higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 4

(10501064)

2 November 2018 (Y-Paper)
13:00–16:00

Nonprogrammable scientific calculators may be used.

This question paper consists of 6 pages, a formula sheet of 2 pages and 1 addendum.

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
5. Answers must be rounded off to THREE decimals, unless stated otherwise.
6. Diagrams are NOT drawn to scale.
7. Start each question on a NEW page
8. Write neatly and legibly.

QUESTION 1

1.1 Given: $Z_1 = -2 + 4i$, $Z_2 = -6 + i$ and $Z_3 = 2i$

Calculate the following and leave your answers in the form $a + bi$:

1.1.1 $Z_1 \times Z_3$ (2)

1.1.2 $\dfrac{Z_2}{Z_1}$ (3)

1.2 Use De Moivre's theorem to calculate $\left(\dfrac{2\sqrt{3} - i}{3 - 2\sqrt{3}i}\right)^6$. Leave your answer in polar form. (5)

1.3 Solve for x if $-x(x-1) = 3$, and leave any complex solutions in the form $a + bi$. (3)

1.4 Solve for x and y if $(yi + x) = (3i - 1)^2$. (4)

[17]

QUESTION 2

2.1 Determine $f'(x)$ from first principles (definition of derivative), if

$$f(x) = -\frac{4}{x}$$ (5)

2.2 Use differentiation rules to determine the derivatives of the following functions. (Leave your answers in simplified form with POSITIVE exponents and in SURD form, where applicable).

2.2.1 $f(x) = \dfrac{3}{x} + x^3 + 2\cos 4x + \dfrac{3}{2x^2}$ (4)

2.2.2 $f(x) = \dfrac{1}{2}e^{2x} - 4\ln x$ (2)

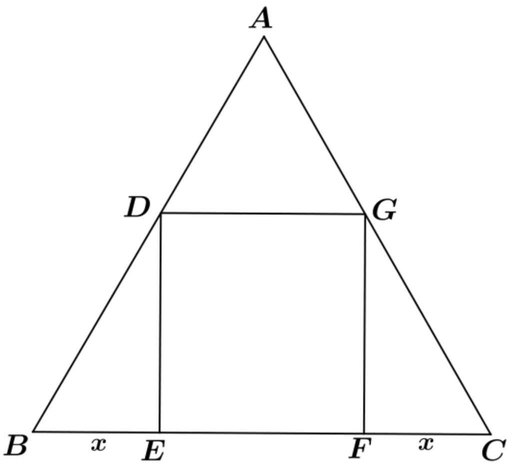
2.3 Differentiate the following functions by using an appropriate rule:

2.3.1 $y = x \ln x$ (3)

2.3.2 $y = \dfrac{x^2}{\sin 3x}$ (3)

2.3.3 $y = (7x^2 - 3)^6$ (3)

2.4 In the diagram Δ ABC is an equilateral triangle with each side measuring a units. D is a point on AB, E and F are points on BC and G is a point on AC such that DEFG is a rectangle. Also, BE = FC = x units.



2.4.1 Prove that the area of the rectangle is given by

$$Area = \sqrt{3}x(a-2x)$$ (3)

2.4.2 Determine the maximum area of the rectangle in terms of a. (4)

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QUESTION 3

3.1 Determine the following integrals. (Leave your answers with POSITIVE exponents and in SURD form, where applicable):

3.1.1 $\int\left(\frac{2}{x}-\frac{x}{2}\right)^2 dx$ (3)

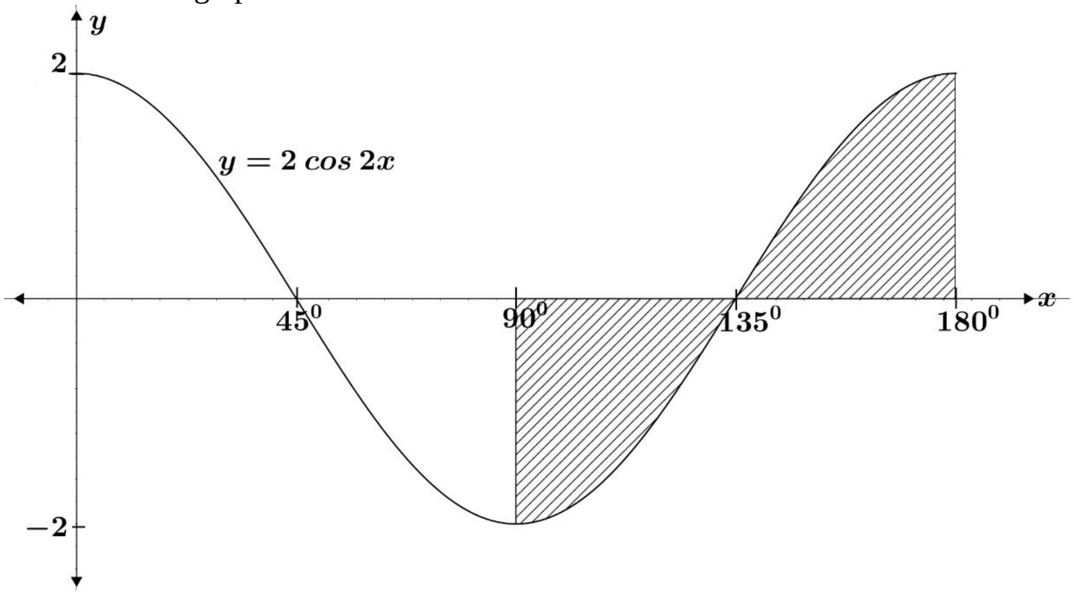
3.1.2 $\int\left(-2x+(2x)^3-3x^{-3}+\frac{1}{x\sqrt{x}}\right)dx$ (4)

3.1.3 $\int\left(\frac{3}{2}e^x+3\ell n2+\frac{1}{2}\sin 4x\right)dx$ (2)

3.2 Evaluate the following definite integral:

$$\int_1^3 e^{2x}dx$$ (3)

3.3 Given below is the graph of the function of $y = 2\cos 2x$, for $0^0 \leq x \leq 180^0$. The area between the graph and the x axis from $x = 90^0$ to $x = 180^0$ has been shaded.



Use integration to calculate the area of the shaded part above. (5)

[17]

QUESTION 4

4.1 Solve for x in the equation: $x(x-1) = 12$ (2)

4.2 Factorise $g(x)$ completely if $g(x) = x^3 - 7x + 6$. (4)

4.3 If $x^2 + x - 6$ is a factor of $f(x) = 2x^3 + px^2 - 17x + q$, determine p and q. (8)

4.4 Given $f(x) = \left(\frac{1}{3}\right)^x$

4.4.1 Write down the inverse of $f(x)$ in the form $x = \cdots$. (1)

4.4.2 Draw the graph of f and f^{-1} on the same system of axes.
Also draw the line $y = x$ on the same axes. (5)

4.4.3 Is $f(x)$ an increasing or decreasing function? Give a reason for your answer. (1)

4.4.4 Write down the domain of f^{-1}. (1)

4.4.5 Write down the range of f. (1)

4.4.6 What is the equation of the horizontal asymptote of $y = 3^{-x}$? (1)

[24]

QUESTION 5

Bina owns a small factory that manufactures two types of cell phones, Bruna phones, and Ondi phones. The quantity of cell phones manufactured per week is subject to the following constraints:

- Each Bruna cell phone requires 10 hours to manufacture while each Ondi cell phone requires 8 hours to manufacture.
- Each Bruna phone requires 3 hours in testing department and each Ondi phone requires 4 hours in the testing department.
- The manufacturing department has a maximum of 800 hours available per week.
- The testing department has a maximum of 360 hours available per week.
- The factory needs to manufacture at least 60 Ondi phones each week.

Let the number of Bruna cell phones manufactured per week be x and the number of Ondi cell manufactured per week be y.

5.1 Write down the constraints, in terms of x and y, that represent the information given above. (3)

5.2 Use the graph paper (ADDENDUM) to represent the constraints graphically. (5)

5.3 Clearly indicate the feasible region by shading it on the graph. (1)

5.4 If the profit on one Bruna cell phone is R200 and the profit on one Ondi cell phone is R250, write down an expression that will represent the total profit P. (1)

5.5 Using a search line, determine the number of Bruna and Ondi cell phones that should be manufactured so that the profit is maximum, assuming that all are sold. Draw the search line on your graph. (3)

5.6 The profit on each type of cell phone is doubled. If the factory is to make a maximum profit, will the number of phones of each type be different from that obtained in question 5.5 above? Give a reason for your answer. (2)

[15]

TOTAL: 100

FORMULA SHEET

1. $Z = r\cos\theta + r\,j\sin\theta$

2. $Z = a \pm bj$ or $Z = a \pm bi$ $\quad$ *where* $i = j = \sqrt{-1}$

3. $r\,\underline{|\,\theta} = r\ cis\ \theta$

4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

5. $\dfrac{d}{dx}x^n = nx^{n-1}$

6. $y = ka^x = ke^{x\ln a}$

7. $\dfrac{d}{dx}k = 0$

8. $\dfrac{dy}{dx} = u.\dfrac{dv}{dx} + v.\dfrac{du}{dx}$ $\quad$ or $\quad$ $\dfrac{d}{dx}\left[f(x).g(x)\right] = f(x)\ g'(x) + f'(x)\ g(x)$

9. $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ $\quad$ or $\quad$ $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\ f'(x) - f(x)\ g'(x)}{[g(x)]^2}$

10. $\dfrac{dy}{dx} = \dfrac{du}{dx} \times \dfrac{dy}{du}$ $\quad$ or $\quad$ $\dfrac{d}{dx}f\left(g(x)\right) = f'\left(g(x)\right)g'(x)$

11. If $y = \ln kx$ then $\dfrac{dy}{dx} = \dfrac{k}{kx}$ $\quad$ or $\quad$ if $f(x) = \ln kx$ then $f'(x) = \dfrac{k}{kx}$

12. If $y = e^x$ then $\dfrac{dy}{dx} = e^x$ $\quad$ or $\quad$ if $f(x) = e^x$ then $f'(x) = e^x$

13. If $y = e^{kx}$ then $\dfrac{dy}{dx} = ke^{kx}$ $\quad$ or $\quad$ if $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$

14. If $y = \sin x$ then $\dfrac{dy}{dx} = \cos x$ $\quad$ or $\quad$ if $f(x) = \sin x$ then $f'(x) = \cos x$

15. If $y = \cos x$ then $\dfrac{dy}{dx} = -\sin x$ $\quad$ or $\quad$ if $f(x) = \cos x$ then $f'(x) = -\sin x$

16 If $y = \tan x$ then $\dfrac{dy}{dx} = \sec^2 x$ $\quad$ or $\quad$ if $f(x) = \tan x$ then $f'(x) = \sec^2 x$

17. If $y=\cot x$ then $\dfrac{dy}{dx}=-\operatorname{cosec}^2 x$ or if $f(x)=\cot x$ then $f'(x)=-\operatorname{cosec}^2 x$

18. If $y=\sec x$ then $\dfrac{dy}{dx}=\sec x\tan x$ or if $f(x)=\sec x$ then $f'(x)=\sec x\tan x$

19. If $y=\operatorname{cosec} x$ then $\dfrac{dy}{dx}=-\operatorname{cosec} x\cot x$ or if $f(x)=\operatorname{cosec} x$ then $f'(x)=-\operatorname{cosec} x\cot x$

20. If $y=\ln\sec x$ then $\dfrac{dy}{dx}=\tan x$ or if $f(x)=\ln\sec x$ then $f'(x)=\tan x$

21. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1}+c$

22. $\int k\,x^n\,dx = k\int x^n\,dx$ where k is a constant.

23. $\int[f(x)\pm g(x)]dx=\int f(x)dx\pm\int g(x)dx$

24. $\int \dfrac{k}{x}\,dx = k\ln x+c$

25. $\int ka^x dx=\dfrac{ka^x}{\ln a}+c$

26. $\int e^{kx}\,dx=\dfrac{e^{kx}}{k}+c$

27. $\int \sin x\,dx=-\cos x+c$

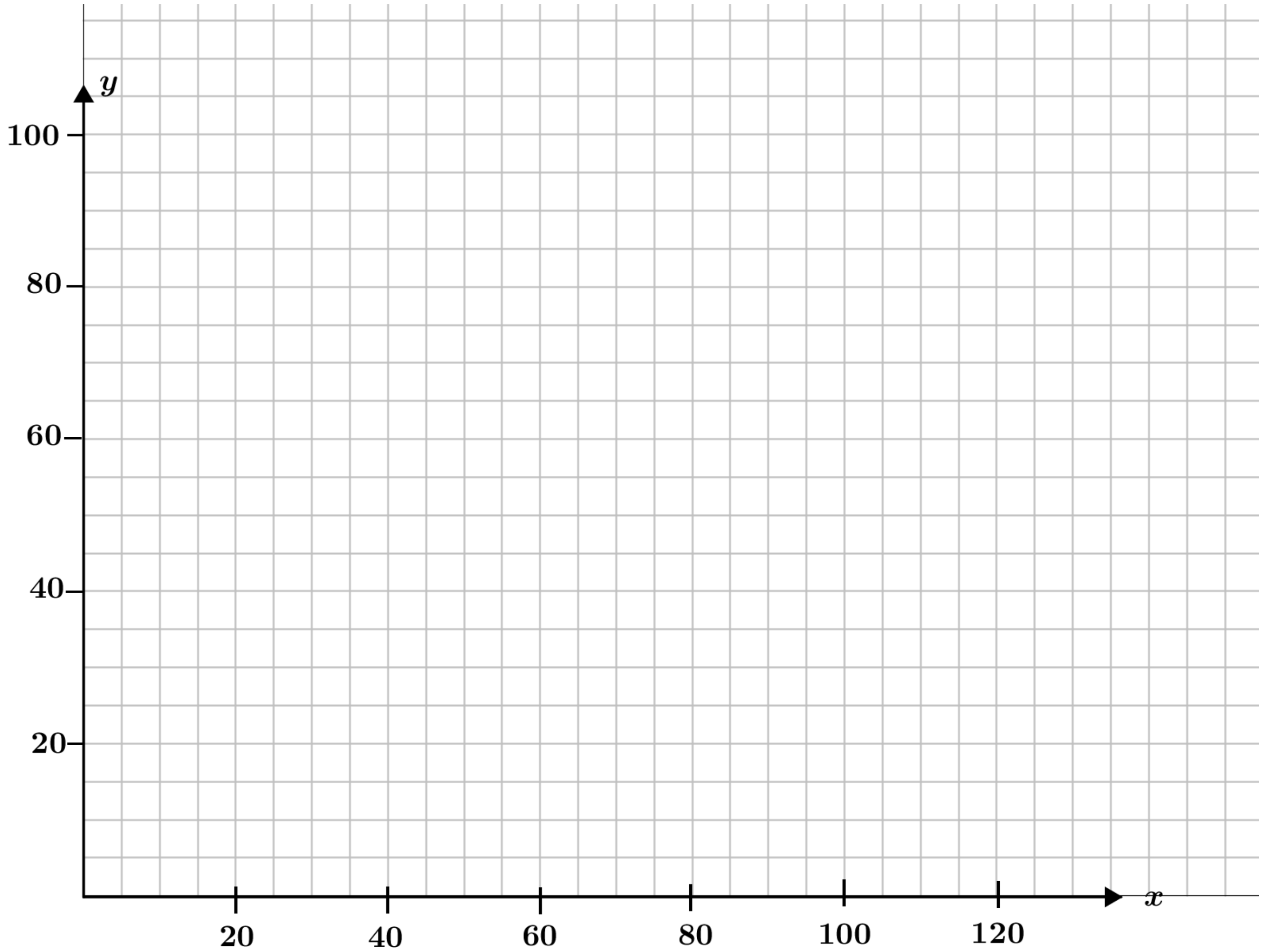
28. $\int \cos x\,dx=\sin x+c$

29. $\int \tan x\,dx=\ln\sec x+c$

30. $[f(x)]_a^b = f(b)-f(a)$

ADDENDUM **EXAMINATION NUMBER:**

QUESTION 5.2, 5.3 and 5.5



HAND IN WITH THE ANSWER BOOK