higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(Second Paper)
NQF LEVEL 3

(10501053)

6 November 2019 (X-Paper)
09:00–12:00

REQUIREMENTS: Graph paper

Nonprogrammable calculators may be used.

This question paper consists of 9 pages , 2 answer sheets and a formula sheet of 2 pages.

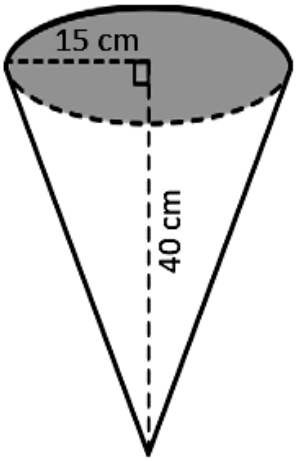
TIME: 3 HOURS
MARKS: 100

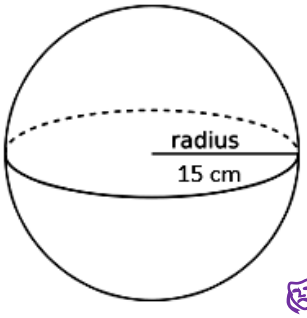
INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. used in determining the answers.
5. Round off answers to THREE decimal places unless stated otherwise.
6. Diagrams are NOT drawn to scale.
7. Write neatly and legibly.

QUESTION 1

1.1 The kindergarten learners at a local school built the following project by using cardboard and a ball. They rolled the cardboard into the shape of a right cone and placed the ball over the circular base. The cone has a radius of 15 cm and a height of 40 cm. The ball has a radius of 15 cm.



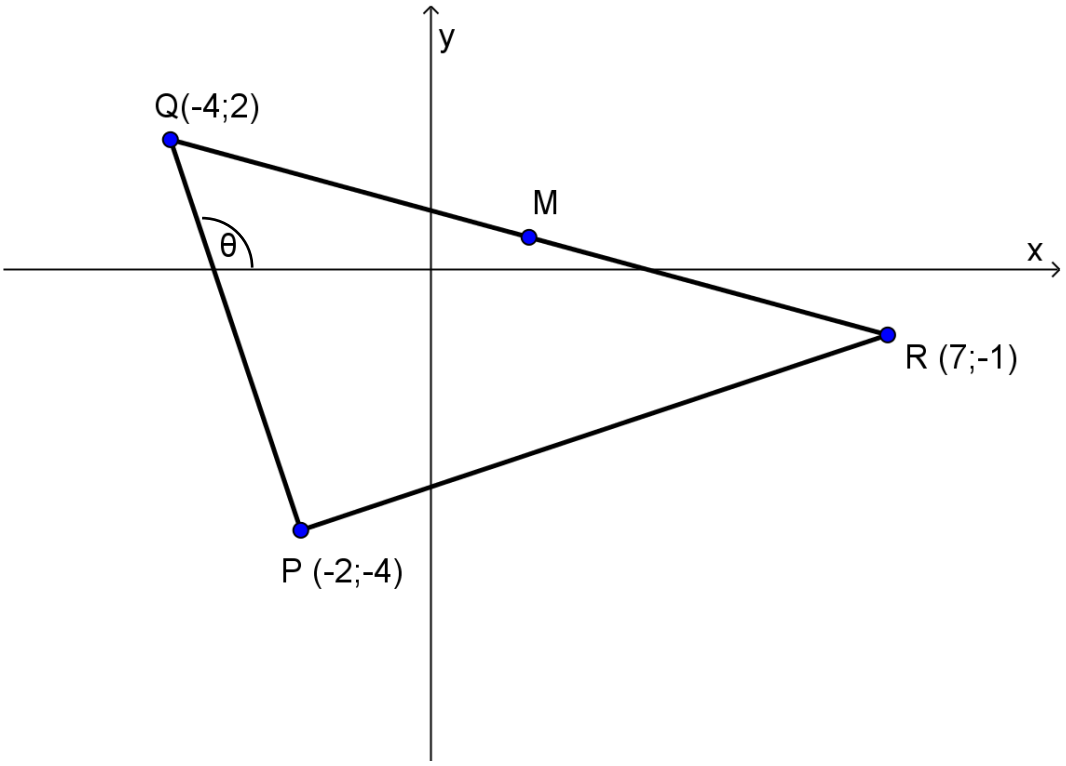


1.1.1 Determine the volume of the ball (sphere). (2)

1.1.2 Determine the volume of the cone. (2)

1.1.3 Determine the surface area of the right cone when closed. (5)

1.2 In the diagram below ΔPQR has vertices P(-2;-4); Q(-4;2) and R(7;-1). The angle of inclination of PQ is θ. Point M is the midpoint of QR.



1.2.1 Prove that ΔPQR is a right-angled triangle. (4)

1.2.2 Calculate the area of ΔPQR. (5)

1.2.3 Calculate the size of θ. (2)

1.2.4 Determine the coordinates of M which is the midpoint of QR. (2)

1.2.5 Hence determine the equation of line MN passing through M which is parallel to PR. (3)

[25]

QUESTION 2

2.1 Simplify the following expression without using a calculator:

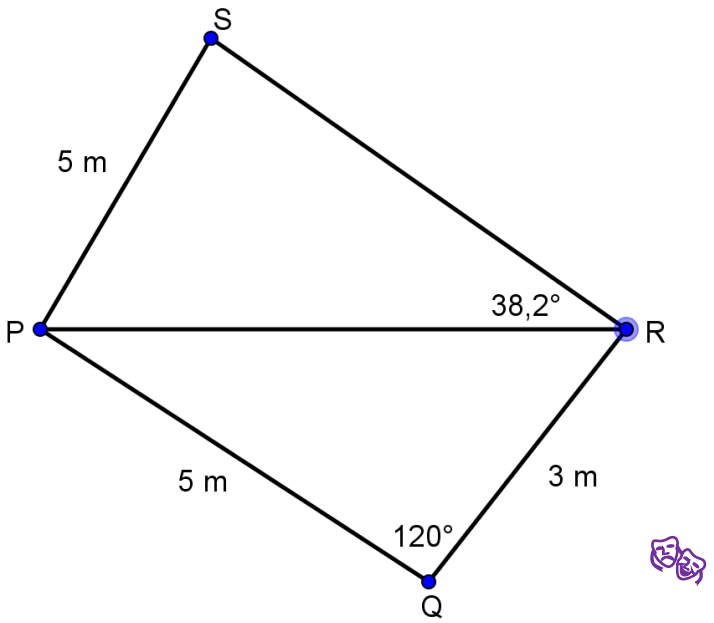
$$\frac{\sin 135°.\sin 315°.\tan 225°}{\cos 330°.\sin 120°}$$ (5)

2.2 Use trigonometric identities to prove the following:

$$\frac{2\sin^2 x}{2\tan x - 2\sin x\cos x} = \frac{\cos x}{\sin x}$$ (5)

2.3 Calculate the value/s of x if $\frac{\tan^2 x+1}{4}=1$ where $x\in[0°;360°]$. (5)

2.4 During a wrestling tournament four wrestlers are positioned at points P, Q, R and S. Their positions are represented by the following diagram where PQ = 5 metres, QR = 3 metres, PS = 5 metres, $P\hat{R}S=38,2°$ and $P\hat{Q}R=120°$.



2.4.1 The wrestler at point P wants to tag his partner at point R.

Determine the distance (PR) he will have to run to tag his partner. (4)

2.4.2 Calculate the magnitude of $\hat{S}$. (3)

2.4.3 Determine the area of triangle PQR. (3)

[25]

QUESTION 3

3.1 The number of hotdogs sold by Joseph in the first 20 days of business is given in the table below.

160	88	80	62	80
16	67	68	70	60
62	48	57	62	66
70	78	76	69	70

3.1.1 Determine the median number of hotdogs sold. (2)

3.1.2 Determine the upper and lower quartiles of the data. (4)

3.1.3 Determine the upper and lower fence values. (4)

3.1.4 Construct a box-and-whisker diagram for the above information showing any outliers. (4)

3.2 Data bundles have become a priority purchase for many South Africans. During a survey of 80 learners on their usage (in gigabytes) per month, the following table was constructed. The class width (c) is 10.

Data usage (in gigabytes)	**Frequency** (f_i)
$0 \le x < 10$	7
$10 \le x < 20$	35
$20 \le x < 30$	12
$30 \le x < 40$	18
$40 \le x < 50$	5
$50 \le x < 60$	3
TOTAL:	80

3.2.1 Complete the following frequency-distribution table on ANSWER SHEET 1 (attached). The first two rows have been done for you. The class width (c) is 10.

Classes (data usage)	**Frequency** (f_i)	**Midpoint** (x_i)	$f_i\,x_i$	**< Cumulative frequency**
$0 \le x < 10$	7	5	35	7
$10 \le x < 20$	35	15	525	42
$20 \le x < 30$	12	25		
$30 \le x < 40$	18	35		
$40 \le x < 50$	5	45		
$50 \le x < 60$	3	55		
TOTAL:	80		$\sum f_i x_i =$	

(4)

Use the table in QUESTION 3.2.1 to answer the following questions:

3.2.2 Calculate the mean for the data used. (2)

3.2.3 Determine the modal value for the data using the formula:

$$Mo = l + \frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}} \times c$$ (3)

3.2.4 Use ANSWER SHEET 2 (attached) to sketch the ogive curve using the less than cumulative frequency and the upper class limit. (5)

3.2.5 Use the ogive curve from QUESTION 3.2.4 to determine the median value for the data used. Show the median value on the ogive curve. (2)

[30]

QUESTION 4

4.1 Westbrook Angling Club is a social club with 80 members that participate in weekly fishing tournaments.

The club expected to receive the following amounts of money for the year ended:

INCOME	
Membership fees	R19 000
Sponsorships	R6 000
Donations	R6 500
TOTAL:	**R31 500**

The club expected to have the following expenditure for the same year:

EXPENDITURE	
Rental for storing boats	R3 000
Cellphone usage	R800
Petrol for generator	R1 400
Catering	R6 000
Prize-giving function	R7 000
Year-end function	R5 000
Refreshments	R4 000
TOTAL:	**R27 200**

Listed below is the actual income and expenditure of the club for the year ended.

INCOME	
Membership fees	R19 000
Sponsorships	R7 000
Donations	R5 500
TOTAL:	**R31 500**

EXPENDITURE	
Rental for storing boats	R3 000
Cellphone usage	R 900
Petrol for generator	R1 400
Catering	R7 000
Prize-giving function	*Will be answered as 4.1.4 in income and expenditure table below*
Year-end function	R7 000
Refreshments	R3 000
TOTAL:	**R29 800**

Use the income and expenditure statements to complete the table below. Write only the answer next to the question number (4.1.1–4.1.7) in the ANSWER BOOK.

ITEM	BUDGETED AMOUNT	ACTUAL AMOUNT	VARIANCE
INCOME/REVENUE			
Membership fees	19 000	**(4.1.1)**	0
Sponsorships	6 000	7 000	**(4.1.2)**
Donations	6 500	5 500	**(4.1.3)**
TOTAL INCOME:	**31 500**	**31 500**	**0**
EXPENSES			
Rental for storing boats	3 000	3 000	0
Cellphone usage	800	900	-100
Petrol for generator	1 400	1 400	0
Catering	6 000	7 000	-1000
Prize-giving function	7 000	**(4.1.4)**	**-500**
Year-end function	5 000	7 000	-2000
Refreshments	4 000	3 000	+1000
TOTAL EXPENSES:	**27 200**	**29 800**	**-2600**
SURPLUS/DEFICIT:	**(4.1.5)**	**(4.1.6)**	**(4.1.7)**

(7 × 1) (7)

4.2 If Moses received R5 000 after a period of two years for money invested at 10,25% per annum compounded quarterly, determine the value of his initial investment. (3)

4.3 When Nelisiwe was born, her father started an investment for her college fund. He invested R50 000 in the bank and was given an interest rate of 12% compounded quarterly. After eight years he had to pay for an emergency operation and withdrew an amount of R10 000. At this time the bank renegotiated his investment and now offered him an interest rate of 10% compounded annually. Six years later (in the 14th year) he deposited an amount of R30 000 in the investment and his investment was now changed to 16% per annum simple interest. The investment continued for a further four years.

4.3.1 Draw a timeline for the investment described in the scenario. (3)

4.3.2 Calculate the value of the investment after 18 years. (7)

[20]

TOTAL: 100

FORMULA SHEET

1. Slant surface area of a pyramid = $\frac{1}{2}a\,l\,n$ or $\frac{1}{2}l\,h_s\,n$ (where n = number of sides)
2. Surface area of triangular pyramid = $\frac{1}{2}bh+\frac{1}{2}pl$ where p = perimeter of the base
3. Surface area of a pyramid with an equilateral triangle as base = $\frac{\sqrt{3}}{4}s^2+\frac{1}{2}pl$
4. Surface area of an equilateral triangular pyramid = $4\times\frac{\sqrt{3}}{4}s^2$
5. Surface area of a square pyramid = $b^2+\frac{1}{2}pl$
6. Surface area of a regular hexagonal pyramid = $\frac{3\sqrt{3}}{2}b^2+\frac{1}{2}pl$
7. Volume of a pyramid = $\frac{1}{3}$(area of base) $\times\perp$ height
8. $s=\frac{1}{2}(a+b+c)$ *and* a,b,c *are the sides of the triangle*
9. $A=\sqrt{s(s-a)(s-b)(s-c)}$
10. Circumference of a circle = $2\pi r$
11. Area of a curved surface of a cone = $\pi r\,l$ or $\pi r\,h_s$
12. Slant height of a cone = $l=\sqrt{h^2+r^2}$ or $h_s=\sqrt{\perp h^2+r^2}$
13. Volume of a cone = $V_{cone}=\frac{1}{3}\pi r^2\times_{\perp} h$
14. Area of a sphere = $A=4\pi r^2$
15. Volume of a sphere = $V=\frac{4}{3}\pi r^3$
16. $m=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$
17. $(x_m\,;y_m)=\left(\frac{x_1+x_2}{2}\,;\frac{y_1+y_2}{2}\right)$
18. $\theta=\tan^{-1}m$
19. Distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

20. $\frac{\sin\theta}{\cos\theta} = \tan\theta$

21. $\sin^2\theta + \cos^2\theta = 1$

22. $\frac{\sin\hat{A}}{a} = \frac{\sin\hat{B}}{b} = \frac{\sin\hat{C}}{c}$

23. $a^2 = b^2 + c^2 - 2bc\ \cos\hat{A}$

24. $A = \frac{1}{2}ab\sin\hat{C}$

25. $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

26. $\bar{x} = \frac{\sum f_i\, x_i}{n}$

27. $Q_{j\ position} = \frac{j}{4}(n+1)$

28. $Q_{IQR} = Q_3 - Q_1$

29. Upper Fence = $Q_3 + 1{,}5(Q_{IQR})$

30. Lower Fence = $Q_3 - 1{,}5(Q_{IQR})$

31. $Me = l + \frac{\left(\frac{n}{2} - F\right)}{f} \times c$

32. $Mo = l + \frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}} \times c$

33. $I = A_0 \times \frac{r}{100} \times t$ or $I = \frac{P\,r\,t}{100}$ or $A_t = P(1+in)$

34. $A_t = A_0\left(1 + \frac{r}{100 \times m}\right)^{t \times m}$ or $A_t = P(1+i)^n$

35. $A_t = A_o(1 - \frac{r}{100})^t$ or $A_t = P(1-i)^n$

ANSWER SHEET 1

EXAMINATION NUMBER:

QUESTION 3.2.1

Classes (data usage)	**Frequency** (f_i)	**Midpoint** (x_i)	$f_i x_i$	**< Cumulative frequency**
$0 \le x < 10$	7	5	35	7
$10 \le x < 20$	35	15	525	42
$20 \le x < 30$	12	25		
$30 \le x < 40$	18	35		
$40 \le x < 50$	5	45		
$50 \le x < 60$	3	55		
TOTAL:	80		$\sum f_i x_i =$	

ANSWER SHEET 2

EXAMINATION NUMBER:

QUESTION 3.2.4