$ $y=2i$

(5 × 1) (5)

1.2 Solve for x in the equation $2x^2+2x+5=0$ and express any non-real solutions in the form $a \pm bi$. (2)

1.3 Simplify and leave the answer in rectangular form, with a rational denominator:

$$\frac{\sqrt{2}\, cis\,(-45^0)}{2i^2+3i+4}$$ (3)

1.4 Apply De Moivre's theorem to calculate the following, leaving your answer in polar form, with a positive argument:

$$\frac{(4-3i)-(i^2+i^3)}{\left(2\,cis\,120^0\right)^5}$$ (5)

1.5 Solve for x and y in the following equation and express any non-real solutions in the form $a \pm bi$:

$$x^2 i^8 - y^2 i^7 + 6xi^6 - xi \;= 9i^2 - 7i$$ (6)

[21]

QUESTION 2

2.1 If $f(x) = 2x^3 + px^2 + 5x + 6$ is divided by $x-2$, it leaves a remainder of -20.

Determine the value of *p*. (2)

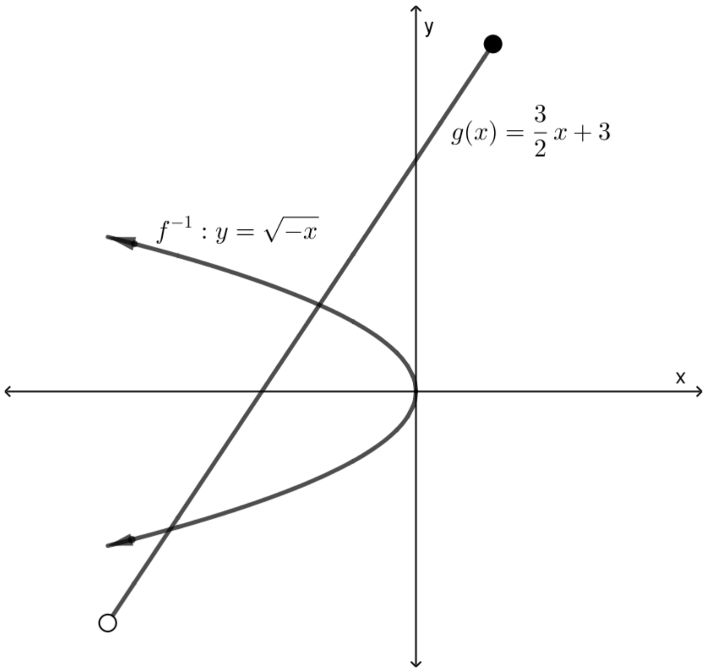
2.2 The expression $-2x^3 - 3x^2 + 13bx + 20$ has the remainder of -56 when it is divided by $x+a$.

If it is also given that $(x-a)$ is a factor of $-2x^3 - 3x^2 + 13bx + 20$, apply the remainder and factor theorems to determine TWO values of *a*. Hence determine the value of b. (5)

2.3 The figure below represents the graphs of the following functions:

$f^{-1}: y = \sqrt{-x}$ and

$g(x) = \frac{3}{2}x + 3$ for $x \in (-4; 2]$ where $x \in R$



GRAPH 1

2.3.1 Does $f^{-1}: y = \sqrt{-x}$ exist at $x = 2$. (1)

2.3.2 Write down the domain of $f^{-1}: y = \sqrt{-x}$. (1)

2.3.3 Write down the range of $g^{-1}(x)$. (1)

2.3.4 Determine the equation of f . (1)

2.3.5 Determine the equation of g^{-1}. (2)

2.3.6 Determine the range $g(x)$. (3)

2.3.7 Sketch g^{-1} on ADDENDUM A and show all intercepts with axes. Also draw the line about which g^{-1} and g is symmetric.

Clearly show the nature of the start and end points of g^{-1} . (3)

[19]

QUESTION 3

3.1 Given the function $f(x) = 5 - 2x^2$. Determine the derivative of $f(x)$ making use of first principles. (3)

3.2 Use differentiation rules to determine the derivatives of the following, leaving your answer with POSITIVE EXPONENTS and in SURD form where applicable:

3.2.1 $f(x) = 3x^2 - \dfrac{\sqrt{x^3}}{3} + 2e^{3x} - \text{cosec}(4x) - 9x$ (5)

3.2.2 $y = \dfrac{3x+1}{3\ln x}$ (3)

3.2.3 $f(t) = (5t)(\sin 5t)$ (3)

3.2.4 $y = \dfrac{1}{\left(x^2 - 2\right)^2}$ (3)

[17]

QUESTION 4

4.1 Determine the following integrals and simplify the answers. Leave the answer with positive exponent and in surd form where applicable:

4.1.1 $\int \left(4x + \sin 4x + \dfrac{4}{x} + 4 \right) dx$ (3)

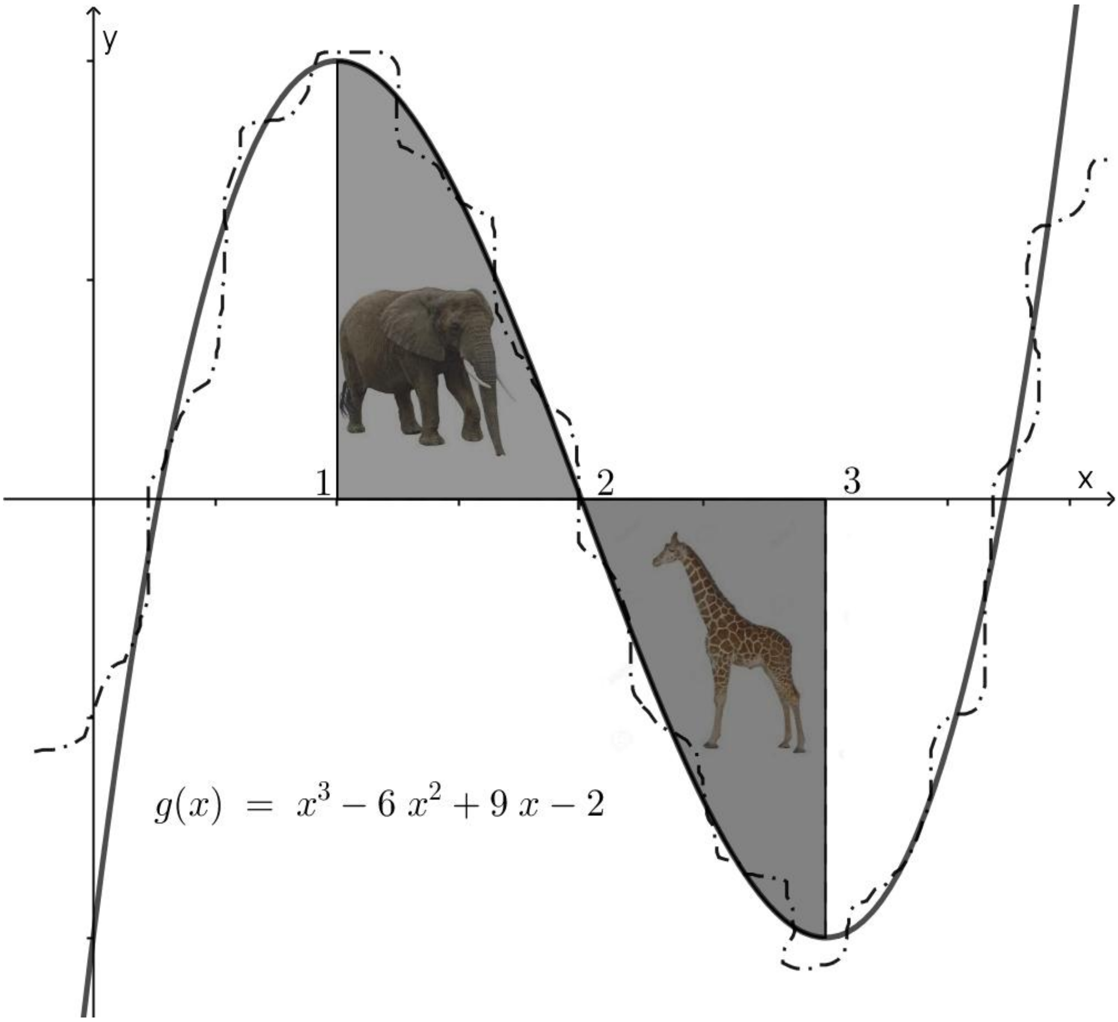
4.1.2 $\int (3.\sqrt[3]{x} - \dfrac{3}{\sqrt{x}} + 4e^{4x} + 3\sec^2 3x)dx$ (4)

4.2 Evaluate the definite integral $\int_{-1}^{2} (4x+3)dx$ (2)

4.3 A game reserve has an irregular shape and a river flow through it. The owner needs to determine the approximate area of the game reserve. The level 4 students used several points to determine the following sample regression equation that will approximate the shape:

$$f(x) = x^3 - 6x^2 + 9x - 2$$

Use this equation for your calculations below:



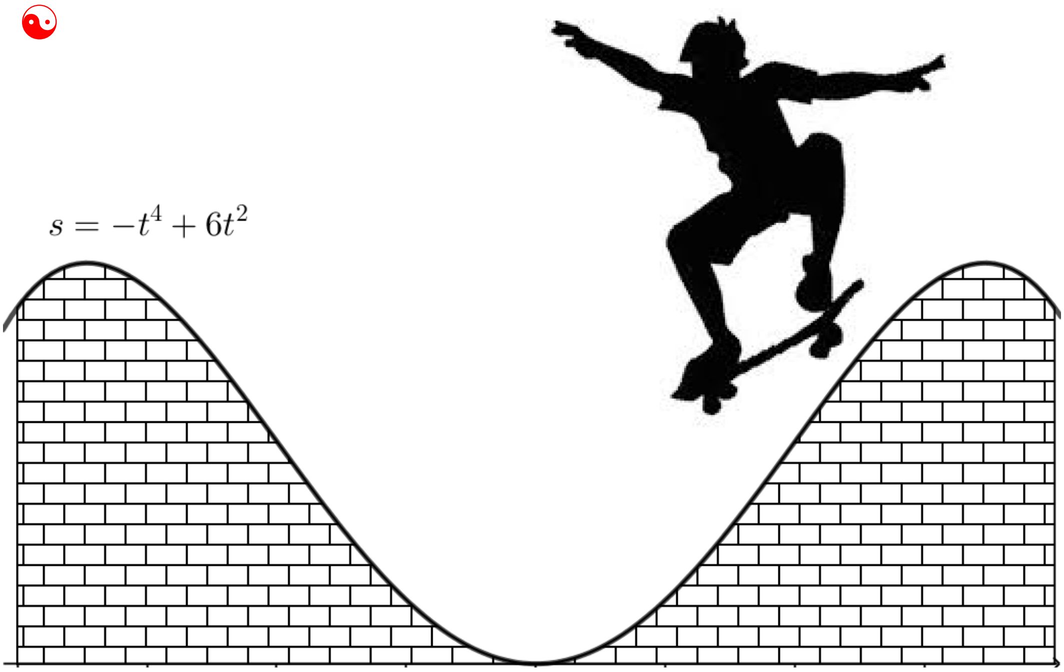
GRAPH 2

Calculate the magnitude of the shaded area by making use of integration. The game reserve is bounded by the graph of f, the x −axis, and the lines $x = 1$ and $x = 3$. (5)

[14]

QUESTION 5

5.1 A skateboarder practices on a ramp shown below. Her equation of motion is given by $s = -t^4 + 6t^2$ where s is the height in metres and t is the time in seconds.



GRAPH 3

5.1.1 Determine the equation that will describe her velocity in terms of t. (1)

5.1.2 Calculate her maximum velocity. (3)

5.1.3 Determine the height at the time when she is at maximum velocity. (1)

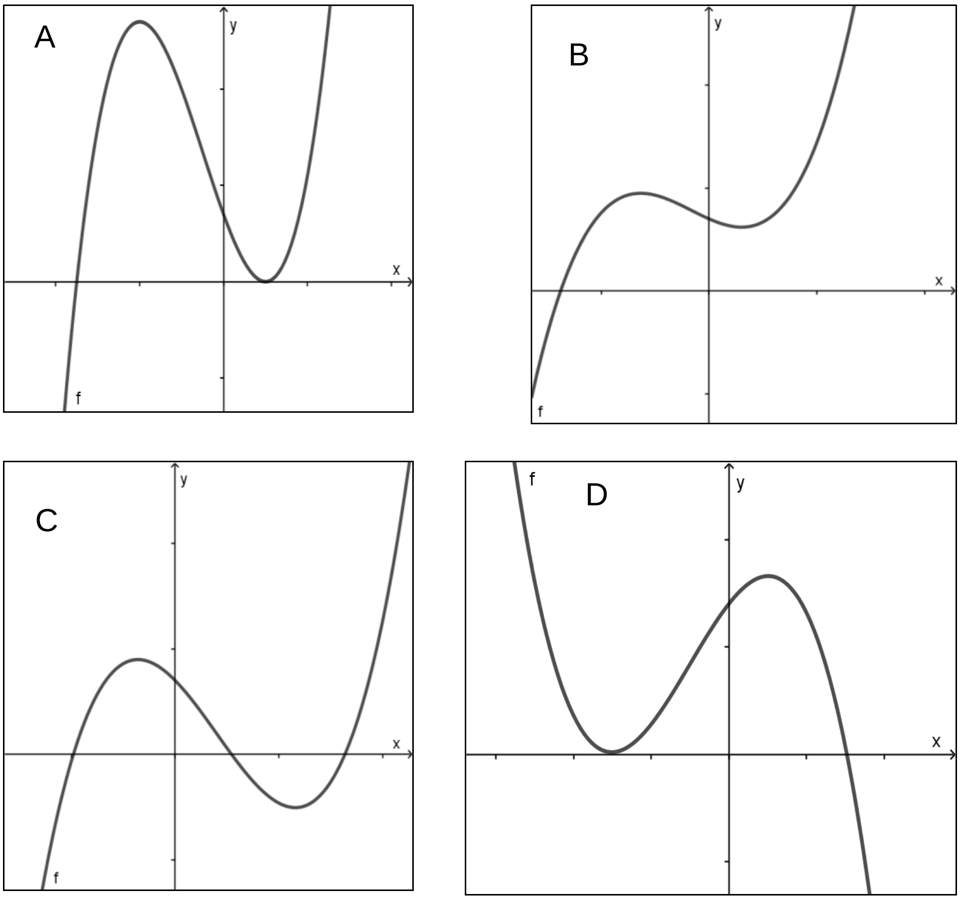
5.1.4 Determine her acceleration at $t = 2$ seconds. (2)

5.2 The third-degree polynomial $f(x) = 2x^3 + 3x^2 - 12x + 7$ is given.

5.2.1 Apply the factor theorem or any other method to factorise $f(x)$ completely. (4)

5.2.2 Refering to QUESION 5.2.1 above choose the correct graph which represents the above third-degree polynomial $f(x) = 2x^3 + 3x^2 - 12x + 7$.

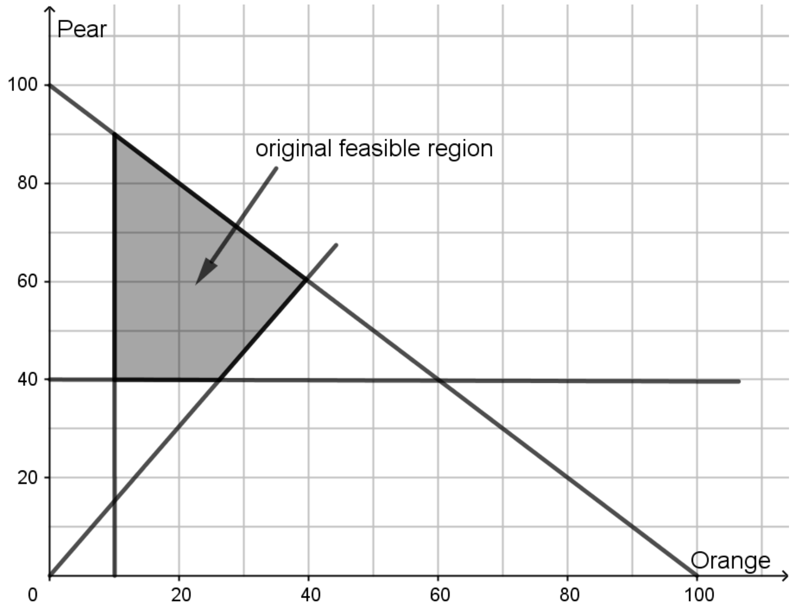
Choose the answer and write only one of the letters (A–D) next to the question number (5.2.2) in the ANSWER BOOK.



(2)
[13]

QUESTION 6

6.1 An entrepreneur sells fresh juice mix in cups. The mixture consists of oranges and pears. Let the quantity of oranges be x and the quantity of pears be y. The shaded area in the diagram represents the feasible region for the conditions under which the juice mixture is made.



GRAPH 4

6.1.1 Referring to the feasible region in graph 4, what is the minimum quantity of pears that could be used in the mixture? (1)

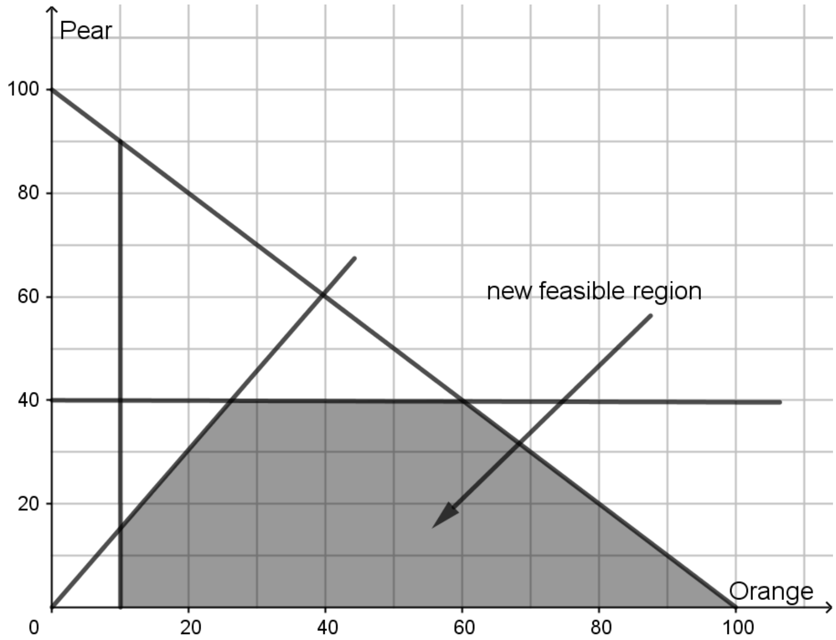
6.1.2 Will a mixture of 60 pears and 20 oranges meet the requirements of the feasible region?

Explain your answer. (2)

6.1.3 The constraints that describe the feasible region on graph 4 above are:

$$y \geq 40;$$
$$x \geq 10;$$
$$y \geq \frac{3}{2}x.$$
$$x + y \leq 100$$

Apply this knowledge and write down the constraints that will describe the new feasible region in Graph 5 below.



GRAPH 5 (3)

6.2 A group of civil engineering students intends to do paving using two different colours of bricks, grey bricks (x batches) and red bricks (y batches). They must use at most 300 batches of grey bricks and at least 150 batches of red bricks. They must use a maximum of 500 batches of bricks for the area they want to pave. For every batch of red bricks there must be at most two batches of grey bricks.

6.2.1 Two of the constraints are $x \geq 0$ and $y \geq 0$. Model the other constraints in respect of the above information in terms of x and y. (2)

6.2.2 Use the attached ADDENDUM B and complete the following questions.

a) Using a scale of 1 unit is 50 bricks, represent graphically the inequalities in QUESTION 6.2.1 (3)

b) Indicate the feasible region on the graph. (1)

c) The students make a profit of R250 per pallet (batch) of the grey bricks and R 300 per pallet of red bricks.

Determine the equation that represents the total profit in terms of x and y. (1)

d) Draw a search line on the graph. (1)

e) Calculate the maximum profit. (2)

[16]

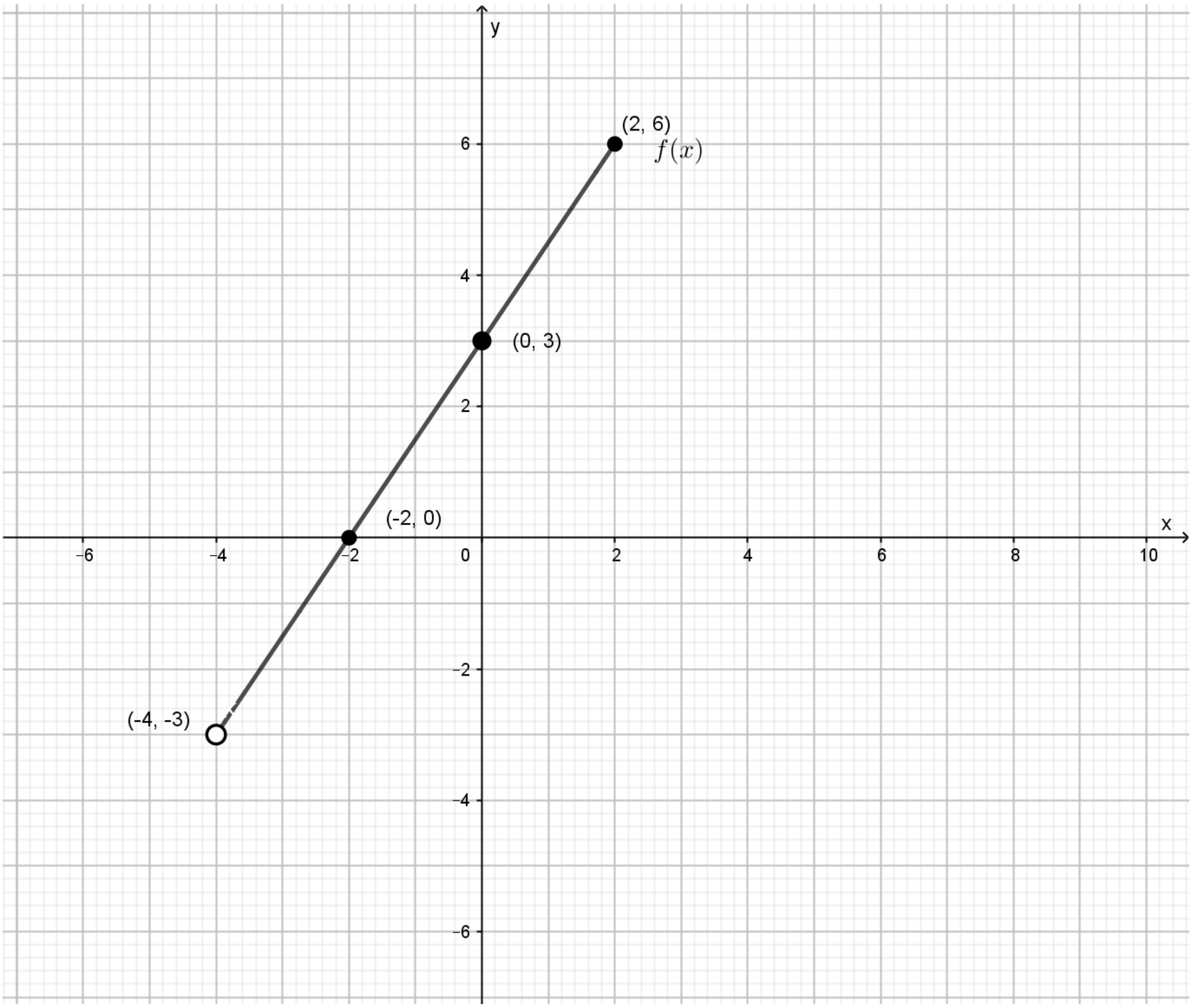
TOTAL: 100

ADDENDUM A

EXAMINATION NUMBER:

CENTRE NUMBER:

QUESTION 2.3.7

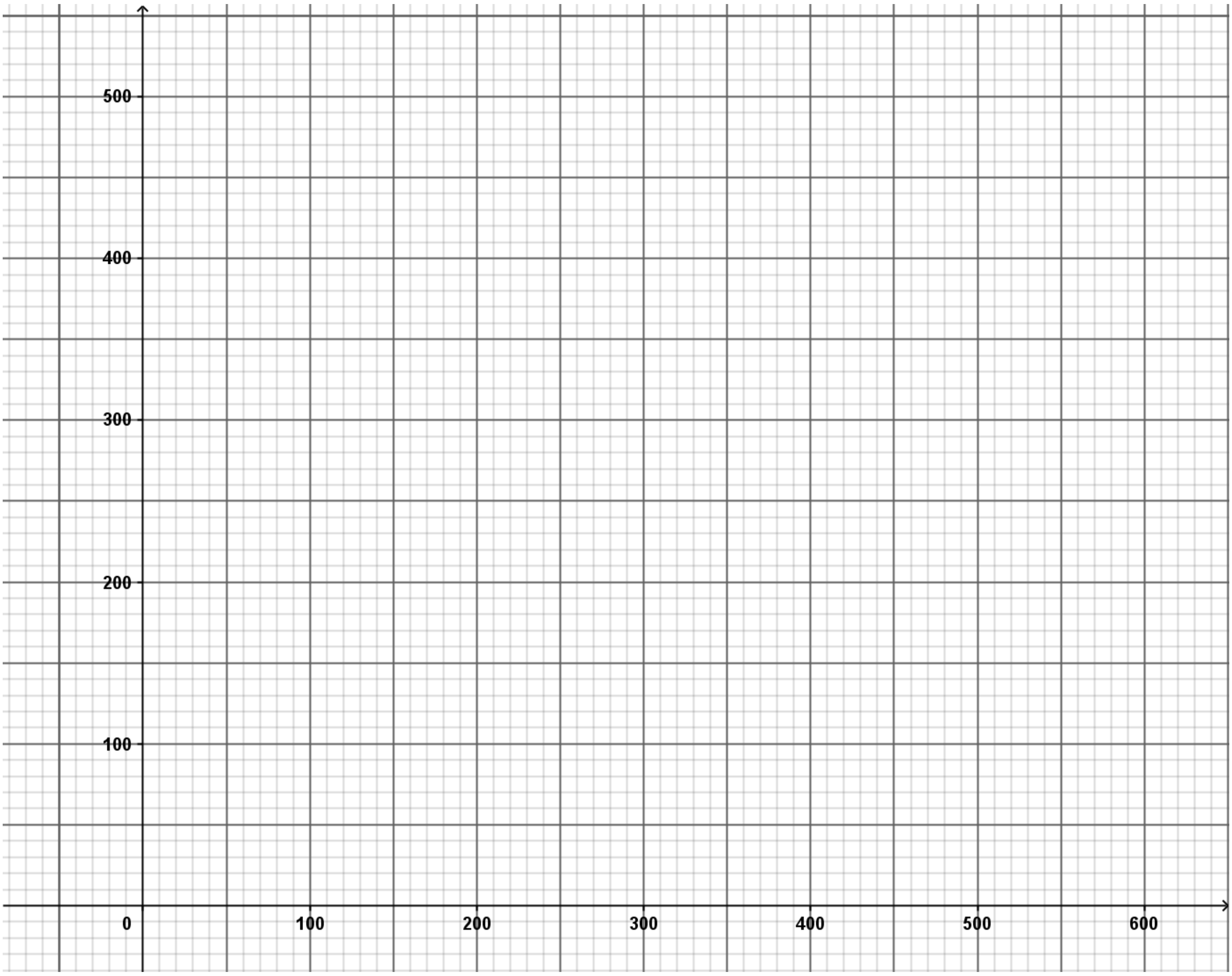


ADDENDUM B

EXAMINATION NUMBER:

CENTRE NUMBER:

QUESTION 6.2.2



FORMULAE SHEET

1. $z = r\cos\theta \pm rj\sin\theta$ or $z = r\cos\theta \pm ri\sin\theta$

2. $z = a \pm bj$ or $z = a \pm bi$ where $i = j = \sqrt{-1}$

3. $r\underline{|\theta} = r\angle\theta = rcis\theta$ where "r" represent modulus and "θ" represent argument

4. $m = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

5. $\dfrac{d}{dx} x^n = nx^{n-1}$

6. $\dfrac{d}{dx} k = 0$

7. $\dfrac{dy}{dx} = u.\dfrac{dv}{dx} + v.\dfrac{du}{dx}$ or $\dfrac{d}{dx}\left[f(x).g(x)\right] = f(x)\ g'(x) + f'(x)\ g(x)$

8. $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ or $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\ f'(x) - f(x)\ g'(x)}{[g(x)]^2}$

9. $\dfrac{dy}{dx} = \dfrac{du}{dx} \times \dfrac{dy}{du}$ or $\dfrac{d}{dx} f\left(g(x)\right) = f'\left(g(x)\right)\ g'(x)$

10. If $y = \left(f(x)\right)^n$ then $\dfrac{dy}{dx} = n\left(f(x)\right)^{n-1}.f'(x)$

11. If $y = \ln kx$ then $\dfrac{dy}{dx} = \dfrac{k}{kx}$ or If $f(x) = \ln kx$ then $f'(x) = \dfrac{k}{kx}$

12. If $y = e^x$ then $\dfrac{dy}{dx} = e^x$ or If $f(x) = e^x$ then $f'(x) = e^x$

13. If $y = e^{kx}$ then $\dfrac{dy}{dx} = ke^{kx}$ or If $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$

14. If $y = \sin x$ then $\dfrac{dy}{dx} = \cos x$ or If $f(x) = \sin x$ then $f'(x) = \cos x$

15. If $y = \cos x$ then $\dfrac{dy}{dx} = -\sin x$ or if $f(x) = \cos x$ then $f'(x) = -\sin x$

16. If $y = \tan x$ then $\dfrac{dy}{dx} = \sec^2 x$ or if $f(x) = \tan x$ then $f'(x) = \sec^2 x$

17. If $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$ or $f(x) = \cot x$ then $f'(x) = -\operatorname{cosec}^2 x$

18. If $y = \sec x$ then $\frac{dy}{dx} = \sec x \tan x$ or $f(x) = \sec x$ then $f'(x) = \sec x \tan x$

19. If $y = \operatorname{cosec} x$ then $\frac{dy}{dx} = -\operatorname{cosec} x \cot x$ or $f(x) = \operatorname{cosec} x$ then $f'(x) = -\operatorname{cosec} x \cot x$

20. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

21. $\int k\, x^n \, dx = k \int x^n \, dx$ where k is a constant value.

22. $\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

23. $\int \frac{k}{x}\, dx = k \ln x + c$

24. $\int e^{kx}\, dx = \frac{e^{kx}}{k} + c$

25. $\int \sin kx \, dx = \frac{-\cos kx}{k} + c = -\frac{1}{k} \cos kx + c$

26. $\int \cos kx \, dx = \frac{\sin kx}{k} + c = \frac{1}{k} \sin kx + c$

27. $\int \sec^2 kx \, dx = \frac{1}{k} \tan kx + c$

28. $A = \int_a^b y\,dx$

29. $[f(x)]_a^b = f(b) - f(a)$