higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 3

(10501053)

1 November 2018 (X-Paper)
09:00–12:00

This question paper consists of 6 pages, 1 formula sheet and 3 answer sheets.

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Write neatly and legibly.

QUESTION 1

1.1 Without the use of a calculator and without conversion to polar form, simplify the given complex numbers to the standard form $a + bi$.

1.1.1 $(2 + 3i) - (5 - 5i)$ (2)

1.1.2 $\dfrac{\sqrt{30}}{\sqrt{-5}}$ (2)

1.1.3 $\left[(2\sqrt{-1})^6\right]^4$ (3)

1.1.4 $\dfrac{24i^8 - 36i^{16} + 48i^{24}}{12i^8}$ (3)

1.1.5 $(3 + 7i) - (1 + 4i)^2$ (3)

1.2 Simplify the following to the form $a + bi$:

$$\frac{1 - \sqrt{-1}}{1 + \sqrt{-1}}$$ (4)

1.3 Let $z = 1 + i$ and $w = 1 - \sqrt{3}i$ be two complex numbers.

1.3.1 Calculate the **modulus** and **argument** of z. (2)

1.3.2 Convert w to the form $\boldsymbol{r}\textbf{cis}\ \boldsymbol{\theta}$, where θ is a positive angle. (3)

1.3.3 Simplify: $z \times w$

Show all working details and leave your answer in polar form. (3)

1.3.4 Determine the value of:

$$\frac{z}{w}$$

Show all working details and leave your answer in polar form with a positive argument. (5)

[30]

QUESTION 2

2.1 Solve for x and show the solution on a number line:

$$2x^2 - 7x - 4 \leq 0 \quad (4)$$

2.2 Solve for x by completing the square:

$$3x^2 + 7x + 2 = 0 \quad (4)$$

2.3 Solve for x and y simultaneously in the following:

$$x - y = 5$$
$$x^2 + y^2 = 13 \quad (6)$$

2.4 Simplify the following:

2.4.1 $$\frac{2a - 2b}{3a + 3b} \div \frac{a^2 - 2ab + b^2}{a^2 - b^2} \quad (3)$$

2.4.2 $$\frac{2x}{x^2 - y^2} + \frac{1}{y - x} \quad (4)$$

2.5 The following constraints are given for a linear programming problem:

$$2 \leq x \leq 16$$
$$y \leq 12$$
$$x + y \leq 18$$
$$\frac{1}{2}x + y \geq 11$$

2.5.1 Use the ANSWER SHEET 1 (attached), to sketch the graph that represents the above constraints. Label each straight line drawn correctly. (5)

2.5.2 Shade in the feasible region on your graph. (1)

2.5.3 Determine the values of x and y that will maximize the objective function $P = 20x + 15y$. (3)

[30]

QUESTION 3

3.1 Given the function: $f(x) = (x-2)^2 - 4$

3.1.1 Write down the coordinates of the turning points. (1)

3.1.2 Determine the coordinates of the *x* and *y* intercepts (3)

3.1.3 Make a neat sketch graph of $y = f(x)$ on the attached ANSWER SHEET 2. Show clearly the *x* intercepts, the *y* intercept and the turning point. (3)

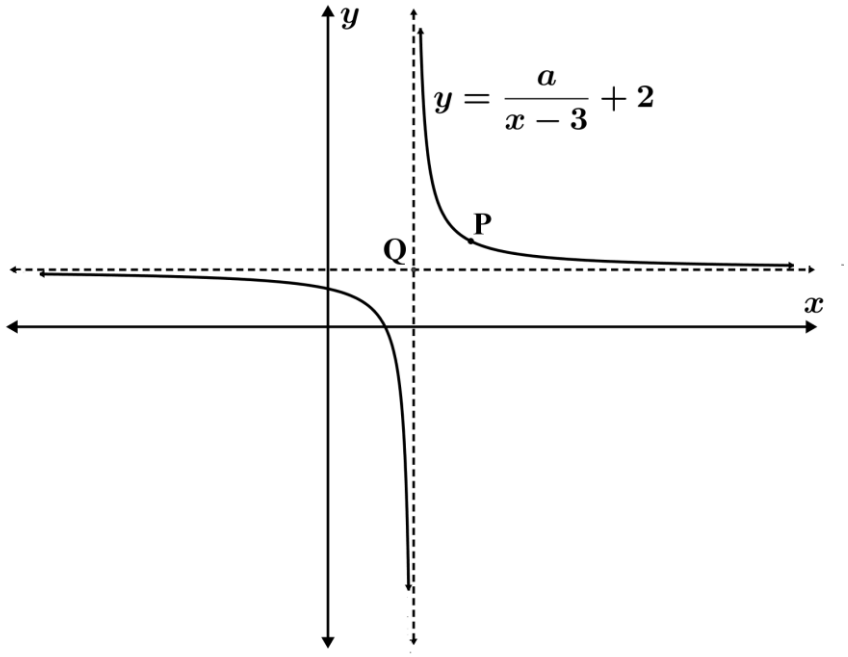
3.1.4 Write down the interval over which the function $y = f(x)$ decreases. (1)

3.1.5 Is $y = -5$ in the range of $y = f(x)$. Explain. (2)

3.2 The figure below (not drawn to scale), shows the graph of

$$y = \frac{a}{x-3} + 2$$

with P a point on the graph. Q is the point of intersection of the two asymptotes. Study the graph and answer the questions.



3.2.1 Calculate the value of *a* if the coordinates of P are $(5; 3)$ (2)

3.2.2 Write down the equation of the vertical asymptote (1)

3.2.3 Write down the coordinates of Q. (1)

3.2.4 The graph has two lines of symmetry. Determine the equation of the line of symmetry with a positive gradient. (2)

3.3 Sketch the graph of $y = -(4^{x-1}) + 4$, showing clearly the asymptote, and the *x* and *y* intercepts. Use ANSWER SHEET 3. (4)

[20]

QUESTION 4

4.1 Given that $f(x) = a + 2px$, calculate:

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$ (4)

4.2 Determine the following limit:

$$\lim_{t\to\infty} \frac{6 - t^3}{8t^3 + 4}$$ (3)

4.3 Use differentiation rules to determine $\frac{dy}{dx}$ for each of the following functions.
Leave your answer with POSITIVE exponent and in SURD form where applicable.

4.3.1 $y = \sqrt{x}(x^4 - x^2 - 2)$ (4)

4.3.2 $y = \frac{x^2 - 4}{x - 2}$ (2)

4.3.3 $y = 1 - 3x^4 - \frac{3}{x}$ (3)

4.4 An analysis of the financial statements of a coal mine indicates that when x tons of coal are extracted per day, the income (in rand) of the mine is

$$I(x) = 1\,210x - 2x^2$$

4.4.1 Determine the value of x that maximizes the income. (3)

4.4.2 Determine the maximum income. (1)

[20]

TOTAL: 100

FORMULA SHEET

1. $z = r\cos\theta + r\,j\sin\theta$

2. $z = a \pm bj$ or $a \pm bi$ where $i = j = \sqrt{-1}$

3. $r = \sqrt{a^2 + b^2}$ or $r = \sqrt{z \times \bar{z}}$

4. $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

5. $r\angle\,\theta = r\ \text{cis}\,\theta$

6. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

7. $y = ax^2 + bx + c$

8. $y = a(x - p)^2 + q$

9. $y = a(x - x_1)(x - x_2)$

10. $y = \frac{a}{(x + p)} + q$

11. $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$

12. $\frac{d}{dx}x^n = nx^{n-1}$

13. $\frac{d}{dx}k = 0$

14. $D_x[kf(x)] = kD_x[f(x)]$

15. $D_x[f(x) \pm g(x)] = D_x[f(x)] \pm D_x[g(x)]$

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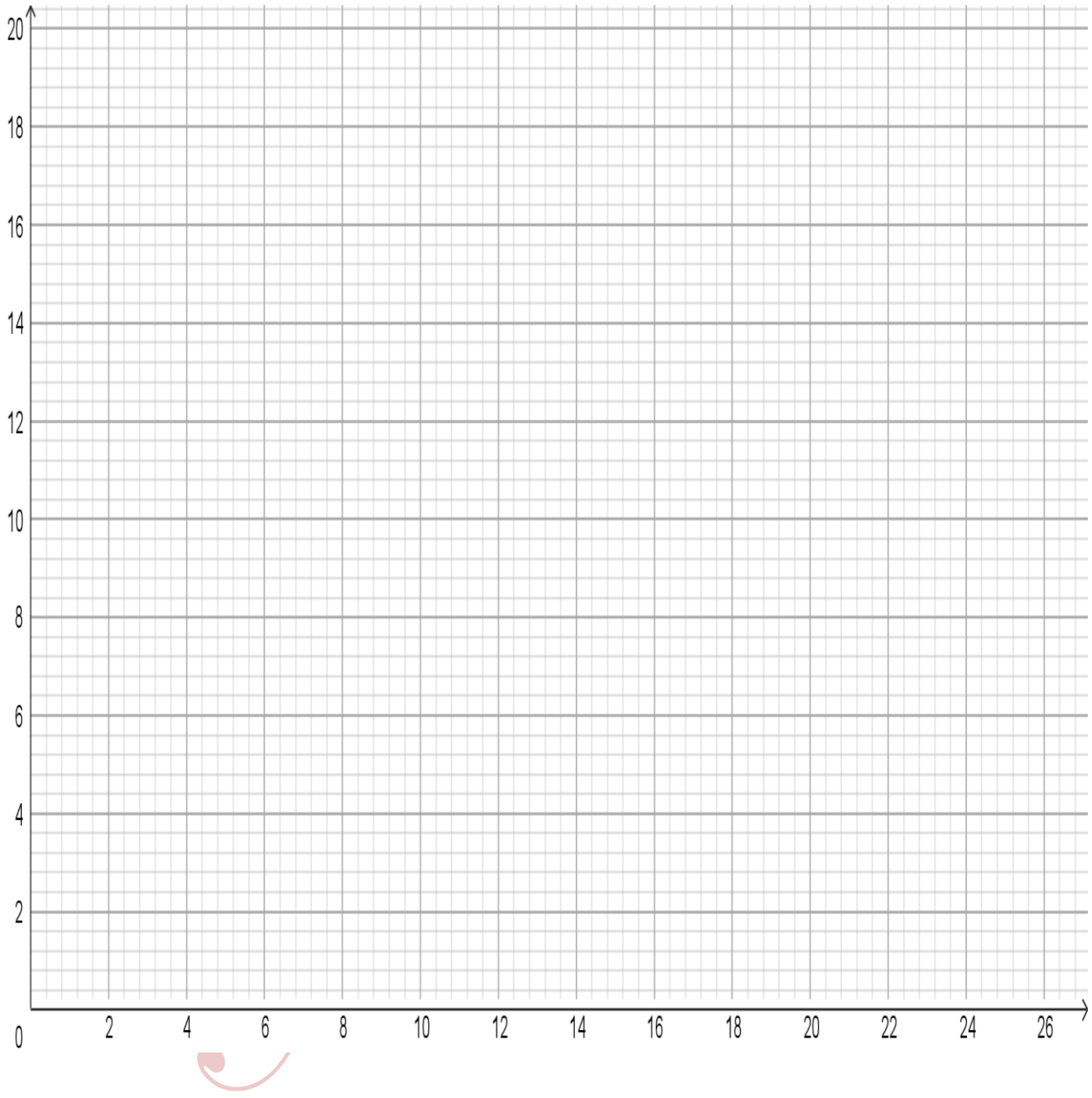
ANSWER SHEET 1

QUESTION 2.5.1



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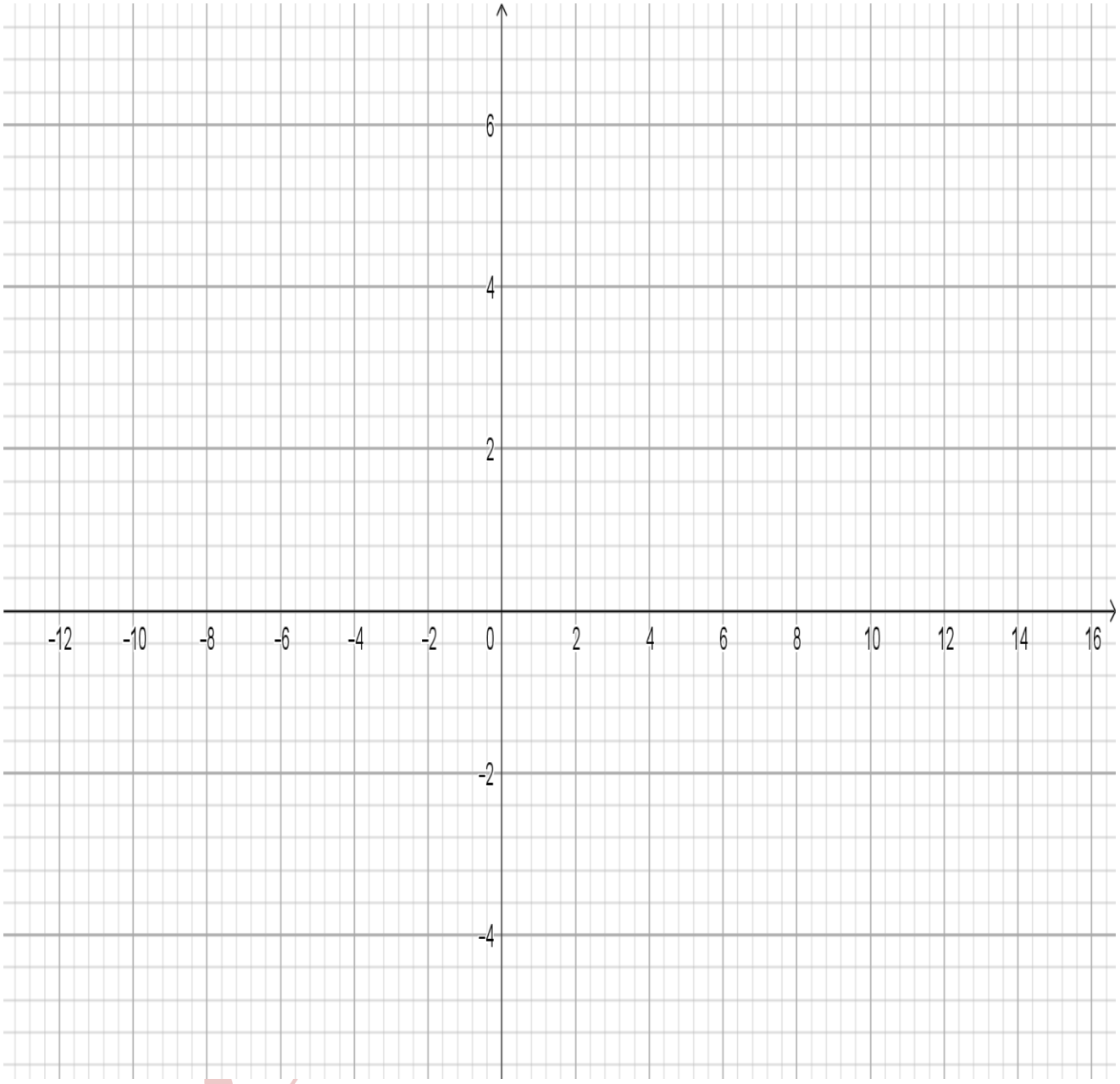
ANSWER SHEET 2

QUESTION 3.1.3



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ANSWER SHEET 3

QUESTION 3.3