higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First paper)
NQF LEVEL 4

21 February 2020

This marking guideline consists of 12 pages.

INSTRUCTIONS TO MARKERS

1. Mathematics lends itself to various problem-solving methods and therefore several alternative answers are possible. Allocate marks for all mathematically correct answers.

2. The answers provided in this marking guideline are not exhaustive and markers are requested to discuss all alternative solutions before marking. The marking guideline strives to minimise discretionary marking to ensure consistency.

3. Implement 'follow-up' marking. In other words, allocate marks for accurate steps following an error, provided that the responses are consistent with the error. This is referred to as consistent accuracy (CA) marks. Be careful of double errors that result in an accurate answer.

4. Marks are allocated per step, but if a student omitted a step and there is evidence or reason to believe that he/she could derive the next step mentally, the mark must still be given to the student.

5. Take note that marks have been allocated for simplification in certain questions. This is to identify high achievers.

6. All steps must be marked and not only the final answer. Ticks must correspond with the total.

7. If the marker comes across a novel or unusual solution please discuss it with the chief marker before proceeding.

8. Ensure that you have marked answers to every subquestion.

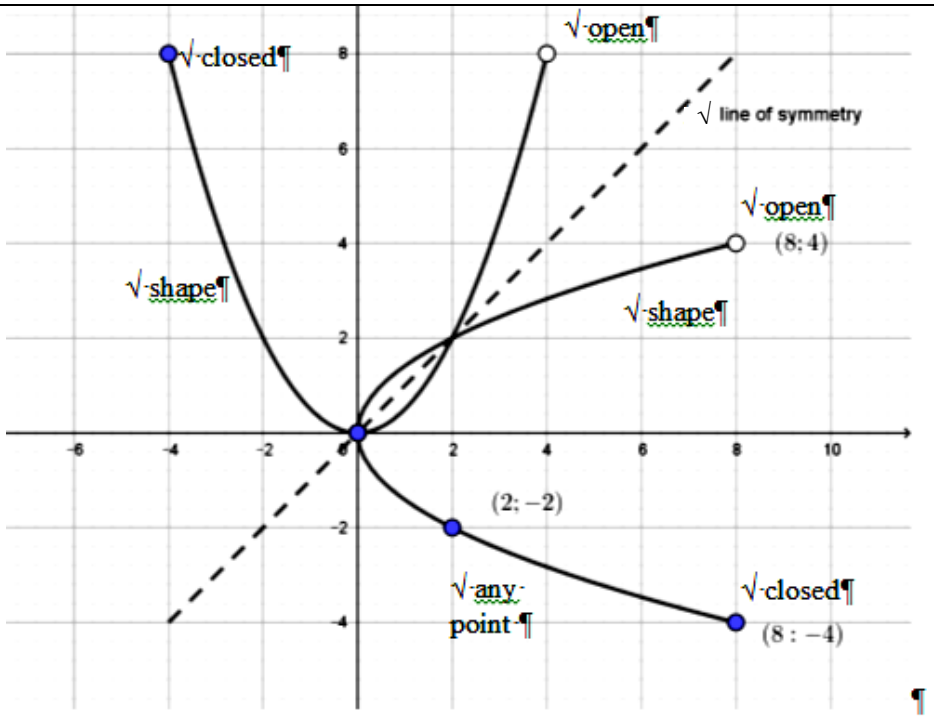
QUESTION 1

1.1	1.1.1	B		
	1.1.2	A		
	1.1.3	C		
	1.1.4	B		
	1.1.5	C		
		(5 × 1)		(5)
1.2	$13\ cis\ (-22{,}62^0) + (5 - 12i)$ $= 12 - 5i + 5 - 12i$✓ $= 17\sqrt{} - 17i\sqrt{}$	1 for changing to rectangular (A) ½ for 17 and ½ for -17i (CA)		(2)
1.3	$\frac{1+i}{1-i}\times\frac{1+i}{1+i}\sqrt{} + \frac{1}{1+i}\times\frac{1-i}{1-i}\sqrt{}$ $= \frac{2i}{2} + \frac{1-i}{2}$✓ $= \frac{i+1}{2} = \frac{1}{2} + \frac{1}{2}i$✓ **Alternative:** $\frac{(1+i)(1+i)+1(1-i)}{(1+i)(1-i)}$ $= \frac{1+2i+i^2+1-i}{2}$ $= \frac{1+i}{2}$ $= \frac{1}{2} + \frac{1}{2}i$	1 for rationalising two denominators (A) 1 for simplifying (A) 1 for answer (CA) 1 for LCD and adding 1 for correctly expanding numerator (CA) 1 for answer (CA)		(3)

1.4	$\frac{(4-3i)^2}{(5cis135^0)^3}$ $=\frac{(5cis-36,87^0)^2}{(5^3 cis(3\times135^0)}$ ✓ $=\frac{25cis-73,74^0}{125cis405^0}$ ✓ $=\frac{25}{125}cis(-73,74^0-405^0)$ ✓ $=\frac{1}{5}\surd cis(-478,74^0)\surd=0,2cis(241,26^0)$ $=\frac{1}{5}\text{cis}(241,26^0)$ ✓	$\frac{(4-3i)(4-3i)}{(5cis135^0)}$ $=\frac{16-24i+9i^2\surd}{5^3 cis(3\times135^0)}$ $=\frac{7-24i\surd}{(125cis405^0)}$ $=\frac{25cis(-73,74^0)}{125cis405^0}$ ✓ $=\frac{25}{125}cis(-73,74^0-405^0)$ ✓ $=\frac{1}{5}\surd cis(-478,74^0)\surd$ $\frac{1}{5}\text{cis}241,26^0$ ✓	Convert polar ✓ or Simplify √ √ Apply De Moivre Division Answer with negative argument Answer with positive argument		(5)
1.5	$yi^5+3i+x^2i^4+yxi^2=-9i^3+14i^6$ $\therefore yi^{\surd}+x^2+3i^{\surd}-yx^{\surd}=-14^{\surd}+9i$ $\therefore yi+3i=9i\surd$ $\therefore y=6\surd$.....1 and $x^2-yx+14=0\surd$ .....2 from 1 : $y=6$ Substitute in ..2 *If* $y=6,\ x^2-6x+14=0$ $\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\therefore x=\frac{6\pm\sqrt{36-4(14)}}{2}$ $x=\frac{6\pm\sqrt{-20}}{2}\surd=\frac{6\pm4,472i}{2}$ $\therefore x=3+2,24i^{\surd}$ *or* $x=3-2,24i^{\surd}$ *OR* $x=3+\sqrt{5}i,\ x=3-\sqrt{5}i$	Simplify i^n: ½ per two correct ½ for equation *i* ½ for correct *y*-value ½ for equation 2 ½ for $x=\frac{6\pm\sqrt{-20}}{2}$ ½ each for two *x*-values			(5)
					[20]

QUESTION 2

2.1	$2x^3-8x^2+4$ $x=2$ $2(2)^3-8(2)^2+4=-12$✓ The remainder is -12.✓		1 for substituting $x=2$ (A) 1 for answer (CA)		(2)
2.2	$2x^3-17x^2+ax+b\div(x-1)$ $\therefore 2(1)^3-17(1)^2+a(1)+b=-9$√ $\therefore -15+a+b=-9$ $\therefore a+b=6$ ...equation.... 1√ $2x^3-17x^2+ax+b\div(x-5)$ $\therefore 2(5)^3-17(5)^2+a(5)+b=15$√ $\therefore 5a+b=190$ equation...2√ Equation $1-2$ $a+b-(5a+b)=-184$ $-4a=-184$ $\therefore a=46$✓ Substitute a in equation 1 $46+b=6$ $\therefore b=-40$✓		½ for substituting $x=1$ (A) ½ for equation 1 (CA) ½ for substituting $x=5$ (A) ½ for equation 2 1 for a-value (CA) 1 for b-value (CA)		(4)
2.3	2.3.1	(a) False (b) False (c) False			
			(3 × 1)		(3)
	2.3.2	Points (-1; 4) and (3; -8) $p=\text{m}=\frac{-8-4}{3+1}=-3$ ✓ $4=-3(-1)+c$ $q=c=1$ ✓ $y=-3x+1$ $x=-3y+1$ √ $y=-\frac{1}{3}x+\frac{1}{3}$ √ $g^{-1}(x)=-\frac{1}{3}x+\frac{1}{3}$ See alternative on next page.	1 for correct p value (A) 1 for correct q-value (A) ½ for inverse as x = … (CA) ½ for answer (CA)		(3)

		Alternative: $m=\frac{-8-4}{3+1}=-3$ √ $\therefore y-4=-3(x+1)$ $\therefore y=-3x+1$ √ $\therefore x=-3y+1$ ✓ $\therefore y=-\frac{x}{3}+\frac{1}{3}$ ✓	*Re-allocate ticks in line with explanation alongside*	1 for correct p or m-value (A) 1 for correct q or c-value (CA) (CA) ½ for inverse as x = … (CA) ½ for answer (CA)		
	2.3.3	$y=a^x$ $(2,9)$ lies on f $9=a^2$ $\therefore a=3$ ✓ $y=3^x$ √ $f^{-1}: x=3^y$ √		1 for a value (A) ½ for equation of f (CA) ½ for inverse (CA)		(2)
	2.3.4	$\{y: 0<y<\infty; y\in R\}$ ✓ or $y>0$ where $y\in R$		1 mark for correct answer (A)		(1)
	2.3.5	$f(x)=g(x)$ at $(0;1)$ ✓		1 mark for correct answer (A)		(1)
2.4	$h(x)=\frac{x^2}{2}$,where $x\in[-4;4)$					
						(4)
						[20]

QUESTION 3

3.1	$f(x)=-2x^3$ $f(x+h)=-2(x+h)^3$ $=-2(x^3+3x^2h+3x\,h^3+h^3)$ $=-2x^3-6x^2h-6x\,h^2-2\,h^3$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-2x^3-6x^2h-6x\,h^2-2\,h^3-(-2x^3)}{h}\surd$ $=\lim_{h\to 0}\frac{-2x^3-6x^2h-6x\,h^2-2\,h^3+2x^3}{h}\surd$ $=\lim_{h\to 0}\frac{-6x^2h-6x\,h^2-2\,h^3\surd}{h}$ $=\lim_{h\to 0}\frac{h(-6x^2-6x\,h-2h^2)}{h}$ $=\lim_{h\to 0}-6x^2-6x\,h-2h^2\ \surd$ $=-6x^2\checkmark$	Show all steps. No steps no marks. Substitution√ **Change** $3xh^3$ to $3xh^2$ Expanding√ Simplifying numerator√ Correct simplification dividing by h√ Correct answer✓			(3)
3.2	3.2.1	$y=6x^2(3x)+e^{4x}+\frac{4}{x}+2^3$ $y=18x^3+e^{4x}+\frac{4}{x}+2^3$ $\frac{dy}{dx}=54x^2\surd+4e^{4x}\surd-\surd\frac{4}{x^2}\checkmark+0\surd$	Allocate marks as indicated (A)		(3)
	3.2.2	$y=2\ln 3x-3e^{5x}-\frac{3x^{-\frac{1}{2}}}{5}+2x^{\frac{3}{2}}$ $\therefore\frac{dy}{dx}=\frac{6}{3x}-15e^{5x}+\frac{3}{10}x^{-\frac{3}{2}}\surd+3x^{\frac{1}{2}}\surd$ $\therefore\frac{dy}{dx}=\frac{2}{x}\surd-15e^{5x}\surd+\frac{3}{10\sqrt{x^3}}\surd+3\sqrt{x}\surd$	Two ½ marks in step 2 should be allocated for conversions in step 1 ½ per term as indicated in step 3 (A) **NOTE:** $\frac{3}{10\sqrt{x^3}}$ may be written as $\frac{3}{10x\sqrt{x}}$		(3)
	3.2.3	$y=\frac{x^3+3}{\sin x}$ $\frac{dy}{dx}=\frac{vu'-uv'}{v^2}\surd$ $\frac{dy}{dx}=\frac{\sin x(3x^2)\checkmark-(x^3+3)\cos x\checkmark}{(\sin x)^2\surd}$	½ for use of quotient rule (A) 1 for first term (A) 1 for second term (A) ½ for v^2 (A)		(3)
					[12]

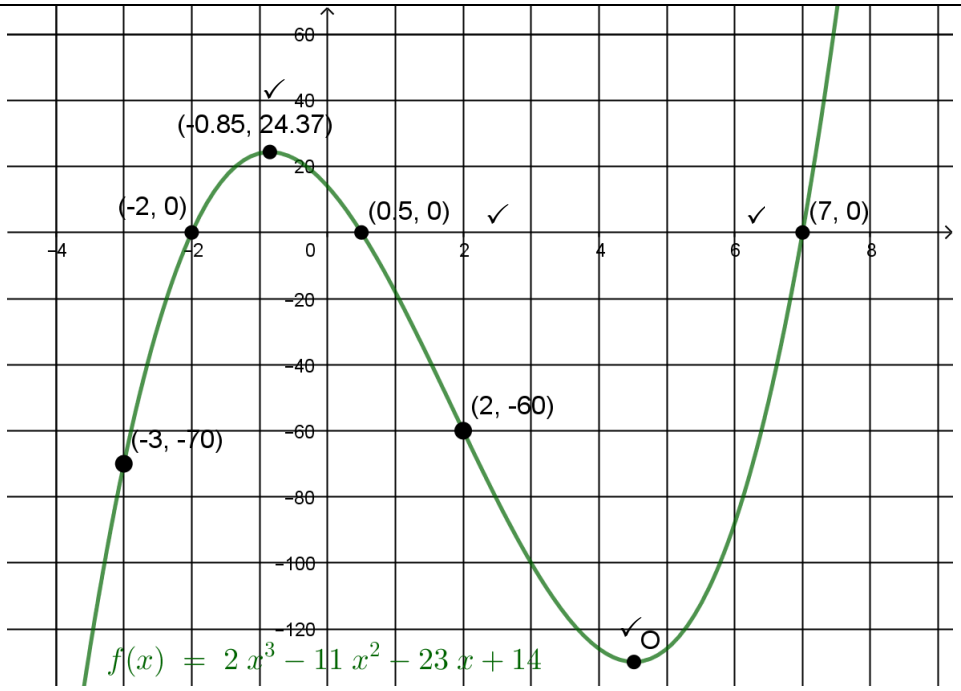
QUESTION 4

4.1	4.1.1	$\int ydx = \frac{1}{3}x^2\sqrt{-\cos x\sqrt{+4\ln x\sqrt{+2x\sqrt{+c}}}}$	-½ if c is not included		(2)
	4.1.2	$\int(4x^2 - \frac{3}{\sqrt{x}} + 4e^{2x} + 9\cos 3x)dx$ $\int(4x^2 - 3x^{-\frac{1}{2}}\sqrt{+4e^{2x}+9\cos 3x)dx}$ $= \frac{4x^3}{3} - 3\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right)\sqrt{+\frac{4e^{2x}}{2}+\frac{9\sin 3x}{3}+c}$ $= \frac{4}{3}x^3\checkmark - 6\sqrt{x}\checkmark + 2e^{2x}\sqrt{+3\sqrt{\sin 3x\sqrt{+c\sqrt{}}}}$	Allocate according to ticks (A)		(5)
4.2	$\int_{-1}^{2}(4)dx$ $= 4x\Big\|_{-1}^{2}\checkmark$ $= 8+4 = 12\checkmark$		1 for integral (A) 1 for answer (CA)		(2)
4.3	4.3.1	$A = \int_a^b -ydx\checkmark$ $A = \int_{-70°}^{70°}(\sec^2 x)\,dx$ $A = \tan x\Big\|_{-70}^{70}\checkmark$ $A = \tan 70^0 - \tan(-70^0)$ $A = 2,747 - (-2,747)$ $A = 5,49\checkmark$	1 for -y (reflecting about x-axis) (A) 1 for integral (A) 1 for answer (CA)		(3)
	4.3.2	Total area $= 4\times 5,49 = 21,96 \approx 22\checkmark$	Allocate marks as indicated.		(1)
					[13]

QUESTION 5

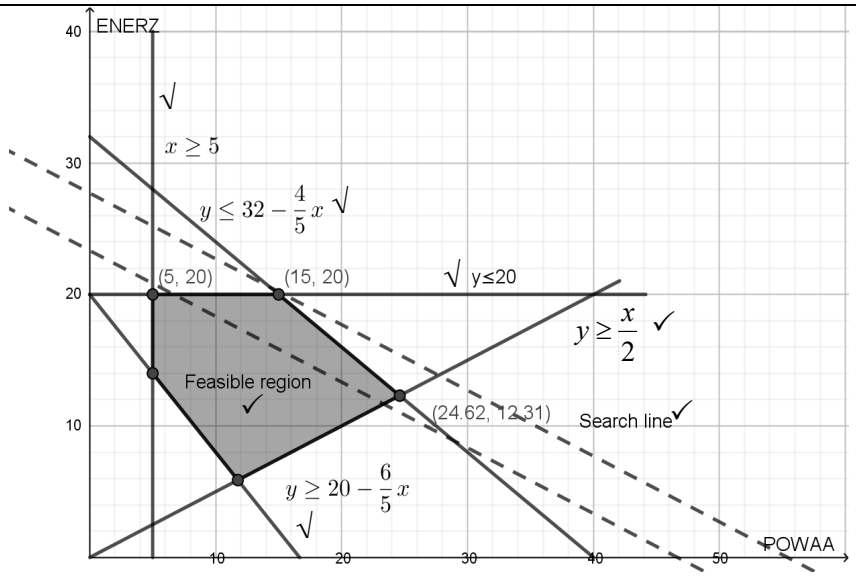
5.1	$s = 2t^3 - 13t^2 + 5t + 6$				
	5.1.1	$(4;-50)\ s = 2(4)^3 - 13(4)^2 + 5(4) + 6 = -54 \neq -50$ He will not hit the trout $(2;-20)\ s = 2(2)^3 - 13(2)^2 + 5(2) + 6 = -20$✓ He will hit the carp✓	1 for calculation (A) 1 for conclusion (A)		(2)
	5.1.2	$t = 1$ $f(t) = 2(1)^3 - 13(1)^2 + 5(1) + 6 = 0 \therefore t - 1$ *is a factor*✓ $\therefore 2t^3 - 13t^2 + 5t + 6 = (t-1)(at^2 + bt + c)$ $\therefore 2t^3 - 13t^2 + 5t + 6 = at^3 - at^2 + bt^2 + ct - bt - c$ Inspection : t^3.....: $2t^3 = at^3 \ \therefore a = 2$ constant :$6 = -c \quad \therefore c = -6$ t^2.....: $-13 = -a + b$ but $a = 2$ $\therefore -13 = -2 + b \therefore b = -11$ $\therefore 2t^3 - 13t^2 + 5t + 6 = (t-1)(2t^2 - 11t - 6)$✓ $\therefore 2t^3 - 13t^2 + 5t + 6 = (t-1)(2t+1)(t-6)$✓ $B = 1$✓ $C = 6$✓ **Alternative:** Use the factor $(t - 1)$ from above $t-1\overline{)\ 2t^3 - 13t^2 + 5t + 6}$ $2t^2 - 11t - 6$ ✓ $2t^3 - 2t^2$ $-11t^2 + 5t$ $11t^2 + 11t$ $-6t + 6$ $-6t + 6$ 0 $\therefore 2t^3 - 13t^2 + 5t + 6 = (t-1)(2t^2 - 11t - 6)$✓ $\therefore 2t^3 - 13t^2 + 5t + 6 = (t-1)(2t+1)(t-6)$✓ $\therefore B = 1$✓ *and* $C = 6$✓ **Alternative:** Division using coefficients only	1 for the first factor (A) 1 for the quadratic factor (A) 1 for factorising quadratic (A) 1 for B and 1 for C (CA) 1 for $t - 1$ 1 for the quadratic factor (A) 1 for factorising quadratic (A) 1 for B and 1 for C (CA)		(5)

	5.1.3	5 seconds✓	1 for answer (CA)		(1)
	5.1.4	$s = 2t^3 - 13t^2 + 5t + 6$ $\frac{ds}{dt} = 6t^2 - 26t + 5\surd$ $\frac{d^2s}{dt^2} = 12t - 26\surd = 0$ $t = \frac{26}{12} = \frac{13}{6} \approx 2{,}17\sec\surd$ $s = 2\left(\frac{13}{6}\right)^3 - 13\left(\frac{13}{6}\right)^2 + 5\left(\frac{13}{6}\right) + 6$ $s \approx -23{,}97\surd$ Point of inflection is $\left(\frac{13}{6}; -23{,}97\right)$	½ for first derivative ½ for second derivative ½ for *t*-value ½ for *s*-value		(2)
	5.1.5	$s = 2t^3 - 13t^2 + 5t + 6$ $\frac{ds}{dt} = 6t^2 - 26t + 5 = 0$✓ $t = \frac{26 \pm \sqrt{(-26)^2 - 4(6)(5)}}{12}$ $t = 2{,}17 \pm 1{,}96$ $t = 0{,}2\surd$ *or* $t = 4{,}1\surd$ $s = 2(4{,}13)^3 - 13(4{,}13)^2 + 5(4{,}13) + 6$ $s = -54{,}2m$✓ Depth = 54,2 m	1 for derivative = 0 (A) ½ each for *t*-values (CA) 1 for depth (CA)		(3)
5.2	5.2.1	$f(x) = 0$ $\therefore (x+2)(x-7)(2x-1) = 0$ $x = -2,\ x = 7\ x = \frac{1}{2}$ $\therefore A = -2\ B = \frac{1}{2}$ and $C = 7$✓	1 mark for all three, ½ for two (A)		(1)
	5.2.2	$f(x) = 2x^3 - 11x^2 - 23x + 14$ $f'(x) = 6x^2 - 22x - 23\surd$ $f''(x) = 12x - 22\surd$ $At\ x = -0{,}85:\ f''(-0{,}85) = 12(-0{,}85) - 22 < 0$ $\Rightarrow$ maximum TP✓ $At\ x = 4{,}52\quad f''(4{,}52) = 12(4{,}52) - 22 > 0$ $\Rightarrow$ minimum TP✓	½ for first derivative (A) ½ for second derivative (CA) 1 for max (CA) 1 for min (CA)		(3)

	5.2.3	 1 mark each for the two turning points ½ each for the three *x*-intercepts. *Insert √ at the x intercepts. Remove two ✓on the x axis* ½ for shape			(4)
					[21]

QUESTION 6

6.1	6.1.1	$y = \frac{60}{40}x$ or $y = \frac{3}{2}x$ ✓	1 for answer (A)		(1)
	6.1.2	$x + y < 100$ ✓, $y \geq 40$ √, $y \leq f(x)$ √	Deduct ½ for $x \geq 10$		(2)
6.2	6.2.1	$300x + 250y \geq 5000$✓ $200x + 250y \leq 8000$✓ $x \leq 2y$ √ $\therefore y \geq \frac{x}{2}$ √ $x \geq 5$ √ $y \leq 20$ √	Allocate according to ticks		(4)

	6.2.2	(a)	 (½ for each of the solid lines, except $y = \frac{x}{2}$ which gets 1 mark)		(3)
		(b)	See graph.		(1)
		(c)	$P = 8x + 16y$✓		(1)
		(d)	See graph.		(1)
		(e)	Max profit at (15;20) $P = 8x + 16y$ $P = 8(15) + 16(20) = 440$		(1)
					[14]

TOTAL: 100