higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 4

(10501064)

21 February 2018 (Y-Paper)
13:00–16:00

Nonprogrammable calculators may be used.

This question paper consists of 9 pages, ONE formula sheet of 2 pages and an addendum of 2 pages.

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Use a BLACK or a BLUE pen.
5. Start each question on a NEW page.
6. Diagrams are NOT drawn to scale.
7. Answers should be rounded off to TWO decimal places, unless stated otherwise.
8. Work neatly.

QUESTION 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5) in the ANSWER BOOK.

1.1.1 The complex number, $z = -3 - 4i$ where $i = \sqrt{-1}$, in polar form is:

A $5cis\ 126{,}870^0$
B $5cis\ 233{,}130^0$
C $5cis\ 53{,}130^0$
D $5cis\ 306{,}870^0$

(1)

1.1.2 The factors of $x^2 + 25$ are …

A $(x-5i)(x-5i)$
B $(x+5)(x+5)$
C $(x-5)(x+5)$
B $(x+5i)(x-5i)$

(1)

1.1.3 Simplify $-4i + i^6 - (-3 + 0i)$ where $i = \sqrt{-1}$:

A $-4i + 3$
B $2 - 4i$
C $2 + 4i$
D $-4 + 6i^5$

(1)

1.1.4 Simplify $(2 + \sqrt{-25})(2 - \sqrt{-25})$:

A 29
B $4 - 25$
C $(2 - \sqrt{-25})^2$
B $4 + 5i^2$

(1)

1.1.5 Solve for *x* and *y* if $3x - 4yi = -6 - 16i$ where $i = \sqrt{-1}$:

A $x = 2$ and $y = -6$
B $x = -2$ and $y = 4$
C $x = 6$ and $y = 12$
D $x = 3$ and $y = 4$

(1)

1.2 Simplify and leave the answer in rectangular form:

$$\frac{1-i}{2+3i}$$ (3)

1.3 Add $z_1 = \sqrt{3}\angle 60^\circ$ *and* $z_2 = 2\angle 45$ and leave the answer in rectangular form $(a+bi)$. (3)

1.4 Apply De Moivre's theorem to calculate the following, leaving the answer in polar form with a positive argument.

$$\frac{\left(\sqrt{9}\angle 230^\circ\right)^4}{\left(2cis330^\circ\right)^5}$$ (4)

1.5 Solve for x and y if $1-i = 2(x+yi)+i^5\left(11i^5-13\right)+3(y+xi)$ (5)

[20]

QUESTION 2

2.1 Use differentiation rules to determine the derivatives of the following, leaving your answer in simplified form with positive exponents and in surd form, where applicable:

2.1.1 $f(x) = 5x^3 - \sqrt{4x} + \frac{2}{5x^3} - 4e^{-2x+1} - \sin 30^0$ (5)

2.1.2 $y = \frac{\sin x}{e^x}$ (3)

2.1.3 $y = \frac{1}{\left(3x^2-5\right)^5}$ (3)

2.1.4 $f(t) = t\left(\frac{3}{t^2} - 5t\right)$ (2)

2.2 Given: $f(x) = \frac{4}{x}$

2.2.1 Determine $f'(x)$ from first principles if $f(x) = \frac{4}{x}$ (3)

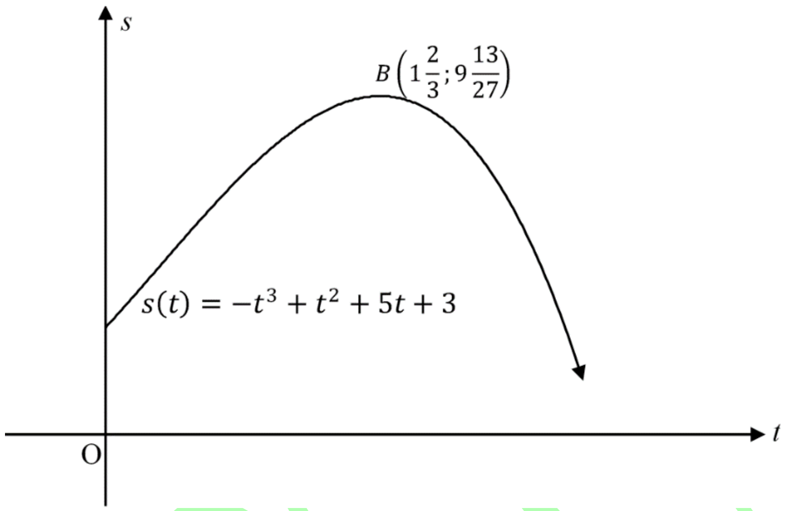
2.2.2 Hence determine the gradient of $f(x)$ at $x = 2$ (1)

2.2.3 Find the equation of the tangent to the curve $f(x) = \frac{4}{x}$ at $x = 2$ (2)

2.3 The graph below represents the motion of an object moving according to the formula

$$s(t) = -t^3 + t^2 + 5t + 3$$

where s is distance in metres and t is time in seconds. The point $B\left(1\frac{2}{3}; 9\frac{13}{27}\right)$ is a turning point of the graph.



2.3.1 Determine the distance travelled after 1 second. (1)

2.3.2 Determine the velocity of the object at $t = 2$. (2)

2.3.3 Use your graph to write down the value(s) of t for which the velocity is negative. (1)

2.3.4 Determine the maximum velocity of the object. (2)

[25]

QUESTION 3

3.1 Determine the following integrals:

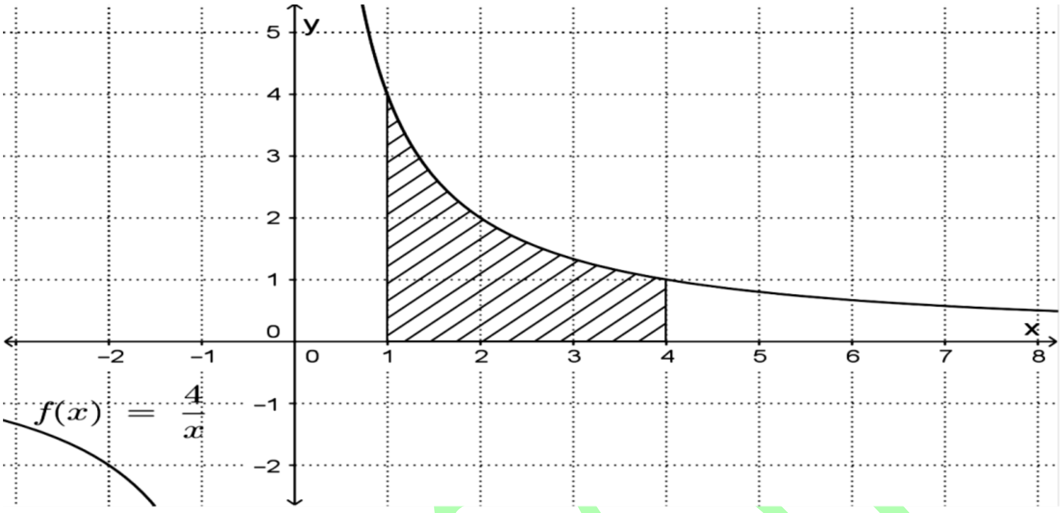
3.1.1 $\int (15x^2 - \frac{3}{2\sqrt{x}} + 4e^{2x} + (2\sec 3x)^2)dx$ (4)

3.1.2 $\int (4\cos 2x + \frac{x}{2} - \frac{2}{x} + 3)\, dx$ (3)

3.2 The velocity equation of an object is given as $v = 3t + 5$ metres per second, where t is the time in seconds.

Determine the distance travelled during the third second. Apply integration to solve the problem. The area below the velocity-time graph gives the distance. (4)

3.3 Given: the graph of $f(x) = \frac{4}{x}$

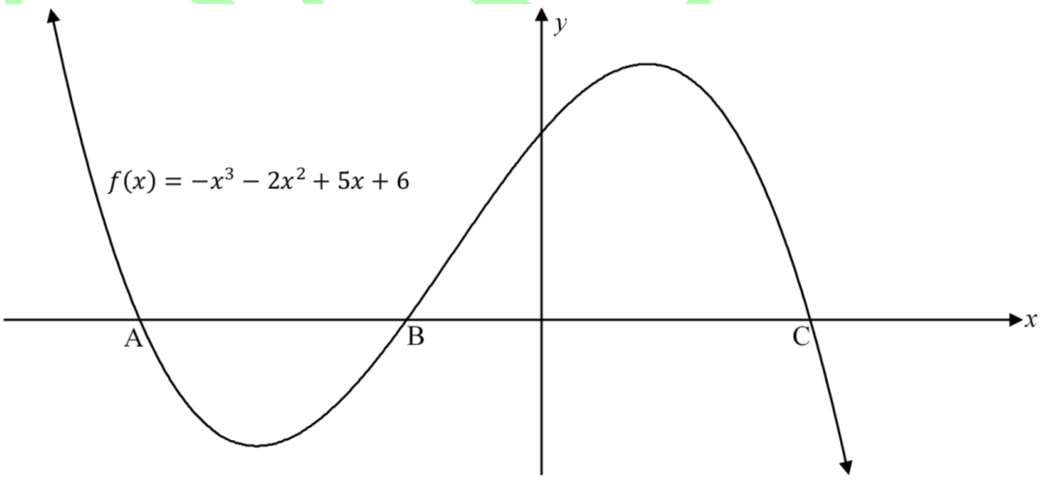


3.3.1 Use ADDENDUM (3.3.1) to show the representative strip that you will use to calculate the shaded area. (1)

3.3.2 Calculate the magnitude of the shaded area. (3)
[15]

QUESTION 4

4.1 The graph represents the third-degree polynomial $f(x) = -x^3 - 2x^2 + 5x + 6$ which cuts the x axis at A, B and C.



Apply the factor theorem and long division (or any other method) to calculate the x coordinates of A, B and C. (5)

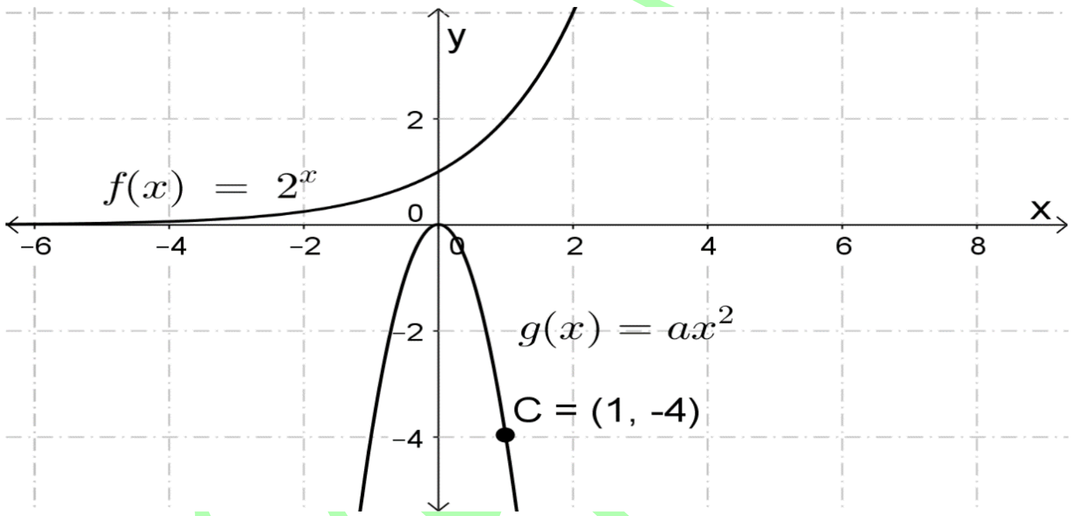
4.2 When $f(x) = x^3 - kx^2 - x + 13$ is divided by $x - 3$, the remainder is 10.

Calculate the value of k using the remainder theorem. (2)

4.3 The expressions $2x^3 - 5x^2 + 2ax + 52$ and $x^3 + 3ax^2 + bx - 6$ have the same remainder when divided by $x - 2$. Also, $x = -2$ is a root of $x^3 + 3ax^2 + bx - 6 = 0$

Apply the remainder theorem, and any other relevant information, to determine the values of a and b. (4)

4.4 The diagram given below shows the graphs $(x) = 2^x$ and $g(x) = ax^2$. The point $C(1; -4)$ lies on the graph of g.



4.4.1 Calculate the value of a.
Hence determine the inverse of g in the form $y = \cdots$. (3)

4.4.2 Write down the domain and the range of $g(x) = ax^2$. (2)

4.4.3 The graph of g is symmetrical about the $x = 0$ line. Write down the equation of the line of symmetry for g^{-1}. (1)

4.4.4 Determine the equation of the inverse of $f(x) = 2^x$ in the form $x = \cdots$. (1)

4.4.5 Use the ADDENDUM A (4.4.5) to sketch the graph of $f^{-1}(x)$, showing clearly the intercept with the x axis and at least one other point. (3)

4.4.6 About which line are $f(x)$ and $f^{-1}(x)$ symmetric? (1)

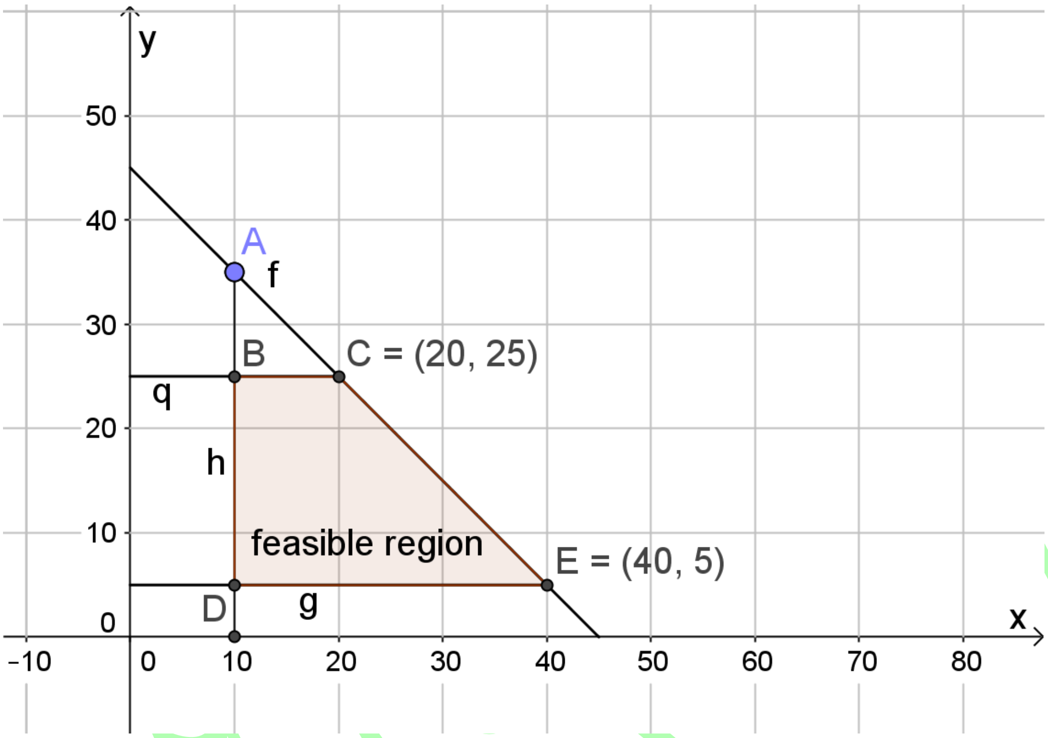
4.4.7 Write down the domain of $f^{-1}(x)$. (1)

4.4.8 A learner gives the range of $f(x) = 2^x$ as $y \epsilon [0; \infty)$ for y a real number.
Explain why the learner is wrong. (2)

[25]

QUESTION 5

5.1 A paint shop mixes two different colours, yellow (x litres) and red (y litres) to get different shades of orange. The shaded area in the diagram represents the feasible region subject to a set of constraints under which they can mix these colours.



5.1.1 Given is the constraint $5 \leq y \leq 25$.

Use the information above to write down inequalities associated with the lines represented by **f** and **h** . (1)

5.1.2 If the paint suppliers only have 30 litres of yellow paint and 15 litres of red paint, will they be able to supply the orange paint that satisfies the prescribed constraints? Verify your answer by applying the inequalities in QUESTION 5.1.1. (2)

5.2 A tourist group wants to stay over for one day at the Eiland Spa Resort and must book their accommodation in advance. The resort has a choice of two types of chalets (cottages).

Type A: These chalets can host a couple and has a double bed.
Type B: These chalets can host a family and has a double bed and two single beds.

In the group there are at least 4 couples who want to stay in the type A chalets and they want not less than 4 of the type B chalets to place families with four members. The ratio of type A to type B must be at least one to two. The group needs 18 chalets at the most.

Let the number of type A chalets be x and the number of type B be y.

5.2.1 Two of the constraints are $x \geq 0$ and $y \geq 0$.

Model the other constraints in respect of the above information in terms of x and y. (2)

5.2.2 The tariff for type A is R1 000/day and type B is R1 500/day.

Write the equation that represents the total cost for the stay in terms of x and y. (1)

5.2.3 Determine the gradient for the search line. Show the steps. (1)

5.2.4 (a) Using a scale of 1 unit = 2 chalets, represent the inequalities graphically on the attached ADDENDUM B (5.2.4). (3)

(b) Indicate the feasible region on the graph. (1)

(c) Draw the search line on the graph and calculate the maximum cost for their stay. (2)

5.2.5 Due to cancellations of tourists, the group on arrival consisted of 10 families who needed type B chalets and 3 couples who required type A chalets.

Will the tour leader be able to meet the requirements of all the tourist in their booked chalets? Give a reason for your answer with reference to the feasible region. (2)
[15]

TOTAL: 100

MATHEMATICS L4

FORMULA SHEET

1. $Z = r\cos\theta + r\,j\sin\theta$

2. $Z = a \pm bj$ or $Z = a \pm bi$ where $i = j = \sqrt{-1}$

3. $r\,\underline{|\theta} = r\ cis\ \theta$

4. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

5. $\dfrac{d}{dx} x^n = nx^{n-1}$

6. $y = ka^x = ka^{x\ln a}$

7. $\dfrac{d}{dx} k = 0$

8. $\dfrac{dy}{dx} = u.\dfrac{dv}{dx} + v.\dfrac{du}{dx}$ or $\dfrac{d}{dx}[f(x).g(x)] = f(x)\ g'(x) + f'(x)\ g(x)$

9. $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ or $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\ f'(x) - f(x)\ g'(x)}{[g(x)]^2}$

10. $\dfrac{dy}{dx} = \dfrac{du}{dx} \times \dfrac{dy}{du}$ or $\dfrac{d}{dx} f(g(x)) = f'(g(x))\ g'(x)$

11. If $y = \ln kx$ then $\dfrac{dy}{dx} = \dfrac{k}{kx}$ or if $f(x) = \ln kx$ then $f'(x) = \dfrac{k}{kx}$

12. If $y = e^x$ then $\dfrac{dy}{dx} = e^x$ or if $f(x) = e^x$ then $f'(x) = e^x$

13. If $y = e^{kx}$ then $\dfrac{dy}{dx} = ke^{kx}$ or if $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$

14. If $y = \sin x$ then $\dfrac{dy}{dx} = \cos x$ or if $f(x) = \sin x$ then $f'(x) = \cos x$

15. If $y = \cos x$ then $\dfrac{dy}{dx} = -\sin x$ or if $f(x) = \cos x$ then $f'(x) = -\sin x$

16 If $y = \tan x$ then $\dfrac{dy}{dx} = \sec^2 x$ or if $f(x) = \tan x$ then $f'(x) = \sec^2 x$

17. If $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$ or if $f(x) = \cot x$ then $f'(x) = -\operatorname{cosec}^2 x$

18. If $y = \sec x$ then $\frac{dy}{dx} = \sec x \tan x$ or if $f(x) = \sec x$ then $f'(x) = \sec x \tan x$

19. If $y = \operatorname{cosec} x$ then $\frac{dy}{dx} = -\operatorname{cosec} x \cot x$ or if $f(x) = \operatorname{cosec} x$ then $f'(x) = -\operatorname{cosec} x \cot x$

20. If $y = \ln \sec x$ then $\frac{dy}{dx} = \tan x$ or if $f(x) = \ln \sec x$ then $f'(x) = \tan x$

21. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

22. $\int k x^n \, dx = k \int x^n \, dx$ where k is a constant.

23. $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$

24. $\int \frac{k}{x} dx = k \ln x + c$

25. $\int k a^x dx = \frac{k a^x}{\ln a} + c$

26. $\int e^{kx} \, dx = \frac{e^{kx}}{k} + c$

27. $\int \sin x \, dx = -\cos x + c$

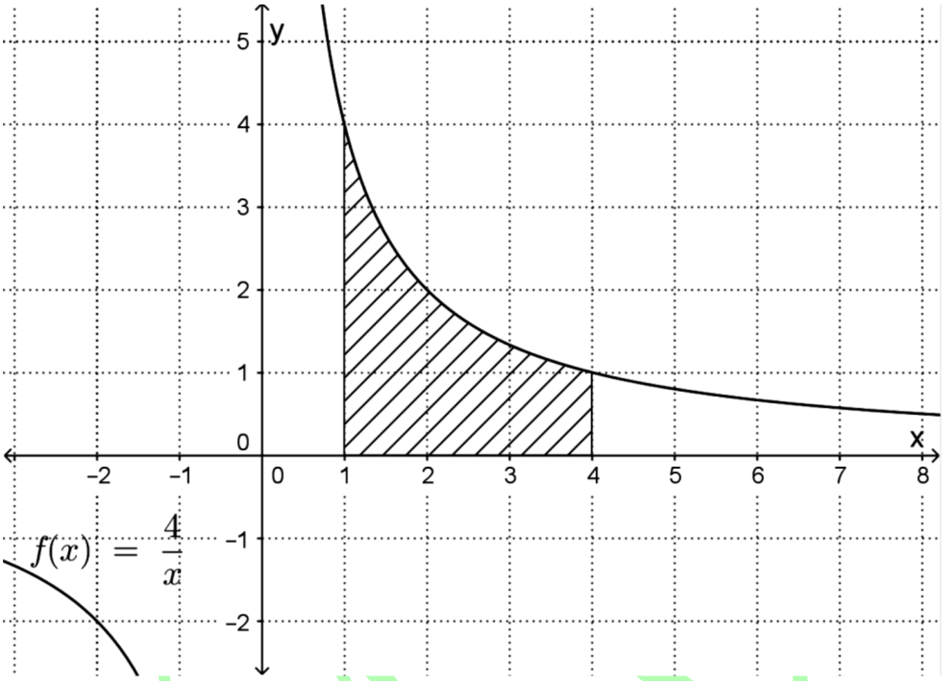
28. $\int \cos x \, dx = \sin x + c$

29. $\int \tan x \, dx = \ln \sec x + c$

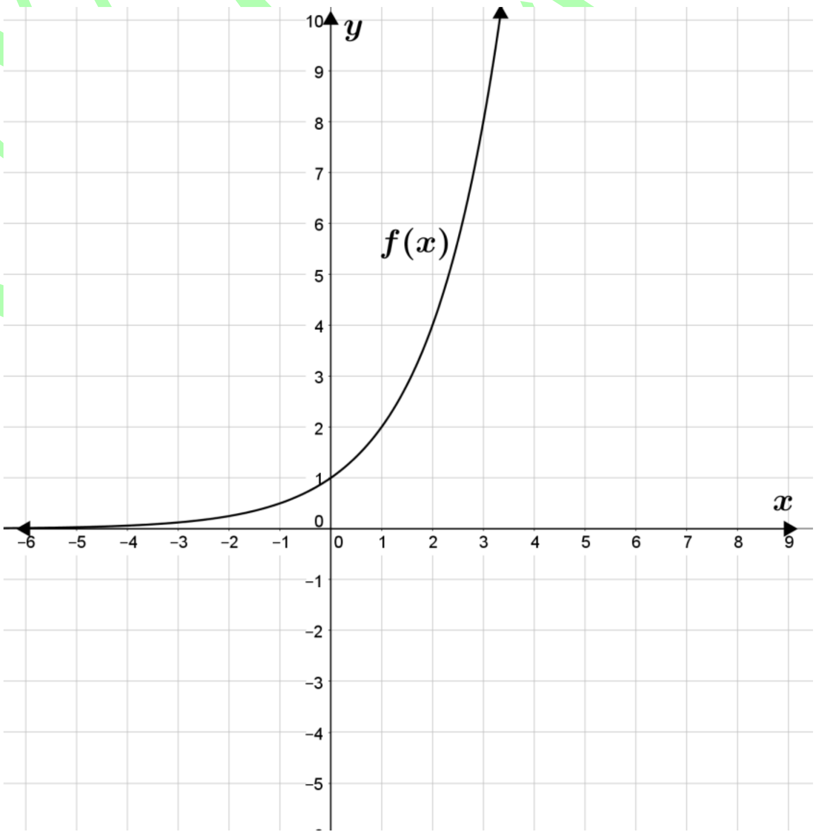
30. $[f(x)]_a^b = f(b) - f(a)$

ADDENDUM A **EXAMINATION NUMBER:**

QUESTION 3.3.1



QUESTION 4.4.5



ADDENDUM B **EXAMINATION NUMBER:**

QUESTION 5.2.4