higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
NQF LEVEL 3
(First Paper)

(10501053)

20 February 2018 (X-Paper)
09:00–12:00

This question paper consists of 6 pages, 1 formula sheet and 3 diagram sheets.

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Use a BLACK or a BLUE pen.
5. Leave at least THREE lines after each question.
6. Start each section on a NEW page.
7. Diagrams are NOT drawn to scale.
8. Answers should be rounded off to THREE decimals places where necessary unless stated otherwise.
9. Write neatly and legibly.

QUESTION 1

1.1 Express the following in form $a+bj$.

$\sqrt{9}+\sqrt{-144}$ (2)

1.2 Given: $z=5-3i$

1.2.1 Determine $\bar{z}$ (the conjugate of z) (1)

1.2.2 Calculate the modulus (r) and argument (θ) of z. (4)

1.2.3 Express $z=5-3i$ in the form $r\ cis\ \theta$ (polar form). (2)

1.3 Simplify the following complex numbers and leave answer in the form a + bi.

Calculators may be used.

1.3.1 $\dfrac{15-\sqrt{-25}}{5}$

1.3.2 $2i^2-4i^3$

1.3.3 $-i(i-4)$

1.3.4 $(3+4i)+(2-7i)$

(4 × 2) (8)

1.4 Rationalise the denominator and express in the form $a+bi$.

$\dfrac{3+4i}{1-2i}$ (4)

1.5 Simplify and leave the answers in standard form.

1.5.1 $(2\ cis\ 50°.3\ cis\ 40°)\div[4cis\ 30°.3cis(-10°)]$ (5)

1.5.2 $2cis\,60^\circ+3cis\,150^\circ$ (4)

[30]

QUESTION 2

2.1 Simplify the following:

2.1.1 $$\frac{3ax+3ay-5bx-5by}{3a-5b}$$ (3)

2.1.2 $$\frac{a}{a^2+10a+21}-\frac{2a}{a^2-2a-15}+\frac{3a}{a^2+2a-35}$$ (4)

2.2 Solve for x by completing the square:

$$\frac{1}{2}x^2+4x-10=0$$ (4)

2.3 Solve for x *and* y algebraically in the following simultaneous equations:

$$y=2x^2+11x+5 \text{ and } y=-2x-6$$ (4)

2.4 Solve the following inequality and give the solution in set builders notation.

$$x^2+3x<5(x+3)$$ (5)

2.5 The following constraints are given in a linear programming problem:

$$x \geq 0$$
$$y \geq 0$$
$$x+y-1 \geq 0$$
$$2x-y+2 \geq 0$$
$$2x+y-6 \leq 0$$

2.5.1 Use DIAGRAM SHEET 1 to sketch the graphs of the above constraints and shade the feasible region clearly. (4)

2.5.2 The objective function is given as $P=-5x-3y+17$. Calculate the maximum value of the function. (6)

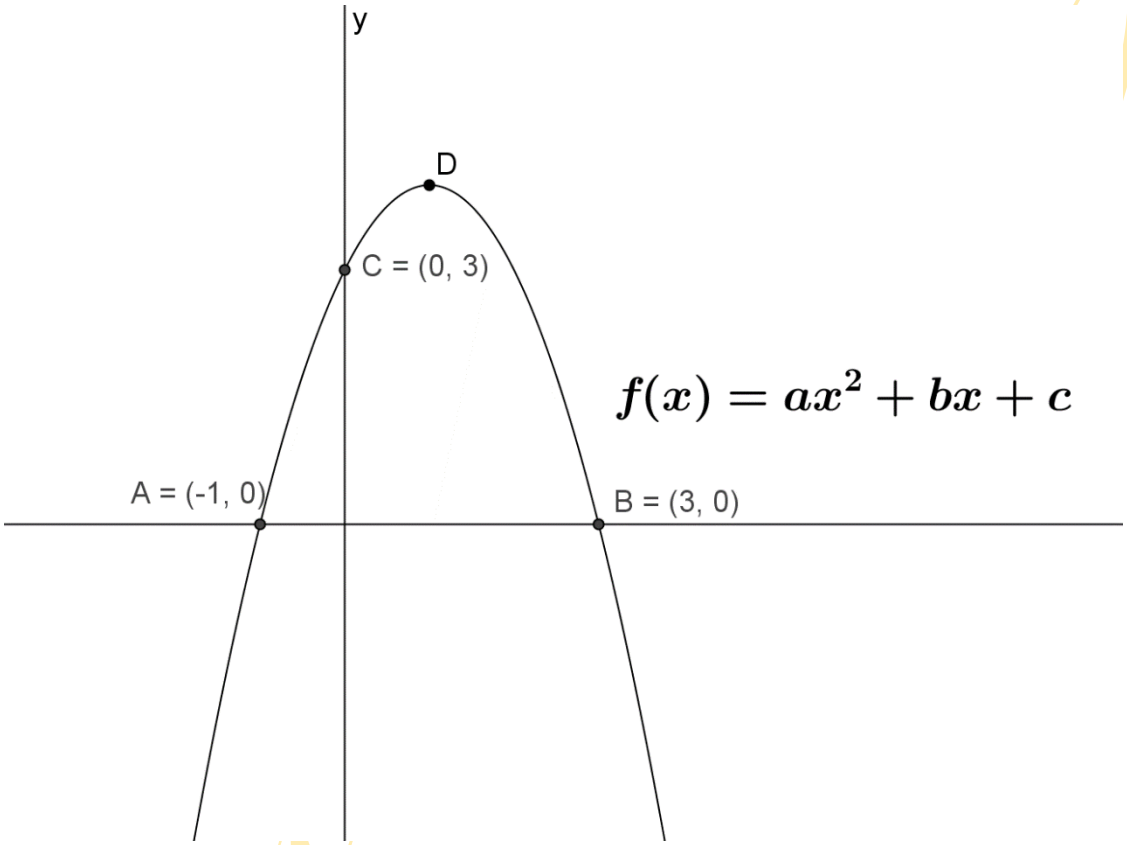
[30]

QUESTION 3

3.1 Given: $xy = 8$ and $y = -\frac{x}{8}$

Use DIAGRAM SHEET 2 to sketch the graphs of the given functions on the same set of axes. Indicate the intercepts on both axes clearly. (6)

3.2 Study the graph of the form $f(x) = ax^2 + bx + c$ and answer the questions.



3.2.1 Determine the equation of the function $f(x)$. (4)

3.2.2 Determine the coordinates of the turning point D. Show all calculations. (4)

3.2.3 Write down the range of the function $f(x)$. (1)

3.2.4 Is the graph of the function $f(x)$ continuous or discontinuous? (1)

3.3 Use DIAGRAM SHEET 3 to sketch the graph of $y = 3^{x-1} + 1$. Show the horizontal asymptote. (4)

[20]

QUESTION 4

4.1 Determine the following:

$$\lim_{x\to\infty}\frac{2x^2-5}{3x^2+x+2}$$ (3)

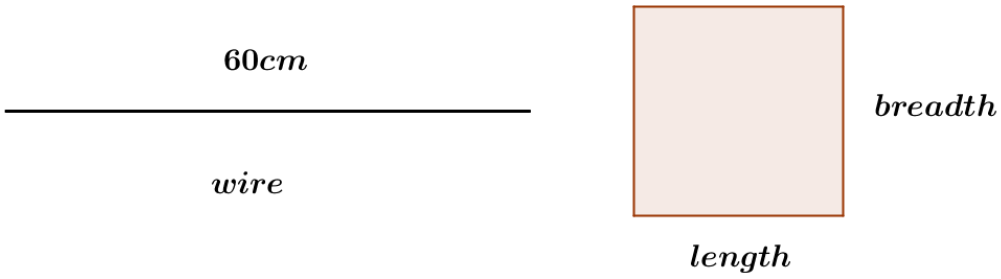
4.2 Determine $f(x)$ from first principles if $y = 3x^2$ (4)

4.3 Use differentiation rules to determine $\frac{dy}{dx}$ of the following. (Leave answers with POSITIVE exponents and in SURD form where applicable)

4.3.1 $$y=\sqrt[3]{x^2}+\frac{5}{2x^2}-(3x)^2$$ (4)

4.3.2 $$y=\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$$ (3)

4.4 As part of a school project, you have to make a rectangular frame using a piece of wire 60 cm long as shown in the diagram.



4.4.1 Let the length (l) of the rectangle be x. Express the breadth (b) in terms of x.

HINT: Perimeter = 2l + 2b (2)

4.4.2 Use calculus to determine the maximum area of the rectangle. (4)

[20]

TOTAL: 100

FORMULA SHEET

1. $z = r\cos\theta + r\ j\sin\theta$

2. $z = a \pm bj$ or $z = a \pm bi$ *where* $i = j = \sqrt{-1}$

3. $r = \sqrt{a^2 + b^2}$ or $r = \sqrt{z \times \bar{z}}$

4. $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

5. $r \angle\theta = r\ cis\ \theta$

6. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

7. $y = ax^2 + bx + c$

8. $y = a(x - p)^2 + q$

9. $y = a(x - x_1)(x - x_2)$

10. $y = \frac{a}{(x + p)} + q$

11. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

12. $\frac{d}{dx}x^n = nx^{n-1}$

13. $\frac{d}{dx}k = 0$

14. $Dx[kf(x)] = kDx[f(x)]$

15. $Dx[f(x) \pm g(x)] = Dx[f(x)] \pm Dx[g(x)]$

DIAGRAM SHEET 1	**EXAMINATION NUMBER:**												

QUESTION 2.5.1

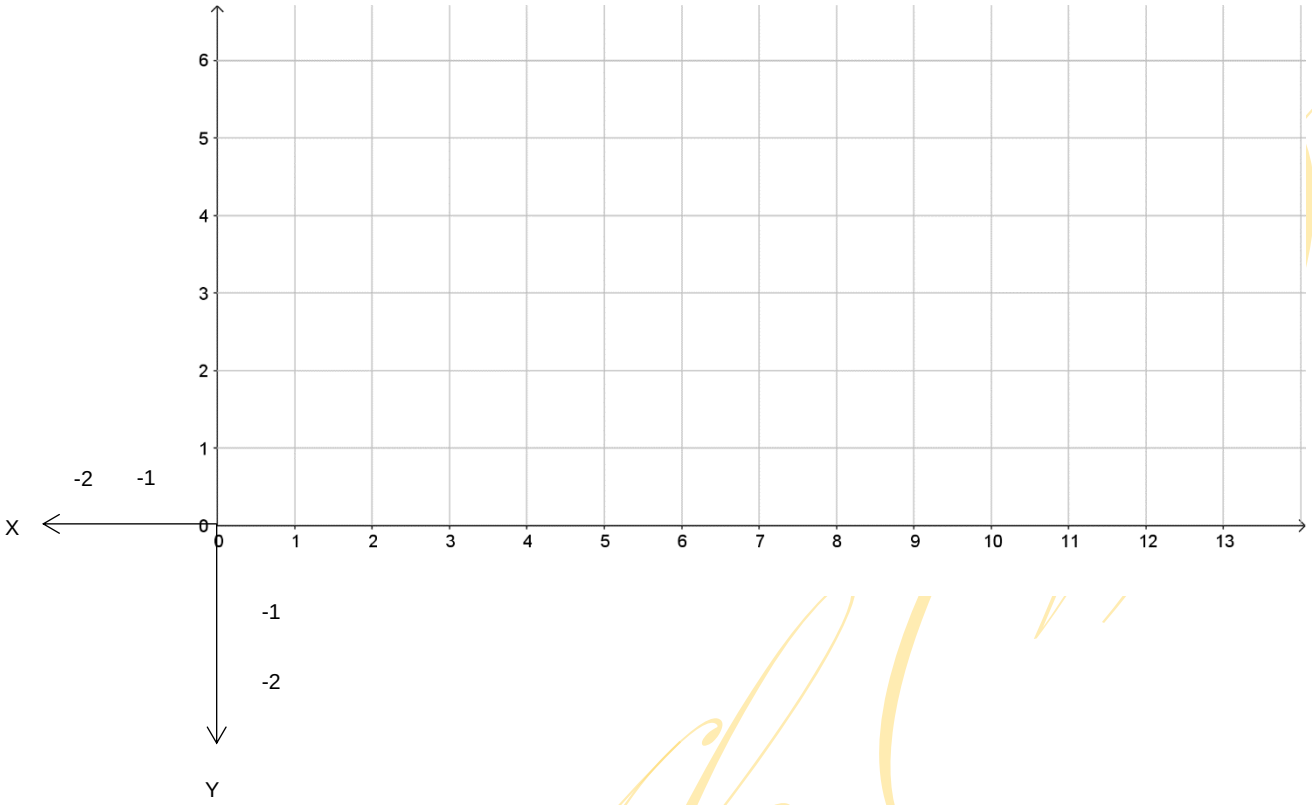


DIAGRAM SHEET 2	**EXAMINATION NUMBER:**												

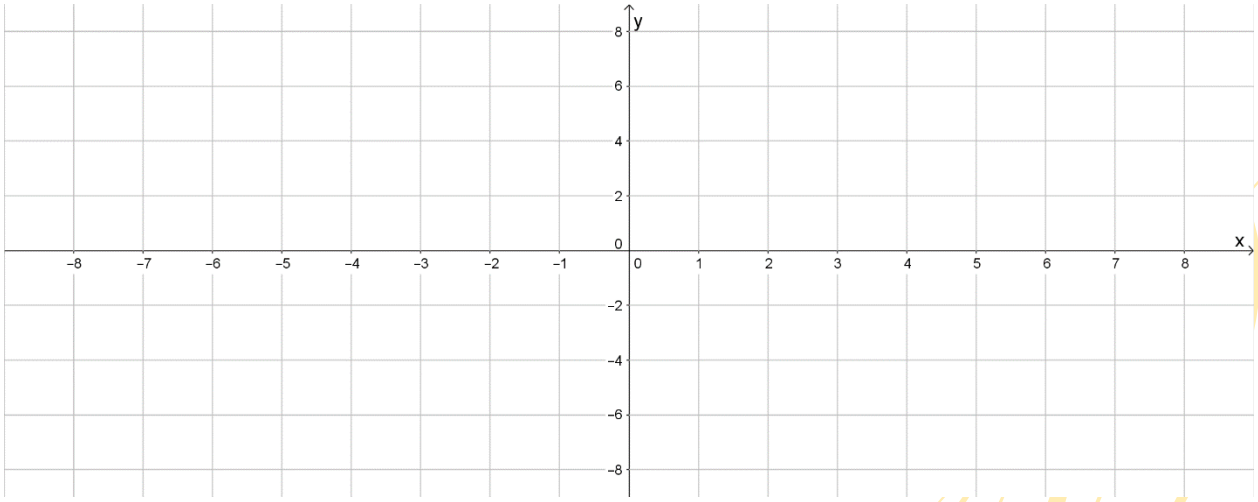


DIAGRAM SHEET 3	**EXAMINATION NUMBER:**											

QUESTION 3.3