N6

Strength of Materials and Structures

Lecturer Guide

Henry T. Wickens

Additional resource material for this title includes:
- Electronic Lecturer Guide
- Exemplar examination paper and memorandum
- PowerPoint presentation
- Past exam papers

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PO Box 13194, Mowbray, 7705
Tel (021) 462 3572
Fax (021) 462 3681
E-mail: info@futuremanagers.com
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Lecturer guidance

1. General aims

This subject builds onto the basic knowledge attained in N5 Strength of Materials and Structures. This subject involves knowledge of various systems and components, hence when presenting modules for the subject, attention should be taken that the students understand each basic scientific principle in such a way that they will be able to integrate this knowledge in their applied subjects.

2. Specific aims

On completion of all the modules in N6 Strength of Materials and Structures, the students should be able to apply the scientific principles mastered to his specific trade theory. Students should be able to apply SI units and derived units correctly. Students should be able to demonstrate understanding of subject content through the application of acquired knowledge. Students should also be able to solve problems by using subject content.

Students should be able to acquire in-depth knowledge of the following content:
1. Thick cylinders
2. Tension in cables
3. Combined bending and twisting of shafts
4. Bending and deflection of beams
5. Combined direct and bending stresses
6. Shear stress in beams
7. Closed-coiled helical springs
8. Transformation of stress
9. Forces in structural frameworks

3. Prerequisites

The student must have a passed N5 Strength of Materials and Structures.

4. Duration

Full-time: 7,5 hours per week. This instructional offering may also be offered part-time or in distance-learning mode.

5. Evaluation

Candidates must be evaluated continually as follows:

5.1 ICASS trimester mark

5.1.1 Two formal class tests for full time and part time students (or Two assignments for distance learning students only)

5.1.2 Obtain a minimum of 40% in order to qualify to write the final examination.
5.1.3 Assessment marks are valid for a period of one year and are referred to as ICASS trimester marks.
5.1.4 Calculation of trimester mark:
Weight of test or assignment 1 = 30% of the syllabus
Weight of test or assignment 2 = 70% of the syllabus.

5.2 Examination

5.2.1 The examination shall consist of 100 % of the syllabus
5.2.2 Duration shall be 3 hours
5.2.3 Minimum pass percentage shall be 40%
5.2.4 Closed book examination
5.2.5 Knowledge, understanding, application and evaluation are important aspects of the subject and should be weighted as follows:

Knowledge	Understanding	Application	Evaluation
60%	20%	15%	5%

5.3 Promotion Mark

The promotion mark consisting of the combination of the Trimester and Examination marks, shall be a minimum of 40%.

6. Weighted values of modules

Modules	Weighting (%)
1. Thick cylinders	12
2. Tension in cables	10
3. Combined bending and twisting of shafts	11
4. Bending and deflection of beams	12
5. Combined direct and bending stresses	11
6. Shear stress in beams	11
7. Closed-coiled helical springs	10
8. Transformation of stress	11
9. Forces in structural frameworks	12
Total	**100**

7. Work schedule

Week	Topic	Content	Exercises	Hours
1	**Module 1** Thick cylinders	1.1 Single cylinders 1.2 Compound cylinders	Group activity 1 Exercise 1.1 Group activity 2 Exercise 1.2	9 hours
2	**Module 2** Tension in cables	2.1 Introduction to simple catenary cables 2.2 Parabolic catenaries	Group activity 3 Exercise 2.1 Group activity 4 Exercise 2.2	9 hours
3	**Module 3** Combined bending and twisting of shafts	3.1 Maximum torque 3.2 Maximum bending moment 3.3 Equivalent torque and bending moment	Group activity 5 Exercise 3.1	9 hours
4	**Module 4** Bending and deflection of beams	4.1 Bending of beams 4.2 Deflection and slope	Group activity 6 Exercise 4.1 Group activity 7 Exercise 4.2	9 hours
5	**Module 5** Combined direct and bending stress	5.1 Introduction to direct and bending stresses 5.2 Combined direct and bending stresses	Group activity 8 Exercise 5.1	9 hours
6	**Module 6** Shear stress in beams	6.1 General shear stress formula 6.2 Shear stress diagrams	Group activity 9 Exercise 6.1	9 hours
7	**Module 7** Closed-coiled helical springs	7.1 Single springs 7.2 Compound springs	Group activity 10 Exercise 7.1	9 hours
8	**Module 8** Transformation of stress	8.1 Introduction to plane stresses 8.2 Mohr's circle for plane stresses	Group activity 11 Exercise 8.1 Group activity 12 Exercise 8.1	9 hours
9	**Module 9** Forces in structural frameworks	9.1 Introduction to sheerlegs and tripods 9.2 Derrick cranes	Group activity 13 Exercise 9.1 Exercise 9.2 Group activity 14 Exercise 9.3 Group activity 15 Exercise 9.4	9 hours
TOTAL				**90 hours**

8. Lesson plan template

LESSON	Content/Outcomes to be covered this week	List of examples to be done in class by the lecturer to explain the outcome/ concept	Facilitation method (Please tick)		Teaching resources/ aids (Please tick)		Student activity (exercise in textbook/additional supporting task) to be done this week
WEEK 1			Lecture		White board/ OHP		
			Group work		Models		
			Demonstration		Handouts		
			Simulation		Multimedia		
			Introduction to lessons				
			Recapping/Reinforcement				

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IESSON	Content/Outcomes to be covered this week	List of examples to be done in class by the lecturer to explain the outcome/ concept	Facilitation method (Please tick)		Teaching resources/ aids (Please tick)		Student activity (exercise in textbook/additional supporting task) to be done this week
WEEK 2			Lecture		White board/ OHP		
			Group work		Models		
			Demonstration		Handouts		
			Simulation		Multimedia		
			Introduction to lessons				
			Recapping/Reinforcement				

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WEEK 3			Lecture		White board/ OHP		
			Group work		Models		
			Demonstration		Handouts		
			Simulation		Multimedia		
			Introduction to lessons				
			Recapping/Reinforcement				

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WEEK 4			Lecture		White board/ OHP		
			Group work		Models		
			Demonstration		Handouts		
			Simulation		Multimedia		
			Introduction to lessons				
			Recapping/Reinforcement				

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WEEK 5			Lecture		White board/ OHP		
			Group work		Models		
			Demonstration		Handouts		
			Simulation		Multimedia		
			Introduction to lessons				
			Recapping/Reinforcement				

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WEEK 6			Lecture		White board/ OHP		
			Group work		Models		
			Demonstration		Handouts		
			Simulation		Multimedia		
			Introduction to lessons				
			Recapping/Reinforcement				

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WEEK 7			Lecture		White board/ OHP		
			Group work		Models		
			Demonstration		Handouts		
			Simulation		Multimedia		
			Introduction to lessons				
			Recapping/Reinforcement				

LESSON	Content/Outcomes to be covered this week	List of examples to be done in class by the lecturer to explain the outcome/concept	Facilitation method (Please tick)		Teaching resources/aids (Please tick)		Student activity (exercise in textbook/additional supporting task) to be done this week
WEEK 8			Lecture		White board/ OHP		
			Group work		Models		
			Demonstration		Handouts		
			Simulation		Multimedia		
			Introduction to lessons				
			Recapping/Reinforcement				

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LESSON	Content/Outcomes to be covered this week	List of examples to be done in class by the lecturer to explain the outcome/ concept	Facilitation method (Please tick)		Teaching resources/ aids (Please tick)		Student activity (exercise in textbook/additional supporting task) to be done this week
WEEK 9			Lecture		White board/ OHP		
			Group work		Models		
			Demonstration		Handouts		
			Simulation		Multimedia		
			Introduction to lessons				
			Recapping/Reinforcement				

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WEEK 10			Lecture		White board/ OHP		
			Group work		Models		
			Demonstration		Handouts		
			Simulation		Multimedia		
			Introduction to lessons				
			Recapping/Reinforcement				

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MODULE

1 *Thick cylinders*

By the end of this module, students should be able to:

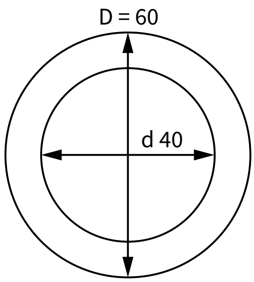
- calculate longitudinal stress;
- apply Lamé's theory to calculate:
  - radial and hoop stresses for internal and external pressure
  - radial and hoop stresses for combined internal and external pressures;
- sketch a stress distribution graph to indicate the values of radial and hoop stresses through the cylinder wall;
- calculate:
  - the strain at the inner and outer diameters
  - the change in diameter at the inner and outer diameters
  - the resultant thickness of the cylinder wall;
- apply Lamé's theory to calculate radial and hoop stresses if a sleeve is shrunk onto a solid shaft;
- sketch a stress distribution graph to indicate the values of radial and hoop stresses in the shaft and sleeve;
- calculate
  - the change in diameters of the shaft and sleeve
  - the shrinkage allowance between the shaft and sleeve
  - the force required to push the sleeve off the shaft and the torque it can transmit without slipping;
- apply Lamé's theory to calculate:
  - the radial and hoop stresses if a sleeve is shrunk onto a hollow shaft
  - the radial and hoop stresses in compound cylinders due to shrinkage;
- sketch a stress distribution graph to indicate the values of radial and hoop stresses in both cylinders;
- calculate:
  - the change in diameters and wall thickness of the cylinders
  - the shrinkage allowance between the cylinders;
- apply Lamé's theory to calculate the resultant radial and hoop stresses in compound cylinders subjected to internal pressure and shrinkage; and
- sketch a stress distribution diagram of the resultant radial and hoop stresses in both cylinders.

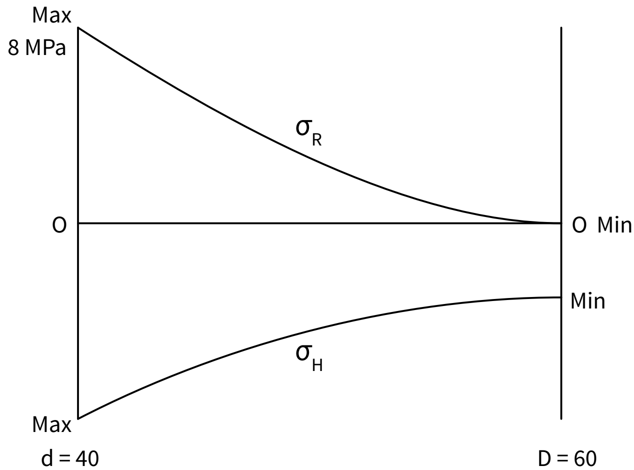
Introduction

In this module, students will become familiar with the application of Lamé's theorem and its uses for calculating radial and hoop stresses for various thick cylinders that have undergone stresses and pressures resulting in deformation such as shrinkage due to change in temperature or a change in pressure.

Exercise 1.1 SB page 17

1.





At D = 60 $\sigma_H = 0 = a + \frac{b}{0{,}06^2}$

$a = 277{,}778\ b \ldots$ ①

At $d = 40$ $\sigma_H = 8\text{M} = a + \frac{b}{0{,}04^2}$

$\therefore 8\text{M} = a + 625b \ldots$ ②

Subsitute ① into ②: $\therefore 8\text{M} = -277{,}778\ b + 625\ b$

$b = 0{,}023\ \text{M} \ldots$ ③

Substitute ③ into ②: $\therefore a = -277{,}778\ (0{,}023\text{M})$

$= -6{,}6389\ \text{M}$

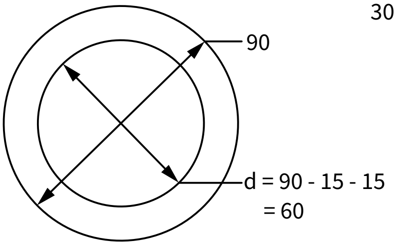
Maximum hoop stress at $d = 40$

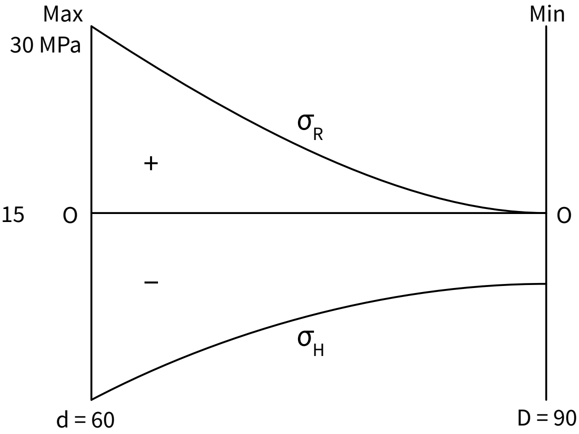
$\therefore \sigma_{H\,max} = a - \frac{b}{0{,}04^2}$

$= \frac{-6{,}389\text{M} - 0{,}023\text{M}}{0{,}04^2}$

$= -20{,}764$ MPa (Tensile)

2.





2.1 Longitudinal stress

$$\therefore \sigma_L = \frac{p_c d^2}{D^2 - d^2}$$

$$= \frac{30M \times 0{,}06^2}{(0{,}09^2 - 0{,}06^2)}$$

$$= 24 \text{ MPa}$$

2.2 Minimum hoop stress

At D = 90 $\quad \therefore \sigma_H = 0 = a + \frac{b}{0{,}09^2}$

$a = -123{,}457\, b \ldots$ ①

At d = 60 $\quad \sigma_H = 30M = a + \frac{b}{0{,}06^2}$

$\therefore 30M = a + 277{,}778\, b \ldots$ ②

Substitute ① into ②: $\quad \therefore 30M = -123{,}457\, b + 277{,}778\, b$

$\therefore b = 0{,}194M \ldots$ ③

Substitute ③ into ①: $\quad \therefore a = -123{,}457\,(0{,}194M)$

$= -23{,}951M$

$\therefore$ At D = 90 $\quad \sigma_{H\,min} = a - \frac{b}{0{,}09^2}$

$= -23{,}951M - \frac{0{,}194M}{0{,}09^2}$

$= -47{,}9$ MPa (Tensile)

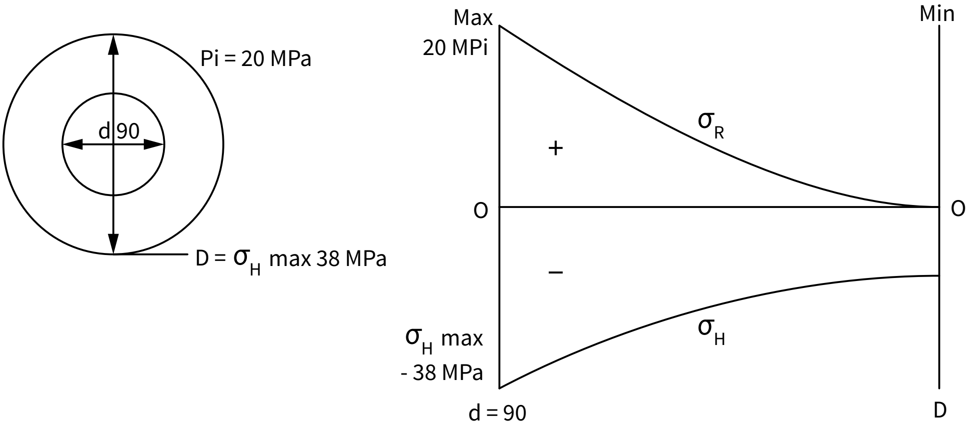
2.3 Maximum hoop stress

At $d = 60\sigma_{H\,max} = a + \frac{b}{0{,}06^2}$

Substitute 'a' and 'b' $\quad \therefore \sigma_{H\,max} = a - 23{,}951M = \frac{0{,}194M}{0{,}06^2}$

$= -77{,}84$ MPa (Tensile)

3.



3.1 Wall thickness

At $d = 90$ $\quad \sigma_R = 20M = a - \frac{b}{0{,}09^2} \ldots$ ①

At $d = 90$ $\quad \sigma_H = -38M = a + \frac{b}{0{,}09^2} \ldots$ ②

Add ① and ②: $\quad \therefore -18M = 2a$

$\therefore a = -9M \ldots$ ③

Substitute ③ into ①: $\quad \therefore 20M = -9M - 123{,}456\, b$

$\therefore b = 0{,}235M$

At D: $\quad \sigma_H = 0 = a + \frac{b}{D^2}$

$= -9M + \frac{0{,}235M}{D^2}$

$\therefore 9M = \frac{0{,}235M}{D^2}$

$D^2 = 0{,}0261$

$D = 0{,}1616$ m

$D = 161{,}6$ mm

$t = \frac{D - d}{2}$

$= \frac{161{,}6 - 90}{2}$

$= 35{,}75$ mm

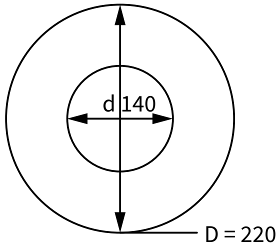
3.2 Hoop stress at D = 161,6

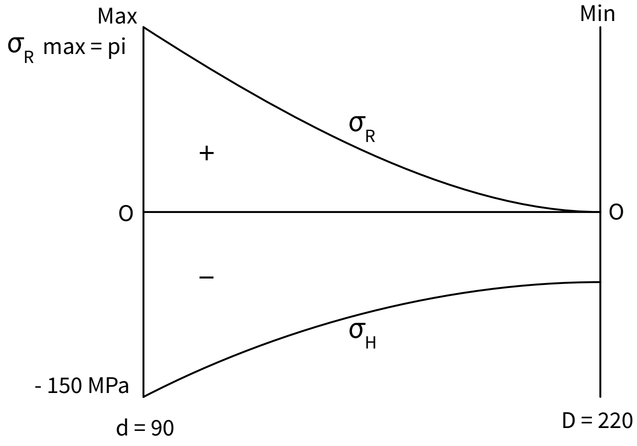
At D: $\quad \sigma_H = a + \frac{b}{0{,}1616^2}$

$= -9M - \frac{0{,}235M}{0{,}1616^2}$

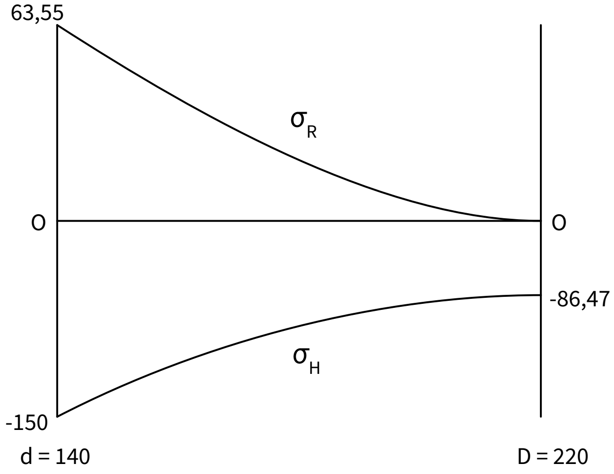
$= -18$ MPa (Tensile)

4.





4.1



Internal pressure

At D = 220 $\quad \sigma_H = -150\text{M} = a - \dfrac{b}{0{,}14^2}$

$$= a - 51{,}02\, b \ldots ①$$

D = 220 $\quad \sigma_R = 0 = a + \dfrac{b}{0{,}22^2}$

$$0 = a + 20{,}661\, b$$

$$a = -20{,}661\, b \ldots ②$$

Substitute ② into ①: $\quad \therefore -150\text{M} = -20{,}661\, b - 51{,}02\, b$

$$\therefore b = 2{,}093\text{M} \ldots ③$$

Substitute ③ into ②: $\quad \therefore a = -20{,}661\,(2{,}093\text{M})$

$$= -43{,}243\text{M}$$

$\therefore$ at D = 140: $\quad pi = \sigma_{R\,max} = a + \frac{b}{0,14^2}$

$= -43,243M + \frac{2,093M}{0,14^2}$

$= +63,55$ MPa (Compressive)

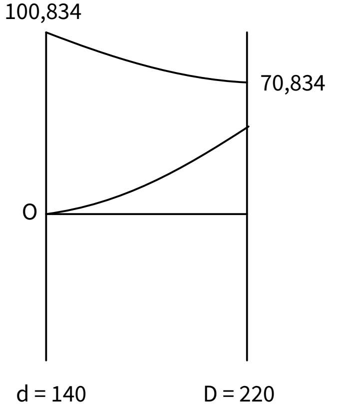
4.2 Main hoop stress $\sigma_{H\,min}$ at D = 220

$\therefore \sigma_{H\,min} = a - \frac{b}{0,22^2}$

$= -43,243M - \frac{2,093M}{0,22^2}$

$= -86,47$ MPa (Tensile)

4.3.1



At d = 140 $\quad \sigma_R = 0 = a + \frac{b}{0,14^2}$

$\therefore a = -51,02b \ldots$ ①

At D = 220 $\quad \sigma_R = 30M = a + \frac{b}{0,22^2} \ldots$ ②

Substitute ① into ②: $\quad 30M = -51,02b + 20,661b$

$b = -988,175\ k$

$a = 50,417M$

At d = 140 $\sigma_{H\,min} = 50,417M - \frac{-988,175k}{0,14^2} = 100,834$ MPa (C)

At D = 220 $\sigma_{H\,max} = 50,417M - \frac{-988,175k}{0,22^2} = 70,834$ MPa (C)

Diameter	Stress	Internal pressure	Outer pressure	Resultant
140	Radial	63,55M	0	63,55 MPa
220	Radial	0	30 M	30 MPa
140	Hoop	–150M	+ 100,834M	–49,166 MPa
220	Hoop	–86,47M	+70,834M	–15,636 MPa

4.3.2 Change in diameter

$$\sigma_L = \frac{p_i d^2}{D^2 - d^2} = \frac{63{,}55M \times 0{,}14^2}{0{,}22^2 - 0{,}14^2} = 43{,}249 \text{ MPa (T)}$$

$$\Delta d = \varepsilon \times d = \frac{1}{E}\left[\sigma_H - \nu\left(\sigma_R + \sigma_L\right)\right] \times d$$

$$= \frac{1}{200G}[-49{,}166M - 0{,}3(63{,}55M - 43{,}249M)]140$$

$$\Delta d = -0{,}039 \text{ mm}$$

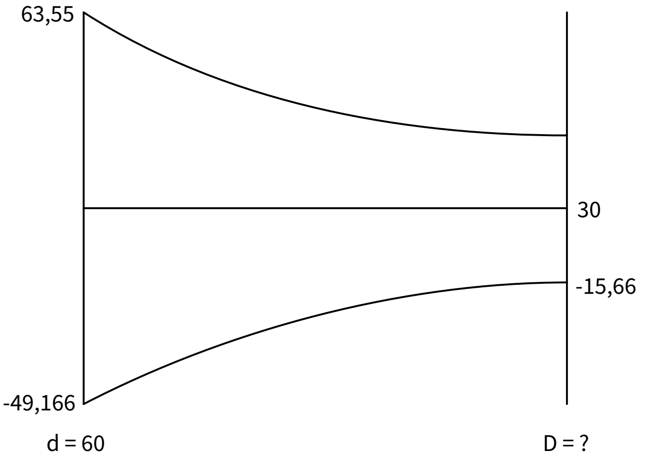
$$\Delta D = \frac{1}{200G}[-15{,}636M - 0{,}3(30M - 43{,}249M)]220$$

$$\Delta D = -0{,}013 \text{ mm}$$

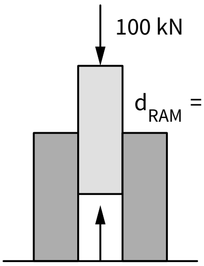
4.3.3 Resultant wall thickness and new wall thickness

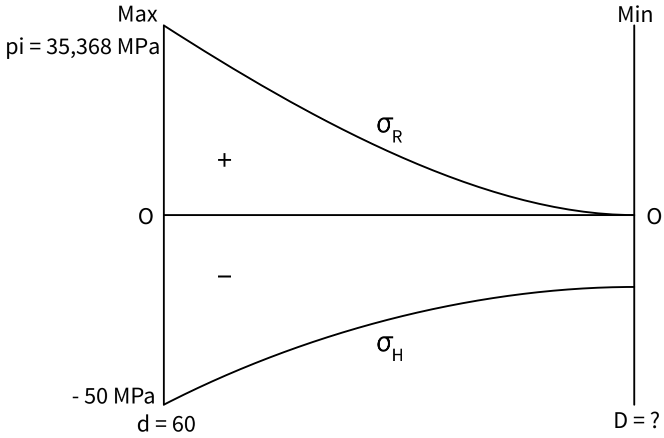
$$t_{new} = \frac{D_{new} - d_{new}}{2} = \frac{220{,}013 - 140{,}039}{2} = 39{,}987 \text{ mm}$$

4.3.4



5.1





Maximum internal pressure $= \frac{F}{A}$

$\frac{100k}{\frac{\pi}{4}0{,}06^2}$

$= 35{,}368$ MPa

At d: $\sigma_R = 35{,}368\text{M} = a + \frac{b}{0{,}06^2}$

$= a + 277{,}778\, b$ … ①

At d: $\sigma_H = -50\text{M} = a - \frac{b}{0{,}06^2}$

$= a - 277{,}778\, b$ … ②

① + ②: $\therefore -14{,}632\text{M} = 2a$

$\therefore a = -7{,}316\text{M}$ … ③

Substitute ③ into ②: $\therefore -50\text{M} = -7{,}316\text{M} - 277{,}77\, b$

$= b = \frac{-50\text{M} + 7{,}316\text{M}}{-277{,}77}$

$= +0{,}154\text{M}$

At D: $Q_R = 0 = a + \frac{b}{D^2}$

$-a = \frac{b}{D^2}$

$D^2 = \frac{b}{-a}$

$= \frac{0{,}154\text{M}}{-(-7{,}316)\text{M}}$

D = 0,14509 m

D = 145,09 mm

Wall thickness $= t = \frac{D - d}{2} = \frac{145{,}09 - 60}{2}$

$= 42{,}55$ mm

5.2 Longitudinal stress

$$\sigma_L = \frac{p_i d^2}{D^2 - d^2}$$

$$= \frac{35{,}368M \times 0{,}06^2}{0{,}14509^2 - 0{,}06^2}$$

$$= 7{,}3 \text{ MPa}$$

6.1 Outer and inner circumferential stress

At $d = 120$ $\qquad \sigma_R = 0 = a + \dfrac{b}{0{,}12^2}$

$a = -69{,}444b \ldots$ ①

At D = 60 $\qquad \sigma_R = 120M = a + \dfrac{b}{0{,}06^2} \ldots$ ②

Substitute ① into ②: $120M = -69{,}444b + 277{,}778b$

$b = 567k$

$a = -40M$

$$\therefore \sigma_{H\ inner} = -40M - \frac{567k}{0{,}06^2} = 200 \text{ MPa (T)}$$

$$\therefore \sigma_{H\ outer} = -40M - \frac{567k}{0{,}12^2} = 80 \text{ MPa (T)}$$

6.2 Longitudinal stress

$$\sigma_L = \frac{p_i d^2}{D^2 - d^2} = \frac{120M \times 0{,}06^2}{0{,}12^2 - 0.06^2} = 40 \text{ MPa (T)}$$

6.3 Circumferential strain inner and outer diameter

At $d = 60$ $\qquad \varepsilon_{inner} = \dfrac{1}{E}\left[\sigma_{H\ inner} - \nu\left(\sigma_{R\ in} + \sigma_L\right)\right]$

$$\varepsilon_{inner} = \frac{1}{195G}[-200M - 0{,}29(120M - 40M)] = -1{,}145 \times 10^{-3}$$

At $d = 120$ $\qquad \varepsilon_{outer} = \dfrac{1}{E}\left[\sigma_{H\ outer} - \nu\left(\sigma_{R\ out} + \sigma_L\right)\right]$

At $d = 120$ $\qquad \varepsilon_{outer} = \dfrac{1}{195G}[-80M - 0{,}29(0 - 40M)] = -3{,}508 \times 10^{-4}$

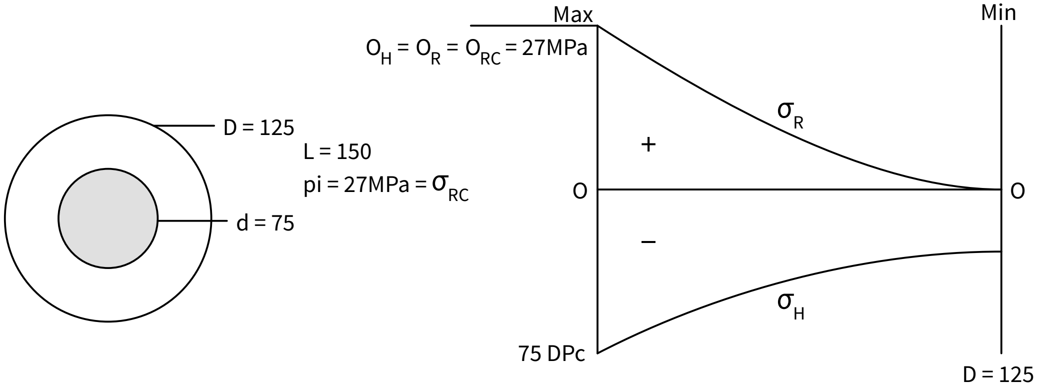
6.4 Change in diameter: inner and outer

Inner $\Delta d = \varepsilon \times d = -1{,}145 \times 10^{-3} \times 60 = -0{,}0687$ mm

Outer $\Delta D = \varepsilon \times D = -3{,}508 \times 10^{-4} \times 120 = -0{,}0421$ mm

Exercise 1.2 **SB page 39**

1.



1.1 Maximum tensile stress hub = $\sigma_{H\,max}$

At $d = 75$: $\sigma_R = 27M = a + \dfrac{b}{0{,}075^2}$

$= a + 177{,}778\,b \ldots$ ①

At $d = 125$: $\sigma_R = 0 = a + \dfrac{b}{0{,}125^2}$

$\therefore a = -64\,b \ldots$ ②

Substitute ② into ①: $\therefore 27M = -64b + 177{,}77\,b$

$\therefore b = 0{,}0237M \ldots$ ③

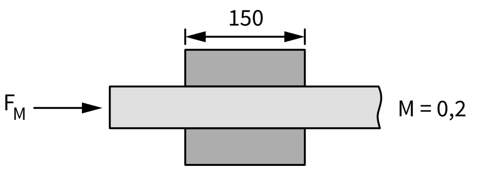
Substitute ③ into ②: $\therefore a = -15{,}168M$

$\therefore \sigma_{H\,max} = a - \dfrac{b}{0{,}075^2}$

$= -15{,}168M - \dfrac{0{,}237M}{0{,}075^2}$

$= 57{,}3$ MPa

1.2



Force to push shaft out = pressure × area × μ

$F\mu = P_o(\pi DL)\mu$

$= 27M\,(\pi \times 0{,}075 \times 0{,}15)0{,}2$

$= 190{,}85$ kN

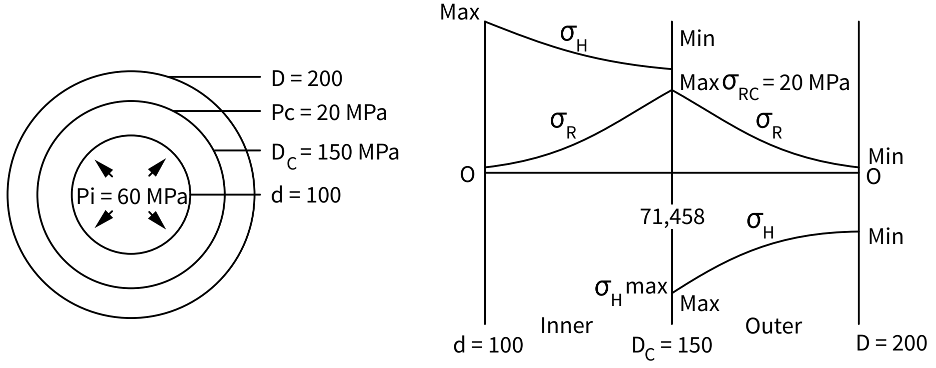
1.3 Torque transmitted = Fμ × R

$$T = F\mu \times \frac{D_c}{2}$$

$$= 190{,}85k \times \frac{0{,}075}{2}$$

$$= 7{,}16 \text{ kNm}$$

2.



For shrinkage

2.1 Maximum hoop stress (outer)

At $d = 200$: $\sigma_R = 0 = a + \dfrac{b}{0{,}2^2}$

$a = -25\,b \ldots$ ①

At $D_C = 150$: $\sigma_{RC} = 20M = a + \dfrac{b}{0{,}15^2}$

$= a + 44{,}444\,b \ldots$ ②

Substitute ① into ②: $20M = -25\,b + 44{,}444\,b$

$b = 1{,}029M \ldots$ ③

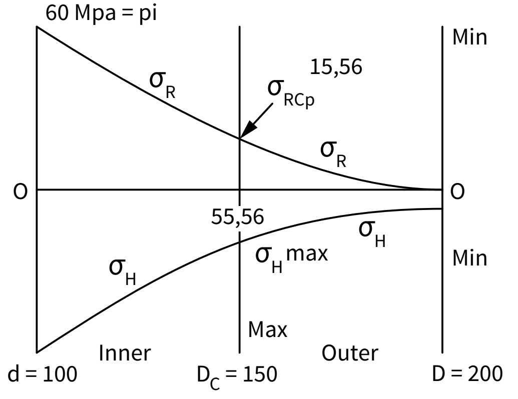
Substitute ③ into ①: $\therefore a = -25{,}725M$

At $d = 150$ $\therefore \sigma_{H\,max} = a - \dfrac{b}{0{,}15^2}$

$$= -27{,}725M - \frac{1{,}029M}{0{,}15^2}$$

$$= 71{,}458 \text{ MPa (Tensile)}$$

2.2 **For internal pressure**



Two cylinders act as one:

a and *b* values are the same for both:

$\therefore$ At D = 200 $\sigma_R = 0 = a + \frac{b}{0{,}2^2}$

$\therefore a = -25\,b \ldots$ ①

At *d* = 100 $\sigma_{RC} = 60\text{M} = a + \frac{b}{0{,}1^2}$

$= a + 100\,b \ldots$ ②

Substitute ① into ②: $\therefore 60\text{M} = -25\,b + 100\,b$

$b = 0{,}8\text{M} \ldots$ ③

Substitute ③ into ②: $\therefore a = -20\text{M}$

$\therefore$ Radial stress at contact diamater due to internal pressre supply

At $D_C = 150$: $\sigma_{RCp} = a + \frac{b}{0{,}15^2}$

$= -20\text{M} + \frac{0{,}8}{0{,}15^2}$

$\sigma_{RCp} = 15{,}56$ MPa (Compressive)

2.3 Hoop stress at D_C

For internal pressure, two cylinders act as one $\therefore$ *a* and *b* values are the same for both.

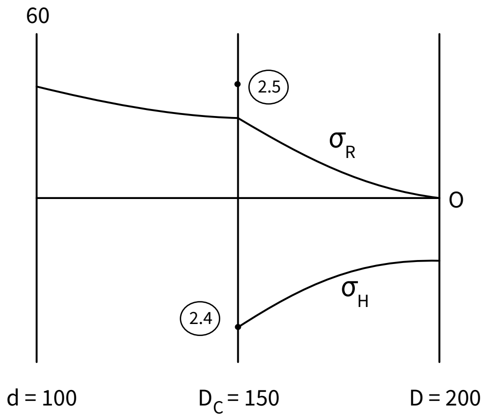
Hoop stress at D_C due to pressure.

At $D_C = 150$ $\therefore \sigma_{H\,max} = a - \frac{b}{0{,}15^2}$

$= -20\text{M} - \frac{0{,}8\text{M}}{0{,}15^2}$

$= 55{,}56$ MPa (Tensile)

Resultant graph



2.4 ∴ Maximum hoop stress in the outer cylinder due to internal pressure and shrinkage:

Resultant: Outer cylinders at D_C

$\therefore \sigma_{H\,max} = \sigma_{H\,max_{shrinkage}} + \sigma_{H\,max_{pressure}}$

$= 71{,}458M + 55{,}56M$

$= 127{,}01$ MPa (Tensile)

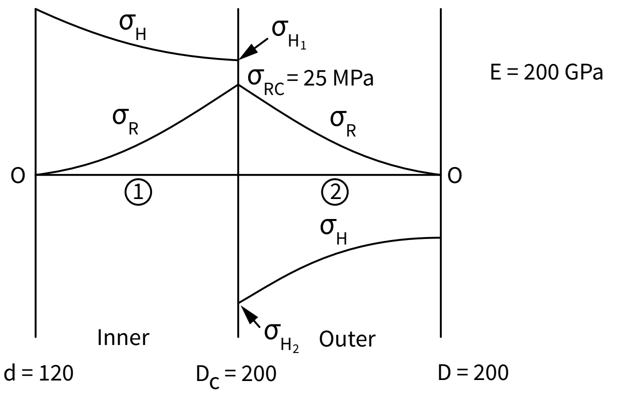
2.5 Radial stress at D_C (resultant)

∴ Resultant radial stress $= \sigma_{RC_R} = \sigma_{RC_{shrinkage}} + \sigma_{RC_{pressure}}$

$\therefore \sigma_{RC_R} = 20M + 15{,}56M$

$= 35{,}56$ MPa (Compressive)

3.



Each cylinder has its own a and b values. Both cylinders are made of the same material.

∴ Shrinkage allowance $= \Delta d = \frac{D_C}{E} = \left(\sigma_{H_1} - \sigma_{H_2}\right)$

Consider the inner cylinder to calculate the hoop stress at the contact diameter = σ_{H_1}

$\therefore$ at $d = 120$ $\qquad \therefore \sigma_R = 0 = a + \frac{b}{0{,}12^2}$

$a = -69{,}444\, b \ldots$ ①

At $D_C = 200$ $\qquad \sigma_R = 25M = a + \frac{b}{0{,}2^2} \ldots$ ②

Substitute ① into ②: $\qquad 25M = -69{,}444\, b + 25\, b$

$\therefore b = -0{,}563M \ldots$ ③

Substitute ③ into ①: $\qquad \therefore a = +39{,}063M$

$\therefore \sigma_{H_1}$ at $D_C = 200$

$\therefore \sigma_{H_1} = a - \frac{b}{0{,}2^2}$

$= +39{,}063M - \frac{-0{,}563M}{0{,}2^2}$

$\sigma_{H_1} = 53{,}138$ MPa (Compressive)

Consider the outer cylinder. To calculate the hoop stress at the contact diameter = σ_{H_1}

$\therefore$ at $D = 260$ $\qquad \sigma_R = 0 = a + \frac{b}{0{,}26^2}$

$a = -14{,}793\, b \ldots$ ①

At DC = 200 $\qquad \sigma_R = 25M = a + \frac{b}{0{,}2^2} \ldots$ ②

Substitute ① into ②: $\qquad 25M = -14{,}793\, b + 25\, b$

$b = 2{,}449M \ldots$ ③

Substitute ③ into ①: $\qquad \therefore a = -36{,}228M$

$\therefore \sigma_{H_2}$ at $D_C = 200$

$\sigma_{H_2} = a - \frac{b}{0{,}2^2}$

$-36{,}228 - \frac{2{,}449M}{0{,}2^2}$

$= -97{,}453$ MPa (Tensile)

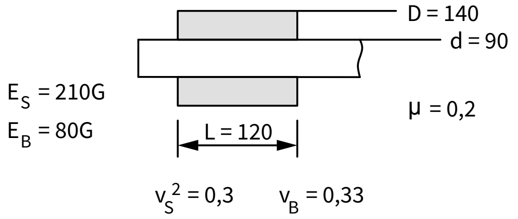
$\therefore$ Shrinkage allowance $= \Delta d = \frac{D_C}{E}\left(\sigma_{H_1} - \sigma_{H_2}\right)$

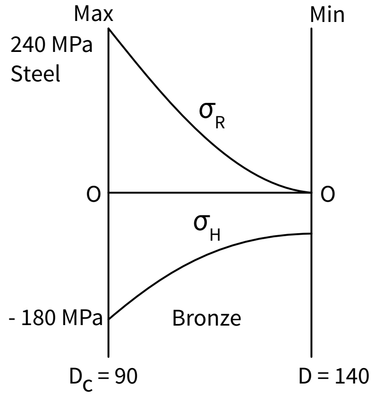
$= \frac{0{,}2}{200G}[53{,}138M - (-97{,}453M)]$

$\therefore \Delta d = 1{,}51 \times 10^{-4}$ m

$= 0{,}151$ mm

4.





Two different materials of which each maximum stress is given. This does not mean that both will reach their maximum stress at the same time when shrunk together.

- One of the materials will reach its maximum and the other one will be below its maximum.
- Consider the maximum of one material and calculate 'a' and 'b' values and check what maximum stress it will cause in the other material.

If the calculated value is less than the maximum given, then this stress with the maximum that was selected is the acting stress.

Consider the smallest of the two maximum stresses, in this case the 180 MPa for bronze, and check what stress will develop in the steel.

4.1 Maximum radial stress at D_C

At $D_C = 90$ $\quad \sigma_{H_B} = -180\text{M} = a - \dfrac{b}{0{,}09^2}$

$= a - 123{,}457\,b \ldots$ ①

At D = 140 $\quad \sigma_R = 0 = a + \dfrac{b}{0{,}14^2}$

$\therefore a = -51{,}02\,b \ldots$ ②

Substitute ② into ①: $\quad \therefore -180\text{M} = -51{,}02\,b - 123{,}457\,b$

$\therefore b = +1{,}032\text{M}$

$\therefore a = -52{,}635\text{M}$

Radial stress in shaft at D_C

At $d = 90$ $\quad \sigma_{H_S} = \sigma_{R_H} = a + \dfrac{b}{0{,}09^2}$ (solid shaft only)

$\therefore \sigma_R = -52{,}635\text{M} + \dfrac{+1{,}032\text{M}}{0{,}09^2}$

$= 74{,}77$ MPa (Compressive)

This is less than 260 MPa if 240 MPa and 0 MPa at D was considered the σ_H if D = 90 will be more than 180 MPa.

$\therefore$ Acting stresses

$\sigma_{H_S} = 74{,}77$ MPa

$\sigma_{H_B} = 180$ MPa

4.2 $\therefore$ Friction force = $F\mu$

$\therefore F\mu = P_o(\pi DL)M$

$= 74{,}77M\ (\pi \times 0{,}09 \times 0{,}12)0{,}2$

$= 507{,}377$ kN

$$\text{Torque} = T = F\mu \times \frac{D_C}{2}$$

$$= 507{,}377\,k \times \frac{0{,}09}{2}$$

$= 22{,}83$ kNm

4.3 The two materials are different

For solid shaft $\sigma_H = \sigma_R = \sigma_{RC}$ of hub

$\therefore$ Shrinkage allowance = Δd

$$\therefore \Delta d = D_C\left(\left[\frac{\sigma_{H_1} - \overbrace{V_1}^{\text{Steel}}\sigma_{RC}}{E_1}\right]\right) - \left(\left[\frac{\sigma_{H_2} - \overbrace{V_2}^{\text{Bronze}}\sigma_{RC}}{E_2}\right]\right)$$

$$\Delta d = 0{,}09\left[\left(\frac{74{,}77M - 0{,}3 \times 74{,}77M}{210G}\right) - \left(\frac{180M - 0{,}033 \times 74{,}77M}{80G}\right)\right]$$

$$= 0{,}09(2{,}492 \times 10^{-4}) - (-2{,}281 \times 10^{-3})$$

$= 0{,}228$ mm

4.4 The new outer diameter of a hub

$\therefore \sigma_{H\,min} = C = 2a = 2 \times (-52{,}635)$

$= -105{,}27$ MPa

Change in diameter

$$\therefore \delta d = \frac{D}{E}(\sigma_H - v\sigma_R)$$

$$= \frac{0{,}14}{80G}[-105{,}27 - (0{,}33 \times 0)]$$

$= 0{,}184$ mm

New diameter = 140 + 0,184

$= 140{,}184$ mm

4.5 Inner diameter of the sleeve

$\therefore$ Change in diameter $= \delta_d = \left(\frac{\sigma_H - \nu\sigma_R}{E}\right)d$

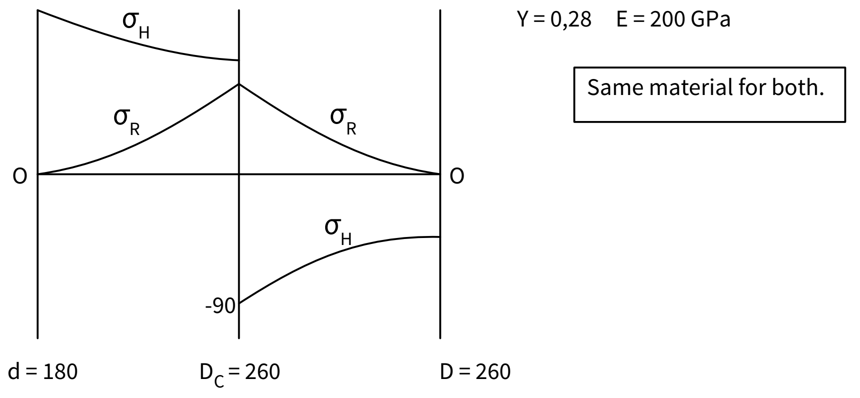
$\therefore$ Change in diameter $= \delta_d = \left(\frac{-180M - 0,33 \times 74,77M}{80G}\right)90 = 0,2303$

$\therefore$ New diameter = 90,2303 mm

4.6 The wall thickness after shrinking

$t_{new} = \frac{140,184 - 90,2303}{2} = 24,977$ mm

5.



5.1 Radial stress at D_C

At $d = 260$ $\qquad \sigma_R = 0 = a + \frac{b}{0,26^2}$

$a = -14,793\, b \ldots$ ①

At $D_C = 220$ $\qquad \sigma_H = -90M = a - \frac{b}{0,22^2} \ldots$ ②

Substitute ① into ②: $\quad \therefore -90M = -14,793\, b - 20,661\, b$

$\therefore b = +2,539M \ldots$ ③

Substitute ③ into ①: $\quad \therefore a = -37,559$ M

$\therefore$ Radial stress at the contact diamater $D_C = 220$ mm

$\therefore \sigma_{R_C} = a + \frac{b}{0,22^2}$

$= -37,559M + \frac{2,539M}{0,22^2}$

$= 14,9$ MPa

5.2 Minimum hoop stress in the outer cylinder at D = 260

At D = 260 $\sigma_{H\,min} = a - \frac{b}{0{,}26^2}$

$= -37{,}559M - \frac{2{,}539M}{0{,}26^2}$

$= 75{,}1$ MPa

5.3 Inner cylinder

At d = 180 $\sigma_R\ 0 = a + \frac{b}{0{,}18^2}$

$a = -30{,}864\,b$ … ①

At D_C = 220 $\sigma_R = 14{,}9M = a + \frac{b}{0{,}22^2}$ … ②

Substitute ① into ②: $\therefore 14{,}9M = -30{,}864\,b + 20{,}661\,b$

$b = -1{,}46M$ … ③

Substitute ③ into ①: $\therefore a = +45{,}072M$

The maximum hoop stress at d = 180

$\therefore \sigma_{H\,max} = a - \frac{b}{0{,}18^2}$

$= 45{,}072M - \frac{-1{,}46M}{0{,}18^2}$

$= 90{,}13$ MPa

The minimum hoop stress at D_C = 220

$\therefore \sigma_{H\,min} = a - \frac{b}{0{,}22^2}$

$= 45{,}072M - \frac{-1{,}46M}{0{,}22^2}$

$= 75{,}24$ MPa

5.4 Circumferential strain at D; outer cylinder

Outer cylinder: D = 260

Circumferential strain $= \epsilon_2 = \frac{\sigma_{H_{260}} - V_2\sigma_{R260}}{\epsilon_2}$

$= \frac{-75{,}1M - 0{,}28 \times 0}{200G}$

$= -3{,}755 \times 10^{-4}$

Stresses must be taken at the diameter where strain had to be calculated.

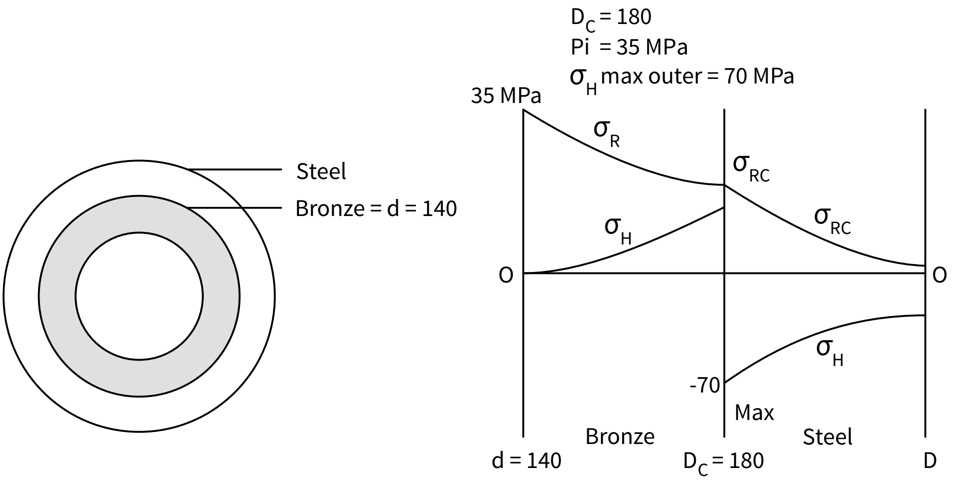
5.5 Circumferential strain the inner diameter d = 180 of inner cylinders

$$\therefore \in = \frac{\sigma_{H180} - V\sigma_{R_{180}}}{\in}$$

$$= \frac{90{,}13M - 0{,}28 \times 0}{200G}$$

$$= 4{,}506 \times 10^{-4}$$

6.



The question refers to a resultant graph.

∴ at d = 140,the pressure (σ_R) is 35 MPa.

Therefore the zero stress referred to is the hoop stress of the inner cylinder at the inner diameter.

When designed on shrink fit, any conditioned can be considered. Therefore, stresses do not have to be the standard case.

6.1 Intermediate pressure at $D_C = \sigma_{R_C}$

∴ For inner cylinder

At d = 140 $\quad \sigma_H = 0 = a - \frac{b}{0{,}14^2}$

∴ a = +51,02 b … ①

At d = 140 $\quad \sigma_R = 35M = a - \frac{b}{0{,}14^2}$ … ②

Substitute ① into ②: ∴ 35M = 51,02 b + 51,02 b

∴ b = 0,342M … ③

Substitute ③ into ①: $\quad$ ∴ a = 17,438M

∴ Internal pressure at D_C = 180

$$\therefore \sigma_{R_C} = a + \frac{b}{0{,}18^2}$$

$$= 17{,}438\text{M} + \frac{0{,}342\text{M}}{0{,}18^2}$$

$$= 28 \text{ MPa}$$

6.2 Maximum hoop stress for bronze (inner cylinders)

The hoop stress is zero at $d = 140$ for the inner cylinder.

∴ The maximum hoop stress must be at $D_C = 180$ mm

At $D_C = 180$ $\quad \sigma_{H\,max} = a + \frac{b}{0{,}18^2}$

$$= 17{,}438\text{M} - \frac{0{,}342\text{M}}{0{,}18^2}$$

$$= 6{,}9 \text{ MPa (Compressive)}$$

The answers will differ. It all depends on the rounding off of values after the comma.

6.3 The wall thickness of steel

Outer cylinder:

At D = 180 $\quad \therefore \sigma_{R_C} = 28\text{M} = a + \frac{b}{0{,}18^2}$

$$= a + 30{,}864\, b \ldots ①$$

At D = 180 $\quad \therefore \sigma_H = -70\text{M} = a - 30{,}864\, b \ldots ②$

Substitute ① into ②: $\quad \therefore -42\text{M} = 2a$

$a = -21\text{M} \ldots ③$

Substitute ③ into ①: $\quad \therefore 28\text{M} = -21\text{M} + 30{,}864\, b$

$\therefore b = 1{,}588$ M

Wall thickness:

At D $\quad \sigma_R = 0 = a + \frac{b}{D^2}$

$$= -21\text{M} + \frac{1{,}588\text{M}}{D^2}$$

$$\therefore D^2 = \frac{1{,}588\text{M}}{21\text{M}}$$

D = 275 mm

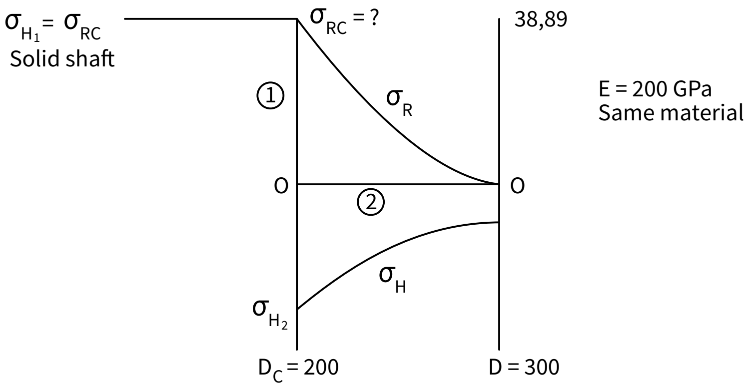
Wall thickness = t

$$\therefore t = \frac{D - d}{2}$$

$$= \frac{275 - 180}{2}$$

$$= 47{,}5 \text{ mm}$$

7.



$\Delta d = 0{,}14$ mm

7.1 $\Delta d = \frac{D_C}{E}\left(\sigma_{H_1} - \sigma_{H_2}\right)$

$\therefore 0{,}14 \times 10^{-3} = \frac{0{,}2}{200G}\left(\sigma_{RC} - \sigma_{H_1}\right)$

$\therefore 140M = \sigma_{RC} - \sigma_{H_2} \ldots$ ①

Outer cylinder:

At $D_C = 200$: $\quad \sigma_{RC} = a + \frac{b}{0{,}2^2}$

$= a + 25\,b \ldots$ ①

At $D_C = 300$: $\quad \sigma_R = 0 = a + \frac{b}{0{,}3^2}$

$a = -11{,}11\,b \ldots$ ②

$\sigma_{RC} = -11{,}11\,b + 25\,b$

$= 13{,}89\,b$

$b = 0{,}072\,\sigma_{RC}$

$a = -0{,}8\,\sigma_{RC}$

$\therefore$ at $D_C = 200 \quad \sigma_{H_1} = a - \frac{b}{0{,}2^2}$

$= -0{,}8\,\sigma_{RC} - \frac{0{,}072\sigma_{RC}}{0{,}2^2}$

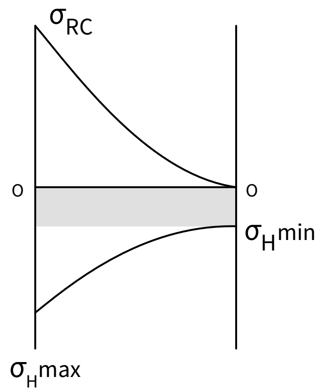
$= \sigma_{H_2} = -2{,}6\sigma_{RC} \ldots$ ②

Substitute ② into ①: $\quad \therefore 140M = \sigma_{RC} - \left(-2{,}6\sigma_{RC}\right)$

$= 3{,}6\,\sigma_{RC}$

$\therefore \sigma_{RC} = 38{,}89$ MPa

7.2 Sleeve



Maximum hoop: $\therefore \sigma_{H_1} = -2,6 \times 38,89$

$\sigma_{H\ max} = -101,11$ MPa (Tensile)

$\sigma_{max} + \sigma_{RC}$ = constant = $\sigma_{H\ min}$

$\sigma_{H\ min} = \sigma_{max} + \sigma_{RC}$

$= -101,11 + 38,89$

$= -62,2$ MPa

7.3 Maximum hoop stress of shaft

σ_{RC} = maximum hoop stress (solid shaft)

Shaft $\therefore \sigma_{H\ max} = 38,89$ MPa

7.4 The force to push the shaft = Fμ

$F\mu = P_o(\pi DL)\mu$

$= 38,89M(\pi 0,2 \times 0,1)0,15$

$= 366,53$ kN

7.5 The wall thickness after shrinking

Change in diameter (inner) = $\delta_d = \left(\frac{\sigma_H - \nu\sigma_R}{E}\right)d$

$\therefore \delta_d = \left(\frac{-101,114M - 0,33 \times 38,89M}{200G}\right)200 = 0,114$ mm

New diameter = 200,114 mm

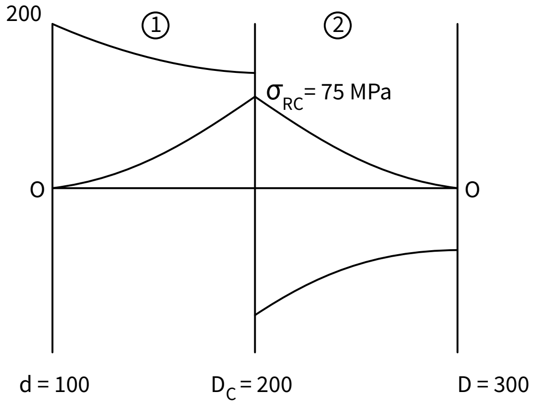
Change in diameter (outer) = $\delta_d = \left(\frac{\sigma_H - \nu\sigma_R}{E}\right)d$

$\therefore \delta_d = \left(\frac{-62,226M}{200G}\right)300 = 0,093$ mm $(\sigma_R = 0)$

$\therefore$ New diameter = 300,093 mm

New wall thickness = $\frac{300,093 = 200,114}{2} = 49,9895$ mm

8.



Hub length = 150 mm

$V_{shaft} = 0{,}3 = V_{sleeve}$

E = 200 GPa

In this case the question state that the maximum tensile (Hoop) stress must not exceed 200 MPa in the shaft or the hub.

Therefore check the value for σ RC using the maximum hoop stress and zero radial stress for both shaft and sleeve and the one giving the smallest value for σ_{RC} will be the acting stress.

8.1 Check σ_{RC} for shaft:

At $d = 100$: $\sigma_R = 0 = a + \frac{b}{0{,}1^2}$

$a = -100\,b \ldots$ ①

At $d = 100$ $\sigma_H = 200\text{M} = a - \frac{b}{0{,}1^2} \ldots$ ②

$a = -100\,b \ldots$ ②

Substitute ① into ②: $200\text{M} = -100\,b - 100\,b$

$200\text{M} = -200\,b$

$b = -1\text{M} \ldots$ ③

Substitute ③ into ①: $\therefore a = 100\text{M}$

At $D_C = 200$ $\therefore \sigma_{RC} = a + \frac{b}{0{,}2^2}$

$= 100\text{M} + \frac{-1\text{M}}{0{,}2^2}$

$= 75$ MPa

Check σ_{RC} for hub

$\therefore$ at D = 300 $\sigma_{RC} = 0 = a + \frac{b}{0{,}3^2}$

$a = -11{,}111\,b \ldots$ ①

At $D_C = 200$: $\sigma_H = -200M = a - \frac{b}{0,2^2}$

$= a - 25\,b$ … ②

Substitute ① into ②: $-200M = -11,111\,b - 25\,b$

$b = 5,538M$ … ③

Substitute ③ into ①: $\therefore a = 61,538M$

At $D_C = 200$ $\therefore \sigma_{RC} = a + \frac{b}{0,2^2}$

$= -61,538M + \frac{5,538M}{0,2^2}$

$= 76,912$ MPa

$\therefore$ Maximum $\sigma_{RC} = 75$ MPa that can be used.

Then tensile stress in shaft will be 200 MPa, and tensile stress in hub will be less than 200 MPa.

If 76,912 MPa $= \sigma_{RC}$ is used, the tensile stress in the shaft will be more than 200 MPa.

8.2 The change in diameter is at the contact diameter.

The change in diameter of the shaft $= Sd_1$

$$Sd_1 = \frac{D_C}{E}\left(\sigma_{H_1} - V_1\sigma_{R_C}\right)$$

Use the '*a*' and '*b*' values from the shaft.

$\therefore \sigma_{H_1}$ at $D_C = 200$ $\therefore \sigma_{H_1} = a - \frac{b}{0,2^2}$

$= 100M - \frac{-1M}{0,2^2}$

$= 125$ MPa

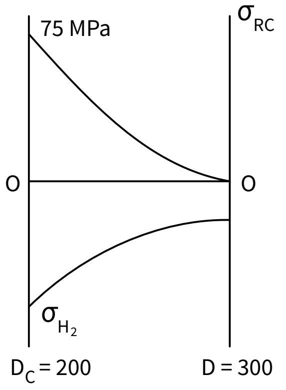
$\therefore Sd_1 = \frac{0,2}{200G}(125M - 0,3 \times 75M)$

$= 1,025 \times 10^{-4}$ m

$= 0,103$ mm

$\therefore$ New diameter $= 200 - 0,103 = 199,897$ mm

8.3 Sleeve



At $D_C = 200$ $\qquad \sigma_{RC} = 75\text{ M} = a + \frac{b}{0{,}2^2}$

$= 75\text{ MPa} = a + 25\,b \ldots$ ①

At D = 300 $\qquad \sigma_R = 0 = a + \frac{b}{0{,}3^2}$

$a = -11{,}111\,b \ldots$ ②

Substitute ② into ①: $\quad 75\text{M} = -11{,}111\,b + 25\,b$

$b = 5{,}4\text{ M} \ldots$ ③

Substitute ③ into ②: $\quad a = -60\text{M}$

∴ Hoop stress σ_{H_2} at D_C

∴ at $D_C = 200$ $\qquad \sigma_{H_2} = a - \frac{b}{0{,}2^2}$

$= -60\text{M} - \frac{5{,}4\text{M}}{0{,}2^2}$

$= -195$ MPa (Tensile)

The change in the reamer for the sleeve:

$8d_2 = \frac{D_C}{E}\left(\sigma_{H_1} - V\sigma_{RC}\right)$

$= \frac{0{,}2}{200\text{G}}(-195\text{M} - 0{,}3 \times 75\text{M})$

$= 2{,}175 \times 10^{-4}$ m

$= -0{,}21$ mm

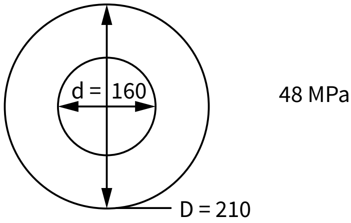
∴ New diagram = 200 + 0,21 = 200,21 mm

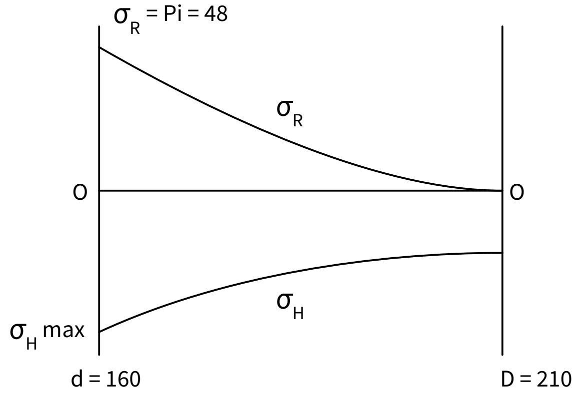
8.4 The material is the same.

∴ Shrinkage allowance $= \Delta d = \frac{0{,}2}{200\text{G}}\left[(125\text{M} - (-195\text{M})\right]$

$= 0{,}31$ mm

9.





There is tensile stress when the cylinder bursts.

$\therefore$ At D = 210 $\sigma_R = 0 = a + \frac{b}{0{,}21^2}$

$\therefore a = -22{,}676\ b\ldots$

At d = 160 $\qquad \sigma_R = 48\text{M} = a + \frac{b}{0{,}16^2}$

$48\text{M} = a + 39{,}063\ b\ldots$

Substitute ① into ②: $\qquad 48\text{M} = -22{,}676\ b + 39{,}063\ b$

$b = 2{,}93\ \text{M}\ldots$③

Substitute ③ into ①: $\qquad a = -66{,}421\ \text{M}$

Tensile stress in the material when the cylinder bursts

At D = 160 $\qquad \therefore \sigma_H = a - \frac{b}{0{,}16^2}$

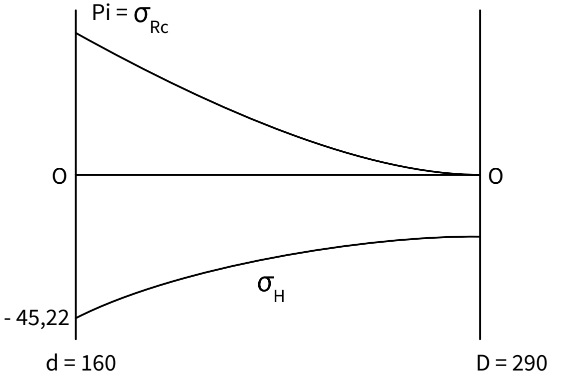
$= -66{,}421\text{M} - \frac{2{,}93\text{M}}{0{,}16^2}$

$= 180{,}87$ MPa

The safe tensile test in material $= \frac{180{,}87}{4}$

$= 45{,}22$ MPa

∴ Safe internal pressure in the cylinder:



At D = 290 $\quad \sigma_R = 0 = a = \frac{b}{0{,}29^2}$

$a = -11{,}891\ b \ldots$ ①

At d = 160 $\quad \sigma_H = -45{,}22\text{M} = a - \frac{b}{0{,}16^2} \ldots$ ②

Substitute ① into ②: $\quad \therefore -45{,}22\text{M} = -11{,}891\ b - 39{,}063\ b$

$\therefore b = +0{,}887\ \text{M}$

$a = -10{,}553\ \text{M}$

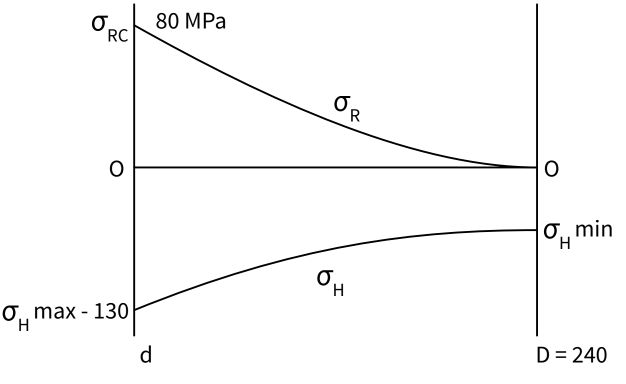
∴ The safe internal pressure:

At D = 160 $\quad \sigma_{RC} = a + \frac{b}{0{,}16^2}$

$= -10{,}552\text{M} + \frac{0{,}887\text{M}}{0{,}16^2}$

$= 24{,}1\ \text{MPa}$

10.



$$\text{Constant} = C = \sigma_H + \sigma_R$$
$$= -130 + 80$$
$$= -50 \text{ MPa}$$

$\therefore$ at D = 240 $\sigma_H = -50M = a + \frac{b}{0{,}24^2}$

$= a - 17{,}361\, b$ … ①

At D = 240 $\sigma_R = 0 = a + \frac{b}{0{,}24^2}$

$a = -17{,}361\, b$ … ②

Substitute ② into ①: $\therefore -50M = -17{,}361\, b - 17{,}361\, b$

$\therefore b = 1{,}44M$

$a = -25M$

At d $\sigma_H = -130M = a - \frac{b}{d^2}$

$\therefore -130M = -25M - \frac{1{,}44M}{d^2}$

$\therefore -105M = \frac{-1{,}44M}{d^2}$

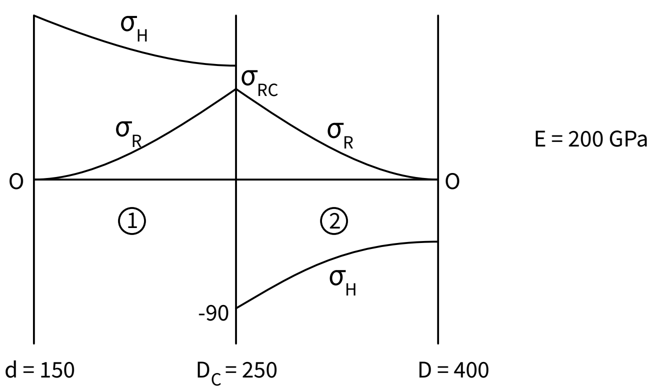
$\therefore d^2 = \frac{-1{,}44M}{-105M}$

$d = 117{,}108$ mm

10.2 Longitudinal stress

$$\sigma_L = \frac{pid^2}{D^2 - d^2}$$
$$= \frac{80M \times 0{,}117108^2}{0{,}24^2 - 0{,}117108^2}$$
$$= 25 \text{ MPa}$$

11.



11.1 Outer cylinder

At D = 400 $\sigma_R = 0 = a + \frac{b}{0{,}4^2}$

$a = -6{,}25\,b$ … ①

At $D_C = 250$ $\sigma_H = -90M = a - \frac{b}{0{,}25^2}$

$= a - 16\,b$ … ②

Substitute ① into ②: $-90M = -6{,}25\,b - 16\,b$

$b = +4{,}045M$

$a = -25{,}281M$

At D_C $\sigma_{RC} = a + \frac{b}{0{,}25^2}$

$= -25{,}281M + \frac{4{,}045M}{0{,}25^2}$

$= 39{,}44$ MPa

11.2 Shrinkage allowance $\Delta d = \frac{D_C}{E}\left(\sigma_{H_1} - \sigma_{H_2}\right)$

The material is the same.

Calculate the minimum σ_H in the outer cylinder = σ_{H1}

$\therefore$ at $D_C = 250$ $\sigma_R = 39{,}44M = a + \frac{b}{0{,}25^2}$

$= a + 16\,b$ … ①

At $d = 150$ $\sigma_R = 0 = a + \frac{b}{0{,}15^2}$

$a = -44{,}444\,b$ … ②

Substitute ① into ②: $\therefore 39{,}44M = -44{,}444\,b + 16\,b$

$b = -1{,}387M$

$a = +61{,}644$ M

At $D_C = 250$

Minimum hoop stress: $\sigma_H = a - \frac{b}{0{,}25^2}$

$= 61{,}44M - \frac{-1{,}387M}{0{,}25^2}$

$= 83{,}836$ MPa (Compressive)

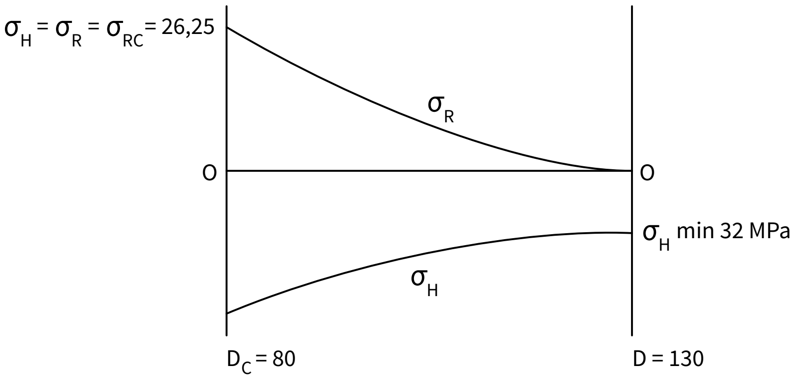
$\therefore \Delta d = \frac{D_C}{E}\left(\sigma_{H_1} - \sigma_{H_2}\right)$

$= \frac{0{,}25}{200G}(-90 - 83{,}836)M$

$= 2{,}173 \times 10^{-4}$ m

$= 0{,}2173$ mm

12.



12.1 The strain at D = 130 is $1{,}6 \times 10^{-4}$

Radial stress at D_C

$\mu = 0{,}2$ E = 200 GPa

$$\text{Strain at} = E = \frac{\sigma_H - \vartheta\sigma_R}{E} \text{ at } D = 130$$

$$\therefore 1{,}6 \times 10^{-4} = \frac{(\sigma_{H\,min} - 0{,}2 \times 0)}{200G}$$

$$\therefore \sigma_{H\,min} = 1{,}6 \times 10^{-4} \times 200G$$

$$= 32 \text{ MPa}$$

Radial stress at D_C

At $d = 130$ $\quad \sigma_R = 0 = a + \dfrac{b}{0{,}13^2}$

$a = -59{,}172\, b$ … ①

At D = 130 $\quad \sigma_H = -32M = a - \dfrac{b}{0{,}13^2}$

$= a - 59{,}172\, b$ … ②

Substitute ① into ②: $\quad \therefore -32M = -59{,}172\, b - 59{,}172\, b$

$b = +0{,}27M$

$a = -15{,}976$ M

At $D_C = 80$ $\quad \sigma_{RC} = a + \dfrac{b}{0{,}08^2}$

$= -15{,}976 + \dfrac{0{,}27M}{0{,}08^2}$

$= 26{,}25$ MPa

12.2 The force to press the shaft out

$$F\mu = P_o(\pi DL)M$$
$$= 26{,}25M(\pi \times 0{,}08 \times 0{,}07)0{,}2$$
$$= 92{,}363 \text{ kN}$$

12.3 Maximum hoop stress in the ring

At $D_C = 80$ $\quad \sigma_{H\,max} = a - \frac{b}{0{,}08^2}$

$$= -15{,}976 - \frac{0{,}27M}{0{,}08^2}$$
$$= 58{,}16 \text{ MPa}$$

MODULE

2 Tension in cables

By the end of this module, students should be able to:

- calculate the minimum and maximum tensions in or at: supports on the same or different levels:
  - the cables
  - the tensions
  - the supports
  - certain points in a cable;
- calculate for supports on the same or different levels:
  - the position in a cable when tension is known
  - the slope of a cable
  - the slope of the cable at both supports
  - the length of a cable
  - the diameter of a cable for a given stress limit
  - the stress in each material for a compound cable
  - the diameter of cable for a given stress limit
  - the tension in the anchor cables when a cable is supported by a frictionless pulley or frictionless rollers
  - the vertical and horizontal reactions at the supports when a cable is supported by a frictionless pulley or frictionless rollers
  - the bending moment on the support(s); and
  - the position of the turning point of a cable.

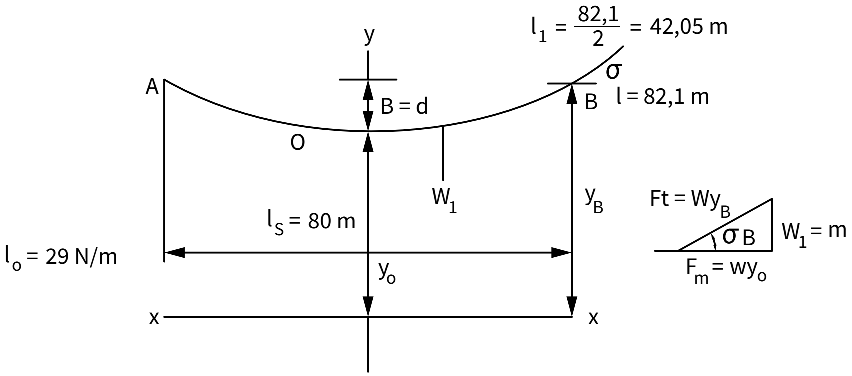
Introduction

When comparing the bending of cables and chains with solid beams, it is assumed that there will be no resistance against bending in a cable and that the force in the cable will always be a tangential force at any point on the cable.

A cable suspended between two supports will either hang in a parabolic shape or a catenary.

Exercise 2.1 SB page 58

1.



1.1 Determine the vertical distances from the *xx*-axis to the cable.

$y_B = y_o + d$

$= y_o + 8 \ldots$ ①

From Δ of forces: $y_B^2 = y_o^2 + l_1^2 \ldots$ ②

Substitute ① into ②: $(y_o + 8)^2 = y_o^2 + 42{,}05^2$

$\therefore y_o^2 + 16y_o = 1\,621{,}103$

$y_o = 101{,}32$ m

$\therefore y_B = 101{,}32 + 8$

$= 109{,}32$ m

∴ Maximum tension at A or B = F*t*

$\therefore Ft = W_{y_B}$

$29 \times 109{,}32$

$= 3{,}17$ kN

1.2 Minimum tension in the cable

At the turning point 'O':

$F_H = wy_o$

$= 29 \times 101{,}32$

$= 2{,}94$ kN

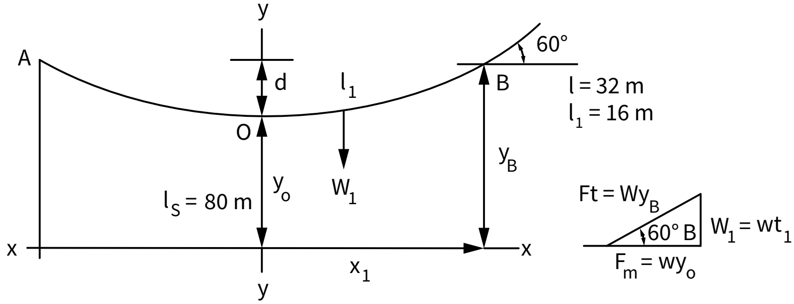
1.3 Maximum slope: from Δ

$\cos\theta = \frac{F_H}{F_t}$

$= \frac{2{,}94}{3{,}17}$

$\therefore \theta = 21{,}96°$

2.



2.1 Sag of chain = d

Determine y_B: $\therefore \sin 60° = \frac{16}{y_B}$

$y_B = 18{,}475$ m

Determine $y_o = \tan 60° = \frac{16}{y_o}$

$\therefore y_o = 9{,}238$ m

$\therefore d = y_B - y_o = 18{,}475 - 9{,}238$

$= 9{,}24$ m

2.2 Distance between the supports

$$x_1 = y_o \ln\left(\frac{y_B + l_1}{y_o}\right)$$

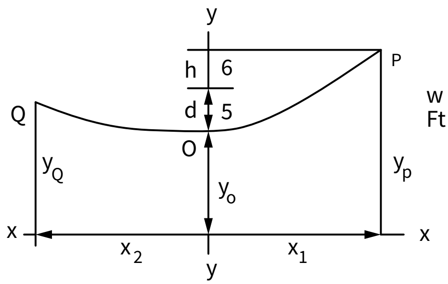
$$= 9{,}328 \ln\left(\frac{18{,}475 + 16}{9{,}238}\right)$$

$= 1\ 217$ m

$\therefore$ The distance between A and B = $2 \times 12{,}17$

$= 24{,}34$ m

3.



3.1 Support P

$y_p = y_o + 5 + 6 \therefore (d + h)$

$= y_o + 11 \ldots$ ②

Ft min $= F_H = wy_o$

$\therefore 200 = 8y_o$

$y_o = 25$ m

$y_p = 25 + 11 - 36$ m

$y_q = y_o + d$

$= 25 + 5$

$= 30$ m

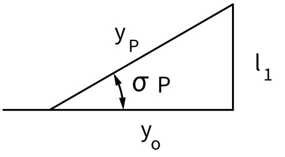
$\therefore$ Ft at P $= y_p w$

$= 36 \times 8 = 288$ N

$\therefore$ Ft at Q $= y_q \times 8$

$= 30 \times 8 = 240$ N

3.2



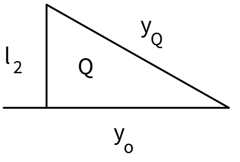
From Δ of the force at P:

$y_p^2 = l_1^2 + y_o^2$

$\therefore l_1 = \sqrt{y_p^2 - y_o^2}$

$= \sqrt{36^2 - 25^2}$

$l_1 = 25{,}9$ m



From Δ at Q:

$\therefore y_Q^2 = y_o^2 + l_2^2$

$$l_2 = \sqrt{y_Q^2 - y_o^2}$$
$$= \sqrt{30^2 - 25^2}$$
$$= 16{,}58 \text{ m}$$

$\therefore$ Length of cable $= l_1 + l_2$
$$= 25{,}9 + 16{,}58$$
$$= 42{,}48 \text{ m}$$

3.3 Maximum slope at the longest support P

From Δ of the force at P:

$$\therefore \sin\theta = \frac{l_1}{y_p}$$
$$= \frac{25{,}9}{36}$$
$$\therefore \theta = 46°$$

3.4 Distance between supports

$$\therefore L = x_1 + x_2$$

x_1 = from Q to P

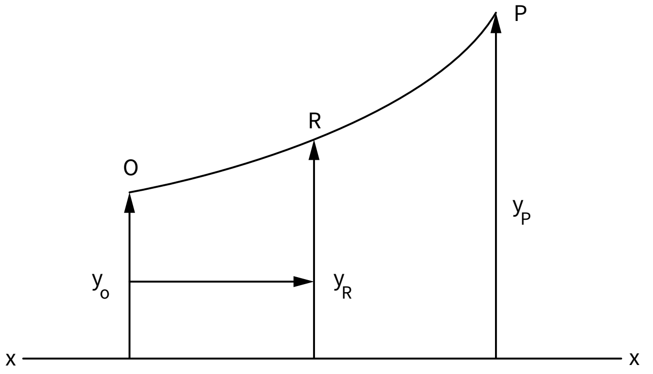
$$\therefore x_1 = y_o \ln\left(\frac{y_{P_{l_1}}}{y_o}\right) = 25 \ln\left(\frac{36 + 25{,}9}{25}\right)$$
$$\therefore x_1 = 22{,}67 \text{ m}$$

x_2 = from O to Q

$$\therefore x_2 = y_o \ln\left(\frac{y_Q + l_2}{y_o}\right) = 25 \ln\left(\frac{(30 + 25{,}9)}{25}\right)$$
$$\therefore x_2 = 15{,}56 \text{ m}$$

$\therefore$ The distance from Q to P $= x_1 + x_2$
$$= 22{,}67 + 15{,}56$$
$$= 38{,}23 \text{ m}$$

3.5



The distance from Q to P

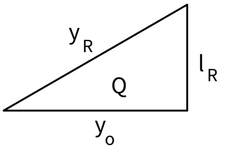
Tension in the cable at R = 260 N

$\therefore 260 = wy_R$

$y_R = \frac{260}{8}$

$= 32{,}5$ m

Distance for Q to R= $x_R = y_o \ln\left(\frac{y_R + l_{OR}}{y_o}\right)$ … ①



From Δ of the force at R:

$l_R^2 = y_R^2 - y_o^2$

$l_R = \sqrt{32{,}5^2 - 25^2}$

$= 20{,}77$ m

Substitute into ①: $x_R = 25 \ln\left(\frac{32{,}5 + 20{,}77}{25}\right)$

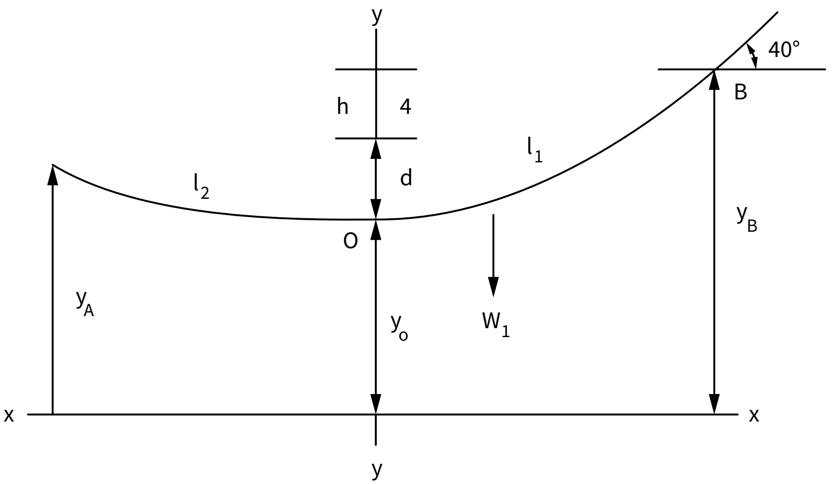
$= 18{,}9$ m

$\therefore$ R from P $= x_1 - x_R$

$= 22{,}67 - 18{,}9$

$= 3{,}77$ m

4.

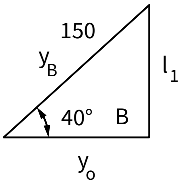


4.1 Maximum tension in the cable at the highest support B

$Ft = 4\,500 = wy_B$

$y_B = \frac{4\,500}{30}$

$= 150$ m



$\therefore \Delta$ of the forces at B

$\cos 40° = \frac{y_o}{150}$

$y_o = 150 \cos 40°$

$= 114{,}91$ m

$\therefore y_B = y_o + d + h$

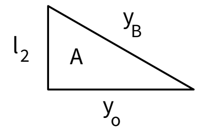
$150 = 114{,}9 + d + 4$

$\therefore$ Sag $= d = 31{,}08$ m

4.2 Length of the cable = $l_1 + l_2$

$\therefore$ From Δ of the force at B

$$l_1 = \sqrt{y_B^2 - y_o^2}$$
$$= \sqrt{150^2 - 114{,}91^2}$$
$$= 96{,}41 \text{ m}$$



$$y_A = y_B - h$$
$$= 150 - 4$$
$$= 146 \text{ m}$$

From Δ of the forces at A:

$$l_2 = \sqrt{y_A^2 - y_o^2}$$
$$= \sqrt{146^2 - 114{,}91^2}$$
$$= 90{,}06 \text{ m}$$

Length of the cable = $l_1 + l_2$

$$= 96{,}41 + 90{,}06$$
$$= 186{,}5 \text{ m}$$

4.3 Stress in aluminium and steel:

$A_{AL} = SA_s \ldots$ ①

$A_T = A_{AL} + A_s \ldots$ ②

Substitute ① into ②: $1\,924 = SA_s + A_s = 6A_s$

$\therefore A_s = 320{,}67 \text{ mm}^2$

$A_{AL} = 320{,}67 \times 5 = 1\,603{,}33 \text{ mm}^2$

$F_{t\max} = F_s + F_{al}$

$\therefore 4\,500 = \sigma_s A_s + \sigma_{AL} A_{AL}$

$= 320{,}67 \times 10^{-6} \sigma_s + 1\,603{,}33 \times 10^{-6}\, \sigma_{AL} \ldots$ ①

But the strain in steel = the strain in aluminium

$$\therefore \frac{\sigma_s}{E_s} = \frac{\sigma_{AL}}{E_{AL}}$$

$E_s = 3E_{al} \ldots$ ②

$\therefore \frac{\sigma_s}{3E_{AL}} = \frac{\sigma_{al}}{E_{al}}$

$\therefore \sigma_s = 3\sigma_{al} \ldots$ ③

Substitute ③ into ①:

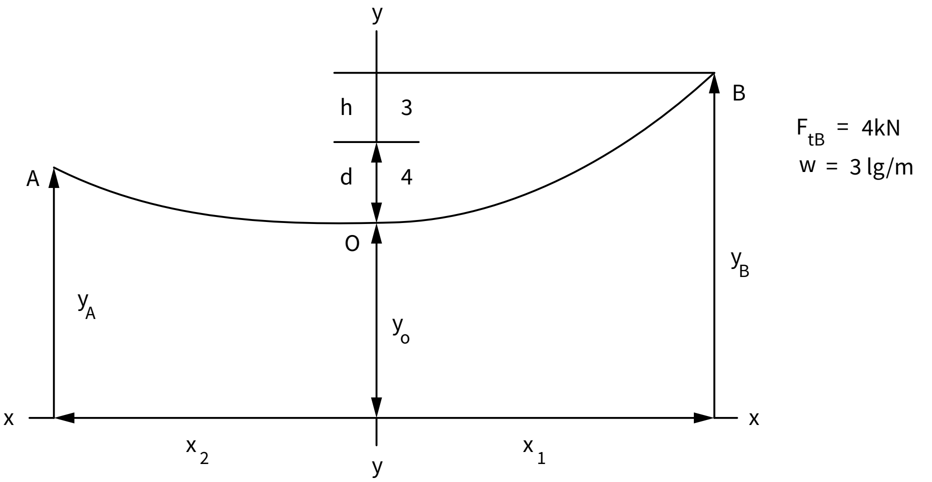
$\therefore 4\,500 = 320{,}67 \times 10^{-6} \times 3\sigma_{AL} + 1\,603{,}33 \times 10^{-6}\sigma_{AL}$

$= 2{,}565 \times 10^{-3}\sigma_{AL}$

$\therefore \sigma_{AL} = 1{,}75$ MPa

$\therefore \sigma_s = 3\sigma_{AL} = 3 \times 1{,}75M = 5{,}25$ MPa

5.



5.1 Tension at A in the cable = F_{tA}

$\therefore F_{tB} = 4\,000 = w_{yB}$

$y_B = \frac{4\,000}{3 \times 9{,}81}$

$= 135{,}92$

$\therefore y_A = y_B - h$

$= 135{,}9 - 3$

$= 132{,}92$ m

Tension at A = $F_{tA} = wy_A$

$= 3 \times 9{,}81 \times 132{,}92$

$= 3{,}91$ kN

5.2 Minimum tension at the turning point 'O'

$F_{t\,min} = F_H = wyo$

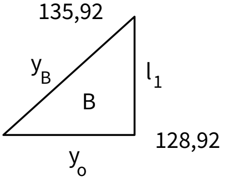
$y_o = y_B - (h + d) = 135{,}92 - (3 + 4)$

$= 128{,}92$ m

$\therefore F_{Hmin} = 3 \times 9{,}81 \times 128{,}92$

$= 3{,}79$ kN

5.3 Length of the cable $= l_1 + l_2$



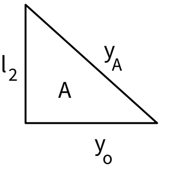
$\therefore l_1$ from ΔB

$l_1 = \sqrt{y_B^2 - y_o^2}$

$= \sqrt{135{,}92^2 - 128{,}92^2}$

$= 43{,}06$ m

l_2 from ΔA



$l_2 = \sqrt{y_A^2 - y_o^2}$

$= \sqrt{132{,}92^2 - 128{,}92^2}$

$= 32{,}36$ m

Length of the cable $= 43{,}06 + 32{,}36$

$= 75{,}42$ m

5.4 Distance between A and B $= x_1 + x_2$

O to B $\therefore x_1 = y_o \ln\left(\frac{y_B + l_1}{y_o}\right)$

$= 128{,}92 \ln\left(\frac{135{,}92 + 43{,}06}{128{,}92}\right)$

$= 42{,}3$ m

O to A $\therefore x_2 = y_o \ln\left(\frac{y_A + l_2}{y_o}\right)$

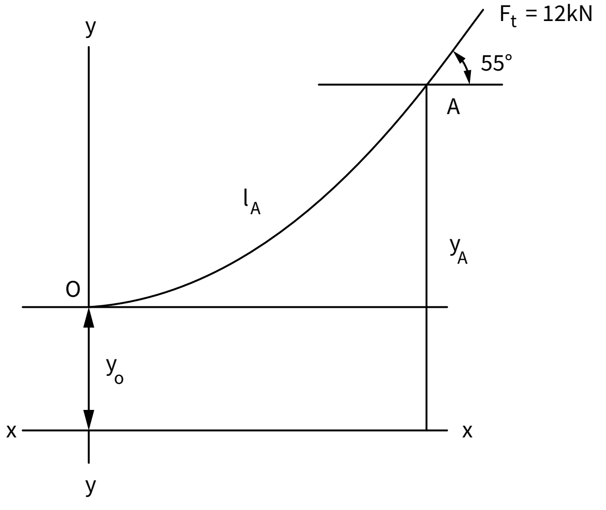
$= 128{,}92 \ln\left(\frac{135{,}92 + 32{,}36}{128{,}92}\right)$

$= 32{,}03$ m

$\therefore$ The distance A to B $= 42{,}3 + 32{,}03$

$= 74{,}33$ m

6.

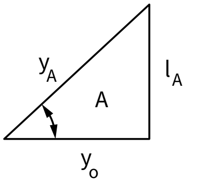


6.1 Length of the rope on the ground $= l - l_A$

$= 100 - l_A$

$\therefore F_{t\,max} = 1\,200 = wy_o$

$y_A = \frac{1\,200}{120} = 100$ m



$\therefore$ from ΔA:

$\cos 55° = \frac{y_o}{y_A}$

$y_o = 100 \cos 55°$

$= 57{,}36$ m

and $\sin 55° = \frac{l_A}{y_A}$

$\therefore l_A = 100 \sin 55°$

$= 81{,}91$ m

$\therefore$ Length on the ground $= 100 - 81{,}91$

$= 18{,}09$ m

6.2 Height of the tower $= y_A - y_o$

$= 100 - 57{,}36$

$= 42{,}64$ m

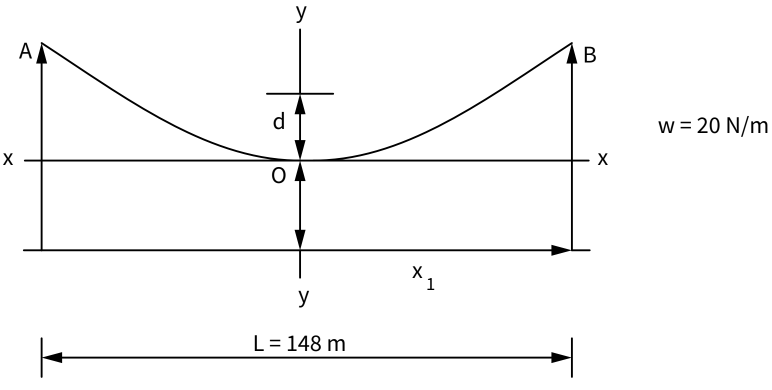
6.3 Minimum tension $= F_H = wy_o$

$= 120 \times 57{,}36$

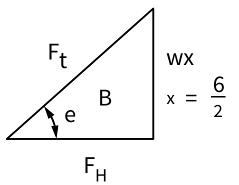
$= 6{,}883$ kN

Exercise 2.2 SB page 78

1.



1.1 Safe tension = maximum tension $= Ft_B = Ft_A$



Minimum tension at 'O' $= F_H$

From ΔB:

$\therefore$ Maximum tension at 'O' $= F_H$

$$\therefore F_H = \sqrt{F_C^2 - \left(\frac{wL}{2}\right)^2}$$

$$= \sqrt{5\,000^2 - \left(20 \times \frac{148}{2}\right)^2}$$

$$= \sqrt{5\,000^2 - 2\,190{,}4}$$

$\therefore F_H = 4{,}78$ kN

1.2 Sag of the cable

$F_H = \frac{wl^2}{8d}$

$\therefore 4\,780 = \frac{20 \times 148^2}{8d}$

$\text{Sag} = d = \frac{20 \times 148^2}{8 \times 4\,780}$

$= 11{,}5 \text{ m}$

1.3 Maximum slope: from ΔB

$\cos\theta = \frac{F_H}{Ft} = \frac{4\,780}{5\,000}$

$\therefore \theta = 17{,}06°$

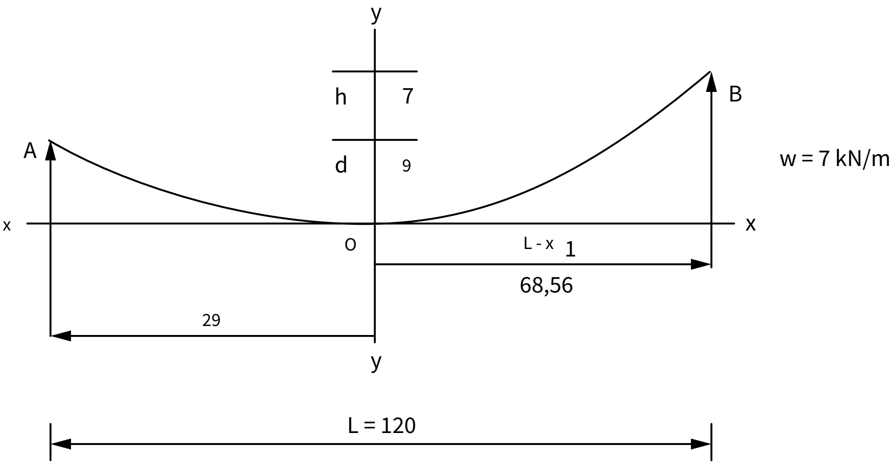
1.3 Length of cable = l

$\therefore l = L + \frac{8d^2}{3L}$

$= 148 + \frac{8 \times 11{,}5^2}{3 \times 148}$

$= 150{,}38 \text{ m}$

2.



2.1 Calculate the position of the turning point

$F_{AA} = F_{HB}$

$\therefore \frac{wx_1^2}{2d} = \frac{w(L - x_1)^2}{2(d + h)}$

$\times \frac{2}{w} \therefore \frac{x_1^2}{d^2} = \frac{L - x_1^2}{d + h}$

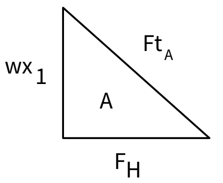
$\therefore \frac{x_1^2}{9} = \frac{(120 - x)^2}{7 + 9}$

$1{,}778x_1^2 = (120 - x_1)^2$

Take the square root $\therefore$ $1{,}333x_1 = 120 - x_1$

$2{,}333x_1 = 120$

$x_1 = 51{,}44$ m



For both cables:

$$F_{min} = F_H = \frac{wx_1^2}{2d}$$

$$F_H = \frac{7k \times 51{,}44^2}{2 \times 9}$$

$= 1{,}029$ MN for both cables.

$\therefore$ Minimum tension/cable $= \frac{1{,}029M}{2}$

$= 514{,}5$ kN

Maximum tension Ft_B at the highest support for both cables.

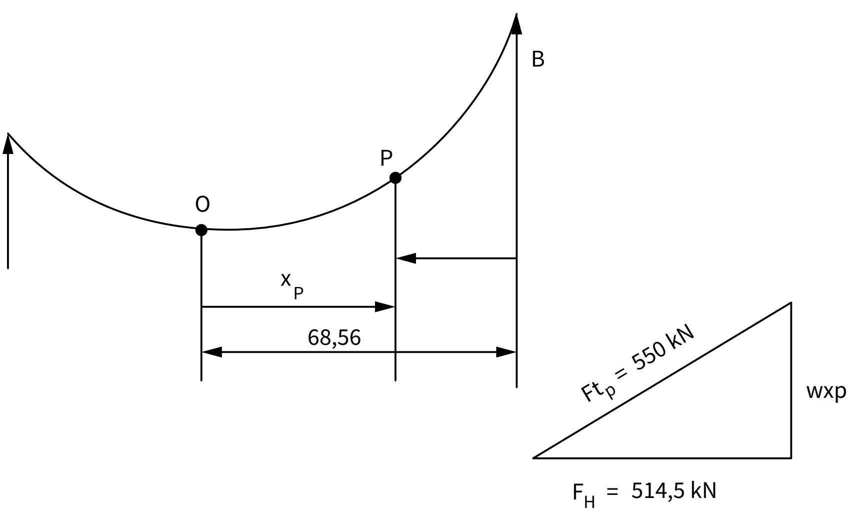
$$Ft_B = \sqrt{F_H^2 + [w(L - x_1)]^2}$$

$$= \sqrt{(1{,}029M)^2 + (7k \times 68{,}56)^2}$$

$= 1{,}135$ MN

Tension/cable $= 567{,}5$ kN

2.2



Force Δ at P:

w per cable $= \frac{7\text{ kN}}{2} = 3{,}5$ kNm

$\therefore F_{tp}^2 = F_H^2 + (wx_p)^2$

$\therefore wx_p = \sqrt{(550k)^2 - (514{,}5k)^2}$

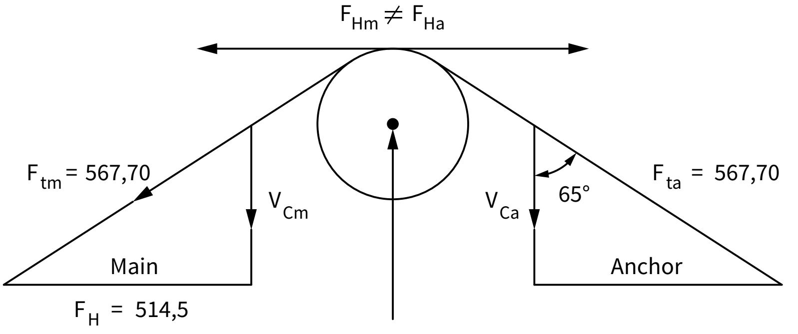
$\therefore x_p = \frac{194{,}396k}{3{,}5k}$

$= 55{,}54$ m

$\therefore p$ from the longest support $= 68{,}56 - 55{,}54$

$= 13{,}02$ m

2.3 Reaction in B (longest)



Reaction at B $= V_{C\text{ main}} + V_{C\text{ anchor}}$

$\therefore V_{C\text{ main}}$

From Δmain $V_{CM} = \sqrt{(567{,}5^2) - (514{,}5)^2}$

$= 239{,}47$ kN

$V_{C\text{ anchor}}$ from Δ_{anchor}

$\therefore \cos 65° = \frac{V_{Ca}}{567{,}5}$

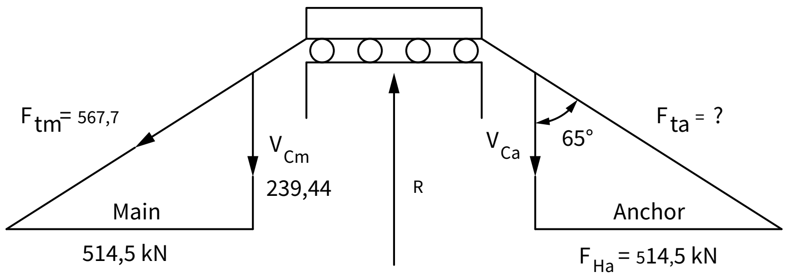
$\therefore V_{Ca} = 239{,}83$ kN

$\therefore$ Reaction in B $= V_{Cm} + V_{Ca}$

$= 239{,}47 + 239{,}83$

$= 479{,}3$ kN

2.4



The vertical component for the main cable remains the same as in 2.3.

$V_{CM} = 239{,}47$ kN

$V_{C\ anchor}$:

$\therefore \tan 65° = \frac{514{,}5k}{\tan 65°}$

$\therefore V_{Ca} = \frac{514{,}5k}{\tan 65°}$

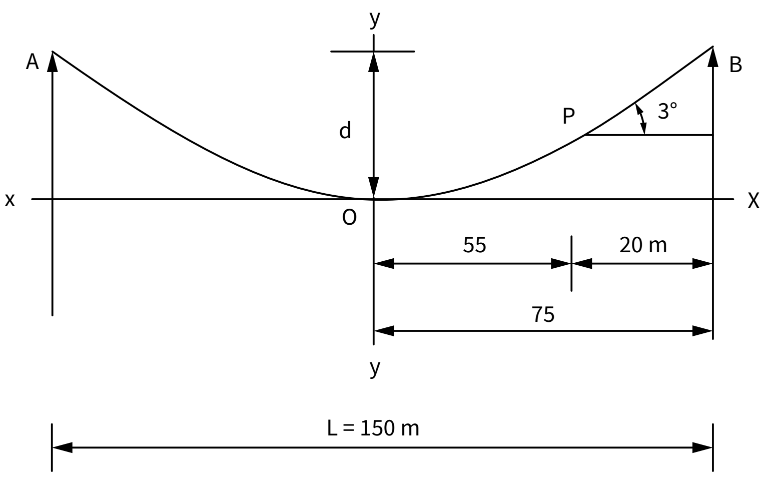
$= 239{,}91$ kN

$\therefore$ Reaction $= V_{Cm} + V_{Ca}$

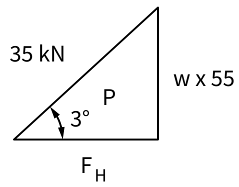
$= 239{,}47 + 239{,}91$

$= 479{,}38$ kN

3.



3.1 From ΔP



$\cos 3° = \frac{F_H}{35k}$

$\therefore$ Minimum tension = F_H = 34,95 kN

From ΔP:

$\sin 3° = \frac{w55}{35k}$

$\therefore w55$ = 1 831,75 N

w = 33,3 Nm

$\frac{\text{mass}}{\text{m}} = \frac{33,33}{9,81}$ = 3,395 kg/m

3.2 Maximum tension at A and B.

$\therefore F_{max} = \sqrt{(F_H^2) + \left(w\frac{L}{2}\right)^2}$

$= \sqrt{34,95k^2 + \left(33,3 \times \frac{150}{2}\right)^2}$

= 35,039 kN

3.3 Sag of the cable = d

$F_H = \frac{wL^2}{8d} = \frac{33,3 \times 150^2}{8d} = 34\ 950$

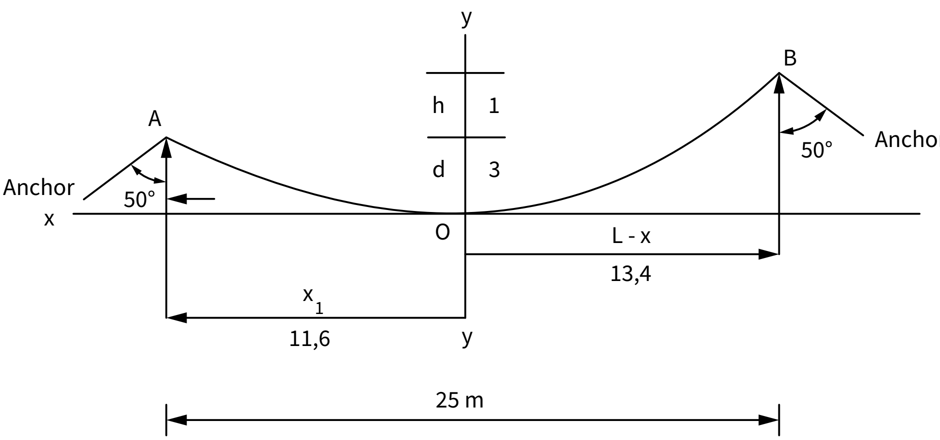
$\therefore$ Sag $d = \frac{33,3 \times 150^2}{8 \times 34\ 950}$

= 2,7 m

3.4 Maximum tensile stress in the cable

$\text{Stress} = \frac{\text{maximum tension}}{\text{area}} = \frac{35\ 039 \times 4}{\pi 0,04^2}$ = 27,883 MPa

4.



4.1 Position of the turning point

$F_{HA} = F_{HB}$

$\therefore \frac{wx_1^2}{2d} = \frac{\left(w(L - x_1)^2\right)}{2(d + h)}$

$\therefore \frac{x_1^2}{3} = \frac{(L - x_1)^2}{3 + 1}$

$\therefore 1{,}33x_1^2 = (L - x_1)^2$

$1{,}155x_1 = 25 - x_1$

$x_1 = \frac{25}{2{,}155}$

$= 11{,}6$ m

Minimum tension $= F_H = \frac{wx_1^2}{2d}$

$= \frac{(18k \times 11{,}6)^2}{2 \times 3}$

$= 403{,}68$ kN

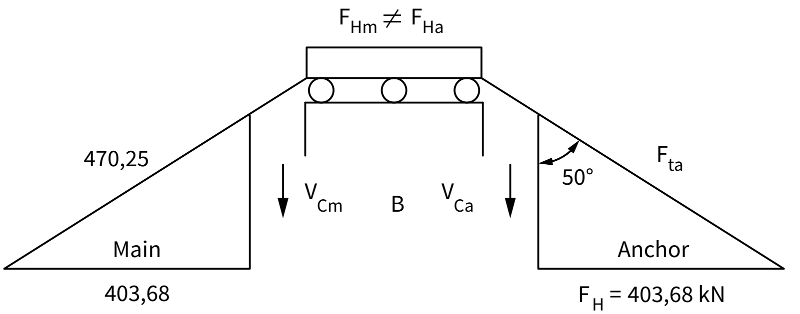
Maximum tension at B

$\therefore F_{tmax} = \sqrt{\left(F_H{}^2 + [w(L - x_1)]^2\right)}$

$= \sqrt{403{,}68k^2 + (18k + 13{,}4)^2}$

$= 470{,}25$ kN

4.2



Because F_H is constant at any point and anchor cables at both supports make the same angle of 50°, therefore at both supports the tension in the anchor cables will be the same.

From Δ anchor

$\therefore \sin 50° = \frac{FH_a}{Fta}$

Tension in the anchor cable $= Fta = \frac{403{,}68k}{\sin 50°}$

$= 526{,}97$ kN

4.3 Reaction at each support

The vertical component force for both anchor cables will be the same. This is for the same reason as above.

From anchor Δ:

$$\therefore \tan 50° = \frac{F_H}{V_{Ca}}$$

$$\therefore V_{Ca} = \frac{403{,}68k}{\tan 50°}$$

$$= 338{,}73 \text{ kN}$$

$\therefore V_{C\,main}$ at support B

From Δ main:

$$V_{cm} = \sqrt{(470{,}25k^2 - 403{,}68k^2)}$$

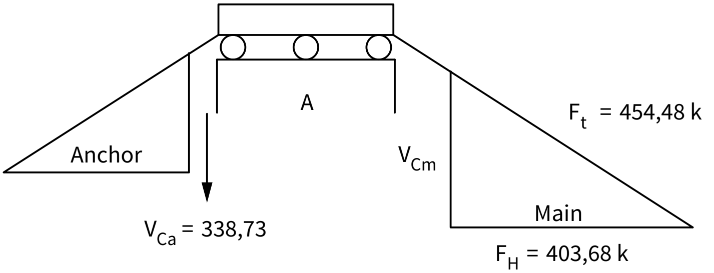
$$= 241{,}2 \text{ kN}$$

$$\therefore \text{Reaction in B} = V_{cm} + V_{ca}$$

$$= 241{,}2k + 338{,}73k$$

$$= 579{,}93 \text{ kN}$$

At support A:



The minimum tension in the cable at support A.

$$Fta = \sqrt{403{,}68k^2 + (18k \times 11{,}6)^2}$$

$$= 454{,}48 \text{ kN}$$

From Δ main V_{cm}

$$\therefore V_{cm_A} = \sqrt{454{,}48k^2 - 403{,}68k^2}$$

$$= 208{,}79 \text{ kN}$$

$$\therefore \text{Reaction in support A} = V_{cm_A} + V_{ca_A}$$

$$\text{Reaction A} = 208{,}79 + 338{,}73$$

$$= 547{,}52 \text{ kN}$$

4.4 Slope at each support

Slope at highest support:

$\cos\theta = \frac{403,68}{470,25}$

$\therefore$ Slope $= \theta = 30,858°$

Slope at shortest support:

$\cos\theta = \frac{403,68}{454,48}$

$\therefore$ Slope $= \theta = 27,349°$

4.5 Diameter of the cable

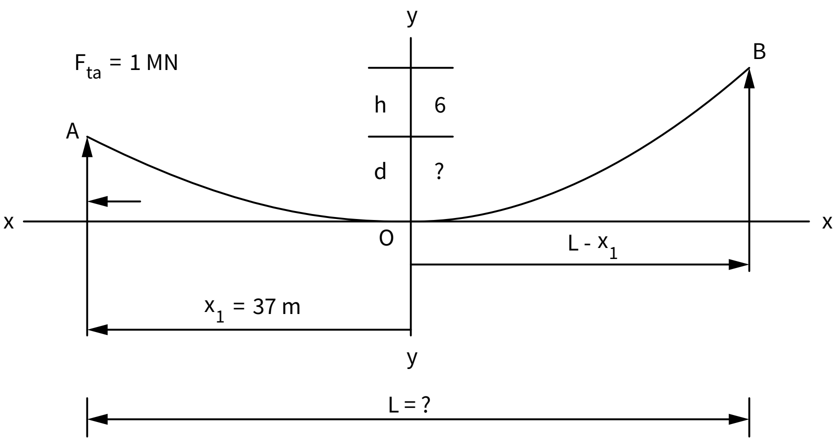
Safe stress $= \frac{\text{ultimate stress}}{\text{FOS}} = \frac{450M}{4} = 112,5$ MPa

Mean area of cable $= \frac{\text{maximum tension}}{\text{stress}} = \frac{470,25k}{112,5m} = 4,18 \times 10^{-3}\ m^2$

Area $= \frac{\pi D^2}{4}$

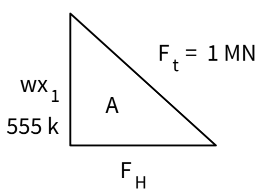
$\therefore$ Diameter $= \sqrt{\frac{4 \times 4,18 \times 10^{-3}}{\pi}} = 72,953$ mm

5.



5.1 Minimum tension in the cable $= F_H$

$\therefore F_H = \frac{wx_1^2}{2d}$



Weight of the cable from 'O' to 'A'

$\therefore w_H = wx_1$

$= 15k \times 37$

$= 555$ kN

From

$\therefore F_H = \sqrt{Ft^2 - (wx_1)^2}$

$= \sqrt{1\ M^2 - 555k^2}$

$= 831{,}85$ kN

5.2 Sag of cable

$$\therefore F_H = \frac{wx_1^2}{2d}$$

$$\text{Sag} = d = \frac{15k \times 37^2}{2 \times 831{,}85k}$$

$= 12{,}34$ m

5.3 Distance between the supports $= L = x_1 + (L - x_1)$

$F_{HA} = F_{HB}$

$$\frac{wx_1^2}{2d} = \frac{(w(L - x_1)^2)}{2(d + h)}$$

$$\frac{(d + h)x_1^2}{2d} = (L - x_1)^2$$

$$\therefore \frac{(6 + 12{,}34)37^2}{12{,}34} = (L - x_1)^2$$

$(1{,}486 \times 37)^2 = (L - x_1)^2$

$\therefore 1{,}219 \times 37 = (L - x_1)$

$45{,}103\ \text{m} = (L - x_1)$

$\therefore$ Resistance between the supports

$= 37 + 45{,}103$

$= 82{,}103$ m

5.4 Maximum tension in the cable at B

$\therefore F_{tB\,max} = \sqrt{(F_H)^2 - [wk - x_1]^2}$

$= \sqrt{831{,}85k^2 + [15k \times 45{,}103]^2}$

$= 1{,}072$ MN

5.5 Length of the cable

$l = l_A + l_B$

$\therefore l_A = x_1 + \frac{2d^2}{3x_1}$

$= 37 + \frac{(2 \times 12{,}34^2)}{3 \times 37}$

$= 39{,}74$

$l_B = (L - x_1) + \frac{2(d+h)^2}{3(L - x_1)}$

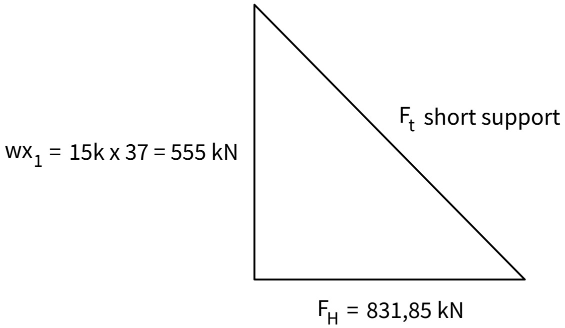
$l_B = 45{,}103 + \frac{2(6 + 12{,}34)^2}{3 \times 45{,}103}$

$= 50{,}075$ m

Length of the cable = 39,74 + 50,075

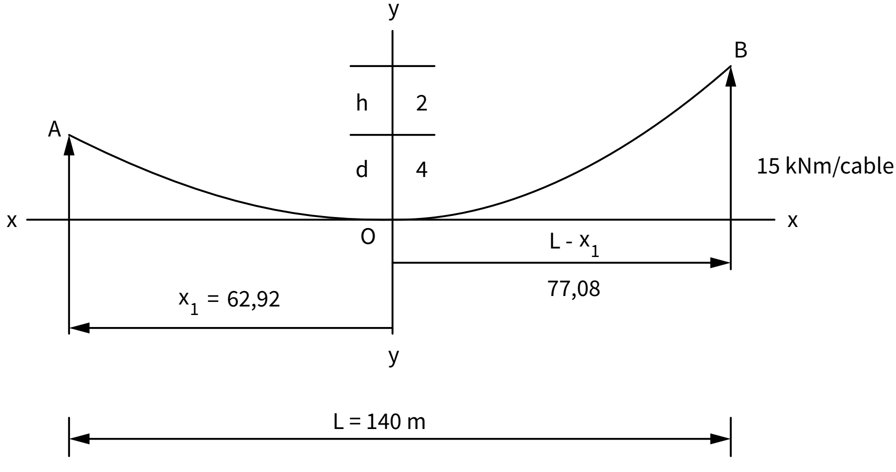
= 89,815 m

5.6 Tension in cable at shortest support



Tension in the cable $F_{t\ short} = \sqrt{(555k)^2 + (831{,}85k)^2} = 1$ MN

6.



6.1 Maximum tension of the cable at B.

Determine the position of the 'O' turning point.

$$\frac{wx_1^2}{2d} = \frac{\left(w(L - x_1)^2\right)}{2(d+h)}$$

$$\frac{(d+h)x_1^2}{2d} = (L - x_1)^2$$

$$\therefore \frac{6x_1^2}{4} = (L - x_1)^2$$

$$1{,}5\ x_1^2 = (L - x_1)^2$$

$$\therefore 1{,}225x_1 = (L - x_1)$$

$$1{,}225\ x_1 = 140$$

$$x_1 = 62{,}92\ \text{m}$$

$\therefore$ Minimum tension in the cable

$$\text{Horizontal constant FOR} = F_H = \frac{wx_1^2}{2d}$$

$$= \frac{15k \times 62{,}92^2}{2 \times 4}$$

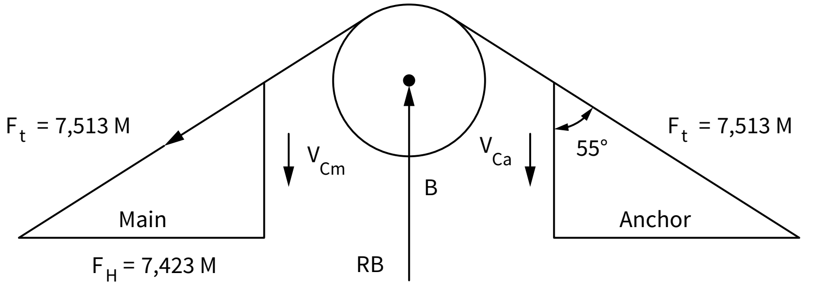
$$= 7{,}423\ \text{MN}$$

$$F_{t\,max} \text{ at B} = F_{tB} = \sqrt{(F_H)^2 + [w(L - x_1)]^2}$$

$$= \sqrt{7{,}423\ \text{M}^2 + (15k \times 77{,}08)^2}$$

$$= 7{,}513\ \text{MN}$$

6.2



The vertical reaction in the main cable

From Δ main:

$$V_{cm} = \sqrt{7{,}513^2 - 7{,}423^2}$$

$$= -1{,}159\ \text{MN}$$

From Δ anchor

$$\cos 55° = \frac{V_{ca}}{7{,}413\text{M}}$$

$$\therefore V_{ca} = 4{,}309\ \text{MN}$$

$\therefore$ Reaction in support B $= V_{cm} + V_{ca}$

$= 1{,}159\text{M} + 4{,}309$

$= 5{,}46 \text{ MN}$

6.3 $F_{Hm} \neq F_{Ha}$

$\therefore F_{Ha}$ from Δ anchor

$\therefore F_{Ha} = \sqrt{Fta^2 - V_{ca}^2}$

$= \sqrt{7{,}513\text{M}^2 - 4{,}309\text{M}^2}$

$= 6{,}154 \text{ MN}$

$\therefore$ Resultant force at the pulley

$F_{res} = F_{H\,main} - F_{H\,anchor}$

$= 7{,}423\text{M} - 6{,}154\text{M}$

$= 1{,}269 \text{ MN}$

$\therefore$ Bending moment in support B

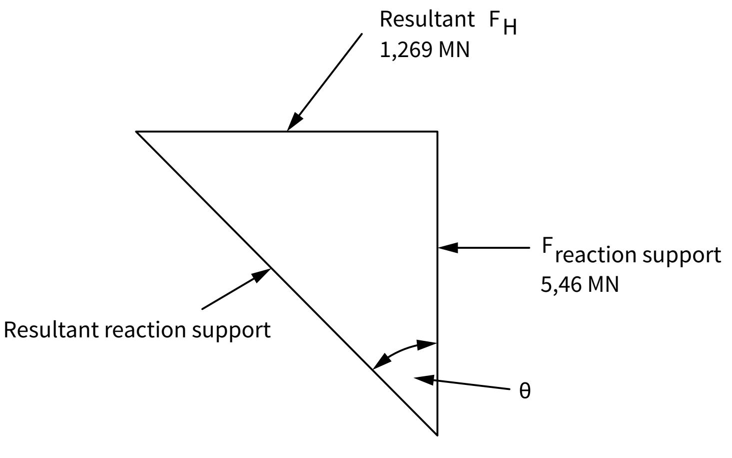
$\therefore BM_B = 1{,}269\text{M} \times 9$

$= 11{,}42 \text{ MNm}$

6.4 Resultant reaction in the longest support

From Question 6.2, the reaction in the support = 5,46 MN.

And, from Question 6.3, the resultant horizontal component force = 1,269 MN to the west.

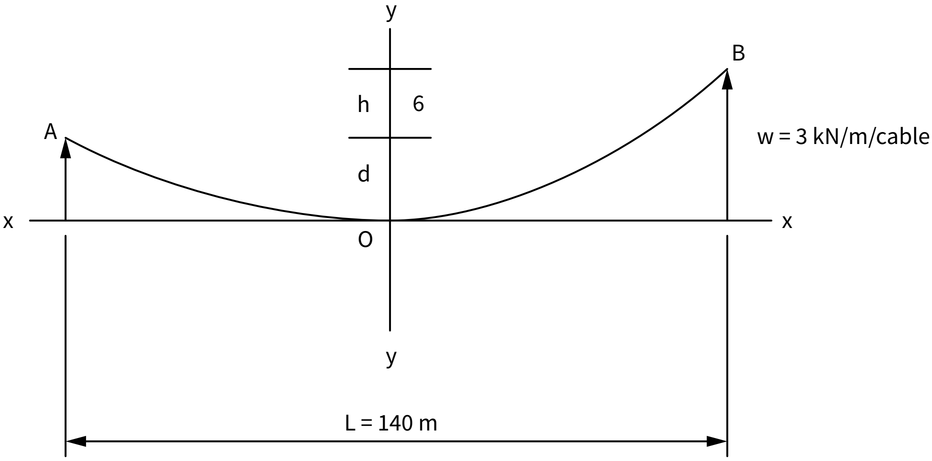


$\therefore \tan\theta = \frac{1{,}269}{5{,}46} = 0{,}232$

$\theta = 13{,}084°$ with the vertical

Resultant reaction $= \sqrt{1{,}269\text{M}^2 + 5{,}46\text{M}^2} = 5{,}606 \text{ MN}$

7.



7.1 $\frac{F_{min}}{MN} = F_H$

Position of 'O':

$F_H = \frac{wx_1^2}{2d}$ $\quad \therefore 1M = \frac{3kx_1^2}{2d}$

$\therefore d = \frac{3kx_1^2}{2 \times 1\,M}$

$= 1{,}65 \times 10^{-6}\,x_1^2 \ldots$ ①

and $F_H = \frac{\left(w(L - x_1)^2\right)}{2(d + h)}$

$1M = \frac{\left(3k(L - x_1)^2\right)}{2(d + 6)}$

$\therefore \frac{2 \times 1\,M(d+)}{3k} = (140 - x_1)(140 - x_1)$

$666{,}667d + 4\,000 = 19\,600 - 280x_1 + x_1^2 \ldots$ ②

Substitute ① into ②:

$666{,}667\left(1{,}65 \times 10^{-6}x_1^2\right) = 19\,600 - 4\,000 - 280x + x_1^2$

$\therefore 1{,}1 \times 10^{-3}x_1^2 = 15\,600 - 280x_1 + x_1^2$

$\therefore -0{,}999x_1^2 + 280x_1 - 15\,600 = 0$

$\div 0{,}999 \therefore -x_1^2 + 280{,}31x - 15\,617{,}179 = 0$

$\therefore x_1 = \frac{-280{,}31 \pm \sqrt{280{,}31^2 - 4(-1)(-15\,617{,}179)}}{-2}$

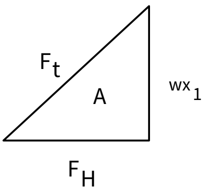
$= \frac{-280{,}31 \pm 126{,}91}{-2}$

$x_1 = 76{,}695$ m

$Ft_A = \sqrt{(F_H)^2 + (wx_1^2)}$

$= \sqrt{1\ M^2 + (3k \times 76{,}695)^2}$

$= 1{,}026$ MN



$Ft_B = \sqrt{F_H + [w(L - x_1)]^2}$

$= \sqrt{1\ M^2 + [3k \times 63{,}305]^2}$

$= 1{,}018$ MN

7.2 Position where Ft = 1,01 MN

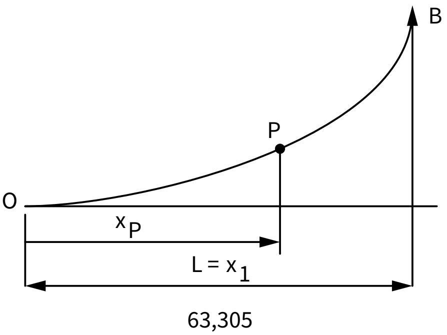
From ΔP:

$wx_p = \sqrt{1{,}01\ M^2 - 1\ M^2}$

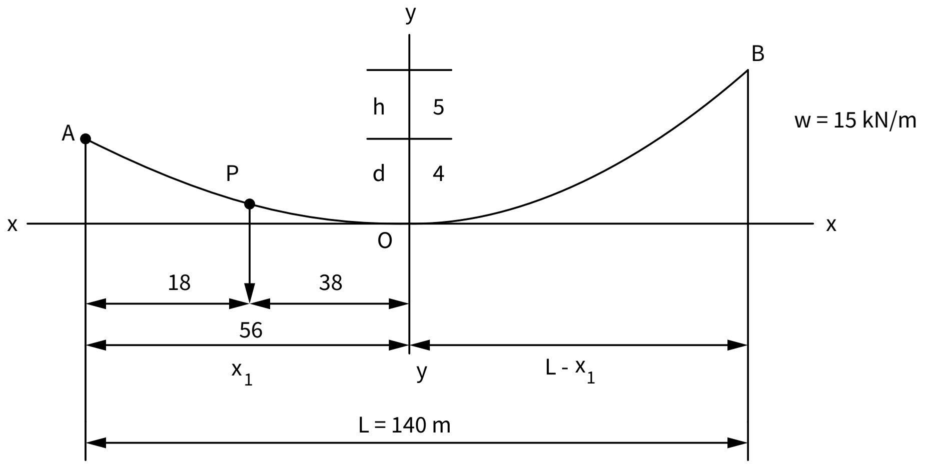
$wx_p = 141\ 774{,}47$

$x_p = \frac{141\ 774{,}47}{3k}$

$= 47{,}26$ m from O



8.



8.1 Position of the turning point 'O'

$F_{HA} = F_{HB}$

$\therefore \frac{wx_1^2}{2d} = \frac{w(t - x_1)^2}{2(d + h)}$

$\frac{(d + h)x_1^2}{d} = (L - x_1)^2$

$\frac{9}{4}x_1^2 = (L - x_1)^2$

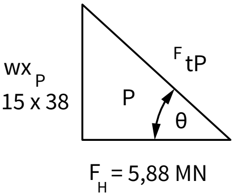
$\sqrt{2{,}5x_1^2} = 140$

$x_1 = 56$ m

$\text{Minimum tension} = F_H = \dfrac{wx_1^2}{2d}$

$= \dfrac{15k \times 56^2}{2 \times 4}$

$= 5{,}88$ MN



From ΔP tension in the cable at P:

$Ft_p = \sqrt{(F_H)^2 + (wx_p)^2}$

$= \sqrt{5{,}88\ M^2 + (15k \times 38)^2}$

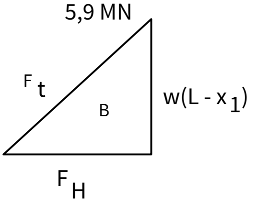
$= 5{,}907$ MN

8.2 Slope at P from ΔP

$\therefore \tan\theta = \dfrac{wx_p}{F_\mu} = \dfrac{15k \times 38}{5{,}88\ M}$

$\theta = 5{,}54°$

9. From ΔB



$Ft^2 = F_H^2 + [w(L - x_1)]^2$

$\therefore (5{,}9\ M)^2 = \left[\dfrac{w(L - x_1^2)}{2(d+h)}\right]^2 = [w(L - x_1)]^2$

$= \left[\dfrac{14k(L - x_1)^2}{2(4+5)}\right]^2 + [14k(L - x_1)]^2$

$= [777{,}778(L - x_1)^2]^2 + (14k)^2(L - x_1)^2$

$(5{,}9\ M)^2 = 60\ 938{,}271\ (L - x_1)^4 + 196M\ (L - x_1)^2$

$\div 604\ 938{,}271$

$\therefore (L - x_1)^4 + 324\ (L - x_1)^2 - 57\ 543\ 061{,}28 = 0$

Replace $(L - x_1)^4$ by y^2 and $(L - x_1)^2 = y$

$\therefore y^2 + 324\ y - 57\ 543\ 061{,}287 = 0$

$\therefore y = \dfrac{-324 \pm \sqrt{324^2 - 4(-57\ 543\ 061{,}28)}}{2}$

$= \dfrac{-324 \pm 15\ 174{,}89}{2}$

$y = 7\ 425{,}44$

But $(L - x_1)^2 = y$

$\therefore L - x_1 = \sqrt{y}$

$= 86{,}17$ m

$$\therefore F_H - \left[\frac{\left(w(L - x_1)^2\right)}{2(d + h)}\right] = \frac{14k \times 0{,}08617^2}{18}$$

$$= 5{,}774 \text{ MN}$$

And $F_H = \frac{wx_1^2}{2d} = \frac{14kx_1^2}{2 \times 5}$

$$\therefore x_1 = \frac{\sqrt{5{,}774M \times 10}}{14k}$$

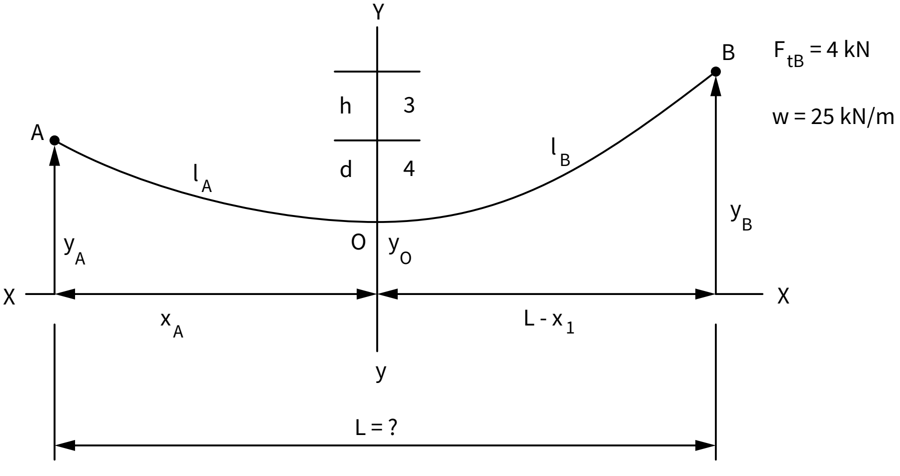
$$= 64{,}22M$$

$$\therefore L = (L - x_1) + x_1$$

$$= 87{,}17 + 64{,}22$$

$$= 150{,}37 \text{ m}$$

10.



10.1 Tension in the cable at A

$\therefore F_{t_B} = wy_B$

$\therefore 4k = 25\ y_B$

$y_B = 160$ m

and $y_B = y_o + d + h$

$\therefore y_O = 160 - 4 - -3 = 153$ m

$\therefore y_A = y_o + d = 153 + 4 = 157$ m

Tension in the cable at A

$\therefore Ft_A = wy_A = 25 \times 157 = 3{,}925$ kN

10.2 Length of the cable = $l = l_A + l_B$

$$\therefore l_A = x_A + \frac{2d^2}{3x_A}$$

But $F_{HA} - \frac{wx_1^2}{2d} = wy_o$

$$\therefore \frac{25x_1^2}{2 \times 4} = 25 \times 153 \qquad (F_H = 3{,}825 \text{ kN})$$

$$\therefore x_A = \frac{\sqrt{(153 \times 2 \times 4 \times 25)}}{25} = 34{,}986 \text{ m}$$

$$\therefore l_A = 34{,}986 + \frac{2 \times 4^2}{3 \times 34{,}986} = 35{,}291\text{M}$$

$$l_B = (L - x_A) + \frac{2(d+h)^2}{3(L - x_A)}$$

$$F_{HB} = \frac{w(L - x_A)^2}{2(d+h)} \therefore (L - x_A) = \frac{\sqrt{3825 \times 2 \times 7}}{25}$$

$= 46{,}282$ m

$$\therefore l_B = 46{,}282 + \frac{2 \times 7^2}{3 \times 46{,}282}$$

$= 46{,}988$ m

$\therefore$ Length of the cable $l = 35{,}291 + 46{,}988$

$= 82{,}279$ m

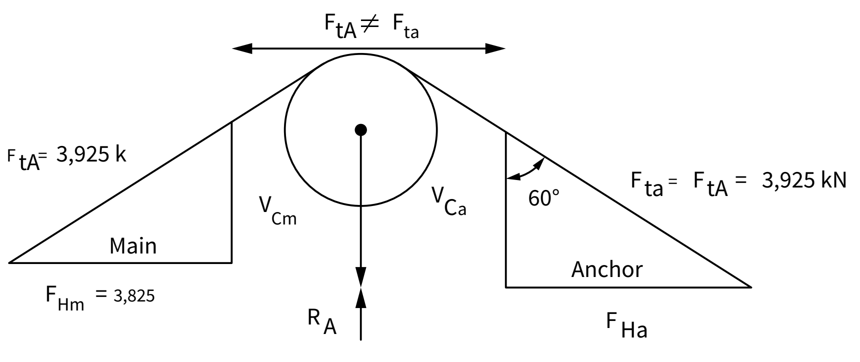
10.3 Horizontal distance between the supports = L

$\therefore L = x_A + (L - x_A)$

$= 34{,}986 + 46{,}282$

$= 81{,}268$ M

10.4 Reaction at support A



Reaction in A = $V_{cm} + V_{ca}$

From Δ main

$V_{cm} = \sqrt{3{,}925k^2 - 3{,}825k^2}$

$= 880{,}341$ N

From Δ anchor:

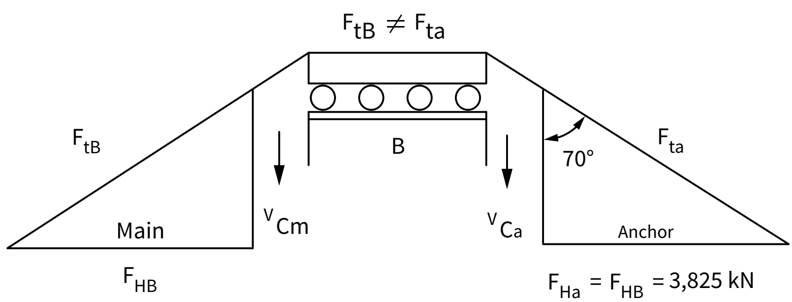
$\cos 60° = \frac{V_{ca}}{3{,}925k}$

$\therefore V_{ca} = \cos 60° \times 3{,}925 = 1{,}963$ kN

$\therefore$ Reaction in A = 880,341 + 1 936

= 2,843 kN

10.5 Tension anchor cable



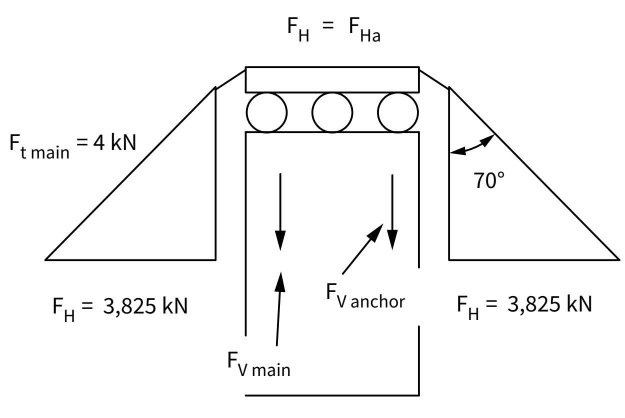
Tension in the anchor cable from Δ anchor:

$\therefore \sin 70° = \frac{3{,}825k}{Fta}$

Tension in the anchor cable = $Fta = \frac{3\ 825}{\sin 70°}$

= 4,07 kN

10.6 The reaction in the longest support



Vertical component force in the main cable

$F_{V\,main} = \sqrt{(4k)^2 - (3{,}825k)^2} = 1{,}17$ kN

Vertical component force anchor cable

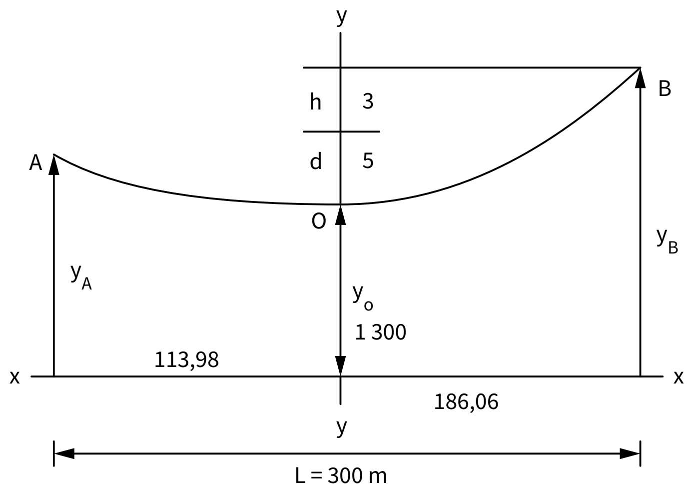
$\tan 70° = \frac{3{,}825k}{F_{V\,anchor}}$

$\therefore F_{V\,anchor} = 1{,}392$ kN

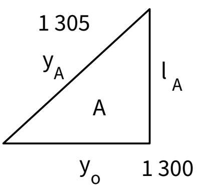
Reaction in support

Reaction in support = $1{,}17k + 1{,}392k = 2{,}562$ kN

11.



11.1 Tension at the supports



Support A:

$y_A = y_o + d$

$= 1\ 300 + 5$

$= 1\ 305$ m

$\therefore F_A = wy_A = 16 \times 1\ 305 = 20{,}88$ kN

Angle for ΔA: $\cos\theta = \frac{1\ 300}{1\ 305}$

$\theta = 5{,}017°$

Support B:

Determine $\therefore x_A = y_o \ln \frac{y_A^B + l_A}{y_o}$

From ΔA $l_A = \sqrt{y_A^2 - y_o^2}$

$= \sqrt{1\ 305^2 - 1\ 300^2}$

$= 113{,}98$

$\therefore l_B = 300 - 113{,}98 = 186{,}02$ m

$\therefore x_B = y_o \ln\left(\frac{y_B + l_B}{y_o}\right)$

$186{,}02 = 1\,300 \frac{\ln(y_B + l_B)}{1\,300}$

$\therefore 0{,}143 = \frac{\ln(y_B + l_B)}{1\,300}$

$e^{0{,}143} = \frac{y_B + l_B}{1\,300}$

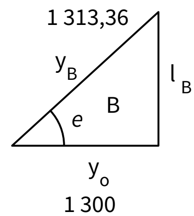
$\therefore 1{,}154 \times 1\,300 = y_B + l_B$

$\therefore l_B = 1\,500{,}2 - y_B \ldots$ ①

> **Note**
> $y = \ln x$
> $\therefore e^y = x$

From ΔB

$l_B^2 = y_B^2 - y_o^2 \ldots$ ②



Substitute ① into ②:

$\therefore (1\,500{,}2 - y_B)^2 = y_B^2 - 1\,300^2$

$\therefore 2\,250\,600{,}04 - 3\,000{,}4\, y_B + y_B^2 - 1\,300^2$

$\therefore 3\,000{,}4\, y_B = 3\,940\,600{,}04$

$\therefore y_B = 1\,313{,}36$ m

$\therefore$ Tension at B = $Ft_B = wy_B$

$= 16 \times 1\,313{,}36$

$= 21{,}013$ kN

The angle at B from ΔB

$\cos\theta = \frac{1\,300}{1\,313{,}36}$

$\theta = 8{,}17°$

11.2 Difference in the height of the support

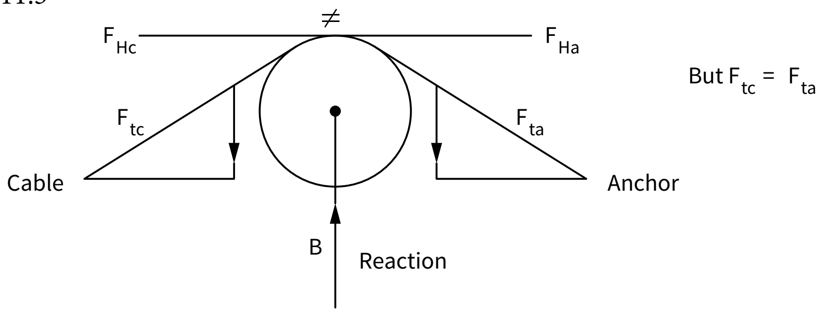
$h = y_B - y_A$

$= 1\,313{,}36 - 1\,305$

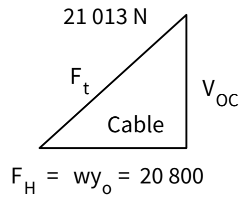
$= 8{,}36$ m

> **Note**
> Make sure that you've covered supports before continuing to Question 11.3.

11.3



Reaction = V_C cable + V_C anchor



$F_{min} = F_H = wy_o$

$= 16 \times 1\,300$

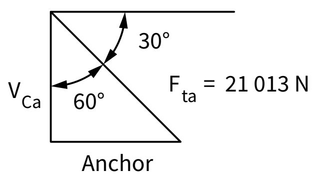
$= 20\,800$ N

$\therefore$ The vertical component force for support B = V_{CC}

From Δ cable $\therefore V_{CC} = \sqrt{21\,013^2 - 20\,800^2}$

$= 2{,}984$ kN

Vertical component's force for anchor:



Cable = V_{Ca} from Δ anchor

For a pulley the tension in main cable is equal to tension in anchor cable.

$\therefore \cos 60° = \frac{V_{Ca}}{21\,013}$

$\therefore V_{Ca} = 10{,}51$ kN

$\therefore$ Reaction $= 10{,}51 + 2{,}984$

$= 13{,}49$ kN

MODULE

3 Combined bending and twisting of shafts

By the end of this module, students should be able to:

- calculate:
  - the maximum torque transmitted by a motor using the power formulae, including the percentage increase due to the starting torque
  - the maximum torque transmitted by a belt drive with given belt tensions
  - the maximum allowable torque that can be transmitted by a solid or hollow shaft, taking both bending and twisting into consideration
  - the actual shear stress in the solid or hollow shaft
  - the dimensions of hollow or solid shafts
  - the weight of a pulley that is eccentrically loaded on the shaft when it is simply supported
  - the maximum bending moment carried by a shaft using the standard formulae used for cantilever and simply supported beams (including include self-weight, flywheel and belt-drive loads)
  - the maximum bending moment carried by a shaft by drawing a shear force diagram to determine the position of the maximum bending moment
  - the equivalent torque using Guest's formula when the shear stress limit is known
  - the equivalent bending moment using Rankine's formula when the principal stress limit is known
  - the dimensions of solid and hollow shafts to satisfy both stress limits
  - the percentage saving in weight if a solid shaft is replaced by a hollow shaft for the same stress limits
  - the maximum torque and bending moment a shaft can transmit to satisfy both stress limits; and
  - the actual shear stress and principal stress in a shaft.

Introduction

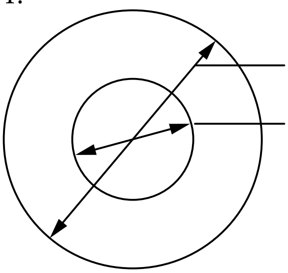
There are many examples where shafts are subjected to bending moments and torque simultaneously. One example is a shaft with a pulley, driven by V-belts or a flat belt.

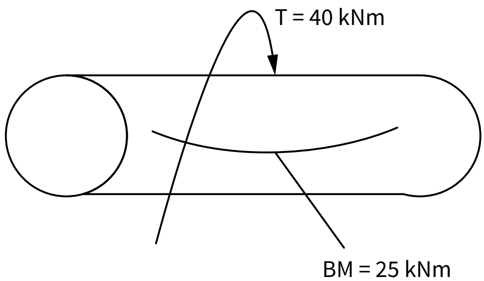
The cause of the bending can be due to:

- the weight of the pulley,
- tension in the belt (T_1 and T_2); and
- the weight of the shaft.

Exercise 3.1 **SB page 108**

1.





1.1 Simple bending stress: Bending stress without the torque

Simple bending equation $\frac{M}{I} = \frac{\sigma}{y}$

$$\sigma_B = \frac{My}{I} = \frac{25k \times 0{,}09}{\frac{\pi}{64}(D^4 - d^4)}$$

$$= \frac{25k \times 0{,}09}{\frac{\pi}{64}(0{,}18^4 - 0{,}12^4)}$$

$$= \frac{2\,250}{4{,}134 \times 10^{-5}}$$

$\therefore$ Bending stress $\sigma_B = 54{,}412$ MPa

1.2 Simple shear stress; shear stress without bending

Simple torque equation

$$\therefore \frac{T}{J} = \frac{\tau}{R}$$

$$\text{Shear stress} = \tau = \frac{TR}{J} = \frac{40k \times 0{,}09}{\frac{\pi}{32}(D^4 - d^4)}$$

$$= \frac{3\,600}{\frac{\pi}{32}(0{,}18^4 - 0{,}12^4)}$$

$\therefore \tau = 43{,}53$ MPa

1.3 Maximium equivalent shear stress

Shear stress when shaft is subjected to bending and torque.

$$\text{Equivalent torque} = Te = \sqrt{M^2 + T^2}$$

$$= \sqrt{25k^2 + 40k^2}$$

$$= 47{,}17 \text{ kNm}$$

Equivalent shear stress:

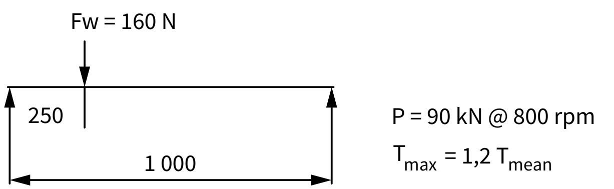
$$Te = \frac{\pi(D^4 - d^4)}{16D}\tau_{max}$$

$$\therefore \tau_{max} = \frac{16TeD}{\pi(D^4 - d^4)}$$

$$= \frac{16 \times 47{,}17k \times 0{,}18}{\pi(0{,}18^4 - 0{,}12^4)}$$

$$\therefore \tau_{max} = 51{,}33 \text{ MPa}$$

2.



2.1 Maximum torque

$$\therefore P = 2\pi NT$$

$$T_{mean} = \frac{P}{2\pi N} = \frac{90k \times 60}{2\pi 800}$$

$$= 1\ 074{,}3 \text{ Nm}$$

$$\therefore T_{max} = 1{,}2\ T_{mean}$$

$$= 1{,}2 \times 1\ 074{,}3$$

$$= 1{,}288 \text{ kNm}$$

2.2 Maximum moment of resistance of the shaft = BM

1 tonne = 1 000 kg × 9,81 = 9,81 kN

No weight was given for the shaft.

$$\therefore \text{Maximum BM} = \frac{Wab}{L}$$

$$= \frac{9{,}81k \times 0{,}25 \times 0{,}75}{1}$$

$$= 1{,}839 \text{ kNM}$$

2.3 Diameter of the shaft

Only the shear stress given ∴ the diameter due to shear stress:

$$\text{Equivalent torque} = Te = \sqrt{\left(M^2 + T^2\right)}$$

$$= \sqrt{1{,}839k^2 + 1{,}288k^2}$$

$$= 2{,}245 \text{ kNm}$$

$$\therefore \text{But } Te = \frac{\pi D^3}{16}\tau nat$$

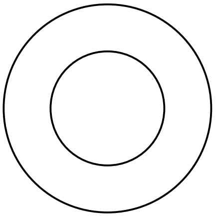
$$D^3 = \frac{16Te}{\pi\tau}$$

$$= \frac{16 \times 2\,245}{\pi \times 38 \text{ M}}$$

$$D = \sqrt[3]{3{,}0088 \times 10^{-4}}$$

$$= 67 \text{ mm}$$

3.1



$\tau = 75$ MPa

150 kW of 250 rpm

$d = ?$ D = 2 $d = 0{,}6D$ $T_{max} = 1{,}12\ T_{mean}$

$d = 0{,}6D \ldots$ ①

Power = P = 2πNT

$$\therefore T_{mean} = \frac{P \times 60}{2\pi N}$$

$$= \frac{150k \times 60}{2\pi 250}$$

$$= 5\,729{,}578 \text{ Nm}$$

$$T_{max} = 1{,}12 \times T_{mean}$$

$$= 1{,}12 \times 5\,729{,}578$$

$$= 6{,}417 \text{ kNm}$$

From single torque equation.

$$\frac{T}{J} = \frac{\tau 2}{D}$$

$$\therefore \frac{6\,417}{J} = \frac{75M \times 2}{D}$$

$$\div 6\,417 \therefore \frac{I}{J} = \frac{23\,375{,}41}{D}$$

$$\therefore \frac{32}{\pi(D^4 - d^4)} = \frac{23\,375{,}41}{D}$$

× D

÷ 32

× π

$$\therefore \frac{D}{D^4 - d^4} = \frac{23\,375{,}41 \times \pi}{32}$$

$$= 2\,294{,}875 \ldots ②$$

Substitute ① into ②: $\therefore \frac{D}{D^4 - (0{,}6D)^4} = 2\,294{,}875$

$$\therefore \frac{1}{0{,}8704\ D^3} = 2\,294{,}875$$

$$0{,}8704\ D^3 = \frac{1}{2\,294{,}875}$$

$$\therefore D^3 = 5{,}0094 \times 10^{-4}$$

$$D = \sqrt{5{,}0094 \times 10^{-4}}$$

$$= 79{,}42 \text{ mm}$$

$$\therefore d = 0{,}6D = 0{,}6 \times 79{,}42$$

$$\therefore = 47{,}65 \text{ mm}$$

3.2 Subjected to BM of 3,2 kNm.

Calculate speed (N) and new torque for equivalent condition.

M = 3,2 kNm

$$Te = \frac{\pi}{16}\frac{(D^4 - d^4)}{D}\tau$$

$$\frac{\pi}{16}\frac{0{,}07942^4 - 0{,}04765^4}{0{,}07942} 75\ M$$

$$= 6{,}421 \text{ kNm}$$

But $T\sqrt{M^2 + T^2}$

$$\therefore (6{,}421k)^2 = (3{,}2k)^2 + T^2$$

$$= 5{,}566 \text{ kNm}$$

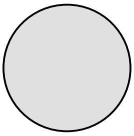
$$\therefore T_{mean} = \frac{5{,}566k}{1{,}12} = 4\,970{,}47 \text{ Nm}$$

$$\therefore P = 2\pi NT$$

$$N = \frac{60 \times 150k}{2\pi \times 4\,970{,}47}$$

$$= 288{,}21 \text{ rpm}$$

4.



D = 100 50 kN = BM T?

$\tau = 65$ MPa $\sigma_B = 70$ MPa

4.1 Maximum torque

Consider $\sigma_B = \sigma n$

$\therefore Me = \frac{\pi D^3 \sigma n}{32}$

$= 6{,}872$ kNm

$\therefore Me = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$

$\therefore 6\ 872 = \frac{1}{2}\left(5k + \sqrt{5k^2 + T^2}\right)$

$\therefore 8\ 744^2 = 5k^2 + T^2$

$T = \sqrt{8\ 744^2 - 5k^2}$

$= 7{,}173$ kNm

Consider the shear stress τ:

$Te = \frac{\pi}{16} D^3 \tau_{max}$

$= \frac{\pi}{16} \times 0{,}1^3 \times 65$ M

$= 12{,}763$ kN

But $Te = \sqrt{M^2 + T^2}$

$= \sqrt{12{,}763k^2 - 5k^2}$

$= 11{,}742$ kNm

Maximum torque = 7,17 kNm

If a torque of 11,742 kNm is used the bending stress will be more than the allowable 70 MPa.

4.2 The actual stress in shaft bending stress is 70 MPa because it determines the maximum torque.

$\therefore$ Shear stress due to a torque of 7,17 kN is applicable.

$$\therefore Te = \sqrt{M^2 + T^2}$$

$$= \sqrt{5k^2 + 7{,}17k^2}$$

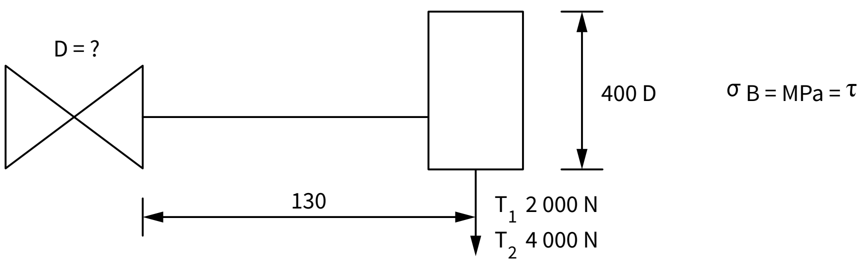
$$= 8{,}741 \text{ kNm}$$

$$Te = \frac{\pi}{16} D^3 \tau_{max}$$

$$\frac{8\,741 \times 16}{\pi \times 0{,}1^3} = \tau_{max}$$

$$\therefore \tau_{max} = 44{,}5 \text{ MPa}$$

5.



Consider as a cantilever

Load on the shaft $= T_1 + T_2$

$= 2\,000 + 400$

$= 2\,400$ N

$\therefore$ BM = WL

$= 2\,400 \times 0{,}13$

$= 312$ Nm

Torque on the shaft $= (T_1 - T_2)\, R_{pulley}$

$$= \frac{(2\,000 - 400)0{,}4}{2}$$

= 320 NM

No name was given to the stress, therefore it had to be considered as bending and shear stress.

Consider the binding stress.

$\therefore Me = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$

$= \frac{1}{2}\left(312 + \sqrt{312^2 + 320^2}\right)$

$= \frac{1}{2}(312 + 446{,}93)$

$= 379{,}46$ Nm

$Me = \frac{\pi}{32}D^3\sigma n$

$\sqrt[3]{\frac{379{,}44 \times 32}{\pi \times 40\ M}} = D$

$\therefore D = 45{,}89$ mm

Consider the shear stress

$\therefore Te = \sqrt{M^2 + T^2}$

$= 446{,}93$ (From Me)

$\therefore Te = \frac{\pi}{16}D^3\tau$

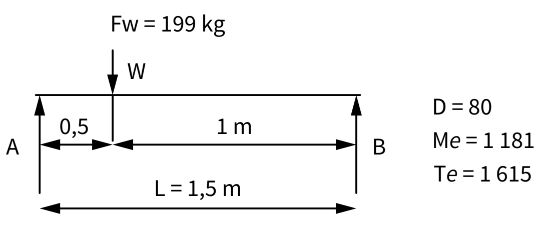
$\sqrt[3]{\frac{446{,}93 \times 16}{\pi \times 40\ M}} = D$

$D = 38{,}46$ mm

Use a shaft with D = 45,9 mm. A shaft with D = 38,46 mm will not be able to take a bending stress of 40 MPa.

For a shaft D 38,46 m: $\sigma_D = 59{,}17 > 40$ MPa allowable.

6.



Weight of shaft/metre = Volqg

$\frac{\pi}{4}0{,}08^2 \times 7\ 800\ g$

$= 384{,}62$ Nm

$\therefore$ Reaction in A: mounts about B

$\therefore 1{,}5A = w \times 1 + 384{,}62 \times 1{,}5 \times \frac{1{,}5}{2}$

$= w + 432{,}6975$

$A = 0{,}667w + 288{,}465$

Consider the maximum BM at position of the flywheel as flywheel mass is unknown.

$\therefore M_{max} = 0{,}5(0{,}667w + 288{,}465) - 384{,}62 \times 0{,}5 \times \frac{0{,}5}{2}$

$\therefore M_{max} = 0{,}3335w + 96{,}155$

Consider $Me = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$

$\frac{1}{2}(M + Te)$

$1\,181 = \frac{1}{2}(M + 1\,615)$

$2\,362 = M + 1\,615$

$\therefore M = 747$ Nm

$\therefore 747 = 0{,}3335w + 96{,}155$

$\therefore w = 1\,951{,}55$ N

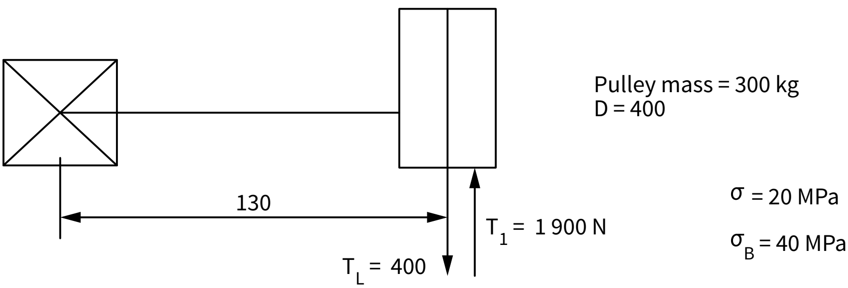
Mass of the flywheel $= \frac{\text{weight}}{g}$

$= \frac{1\,951{,}559}{9{,}81}$

$= 198{,}9$ kg

The sag is 199 kg.

7.



7.1 Bending moment for the shaft

$\therefore M = WL + L(T_1 + T_2)$

$= (30 \times 9{,}81)0{,}13 + 0{,}13(1\,900 + 400)$

$= 38{,}259 + 299$

$= 337{,}259$ Nm

Torque on the shaft $= (T_1 - T_2)R_{pulley}$

$= \frac{(1\,900 - 400)0{,}4}{2}$

$= 300$ Nm

Consider the bending stress:

$$Me = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$

$$\frac{1}{2}\left(337{,}259 + \sqrt{337{,}259^2 + 300^2}\right)$$

$$Me = \frac{1}{2}(337{,}259 + 451{,}38)$$

$$= 394{,}319 \text{ Nm}$$

and $Me = \frac{\pi}{32}D^3\sigma n$

$$\therefore D = \sqrt[3]{\frac{Me32}{\pi\sigma n}}$$

$$= \sqrt[3]{\frac{394{,}319 \times 32}{\pi \times 40m}}$$

$D = 46{,}48$ mm

Consider the shear stress:

$$Te = \sqrt{M^2 + T^2}$$

$$= 451{,}38 \text{ (from } Me\text{)}$$

$$\therefore Te = \frac{\pi}{16}D^3\tau$$

$$\therefore D = \sqrt{\frac{451{,}38 \times 16}{\pi \times 20 \text{ M}}}$$

$D = 48{,}62$ mm

Use a shaft of 48,62 mm.

7.2 Actual stress

The shear stress is 20 MPa as it determines the shaft diameter.
Calculate the bending stress.

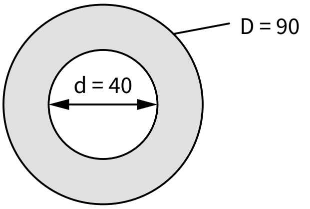
$$\therefore Me = \frac{\pi}{32}D^3\sigma_B$$

$$\therefore \sigma_B = \frac{Me \times 32}{\pi D^3}$$

$$= \frac{394{,}319 \times 32}{\pi \times 0{,}04862^3}$$

$$= 35 \text{ MPa}$$

8.



M = 6 kNm
allowable stress 120 MPa

$\therefore \sigma_B = 120$ MPa

8.1 Consider the bending stress.
Torque in the shaft:

$$Me = \frac{\pi}{32}\frac{D^4 - d^4}{D}\sigma n$$

$$= \frac{\pi}{32}\left(\frac{0{,}09^4 - 0{,}04^4}{0{,}09}\right)120M$$

$$= 8{,}253 \text{ kNm}$$

$$Me = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$

$$8\ 253 = \frac{1}{2}\left(6k + \sqrt{6k^2 + T^2}\right)$$

$$10\ 506{,}451 = \sqrt{6k^2 + T^2}$$

$$10\ 506{,}451^2 = 6l^2 + T^2$$

$$\therefore T = \sqrt{10\ 506{,}451^2 - 6k^2}$$

$$= 8{,}62 \text{ kNm}$$

8.2 Power

$$\text{Mean torque} = \frac{T_{max}}{1{,}12}$$

$$= \frac{8{,}62k}{1{,}12}$$

$$= 7{,}696 \text{ kNm}$$

$$\therefore P = 2\pi NT$$

$$= 2\pi \times \frac{320}{60} \times 7{,}696\ k$$

$$= 258{,}03 \text{ kW}$$

8.3 Shear stress

$$Te = \sqrt{M^2 + T^2}$$

$$= \sqrt{6k^2 + 8{,}62k^2}$$

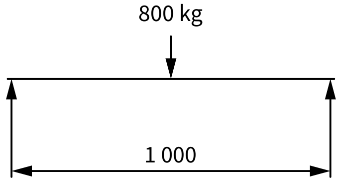
$$= 10{,}503 \text{ kNm}$$

$$Te = \frac{\frac{\pi}{16}(D^4 - d^4)}{D}\tau$$

$$= 1{,}376 \times 10^{-4}\tau$$

$\therefore$ Shear stress $= \tau = 76{,}37$ MPa

9.



$$BM = \frac{WL}{4} = \frac{800 \times 9{,}81 \times 1}{4} = 1\ 962 \text{ Nm}$$

$$P = 2\pi NT = 2\pi \frac{80}{60} T = 100k$$

$$\therefore T = 1\ 193{,}662 \text{ Nm}$$

$$\therefore T_{max} = 1{,}15 \times 1\ 193{,}662 = 1\ 372{,}711 \text{ Nm}$$

Consider σ_B

$$\therefore Me = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$

$$= \frac{1}{2}(1\ 962 + 2\ 394{,}531)$$

$$= 2\ 178{,}266$$

$$Me = \frac{\pi}{32} \frac{D^4 - d^4}{D} \sigma_B$$

$$\therefore \frac{2\ 178{,}266 \times 32}{\pi \times 80\ M} = \frac{D^4 - d^4}{D}$$

$$\frac{D^4 - d^4}{D} = 2{,}773 \times 10^{-4} \ldots ①$$

$$D = 1{,}5d \ldots ②$$

Substitute ② into ①:

$$\frac{(1{,}5d)^4 - d^4}{1{,}5d} = 2{,}773 \times 10^{-4}$$

$$2{,}708\ d^3 = 2{,}773 \times 10^{-4}$$

$$d = \sqrt[3]{\frac{2{,}773 \times 10^{-4}}{2{,}708}}$$

$$= 46{,}78 \text{ mm}$$

$$\therefore D = 1{,}5 \times 46{,}78$$

$$= 70{,}17 \text{ mm}$$

Consider the shear stress:

$$\therefore Te = \sqrt{M^2 + T^2}$$

$$= 2\ 394{,}531 \text{ (from } Me\text{)}$$

$$Te = \frac{\pi}{16} \frac{D^4 - d^4}{D} 80M$$

$$1{,}5246 \times 10^{-4} = 2{,}708\ d^3$$

$$d = \sqrt{\frac{1{,}5246 \times 10^{-4}}{2{,}708}}$$

$= 38{,}32$ mm

$D = 1{,}5 \times 38{,}32$

$= 57{,}48$ mm

Use a shaft:

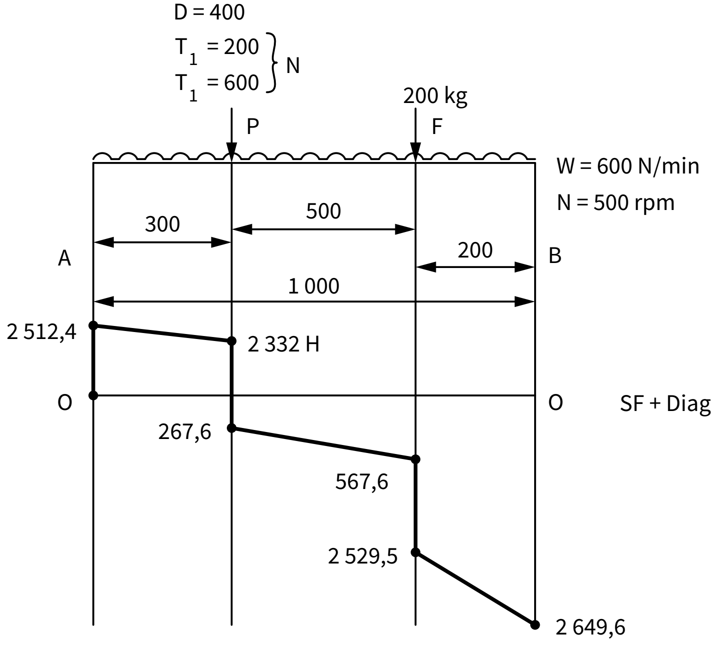
D = 70,14 mm

d = 46,78 mm

11,68 wall thickness

Shear stress shaft wall thickness 9,58 mm.

10.



10.1 Reaction of the bearings

Moment about A:

$\therefore 1B = 0{,}3(2\ 000 + 600) + 0{,}8(200 \times 9{,}81) + \left(600 \times 1 \times \frac{1}{2}\right)$

$= 780 + 1\ 569{,}6 + 300$

B = 2 649,6 N

Moment about B:

$\therefore 1A = 0{,}2(200 \times 9{,}81) + 0{,}7(2\ 600) + \left(600 \times 1 \times \frac{1}{2}\right)$

$A = 392{,}4 + 1\ 820 + 300$

$= 2\ 512{,}4$ N

10.2 Maximum BM at the pulley

$\therefore M = (2\ 512{,}4 \times 0{,}3) - \left(600 \times 0{,}3 \times \frac{0{,}3}{2}\right)$

$= 753{,}72 - 27$

$= 726{,}72$ Nm

10.3 Maximum torque $= (T_1 + T_2)R_{pulley}$

$= (2\ 600)\frac{0{,}4}{2}$

$= 520$ Nm

The allowable stress is 65 MPa.

Shaft diameter – consider σB

$\therefore Me = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$

$= \frac{1}{2}\left(726{,}72 + \sqrt{726{,}72^2 + 520^2}\right)$

$= \frac{1}{2}(726{,}72 + 893{,}6)$

$= 809{,}66$ Nm

$\therefore Me = \frac{\pi}{32}D^3\sigma_n$

$= \sqrt[3]{\frac{Me \times 32}{\pi\sigma_n}}$

$= \sqrt[3]{\frac{806{,}66 \times 32}{\pi \times 65\ M}}$

$= 50{,}24$ mm (49)

Consider τ

$Te = 893{,}6$ Nm (From Me)

$Te = \frac{\pi}{16}D^3\tau$

$D = \sqrt[3]{\frac{Te \times 16}{\pi\tau}}$

$D = \sqrt[3]{\frac{893{,}6 \times 16}{\pi \times 65\ M}}$

$= 41{,}22$ mm

Use a shaft diameter of 50,24 mm.

10.4 $d = 0{,}6D$ … ①

Hollow shaft dimensions

Consider σ_B:

Solid shaft $D^3 = \frac{D^4 - d^4}{D}$ hollow shaft

$$\therefore 0{,}05024^3 = \frac{D^4 - 0{,}6^4\, D^4}{D}$$

$$= 0{,}8704D^3$$

$$\therefore D = \sqrt{\frac{0{,}05024^3}{0{,}8704}}$$

$$= 52{,}62 \text{ mm}$$

$d = 0{,}6$ D

$= 31{,}57$ mm

Consider τ:

$$D^3 = \frac{D^4 - d^4}{D}$$

$$\therefore 0{,}04122^3 = 0{,}8704D^3$$

$$\therefore D = \sqrt[3]{\frac{0{,}4122^3}{0{,}8704}}$$

$$= 43{,}17 \text{ mm}$$

$d = 0{,}6$ D

$= 25{,}9$ mm

Use shaft D = 52,62 mm d 31,57 mm.

10.5 Percentage saving in weight

$$\therefore \% \text{ saving} = \frac{A_s - A_s}{A_s} \times \frac{100}{1}$$

$$= \left(\frac{D^2 - (D^2 + d^2)}{D^2}\right) \times \frac{100}{1}$$

$$= \left(\frac{50{,}24^2 - (52{,}62^2 - 31{,}57^2)}{50{,}24^2}\right) \times \frac{100}{1}$$

$$= 29{,}78\%$$

MODULE

4 *Bending and deflection of beams*

By the end of this module, students should be able to:

- calculate:
  - the bending moment for standard cantilever and simply supported beams
  - the position and value of the maximum bending moment for an eccentric lateral load on a simply supported beam
  - the second moment of area for standard beams and built-up beams consisting of no more than two profiles;
- understand and apply the simple bending equation;
- select profiles according to the profile modulus from the steel tables if the bending stress limit is known;
- understand and apply deflection and slope equations;
- calculate:
  - the maximum slope and maximum deflection for simply supported beams carrying a UDL over the full length
  - the maximum slope and maximum deflection for simply supported beams carrying a concentrated load at mid-span
  - the maximum slope and maximum deflection for simply supported beams carrying a UDL over the full length as well as a concentrated load at mid-span
  - the force in a prop placed at the mid-span to prevent all or some of the deflection
  - the maximum slope and maximum deflection for cantilever carrying a concentrated load at the free end
  - the maximum slope and maximum deflection for cantilevers carrying a concentrated load which is not at the free end
  - the maximum slope and maximum deflection for cantilevers carrying a UDL over the full length
  - the maximum slope and maximum deflection for cantilevers carrying a UDL from the fixed point, but not for the full length
  - the maximum slope and maximum deflection for cantilevers carrying a UDL from the free end, but not all the way to the fixed point
  - the force in a prop placed at the free end to prevent all or some of the deflection;

- select profiles according to the second moment of area from the steel tables if the deflection limit is known;
- select profiles from the steel tables if the bending stress limit and deflection limit are both known; and
- calculate:
  - the actual stress and deflection of a selected beam
  - the maximum allowed length of the beam if the bending stress limit and deflection limit are both known
  - the maximum allowed load that the beam may carry if the bending stress limit and deflection limit are both known.

Introduction

A beam may be strong enough to safely resist the bending moments due to the applied load and yet not be structurally suitable because its deflection is too great.

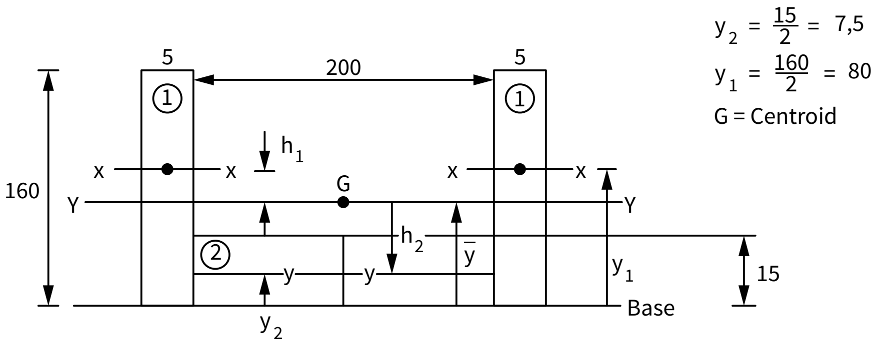
The deflection of a beam should not exceed $\frac{1}{360}$ of its length, depending on where it is used and how critical the condition is.

In other words, deflection can be more than $\frac{1}{360}$ of its length and will not cause damage if the condition is not critical.

Exercise 4.1 SB page 129

This exercise is useful for revision of the N5 learning outcomes.

1.



Position of G:

$\therefore$ Area moments about the base:

$\therefore A_T\bar{y} = 2(A_1y_1) + A_2y_2$

$\therefore \left[2(5| \times 160) +)200 \times 15)\right]\bar{y} = 2[(5 \times 160) \times 80] + \left[(200 \times 15) \times 7{,}5\right]$

$\therefore (1\ 600 + 3\ 000)\bar{y} = 128\ 000 + 22\ 500$

$\bar{y} = \frac{150\ 500}{4\ 600}$

$= 32{,}72$ mm from base

Moment of inertia about yy-axis

$\therefore I_{yy} = 2\left[I + 1 + A_1 h_1^2\right] + \left[I_2 + A_2 h^{22}\right]$

$\therefore h_1 = y_1 - \bar{y} = 80 - 32{,}72 = 47{,}28$ mm

$h_2 = \bar{y} - y_2 = 32{,}72 - 7{,}5 = 25{,}22$ mm

$$\therefore Iyy = 2\left[\frac{0{,}005 \times 0{,}16^3}{12} + \left(8 \times 10^{-4} \times 0{,}04728^2\right)\right]$$

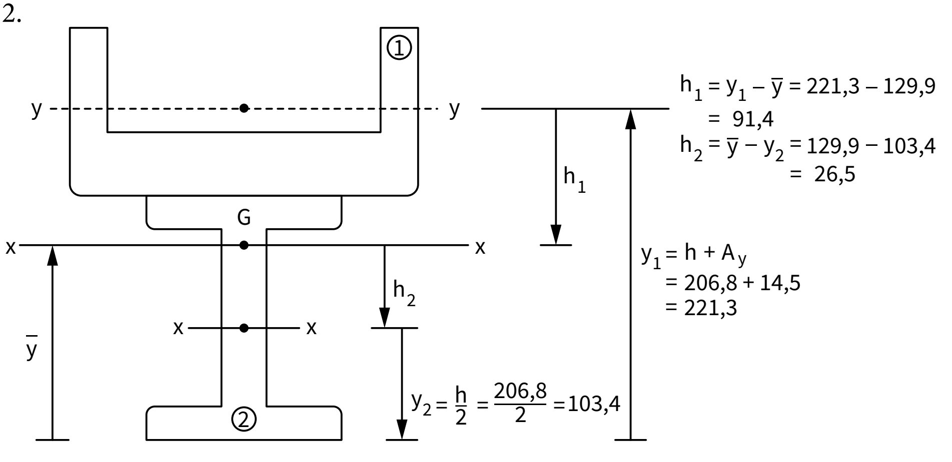
$$+ \left[\frac{0{,}2 \times 0{,}015^3}{12} + \left(3 \times 10^{-3} \times 0{,}02522^2\right)\right]$$

$$= 2\left[1{,}707 \times 10^{-6} + 1{,}7883 \times 10^{-6}\right] + \left[5{,}625 \times 10^{-8} + 1{,}908 \times 10^{-6}\right]$$

$$= 6{,}991 \times 10^{-6} + 1{,}965 \times 10^{-6}$$

$$= 8{,}956 \times 10^{-6}\ \text{m}^4$$

2.



2.1 Calculate the position of G

$\therefore A_T\bar{y} = A_1 y_1 + A_2 y_2$

$\therefore \left(1{,}102 \times 10^{-3} + 3{,}8 \times 10^{-3}\right)\bar{y} = \left(1{,}102 \times 10^{-3} \times 0{,}2213\right)$
$+ \left(3{,}8 \times 10^{-3} \times 0{,}1034\right)$

$\therefore 4{,}902 \times 10^{-3}\bar{y} = 2{,}439 \times 10^{-4} + 3{,}9292 \times 10^{-4}$

$\bar{y} = \frac{6{,}3682 \times 10^{-4}}{4{,}902 \times 10^{-3}}$

$= 129{,}9$ mm

2.2 Moment of inertia about the *xx*-axis

$$Ixx = \left[Iyy_1 + A_1h_1^2\right] + \left[Ixx_2 + A_2h_2^2\right]$$

$$= \left[0{,}1936 \times 10^{-6} + \left(1{,}102 \times 10^{-3} \times 0{,}0914^2\right)\right]$$

$$+ \left[28{,}88 \times 10^{-6} + \left(3{,}8 \times 10^{-3} \times 0{,}0265^2\right)\right]$$

$$\therefore Ixx = 9{,}3997 \times 10^{-6} + 3{,}1549 \times 10^{-5}$$

$$= 40{,}948 \times 10^{-6}\ m^4$$

2.3 Moment of inertia about the *yy*-axis

$$I_{yy} = \left[I_{xx1} + \left(A_1h_{1x}^2\right)\right] + \left[I_{yy2} + \left(A_2h_{2x}^2\right)\right]$$

$$I_{yy} = 1{,}059 \times 10^{-6} + 3{,}838 \times 10^{-6} = 4{,}897 \times 10^{-6}\ m^4$$

2.4 The maximum stress in the beam

y-distance top and bottom

y-bottom = 129,9 mm

y-top = *h* – I-section + *b*-channel – 129,9

y-top = (206,8 + 45) – 129,9 = 121,9 mm

Maximum stress at the maximum *y*-distance

$$\sigma_{max} = \frac{My_{max}}{I_{xx}} = \frac{WLy_{max}}{4I_{xx}}$$

$$\sigma_{max} = \frac{WLy_{max}}{4I_{xx}} = \frac{100k \times 8 \times 0{,}1299}{4 \times 40{,}948 \times 10^{-6}} = 634{,}463\ MPa$$

2.5 Minimum stress in beam

$$\sigma_{min} = \frac{WLy_{min}}{4I_{xx}} = \frac{100k \times 8 \times 0{,}1219}{4 \times 40{,}948 \times 10^{-6}} = 595{,}389\ MPa$$

2.6 Maximum stress about the *yy*-axis for the same BM

$$x\text{-distance} = \frac{h\text{-channel}}{2} = \frac{40}{2} = 20\ mm$$

$$x\text{-distance} = \frac{b\ \text{I-section}}{2} = \frac{133{,}8}{2} = 66{,}9\ mm\ \text{maximum distance}$$

$$\sigma_{max} = \frac{WLx_{max}}{4I_{yy}} = \frac{100k \times 8 \times 0{,}0669}{4 \times 4{,}897 \times 10^{-6}} = 2{,}732\ GPa$$

2.7 Minimum stress about the *yy*-axis

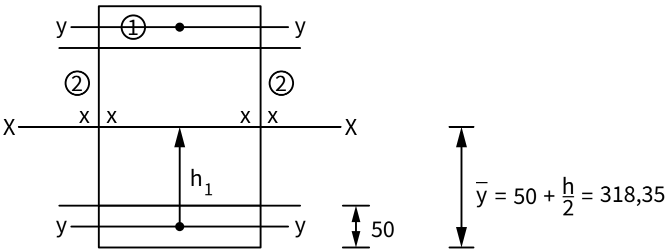
$$\sigma_{min} = \frac{WLx_{min}}{4I_{yy}} = \frac{100k \times 8 \times 0{,}020}{4 \times 4{,}897 \times 10^{-6}} = 816{,}827\ MPa$$

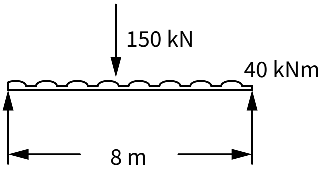
2.8 Replace the beam with an I-section

305 × 102 × 24,5 kg/m

3.

σ = 95 MPa





101 kg/m

$h = 536{,}7$

$A = 12{,}92 \times 10^{-3}$

$Ixx = 616{,}5 \times 10^{-6}$

$\bar{y} = 50 + \frac{h}{2} = 318{,}35$

$h_2 = 0$

$h_1 - \bar{y} - \frac{50}{2} = 293{,}35$

$h_2 = 0 \therefore A_2 h^{22}$

Ixx for the beam: $\frac{M_{max}}{Ixx} = \frac{\sigma}{y} \ldots$ ①

$\therefore y = \bar{y} = 318{,}35$ mm

$$M_{max} = M_{UDC} = \frac{WL}{4} + \frac{wL^2}{8}$$

$$= \frac{150k \times 8}{4} + \frac{40k \times 8^2}{8}$$

$$= 300k + 320k$$

$M_{max} = 620$ kNm

$\therefore$ From ①: $Ixx = \frac{M_{max} y}{\sigma}$

$$= \frac{620k \times 0{,}31835}{95 \times 10^6}$$

$$= 2{,}0777 \times 10^{-3}\ \text{m}^4$$

But $Ixx = 2\left[I_{1yy} + A_1 h_2^2\right] + 2\left[I_{2xx} + A_2 h_2^2\right]$

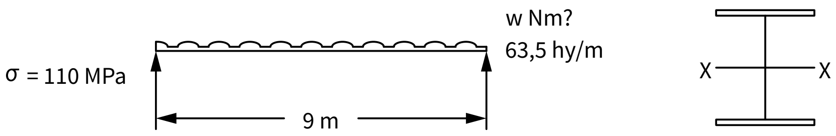
$$= 2\left[\frac{W \times 0{,}05^3}{12} + \left(W \times 0{,}05 \times 0{,}29335^2\right)\right] + 2\left[616{,}5 \times 10^{-6}\right]$$

$$\therefore 2{,}0777 \times 10^{-3} = 2\left[1{,}0417 \times 10^{-5}\text{W} + 4{,}3027 \times 10^{-3}\text{W}\right] + 1{,}233 \times 10^{-3}$$

$$\therefore 4{,}2235 \times 10^{-4} = 4{,}3131 \times 10^{-3}\text{W}$$

$$\therefore \text{W} = 97{,}92 \text{ mm}$$

4.



$305 \times 152 \times 65{,}5$ kg/m

$h = 305$ mm

$\text{A} = 8{,}385 \times 10^{-3}\ \text{m}^2$

$\text{I}xx = 131{,}8 \times 10^{-6}$

$$y = \frac{h}{2} - \frac{305}{2} = 152{,}5$$

Calculate maximum BM for the beam

$$\therefore \frac{\text{M}_{max}}{\text{I}xx} = \frac{\sigma_{max}}{y}$$

$$\text{M}_{max} = \frac{\sigma_{max} \times \text{I}xx}{y}$$

$$= \frac{110\text{M} \times 131{,}8 \times 10^{-6}}{0{,}1525}$$

$$= 95{,}069 \text{ kNm}$$

Important

The weight of a STANDARD section must always be considered for ALL N6 problems in ALL chapters where possible, whether the question asks for it or not.

But $\text{M}_{max} = \text{M}_{\text{UDL}} + \text{M}_{\text{weight}}$

$$\therefore 95{,}069k = \frac{w\text{L}^2}{8} + \frac{w\text{L}^2}{8}$$

$$= \frac{w9^2}{8} + \frac{(65{,}5 \times 9{,}81)9^2}{8}$$

$$\therefore w = \frac{w9^2}{8} + 6{,}506k$$

$$\therefore w = \frac{88{,}59 \times 8}{9^2}$$

$= 8{,}747$ kNm safe load on the beam.

Note

If the weight of the beam is not considered, the UDL will be too great and will cause the stress to be more than the allowable stress of 110 MPa, and the beam will fail. (This can result in death in real life).

4.2 Value of the PL at the middle

$$\text{Maximum} = M_{PL} + M_{weight}$$

$$\therefore M_{max} = \frac{WL}{4} + \frac{wL^2}{2}$$

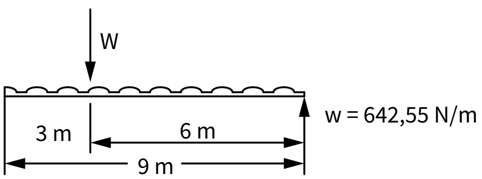
$$\therefore 950{,}069k = \frac{W9}{4} + \frac{(65{,}5 \times 9{,}81)9^2}{8}$$

$$= 2{,}25\ W + 6{,}506k$$

$$\therefore W = \frac{88{,}563}{2{,}25}$$

$$= 39{,}361 \text{ kN point load}$$

4.3



Note

For a point load not in the middle, the maximum BM equation is $\frac{Wab}{L}$; BUT, this only applies if NO other load is applied.

$$\therefore \text{Maximum BM is NOT} = \frac{Wab}{4} + \frac{wL^2}{8}$$

Determine left-hand reaction

Important

In a case like this with an unknown load, consider the maximum BM at the point load on the beam. If all loads are known, a shear force diagram must be drawn to find the point of maximum BM on the beam.

Moments about R:

$\therefore 9L = 6W + (642{,}55 \times 9) \times \frac{9}{2}$

$= 6W + 26{,}023k$

$\therefore L = 0{,}667\ W + 2{,}891k$

Maximum BM PL – moment to the left

$\therefore M_{max} = L \times 3 + (642{,}55 \times 3) \times \frac{3}{2}$

$= 3L + 2{,}891k$

$\therefore M_{max} = 95{,}069k = 3(0{,}667W + 2{,}891k) + 2{,}891k$

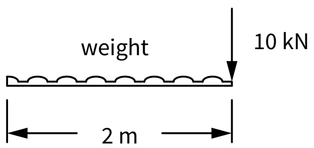
$\therefore 92{,}178k = 3(0{,}667W + 2{,}891k)$

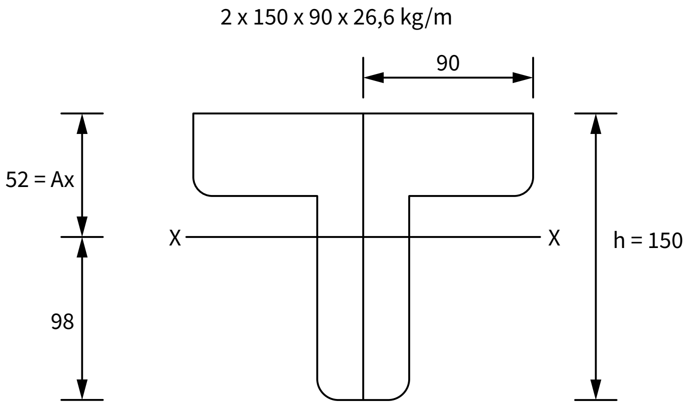
$\therefore 0{,}667\ W = 30{,}726k - 2{,}891k$

$W = \frac{27{,}835k}{0{,}667}$

$= 41{,}732\ kN$

5.





$Ixx = 7{,}611 \times 10^{-6} \times = 15{,}222 \times 10^{-6}\ m^4$

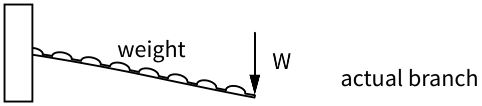
$Ax = 52$

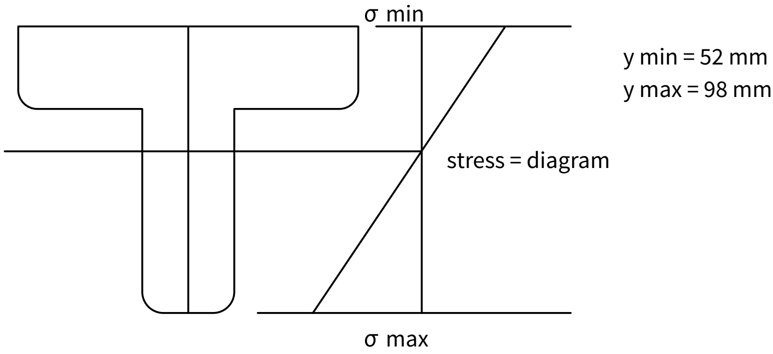
$\text{Maximum BM} = M_{max} = WL + \frac{wL^2}{2}$

$= 10k \times 2 + \frac{((2 \times 26{,}6 \times 9{,}81)2^2)}{2}$

$= 20k + 1{,}044k$

$= 21{,}044\ kNm$





$\therefore$ Maximum stress: $\dfrac{M_{max}}{Ixx} = \dfrac{\sigma_{max}}{y_{max}}$

$$\therefore \sigma_{max} = \frac{M_{max} y_{max}}{y_{max}}$$

$$\therefore \sigma_{max} = \frac{M_{max} y_{max}}{Ixx}$$

$$= \frac{21{,}044k \times 0{,}098}{15{,}222 \times 10^{-6}}$$

$$= 135{,}48 \text{ MPa (Compressive)}$$

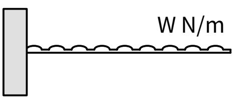
Minimum stress: $\dfrac{M_{max}}{Ixx} = \dfrac{\sigma_{min}}{y_{min}}$

$$\therefore \sigma_{min} = \frac{21{,}044k \times 0{,}052}{15{,}222 \times 10^{-6}}$$

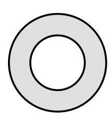
$$= 71{,}89 \text{ MPa (Tensile)}$$

Exercise 4.2 SB page 147

1.



$p = 7\,800$ hg/m^3
E = 200 GPa
σ = 120 MPa



D = 60
d = 25
$y = \frac{60}{2} = 30$

1.1 Weight/m = Volqg

$$= \frac{\pi}{4}(D^2 \times d^2) \times 1 \times 7\,800 \times 9{,}81)$$

$$= \frac{\pi}{4}(0{,}06^2 - 0{,}025^2)76\,518$$

$$= 178{,}789 \text{ Nm}$$

Moment of inertia of the shaft:

$\therefore Ixx = \frac{\pi}{64}(D^4 - d^4)$

$= \frac{\pi}{64}(0{,}06^4 - 0{,}025^4)$

$= \frac{\pi}{64}(1{,}257 \times 10^{-5})$

$= 6{,}17 \times 10^{-7}\ m^4$

$\therefore \frac{M}{I} = \frac{\sigma}{y} \qquad \therefore M = \frac{\sigma I}{y}$

$= \frac{120M \times 6{,}17 \times 10^{-17}}{0{,}03}$

$= 246{,}8$ Nm

But $M = \frac{wl^2}{2} \qquad \therefore 2\ 468 = \frac{178{,}789L^2}{2}$

$L^2 = 27{,}608$

$L = 5{,}25$ m

1.2 Maximum slope $= \theta = \frac{wL^3}{8EI}$

$= \frac{178{,}789 \times 5{,}25^3}{6 \times 200G \times 6{,}17 \times 10^{-7}}$

$= 0{,}0349$ rad

$= 1{,}999°$

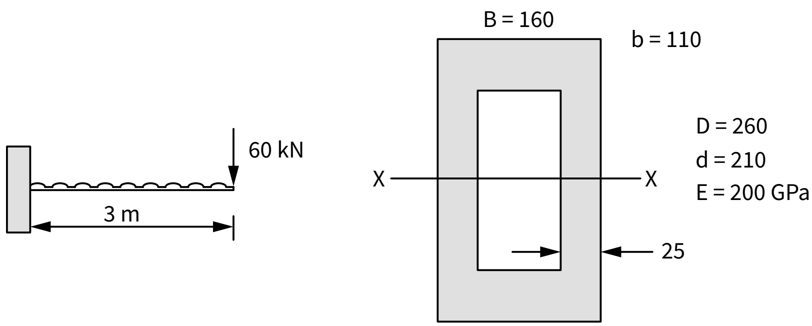
1.3 Maximum deflection $= \Delta = \frac{wL^4}{8EI}$

$= \frac{178{,}789 \times 5{,}25^4}{8 \times EI}$

$= 0{,}13759$ m

$= 137{,}59$ mm

2.



2.1 Total UDL = 150 kN

$\therefore$ UDL $= \frac{150}{3} = 50$ kNm

$$Ixx = \frac{1}{12}[BD^3 - bd^3]$$

$$= \frac{1}{12}[(0{,}16 \times 0{,}26^3) - (0{,}11 \times 0{,}21^3)]$$

$$= \frac{1}{12[2{,}812 \times 10^{-3} - 1{,}019 \times 10^{-3}]}$$

$$= 1{,}494 \times 10^{-4}\ \text{m}^4$$

Deflection at the free end:

$$\therefore \Delta = \Delta_{UDL} + \Delta_{PL}$$

$$= \frac{wL^4}{8EI} + \frac{WL^3}{3EI}$$

$$= \frac{1}{EI}\left[\frac{wL^4}{8} + \frac{WL^3}{3}\right]$$

$$= \frac{1}{200G \times 1{,}494 \times 10^{-4}}\left[\frac{50k \times 3^4}{8} + \frac{60k \times 3^3}{3}\right]$$

$$= \frac{1}{EI}[506{,}25k + 540k]$$

$$= 0{,}035\ \text{m}$$

$$= 35\ \text{mm}$$

2.2 Free end maximum slope $\theta = \theta_{UDL} + \theta_{PL}$

$$\therefore \theta_{max} = \frac{wL^3}{6EI} + \frac{WL^2}{2EI}$$

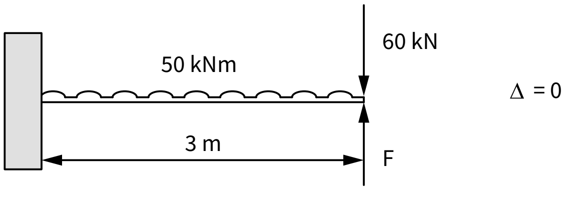
$$= \frac{1}{EI}\left[\frac{wL^3}{6} + \frac{WL^2}{2}\right]$$

$$= \frac{1}{EI}\left[\frac{50k \times 3^3}{6} + \frac{60k \times 3^2}{2}\right]$$

$$= \frac{1}{EI}[225k + 270k]$$

$$= 0{,}0166\ \text{rad}$$

2.3



Deflection at the free end $= 0 = \Delta_{down} - \Delta_{up}$

$$\therefore \Delta_{up} = \Delta_{down}$$

Take as PL the deflection up $\therefore \frac{WL^3}{3EI} = 0{,}035$

$\therefore$ Force in the support $= W = \frac{0{,}035 \times 3EJ}{3^3}$

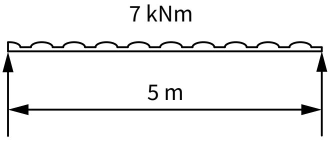
$= 116{,}2$ kN

Note

Stress and deflection

Enough information is given to calculate A: The I-value for deflection and A Z value for stress. Therefore, a section must be calculated for each (deflection and stress) to find a section that is strong enough for the condition to satisfy both.

3.



$Z = \frac{M}{\sigma}$

E = 21 GPa

I = section ?

$\sigma = 90$ MPa

$\Delta = \frac{1}{360} \times L$

$= \frac{5}{360}$

A. Consider deflection:

$= \frac{5 \times 7k \times 5^4 \times 360}{384 \times 210G \times 5}$

$Ixx = 19{,}532 \times 10^{-6}\ m^4$

Biggest nearest $Ixx = 22{,}97 \times 10^{-4}\ m^4$

Section = 203 × 102 × 25,3 k/m

$Zx = 226{,}1 \times 10^{-6}\ m^3$

B. Consider stress

$M = \frac{wL^2}{8} = \frac{7k \times 5^2}{8} = 21{,}875$ kNm

$\therefore$ Section modulus $= Z = \frac{M}{\sigma}$

$= \frac{21\ 875}{90M}$

$Zxx = 243{,}06 \times 10^{-6}\ m^3$

Nearest Z = $286{,}4 \times 10^{-6}$ m^3

Section = 205 × 102 × 24,5 kg/m

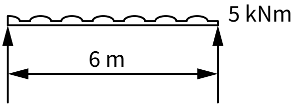
Use 305 × 102 × 24,5 kg/m

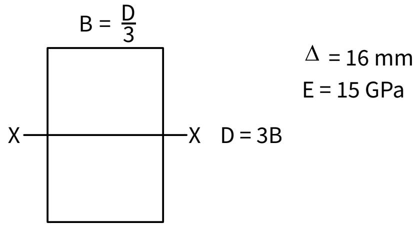
The stronger one:

Z for σ is > Z for Δ

4. Total UDL = 30 kN

$\therefore \frac{UDL}{m} = \frac{30}{6} = 5$ kNM





Maximum Δ at the middle

$\therefore \Delta = \frac{5wL^4}{384EI}$

$\therefore Ixx = \frac{5wL^4}{384E\Delta}$

$= \frac{5 \times 5k \times 6^4}{384 \times 15G \times 0{,}016}$

$= 3{,}516 \times 10^{-6}$ m^4

But $Ixx = \frac{BD^3}{12}$ … ①

D = 3B … ②

Substitute ② into ①:

$\therefore 3{,}516 \times 10^{-4} = \frac{B(3B)^2}{12}$

$4{,}2192 \times 10^{-3} = 27\ B^4$

$B^4 = 1{,}5627 \times 10^{-4}$

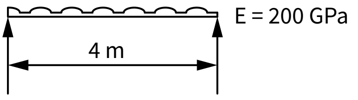
$B = \sqrt[4]{1{,}5627 \times 10^{-4}}$

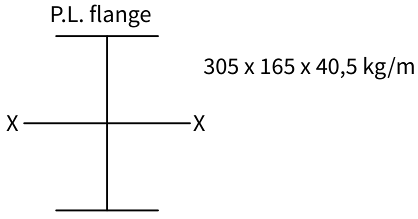
= 111,8 mm

D = 3B

= 335,4 mm

5. The stress and deflection are given ∴ there is enough information given to calculate a UDL for stress and deflection.





$\sigma = 110$ MP

$\frac{M}{I} = \frac{\sigma}{y} \qquad \Delta_{max} = \frac{5wL^4}{3EI} + \Delta$ weight

and $M = \frac{wL^2}{8} + \frac{wL^2}{8}$ weight

$305 \times 165 \times 40{,}5$ kg/m

$Ixx = 85{,}51 \times 10^{-6}$

40,5 kg/m

$y = \frac{h}{2} = \frac{303{,}8}{2}$

$= 151{,}9$

Consider deflection $\qquad \therefore \Delta_{max} = \Delta_{load} + \Delta_{weight}$

$\frac{4}{360} = \frac{5wL^4}{384EI} + \frac{5wL^4}{384EI}$

$\times$ EI $\therefore \frac{4 \times 200G \times 85{,}51 \times 10^{-6}}{360} = \frac{5wL^4}{384} + \frac{5(40{,}5 \times 9{,}81)4^4}{384}$

$\therefore 190{,}022k = \frac{5w \times 4^4}{384} + 1{,}3244k$

$\therefore 188{,}6976 = \frac{5w4^4}{384}$

$w = \frac{384 \times 188{,}6976k}{5 \times 4^4}$

$= 56{,}61$ Nm

Consider stress:

$\therefore M = \frac{\sigma I}{y}$

$= \frac{(110M \times 85{,}51 \times 10^{-6})}{0{,}1519}$

$= 61{,}923$ kNm

$\therefore M_{max} = M_{load} + M_{weight}$

$= \frac{wL^2}{8} + \frac{wL^2}{8}$

$$\therefore 61\,923 = \frac{w \times 4^2}{8} + \frac{(40{,}5 \times 9{,}8)4^2}{8}$$

$$\therefore \frac{w4^2}{8} = 61\,923 - 794{,}61$$

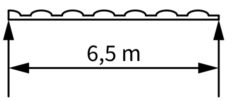
$$w = \frac{61\,128{,}39 \times 8}{4^2}$$

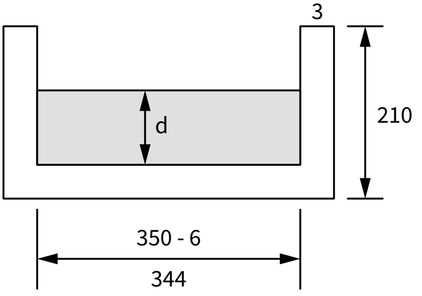
$$= 30{,}56 \text{ kNm}$$

Answer

The safe load is 30,56 kNm, the stress will be 110 MPa and deflection will be less than $\frac{4}{360}$. If 56,61 kNm is used, deflection will be $\frac{4}{360}$, but stress will be more than 110 MPa and beam will fail.

6. Stress and deflection are given as a limit, therefore both must be considered to find first the maximum depth of slime.





q_{slime} 1 100 kg/m^3

q_{metal} 7 800 kg/m

$\Delta = 6$ mm

$\sigma = 38$ MPa

E = 200 GPa

Consider deflection:

$$\Delta_{max} = \Delta_{slime} + \Delta_{metal}$$

$$= \frac{5wL^4}{384EI} + \frac{5wL^4}{384EI}$$

Weight/m for slime = Volqg

$$= (0{,}344 \times d) \times 1 \times 1\,100 \times 9{,}81$$

$w = 3\,712{,}104\, d$ Nm … ①

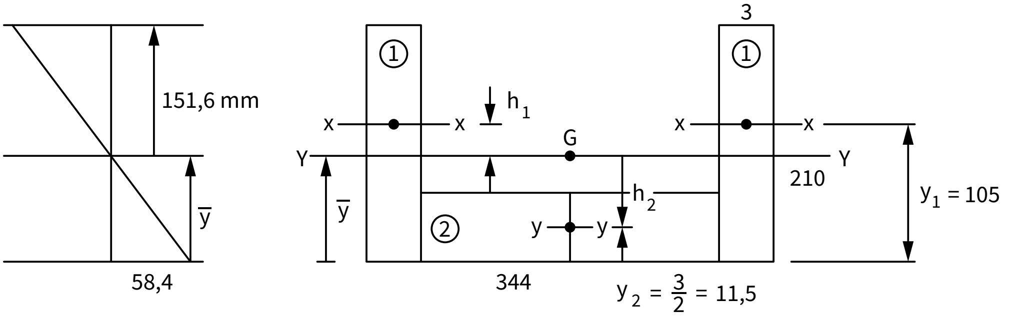
Weight of channel/m (metal)

$w = \text{Volqg}$

$= [(0{,}003 \times 0{,}21 \times 2) + (0{,}003 \times 0{,}344)] \times 1 \times 7\,800 \times 9{,}81$

$= \left[1{,}26 \times 10^{-3} + 1{,}032 \times 10^{-3}\right] 1 \times 7\,800 \times 9{,}81$

$= 175{,}38 \text{ M/m}$



$h_1 = y_1 - \bar{y} = 105 - 58{,}4 = 46{,}6 \text{ mm}$

$h_2 = \bar{y} - y_2 = 58{,}4 - 1{,}5 = 56{,}9 \text{ mm}$

Determine G: $\bar{y}A_T = A_1 g_1 + A_2 y_2$

$\bar{y}\left(1{,}26 \times 10^{-3} + 1{,}032 \times 10^{-3}\right) = 2\left(6{,}3 \times 10^{-4} + 0{,}105\right)$

$+ \left(1{,}032 \times 10^{-3} \times 0{,}0015\right)$

$\bar{y}2{,}292 \times 10^{-3} = 1{,}323 \times 10^{-4} + 1{,}548 \times 10^{-6}$

$\bar{y} = 58{,}4 \text{ mm}$

Moment of inertia:

$\text{I}yy = \left[\text{I}_{xx_1} + A_1 h_1^2\right] 2 + \left[\text{I}_{yy_1} + A_2 h_2^2\right]$

$\therefore \text{I}yy = \left[\frac{0{,}003 \times 0{,}21^2}{12} + \left(6{,}3 \times 10^{-4} \times 0{,}0466^2\right)\right] 2 + \left[\frac{0{,}344 \times 0{,}003^3}{12}\right.$

$\left. + \left(1{,}032 \times 10^{-3} \times 0{,}0569^2\right)\right]$

$\therefore \text{I}yy = \left[2{,}315 \times 10^{-6} + 3{,}342 \times 10^{-6}\right.$

$= 7{,}366 \times 10^{-6} + 3{,}342 \times 10^{-6}$

$= 10{,}708 \times 10^{-6} \text{ m}^4$

Slime depth for deflection

$\therefore \Delta_{max} = \left(\frac{5wL^4}{384EI}\right)_{slime} + \left(\frac{5wL^4}{384EI}\right)_{weight} \times 384EI$

$0{,}006 \times 384EI = 5w \times 6{,}5^4 + 5 \times 175{,}38 \times 6{,}5^4$

$\therefore 4{,}934\text{M} = 8\,925{,}313\, w + 1{,}565\text{M}$

$w = \dfrac{3{,}369\text{M}}{8\,925{,}313}$

$= 377{,}466$ Nm … ②

Substitute ② into ①: $\therefore 377{,}466 = 3\,712{,}104\, d$

Depth of slime $d = 101{,}7$ mm

Consider the stress:

$\therefore \dfrac{\text{M}}{\text{I}} = \dfrac{\sigma}{y}$

$\text{M} = \dfrac{\sigma \text{I}}{y_{\text{max}}}$

$= \dfrac{38 \times 10^{6} \times 10{,}708 \times 10^{-6}}{0{,}1516}$

$= 2\,684{,}06$ Nm

$\text{Maximum} = \left(\dfrac{w\text{L}^2}{8}\right)_{\text{slime}} + \left(\dfrac{w\text{L}^2}{8}\right)_{\text{metal}}$

$2\,684{,}06 = \dfrac{w \times 6{,}5^2}{8} + \dfrac{175{,}38 \times 6{,}5^2}{8}$

$\times 8 \therefore 21\,472{,}51 = 6{,}5^2 w + 7\,409{,}81$

$\therefore 6{,}5^2 w = 14\,062{,}705$

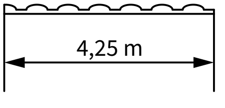
$w = 332{,}84$ Nm … ③

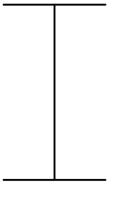
Substitute ③ into: $332{,}84 = 3\,712{,}104\, d$

Depth of the slime = $d = 89{,}7$ mm

$\therefore$ Depth of the slime for the channel to be safe is 89,7 mm.

7. 7.1 Stress and deflection are given. Consider each one for the safe load:





$\sigma = 100$ MPa PL flange $203 \times 133 \times 25{,}3$ kg/m

$\Delta = \dfrac{\text{L}}{360} = \dfrac{4{,}25}{360}$ $h = 203{,}2$

$\text{E} = 200$ GPa $\text{A} = 3{,}219 \times 10^{-3}$

$\text{I}xx = 23{,}49 \times 10^{-6}$

$y = \dfrac{h}{2} = \dfrac{203{,}2}{2}$

$= 101{,}6$

Consider the stress:

$$\therefore \frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{\sigma I}{y} = \frac{100M \times 23{,}49 \times 10^{-6}}{0{,}1016}$$

$$= 23\ 120{,}078\ \text{Nm}$$

$$M = \frac{wL^2}{8}$$

$$23\ 120{,}078 = \frac{w \times 4{,}25^2}{8}$$

$$\therefore w = 10{,}24\ \text{kNm}$$

Consider the deflection:

$$\Delta = \frac{5wL^4}{384EI}$$

$$\therefore \frac{4{,}25}{360} = \frac{5w \times 4{,}25^4}{384 \times 200G \times 23{,}49 \times 10^{-6}}$$

$$\therefore w = \frac{4{,}25 \times 384 \times 200G \times 23{,}497 \times 10^{-6}}{360 \times 5 \times 4{,}25^4}$$

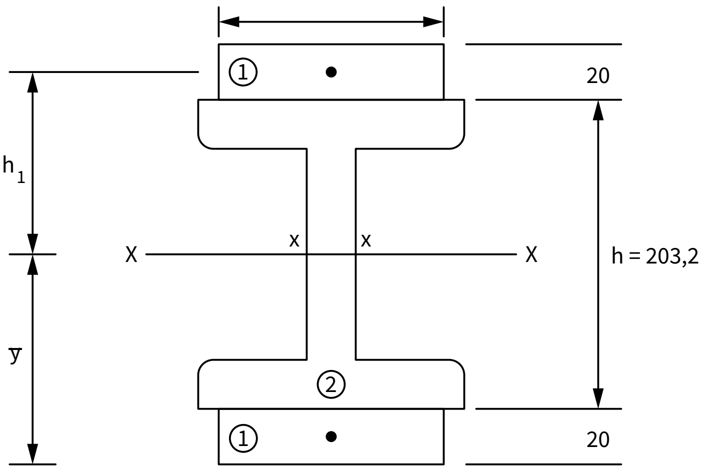
Load/m = w = 13,06 kNm

$\therefore$ Safe load = 10,24 kNm

7.2 Load/m = 2 × 10,24

= 20,48 kNm

E = 200 GPa



Deflection for 10,24 kNm E = GPa

$$\therefore \Delta = \frac{5wL^4}{384EI}$$

$$= \frac{5 \times 10{,}24k \times 4{,}25^4}{384E \times 23{,}49 \times 10^{-6}}$$

$$= 9{,}26 \times 10^{-3}$$

$\therefore$ New moment of inertia:

$$\Delta = \frac{5wL^4}{384EI}$$

$$\therefore 9{,}26 \times 10^{-3} = \frac{5 \times 20{,}48k \times 4{,}25^4}{384 \times 9{,}6 \times 10^{-3}}$$

$$= 4{,}698 \times 10^{-5}\ \text{m}^4$$

$$y = 20 + \frac{203{,}2}{2} = 121{,}6$$

$$h_1 = \frac{203{,}2}{2} + \frac{20}{2} = 111{,}6\ (h_2 = 0)$$

$$A_1 = 0{,}02\ w$$

$$A_2 = 3{,}219 \times 10^{-3}$$

$$\therefore Ixx = 2\left[I_{yy_1} + A_1h_1^2\right] + \left[I_{xx_1} + A_2h_2^2\right]$$

$$\therefore 4{,}698 \times 10^{-5} = 2\left[\frac{w \times 0{,}02^3}{12} + 0{,}02W \times 0{,}1116^2\right] + \left[23{,}49 \times 10^{-6}\right]$$

$$\therefore 2{,}349 \times 10^{-5} = 2\left[6{,}667 \times 10^{-7}W + 2{,}491 \times 10^{-4}W\right]$$

$$\therefore 2{,}498 \times 10^{-4}W = 1{,}1745 \times 10^{-5}$$

$$W = 47{,}01\ \text{mm}$$

Note

Also, if the load doubles and deflection remains constant, then the I-value must also be doubled.

$\therefore$ Ixx (new) was unnecessary to calculate.

$$\text{New } Ixx = 2 \times 23{,}49 \times 10^{-6} = 4{,}698 \times 10^{-5}\ \text{m}^4$$

8. Stress and deflection are given, but very little other information. However, both obtain the same values.

   $\therefore$ Stress:

   $$\therefore \frac{M}{I} = \frac{\sigma}{y}$$

   $$\therefore \frac{wL^2}{8I} = \frac{50M \times 2}{D}$$

$\therefore I = \frac{DwL^2}{800\ M} \ldots$ ①

$\therefore$ Deflection: $\Delta = \frac{5wL^4}{384EI}$

$\therefore \frac{L}{500} = \frac{5wL^4}{384EI}$

$\div L \therefore 384EI = 500 \times 5wL^3$

$I = \frac{2\ 500wL^3}{384 \times 210G}$

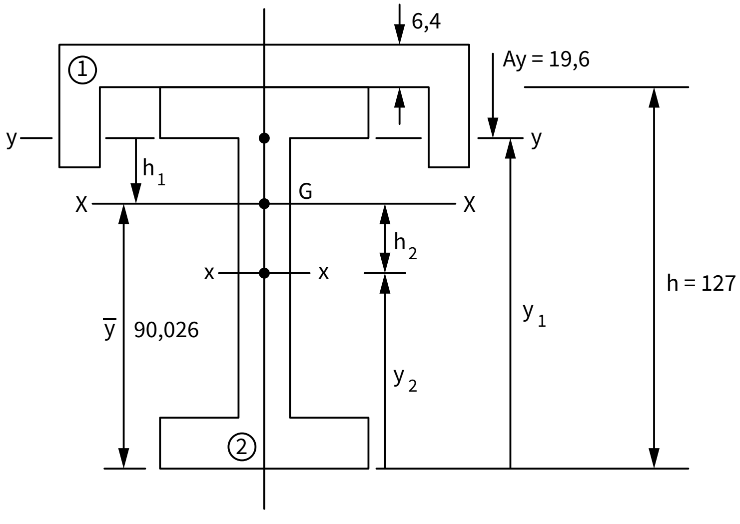
$= 3{,}1 \times 10^{-11}wL^3 \ldots$ ②

Substitute ② = ①: $3{,}1 \times 10^{-11}wL^3 = \frac{DwL^2}{800\ M}$

$\div wL^2 \therefore 0{,}0248L = D$

$\therefore L = 40{,}32\ D$

9. 9.1



Channel: $A_1 = 1{,}898 \times 10^{-3}$ $Iyy = 0{,}6845 \times 10^{-6}$

I-section $A_2 = 1{,}701 \times 10^{-3}$ $Ixx = 4{,}76 \times 10^{-6}$

$y_1 = h + t_1 = Ay$

$y_1 = (127 + 6{,}4 - 19{,}6) = 113{,}8$

$y_2 = \frac{h}{2} = \frac{127}{2} = 63{,}5$

$h_1 = 23{,}774$

$h_1 = y_1 - \bar{y} = 113{,}8 - 90{,}026$

$h_2 = \bar{y} - y_2 = 90{,}026 - 63{,}5$

$= 26{,}526$

Determine G:

$A_T y = A_1 y_1 + A_2 y_2$

$\therefore \bar{y}(1{,}898 + 1{,}701)10^{-3} = (1{,}898 \times 10^{-3} \times 0{,}1138)$

$+ (1{,}701 \times 10^{-3} \times 0{,}0635)$

$3{,}599 \times 10^{-3}\bar{y} = 2{,}1599 \times 10^{-4} + 1{,}0801 \times 10^{-4}$

$y = 90{,}026$ mm

Moment of inertia Ixx

$\therefore Ixx = \left[I_{yy_1} + A_1 h_1^2\right] + \left[I_{xx_1} + A_2 h_2^2\right]$

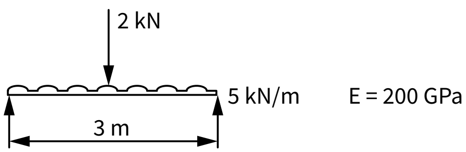
$\therefore Ixx = \left[0{,}6845 \times 10^{-6} + \left(1{,}898 \times 10^{-3} \times 0{,}023774^2\right)\right]$

$+ \left[4{,}76 \times 10^{-6} + \left(1{,}701 \times 10^{-3} \times 0{,}026526^2\right)\right]$

$\therefore Ixx = \left[0{,}6845 \times 10^{-6} + 1{,}0728 \times 10^{-6}\right] + \left[4{,}76 \times 10^{-6} + 1{,}1969 \times 10^{-6}\right]$

$= 1{,}7573 \times 10^{-6} + 5{,}9569 \times 10^{-6}$

$= 7{,}7142 \times 10^{-6}\ m^4$



$\Delta_{max} = \Delta_{PL} + \Delta_{UDL}$

$= \frac{WL^3}{48EI} + \frac{5wL^4}{384EI}$

$= \frac{2k \times 3^3}{48 \times 200G \times 7{,}7142 \times 10^{-6}} + \frac{5 \times 5k \times 3^4}{384EI}$

$= 7{,}292 \times 10^{-4} + 3{,}418 \times 10^{-3}$

$= 4{,}14$ mm

9.2 Maximum slope

$\theta_{max} = \theta_{PL} + \theta_{UDL}$

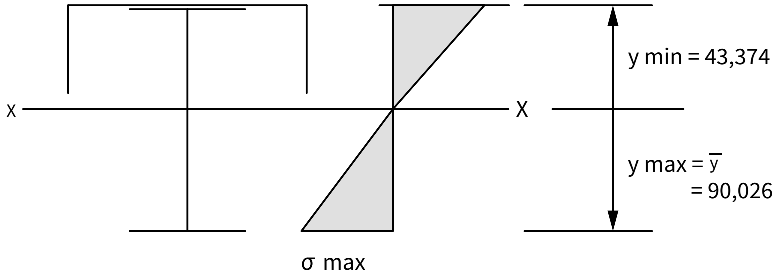
$= \frac{WL^2}{16EI} + \frac{wL^3}{24EI}$

$= \frac{2k \times 3^2}{16EI} + \frac{5k \times 3^3}{24EI}$

$= 7{,}292 \times 10^{-4} + 3{,}646 \times 10^{-3}$

$= 4{,}375 \times 10^{-3}$ rad

9.3 Maximum stress



$$\frac{M_{max}}{Ixx} = \frac{\sigma_{max}}{y_{max}}$$

$$\therefore M_{max} = M_{PL} + M_{UDL}$$

$$= \frac{WL}{4} + \frac{wL^2}{8}$$

$$= \frac{2k \times 3}{4} + \frac{5k \times 3^2}{8}$$

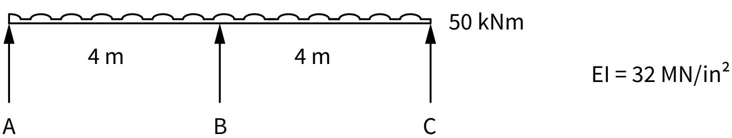
$$= 1{,}5k + 5{,}625$$

$$= 7{,}125 \text{ kNm}$$

$$\therefore \sigma_{max} = \frac{My}{I} = \frac{7\ 125 \times 0{,}090026}{7{,}7142 \times 10^{-6}}$$

$$= 83{,}16 \text{ MPa}$$

10.



10.1 Deflection at B = θ = $\Delta_{down} = \Delta_{up}$

$$\therefore \Delta_{down} = \Delta_{up}$$

$$\therefore \frac{5wL^4}{384EI} = \frac{WL^3}{48EI}$$

$$\times \frac{EI}{L^3} \therefore \frac{5wL}{384} = \frac{W}{48}$$

$$\therefore \text{Reaction at B} = W = \frac{5wL48}{384}$$

$$= \frac{5 \times 50k \times 8 \times 48}{384}$$

$$= 250 \text{ kN}$$

10.2 Deflection at B = θ mm

$$= \Delta_{down} = \frac{5 \times 50k \times 8^4}{384 \times 32 \times 10^6}$$

$$= 83{,}33 \text{ mm}$$

$$\therefore \Delta_{up} = 83{,}33 - 30$$

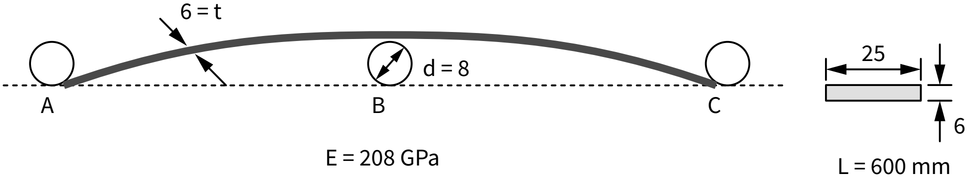
$$= 53{,}33 \text{ mm}$$

$$\text{Reaction in B} = W = \frac{\Delta_{up} \times 48 \times EI}{L^3}$$

$$= \frac{0{,}05333 \times 48 \times 32 \text{ M}}{8^3}$$

$$= 160 \text{ kN}$$

11.

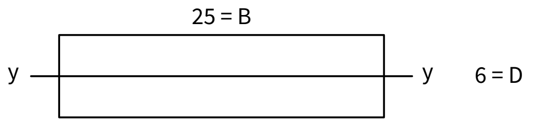


11.1 Deflection at B = $\Delta_B = d + t$

$$= 8 + 6$$

$$= 14 \text{ mm}$$

11.2



$$Iyy = \frac{BD^3}{12} = \frac{0{,}025 \times 0{,}006^3}{12}$$

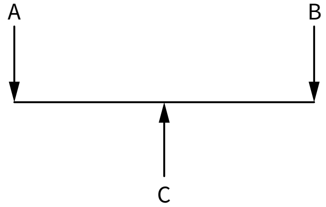
$$= 4{,}5 \times 10^{-10} \text{ m}^4$$

$$\therefore \Delta_B = \frac{WL^3}{48EI}$$

$$\text{Reaction B} = W = \frac{\Delta_B \times 48EI}{L^3} = \frac{0{,}014 \times 48 \times 208G \times 4{,}5 \times 10^{-10}}{0{,}6^3}$$

$$W = 291{,}2 \text{ N}$$

11.3 $F_{up} = F_{down}$



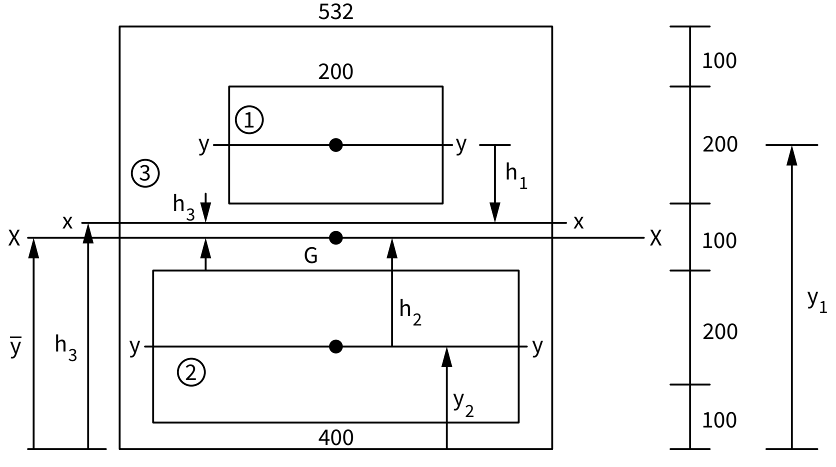
$\therefore F_B = F_A = F_C$

$\therefore 291{,}2 = 2F$

$F = 145{,}6 \; (F_A = F_C \text{ symmetrical beam})$

$\therefore F_A = F_C = 145{,}6$ N

12. 12.1



Area 3 is taken as a solid without the holes 1 and 2.

No.	Area		A	y	Ay
1	0,2 × 0,2	A_1	−0,04	0,5	−0,02
2	0,2 × 0,4	A_2	−0,08	0,2	−0,016
3	0,532 × 0,7	A_3	0,2724	0,35	+0,13034

A_T 0,2524 $\Sigma A_M = 0{,}09434$

Determine G:

$\therefore \bar{y}A_T = \Sigma$ Area moments

$\therefore \bar{y}0{,}2524 = 0{,}09434$

$\bar{y} = 373{,}772$ mm

12.2 Moment of inertia = Ixx

$Ixx = I_3 - I_1 - I_2$

$h_3 = y - y_3 = 373{,}772 - 350 = 23{,}772$

$\therefore I_{3xx} = \frac{BD^3}{12} + A_3 h_3^2$

$= \frac{0{,}532 \times 0{,}7^3}{12} + (0{,}3724 \times 0{,}023772^2)$

$I_3 = 15{,}416 \times 10^{-3}$ m^4

$h_1 = y_1 - \bar{y}$

Square: $I_{1yy} = \frac{5^4}{12} + A_1 h_1^2$ $\quad h_1 = 500 - 373{,}772 = 126{,}228$

$= \frac{0{,}2^4}{12} + (0{,}04 \times 0{,}126228^2)$

$I_1 = 7{,}7067 \times 10^{-4}$ m^4

$\therefore I_{2yy} = \frac{DB^3}{12} + A_2 h_2^2$ $\quad h_2 = \bar{y} - y_2$

$= 373{,}772 - 200$

$= 173{,}772$

$I_2 = \frac{0{,}4 \times 0{,}2^3}{12} + (0{,}08 \times 0{,}173772^2)$

$= 2{,}682 \times 10^{-3}$ m^4

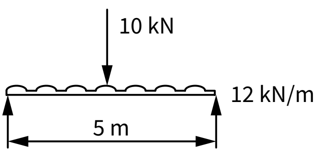
$\therefore Ixx = I_3 - I_1 - I_2$

$= 15{,}416 \times 10^{-3} - 7{,}7067 \times 10^{-4} - 2{,}682 \times 10^{-3}$

$= 11{,}9637 \times 10^{-3}$ m^4

12.3 Maximum stress: $\frac{M_{max}}{Ixx} = \frac{\sigma_{max}}{y_{max}}$

$\therefore \sigma_{max} = \frac{My}{I}$



$$M_{max} = M_{PL} + M_{UDL}$$

$$= \frac{WL}{4} + \frac{wL^2}{8}$$

$$= \frac{10k \times 5}{4} + \frac{12k + 5^2}{8}$$

$$= 125k + 37{,}5k$$

50 kNm

$$\therefore \sigma = \frac{My}{I} = \frac{50k \times 0{,}373772}{11{,}9637 \times 10^{-3}}$$

$$= 1{,}56 \text{ MPa}$$

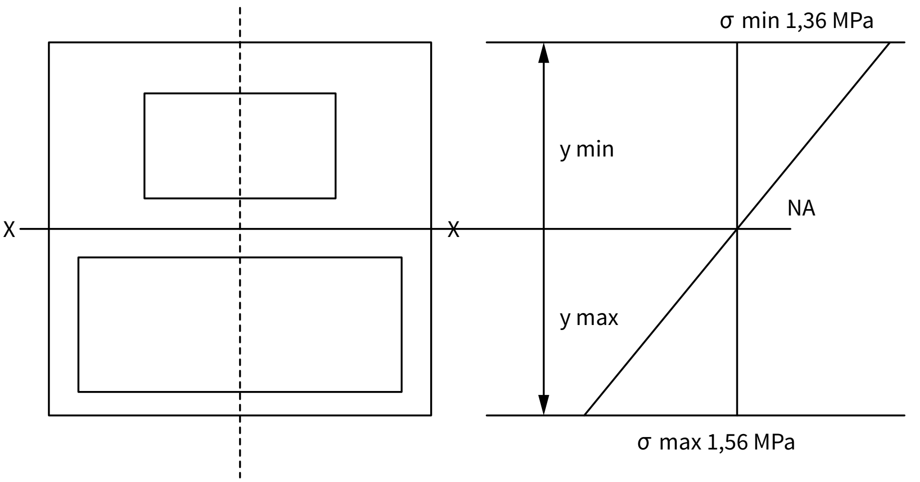
12.4 Minimum $\sigma \therefore \sigma_{min} = \frac{M_{max} y_{min}}{I}$

$$y_{min} = 700 = 373{,}772 = 326{,}228$$

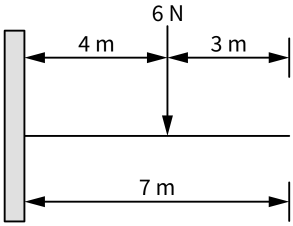
$$\sigma_{min} = \frac{50k \times 0{,}326228}{11{,}9637 \times 10^{-3}}$$

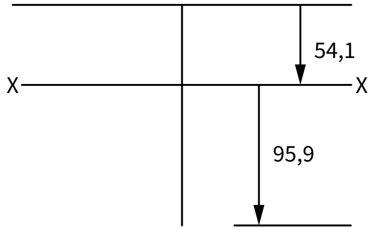
$$= 1{,}36 \text{ MPa}$$

12.5



13. 13.1 Maximum BM





Maximum M PL = Wa = 6 × 4 = 24 Nm

Weigh per meter = 2 × 20,2 × 9,81 = 396,34 Nm

$$M_{weight} = \frac{WL^2}{2} = \frac{396{,}324 \times 7^2}{2} = 9{,}71 \text{ kN}$$

$$M_{max\ total} = 24 + 9{,}71k = 9{,}734 \text{ kNM}$$

13.2 Maximum and minimum stress and the nature of stresses

$$\sigma_{min} = \frac{My_{min}}{I} = \frac{9{,}734k \times 0{,}0541}{2 \times 5{,}89 \times 10^{-6}} = 44{,}704 \text{ MPa (Tensile)}$$

$$\sigma_{max} = \frac{My_{max}}{I} = \frac{9{,}734k \times 0{,}0959}{2 \times 5{,}89 \times 10^{-6}} = 79{,}244 \text{ (Compressive)}$$

13.3 Maximum deflection

$$\Delta_{weight} = \frac{wL^4}{8EI} = \frac{396{,}324 \times 74}{8 \times 200G \times (2 \times 5{,}89 \times 10^{-6})} = 50{,}487 \text{ mm}$$

$$\Delta_{PL} = \Delta_B + (\theta_B \times b)$$

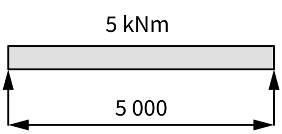
$$\Delta_{PL} = \frac{Wa^3}{3EI} + \left[\frac{Wa^2}{2EI} \times b\right]$$

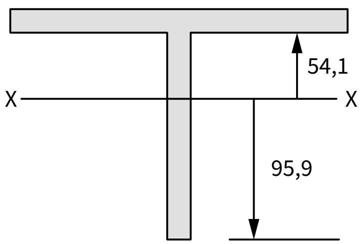
$$\therefore_{PL} = \frac{6 \times 4^3}{3 \times 200G(2 \times 5{,}89 \times 10^{-6})} + \left[\frac{6 \times 4^2}{2EI} \times 3\right]$$

$$\Delta_{PL} = 0{,}054 + 0{,}061 = 0{,}115 \text{ mm}$$

∴ Total deflection = 50,487 + 0,115 = 50,602 mm

14. 14.1 Select unequal leg angles to use as a beam





$$\Delta_{max} = \frac{5wL^4}{384EI}$$

$$\therefore I_{xx} = \frac{5 \times 5k \times 54}{384 \times 209G \times 0{,}018}$$

$$= 10{,}816 \times 10^{-6}\ m^4$$

1 per angle = $5{,}408 \times 10^{-6}\ m^4$

∴ Use angle 150 × 75 × 12 × 20,2 kg/m by 2 off

14.2 Actual deflection

$$\Delta_{actual} = \frac{5 \times 5k \times 54}{384 \times 209G(5{,}89 \times 10^{-6} \times 2)} = 16{,}527\ mm$$

14.3 Safe load

Safe load = Total load – weight of angles

$$= 5k - (2 \times 20{,}2 \times 9{,}81) = 4{,}604\ kNm$$

14.4 Maximum and minimum bending stress and the nature of stresses

$$\text{Maximum bending moment} = \frac{wL^2}{8} = \frac{5k \times 5^2}{8} = 15{,}625\ kNm$$

$$\sigma_{max} = \frac{My_{max}}{I_{xx}} = \frac{15{,}625k \times 0{,}0959}{2 \times 5{,}89 \times 10^{-6}} = 127{,}202\ MPa\ (Tensile)$$

$$\sigma_{min} = \frac{My_{min}}{I_{xx}} = \frac{15{,}625k \times 0{,}0541}{2 \times 5{,}89 \times 10^{-6}} = 71{,}758\ MPa\ (Compressive)$$

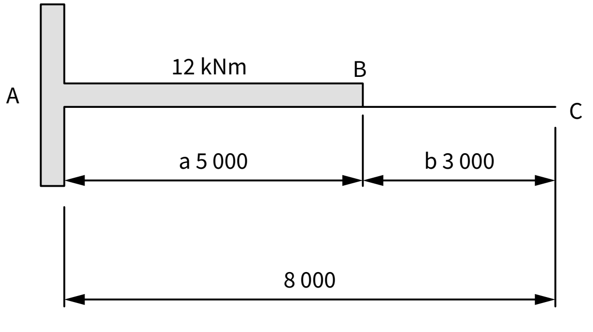
14.5 Force in prop

$$\Delta_{prop} = \Delta_{total} - \text{difference in height} = 16{,}527 - 8 = 8{,}527\ mm$$

$$\Delta_{prop} = \frac{FL^3}{48EI}$$

$$\therefore F = \frac{\Delta_{prop} \times 48 \times 209G(2 \times 5{,}89 \times 10^{-6})}{5^3} = 8{,}062\ kN$$

15. 15.1 Select a suitable beam to satisfy the stress and deflection



$$\text{Maximum M at A} = \frac{wa^2}{2} = \frac{12k \times 5^2}{2} = 150 \text{ kNm}$$

$$\text{Consider stress: } Z = \frac{M}{\sigma} = \frac{150k}{80M} = 1\,875 \times 10^{-6} \text{ m}^3$$

Nearest 457 × 191 × 98,3 kg/m

Lightest 533 × 210 × 92,50 kg/m

$$\text{Consider deflection: } \Delta_C = \Delta_B + (\theta_B \times b)$$

$$\Delta_C = 0{,}015 - \frac{wa^4}{8EI} + \left[\frac{wa^3}{6EI} \times b\right]$$

$$0{,}015 = \frac{12k \times 5^4}{8 \times 210GI} + \left[\frac{12k5^4}{8 \times 210GI} + \left[\frac{12k5^3}{6 \times 210GI} \times 3\right]\right.$$

$$0{,}015 = \frac{4{,}464 \times 10^{-6}}{I} + \frac{3{,}571 \times 10^{-6}}{I}$$

$$\therefore I = 535{,}667 \times 10^{-6} \text{ m}^4$$

The nearest 533 × 210 × 92,5 kg/m is also the lightest. Use this beam as it will satisfy both conditions.

> **Note**
>
> One must select the strongest beam of the two to satisfy both conditions. This is a once-off case that both conditions work out to the same section. Normally they differ and the section with the biggest I-value will be selected as the section to use to satisfy both conditions.

15.2 Actual stress and deflection

$$\Delta_{actual} = \frac{wa^4}{8EI} + \left[\frac{wa^3}{6EI} \times b\right]$$

$$= \frac{12k \times 5^4}{8 \times 210G \times 553{,}3 \times 10^{-6}} + \left[\frac{12k \times 5^3}{6EI} \times 3\right]$$

$$\Delta_{actual} = 14{,}523 \text{ mm}$$

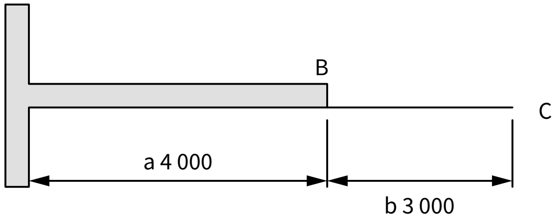
Actual stress

$$\sigma = \frac{M}{Z} = \frac{150k}{2\,076 \times 10^{-6}} = 72{,}254 \text{ MPa}$$

16. 16.1 Total deflection

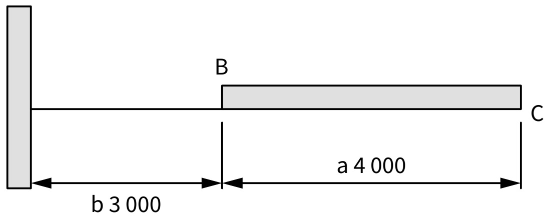
$$\Delta = \frac{wL^4}{8EI} = \frac{15k \times 7^4}{8 \times 340{,}6 \times 10^6} = 13{,}217 \text{ mm}$$

16.2 Deflection at the free end; load from the fixed end for 4 m



$$\Delta_C = \frac{wa^4}{8EI} + \left[\frac{wa^3}{6EI} \times b \times\right] = \frac{15k \times 4^4}{8 \times 340{,}6 \times 10^6} + \left[\frac{15k \times 4^3}{6 \times 340{,}6 \times 10^6} \times 3\right] = 2{,}812 \text{ mm}$$

16.3 Deflection at the free end; load 4 m from the free end to the fixed end



$$\Delta_C = \Delta_{total} - \left[\frac{wa^4}{8EI} + \left(\frac{wa^3}{6EI} \times b\right)\right]$$

$$\Delta_C = 13{,}217 - \left[\frac{15k \times 3^4}{8 \times 340{,}6M} + \left(\frac{15k \times 3^3}{6 \times 340{,}6M} \times 4\right)\right] = 11{,}978 \text{ mm}$$

16.4 Select a parallel I-section

$$BM = \frac{wL^2}{2} = \frac{15k \times 7^2}{2} = 367{,}5 \text{ kNm}$$

$$\therefore Z = \frac{M}{\sigma} = \frac{367{,}5k}{190M} = 19\ 434{,}21 \times 10^{-6} \text{ m}^3$$

Nearest 457 × 191 × 98,3 kg/m

Lightest 533 × 210 × 92,5 kg/m to use

MODULE

5 Combined direct and bending stress

By the end of this module, students should be able to:

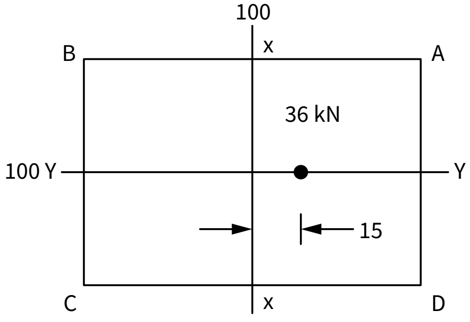
- calculate the direct stress in a column supporting a force;
- calculate the maximum and minimum bending stresses of beams with symmetrical and non-symmetrical profiles;
- calculate the direct stress, bending stress and resultant stresses of:
  - a column supporting its own weight as well as a horizontal force due to wind
  - a column supporting a load that is eccentric from either the *xx*- or *yy*-axis
  - a column supporting a load that is eccentric from both the *xx*-and *yy*-axis
  - a beam supporting a lateral load as well as a horizontal concentric load;
- calculate the position of the neutral axis;
- sketch a stress distribution diagram to show the position of the neutral axis;
- sketch a resultant stress distribution diagram to show the position of the neutral axis;
- calculate the eccentricity of the force if the resultant stress limit is known; and
- calculate the magnitude of the force if the resultant stress is known.

Introduction

The types of stresses at work all depend on how a load is applied to a member and what stresses will develop in the member.

Exercise 5.1 SB page 185

1. 1.1 Stress at the plane BC and AD



Direct stress = $\sigma_D = \frac{36k}{0{,}1^2} = 3{,}6$ MPa

Bending stress = $\sigma_B = \frac{My}{I} = \frac{Fey}{I}$

M = $36k \times 0{,}015 = 540$ NM

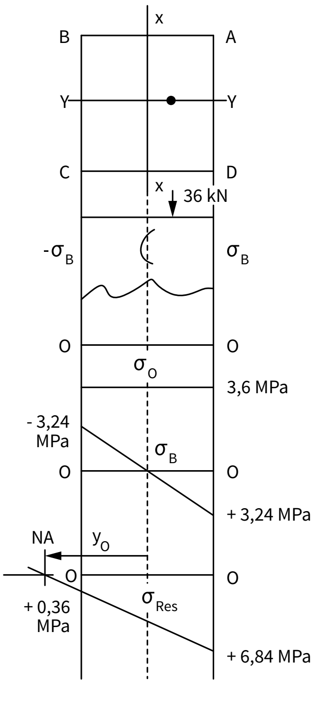
$I_{xx} = \frac{s^4}{12} = \frac{0{,}1^4}{12} = 8{,}333 \times 10^{-6}\ m^4$

Bending stress = $\sigma_B = \frac{Fey}{I} = \frac{540 \times 0{,}05}{8{,}333 \times 10^{-6}} = 3{,}24$ MPa

Stress at BC = $\sigma_{BC} = \sigma_D - \sigma_B = 3{,}6M - 3{,}24M = 0{,}36$ MPa

Stress at AD = $\sigma_{AD} = \sigma_D + \sigma_B = 3{,}6M + 3{,}24M = 6{,}84$ MPa

1.2 Stress diagram

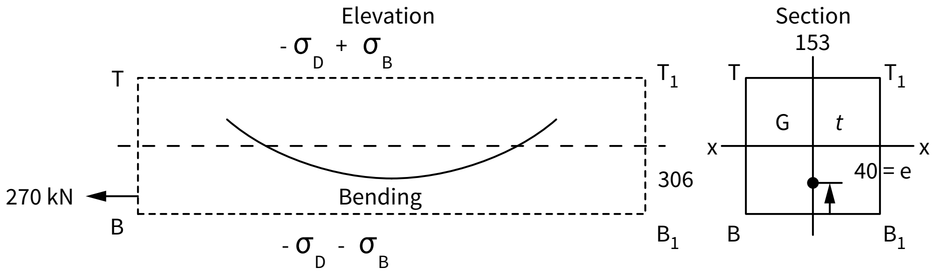


1.3 Position of the NA

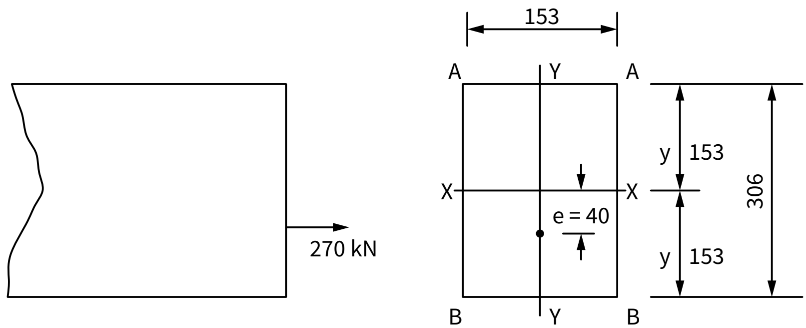
At NA $\sigma_D = \sigma_B$ $\quad \therefore 3{,}6M = \frac{My_0}{I}$

$$y_0 = \frac{3{,}6\ M \times 8{,}333 \times 10^{-6}}{540} = 55{,}553 \text{ mm from centroid}$$

2. Beam



2.1 Maximum and minimum stress in cross-section



Note

Values that are calculated from Mohr's circle will always differ a little bit.

Direct stress= $\sigma_D = \frac{F}{A} = \frac{-270k}{0{,}153 \times 0{,}306} = -5{,}767$ MPa (T)

Bending stress

Bending moment = M = Fe = $270k \times 0{,}04 = 10{,}8$ kNm

$$y_{AA} = y_{BB} = \frac{306}{2} = 153 \text{ mm}$$

$$I_{xx} = \frac{0{,}153 \times 0{,}306^3}{12} = 3{,}653 \times 10^{-4}$$

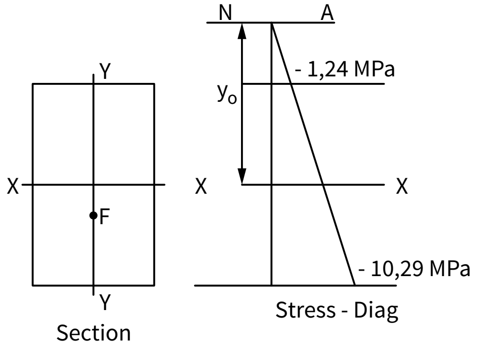
Bending stress = $\sigma_B = \frac{My}{I} = \frac{10{,}8k \times 0{,}153}{3{,}653 \times 10^{-4}} = 4{,}523$ MPa (T)

Stress at top AA

$\sigma_{AA} = -\sigma_D + \sigma_B = -5{,}767M + 4{,}523M = -1{,}24$ MPa (T) minimum stress

$\sigma_{BB} = -\sigma_D - \sigma_B = -5{,}767M - 4{,}523M = -10 - 29$ MPa (T)
maximum stress

2.2 Position of NA



Stress at the neutral axis is zero.

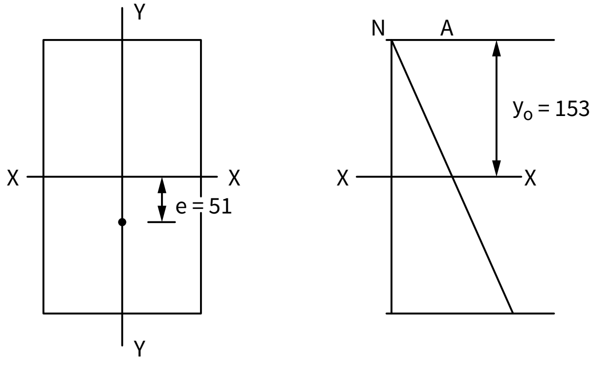
$\therefore \sigma_D = \sigma_B$

$\frac{F}{A} = 5{,}767M = \frac{My_O}{I}$

$Y_O = \frac{5{,}767M \times 3{,}653 \times 10^{-4}}{10{,}8k} = 195{,}07$ mm

NA is 195,07 mm from the centroid of the beam.

2.3 There's a distance of F from G so that the neutral axis is at top of section



$\therefore y_O = 153$ mm and direct stress = bending stress at NA

$5{,}767m = \frac{M \times 0{,}153}{3{,}653 \times 10^{-4}}$

M = 13,77 kNm

and M = Fe $\quad \therefore e = \frac{13{,}77k}{270k} = 51$ mm

The load must be applied 51 mm from the XX-axis on the YY-axis.

3. 3.1 PF section

$A = 3{,}219 \times 10^{-3}$

$t_1 = 5{,}8$

$b = 133{,}4$

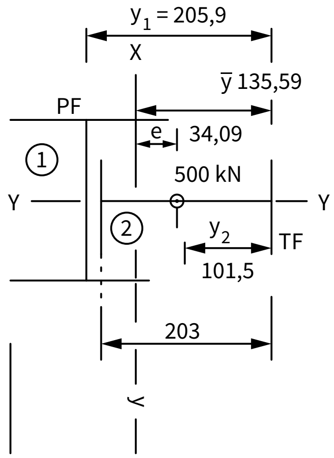
TF section

$A = 6{,}641 \times 10^{-3}$

$Ixx = 47{,}76 \times 10^{-6}$

$h = 203$

$y^2 = \frac{h}{2} = \frac{203}{2} = 101{,}5$



Elevation

$\sigma_D = \frac{F}{A} = \frac{F}{A_1 + A_2} = \frac{500k}{(3{,}219 + 6{,}641)10^{-3}}$

$= 50{,}71$ MPa

Bending stress

Determine G (the position of the xx-axis)

$A_T\bar{y} = A_1y_1 + A_2y_2$

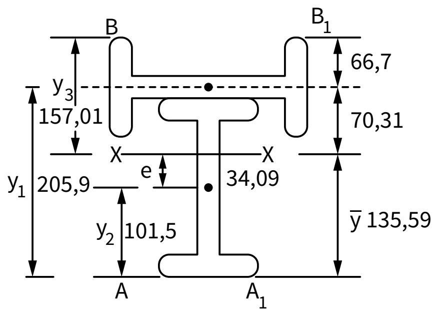
$\therefore \bar{y}(3{,}219 + 6{,}641)^{10^{-3}} = (3{,}219 \times 10^{-3} \times 0{,}2059) + (6{,}641 \times 10^{-3} \times 0{,}1015)$

$\bar{y}9{,}86 \times 10^{-3} = 6{,}628 \times 10^{-4} + 6{,}741 \times 10^{-4}$

$\therefore \bar{y} - 135{,}59$ mm

$h_1 = y_1 - \bar{y} = 205{,}9 - 135{,}59 = 70{,}31 - h_1$

$h_2 = \bar{y} - y_2 = 135{,}59 - 101{,}5 = 34{,}09 = h_2$



$Ixx = \left[I_{yy_1} + A_1h_1^2\right] + \left[I_{xx_2} + A_2h_2^2\right]$

$= \left[3{,}09 \times 0^{-6} + (3{,}219 \times 10^{-3} \times 0{,}0731^2)\right]$

$+ \left[47{,}76 \times 10^{-6} + (6{,}641 \times 10^{-3} \times 0{,}03409^2)\right]$

$= 2{,}0291 \times 10^{-5} + 5{,}5478 \times 10^{-5}$

$= 7{,}5769 \times 10^{-5}$ m^4

$e = \bar{y} - y_2 = 135{,}59 - 101{,}5 = 34{,}09$ mm

Bending stress at AA_1 $\quad \therefore y_{AA_1} = y = 135{,}59$

$$M = Fe$$
$$= 500k \times 0{,}03409$$
$$= 17{,}045 \text{ kNm}$$

$$\therefore \sigma_{AA_1} = \frac{My}{I}$$
$$= \frac{17{,}045k \times 0{,}13559}{7{,}5769 \times 10^{-5}}$$
$$= 30{,}5 \text{ MPa}$$

Bending stress at BB_1 $\quad \therefore y_{BB_1} = \left(y_1 + \frac{b}{2}\right) - \bar{y}$

$$= 205{,}9 + \frac{133{,}4}{2} - 135{,}59$$
$$= 137{,}01 \text{ mm}$$

$$\therefore \sigma_{BB_1} = \frac{My}{I} = \frac{17{,}045k \times 0{,}13701}{7{,}5769 \times 10^{-5}}$$
$$= 30{,}11 \text{ MPa}$$

$$\sigma_{min} \text{ of } BB_1 = \sigma_D - \sigma_{BB_1}$$
$$= 50{,}71 - 30{,}11$$
$$= 20{,}6 \text{ MPa}$$

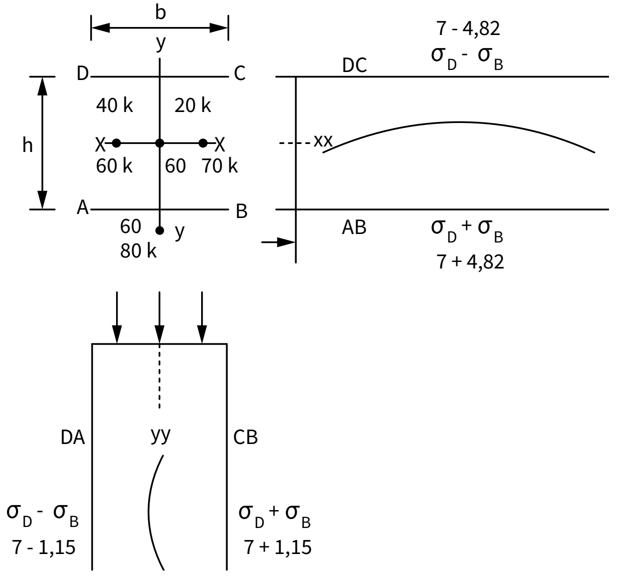
$$\sigma_{max} \text{ of } AA_1 = \sigma_D - \sigma_{B_{AA_1}}$$
$$= 50{,}71 + 30{,}5$$
$$= 81{,}21 \text{ MPa}$$

3.2 Position of the NA

At NA $\sigma_D = \sigma_B$ $\quad \therefore 50{,}71M = \frac{My_0}{I}$

$$y_0 = \frac{50{,}71M \times 7{,}5769 \times 10^{-6}}{17{,}045k} = 225{,}418 \text{ mm from centroid}$$

4. $t_1 = 18{,}5$



$h = 381$

$b = 395$

$A = 29{,}99 \times 10^{-3}$

$Ixx = 791{,}5 \times 10^{-6}$

$Iyg = 310{,}4 \times 10^{-6}$

$356 \times 406 \times 235$ kg/m

$$\text{Direct } \sigma_D = \frac{F}{A} = \frac{(40 + 20 + 70 + 80)10^3}{29{,}99 \times 10^{-3}}$$

$$= 7 \text{ MPa}$$

About *yy*-axis

$$\therefore \sigma_{yy} = \frac{Mx}{Iyy} \qquad M = Fe = (70 - 40)0{,}06$$

$$= \frac{1\,800 \times 0{,}1975}{310 \times 10^{-6}} \qquad = 1\,800 \text{ Nm}$$

$$= 1{,}15 \text{ MPa} \qquad x - \frac{b}{2} = \frac{395}{2}$$

$$= 197{,}5$$

About *xx*-axis

$M = Fe = 80k \times 0{,}2505$ $\qquad e = \frac{b}{2} + 60$

$= 20\ 040 \text{ Nm}$ $\qquad = \frac{381}{2} + 60$

$= 250{,}5$

$\therefore \sigma_{xx} = \frac{My}{Ixx}$ $\qquad y = \frac{h}{2}$

$= \frac{20\ 040 \times 0{,}1905}{791{,}5 \times 10^{-6}}$ $\qquad = \frac{381}{2}$

$= 4{,}82 \text{ MPa}$ $\qquad = 190{,}5 \text{ mm}$

$= 7 + 4{,}82 - 1{,}15$

$= 10{,}67 \text{ MPa}$

$\sigma_B = \sigma_D + \sigma_{Bxx} + \sigma_{Byy}$

$= 7 + 4{,}82 + 1{,}15$

$= 12{,}97 \text{ MPa}$

$\sigma_C = \sigma_D - \sigma_{Bxx} + \sigma_{Byy}$

$= 7 - 4{,}82 + 1{,}15$

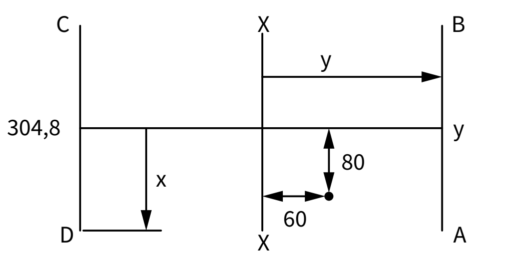
$= 33{,}33 \text{ MPa}$

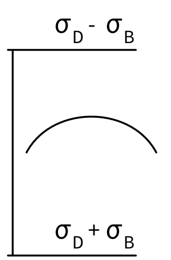
$\sigma_D = \sigma_D - \sigma_{Bxx} = \sigma_{Byy}$

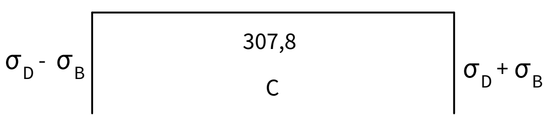
$= 7 - 4{,}82 - 1{,}15$

$= 1{,}03 \text{ MPa}$

5.







5.1 $A = 12{,}33 \times 10^{-3}$

$Ixx = 222 \times 10^{-6}$

$Iyy = 72{,}71 \times 10^{-6}$

$h = 307{,}8$

$b = 304{,}8$

$\sigma_A = 15$ MPa

Direct stress

$\therefore \sigma_D = \frac{F}{A} = \frac{F}{12{,}33 \times 10^{-3}}$

$\sigma_D = 81{,}103$ F … ①

Bending stress about the xx-axis

$y_{CD} = y_{AB} = \frac{307{,}8}{2} = 153{,}9$

$Mxx = F \times Cx = F \times 0{,}06 = 0{,}06$ F

$\sigma_{Bxx} = \frac{Mxxy}{Ixx} = (0{,}06F \times 0{,}1539)$

$= 41{,}595$ F

Bending stress about yg-axis

$x = \frac{304{,}8}{2} = 152{,}4$

$M_{yg} = F \times e_g = F \times 0{,}08$

$= 0{,}08$ F

$\therefore \sigma_{Byg} = \frac{M^x_{yg}}{I_{yg}} = \frac{0{,}08F \times 0{,}1524}{72{,}71 \times 10^{-6}}$

$\sigma_{Byg} = 167{,}6898$ F

$\therefore \sigma_A = \sigma_D + \sigma_{Bxx} + \sigma_{yy}$

$\therefore 15 \times 10^6 = 81{,}103$ F 41,595 F 167,68 F

$= 290{,}378$ F

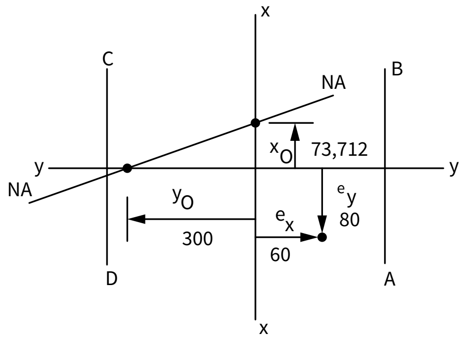
$F = 51{,}66$ kN

5.2 Co-ordinates of neutral axis

Co-ordinate on xx-axis: $\therefore x_o = \frac{I_{yy}}{Ae_y} = \frac{72{,}71 \times 10^{-6}}{12 \times 10^{-3} \times 0{,}08} = 73{,}712$ mm

Co-ordinate on yy-axis: $\therefore y_o = \frac{I_{xx}}{Ae_x} = \frac{222 \times 10^{-6}}{12{,}33 \times 10^{-3} \times 0{,}06} = 300$ mm

5.3 Sketch to show position of the NA:



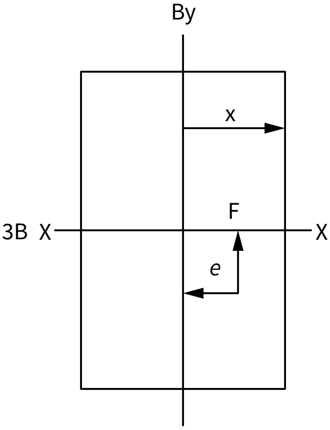
6.1 $= e - 0{,}125$ B

F = 250 kN

$B = \frac{D}{3}$

D = 3B

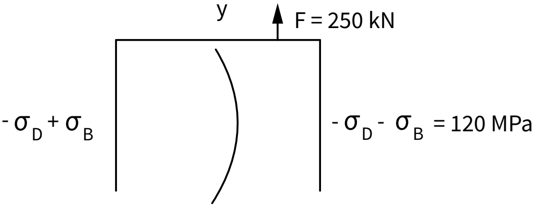
σt = 120 MPa



Direct stress $= \frac{F}{A}$

$= \frac{250k}{B \times 3B}$

$= \frac{250k}{3B^2}$



Bending stress:
about the *yg*-axis

$x = \frac{B}{2} = 0{,}5$ B

$e = 0{,}125$ B

$Myg = Fey = 250k \times 0{,}125$ B

$= 31\,250$ B

$\sigma yg = \frac{Mx}{I} \qquad \therefore I = \frac{3B \times B^3}{12}$

$\therefore \sigma yg = \frac{31\,250B \times 0{,}5B}{0{,}25B^4} = 0{,}25B^4$

$= \frac{62\,500}{B^2}$

$\sigma_{max} = -120 \times 10^6 = \sigma_D = \sigma_B$

$= -\frac{250k}{3B^2} - \frac{62\,500}{B^2}$

$\times -B^2 \qquad \therefore 120 \times 10^6 B^2 = 83\ 333{,}33 + 62\ 500$

$B^2 = \frac{145\ 833{,}33}{120 \times 10^6}$

$= 1{,}2153$

$B = 34{,}86 \text{ mm}$

$D = 3 \times 34{,}86 \text{ mm}$

$= 104{,}58 \text{ mm}$

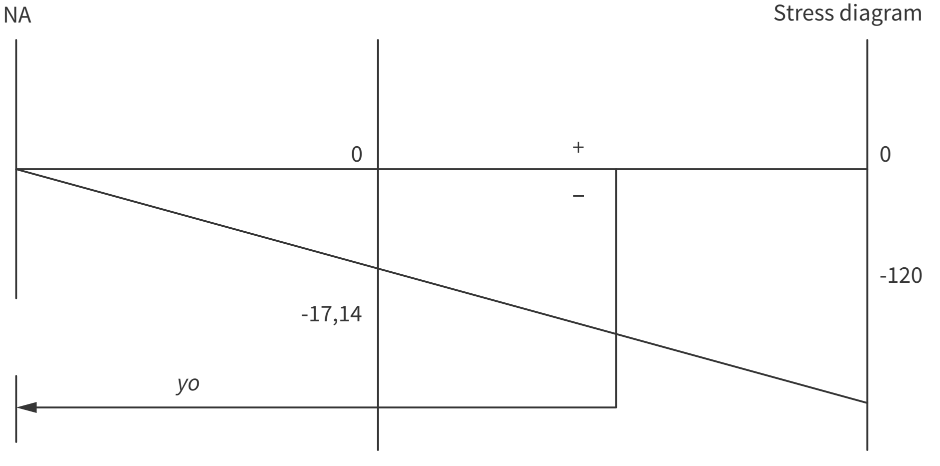
6.2 $\sigma_D = \frac{250k}{3B^2} = \frac{250k}{3 \times 0{,}03486^2} = 68{,}57 \text{ MPa}$

$\sigma_B = \frac{62\ 500}{B^2} = \frac{62\ 500}{0{,}03486^2} = 51{,}43 \text{ MPa}$

$\sigma_{smin} = -\sigma_D + \sigma_B = -68{,}57 + 51{,}43$

$= -17{,}14 \text{ MPa (Tensile)}$

$\sigma_{smax} = -\sigma_D - \sigma_B = -120 \text{ MPa}$



At NA, stress = 0

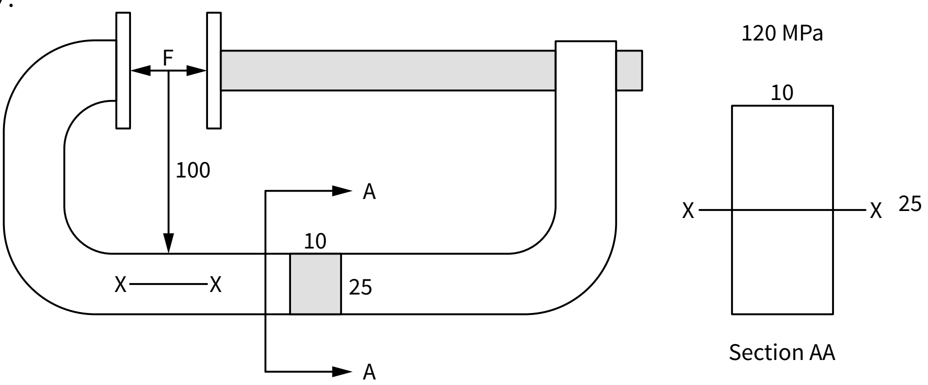
$\therefore \sigma_B = \sigma_D$

$\therefore \frac{My_o}{I} = \sigma_D$

$\therefore \frac{31\ 250 \times 0{,}03486 yo}{0{,}25(B)^4} = 68{,}57 \text{ M}$

$\therefore yo = 23{,}24 \text{ mm}$

7.



7.1 Direct stress: $\sigma_D = \frac{F}{A} = \frac{F}{0,01 \times 0,025}$

$= 4\ 000\ F \ldots$ ①

Bending stress: $e = 100 + \frac{25}{2} = 112,5$ mm

$M = F \times e = F = \times 0,1125$

$Ixx = \frac{bd^3}{12} = \frac{0,01 \times 0,025^3}{12}$

$= 1,3021 \times 10^{-8}\ m^4$

$y = \frac{25}{2} = 12,5$ mm

$\sigma_B = \frac{My}{I}$

$= \frac{0,1125\ F \times 0,0125}{1,3021 \times 10^{-8}}$

$\sigma_B = 107\ 998,618\ F$

$\therefore \sigma_{max} = -\sigma_D - \sigma_B$

$\therefore -120M = -4\ 000\ F - 107\ 998,618\ F$

$\therefore F = \frac{120 \times 10^6}{111\ 998,618}$

$F = 1,071$ kN

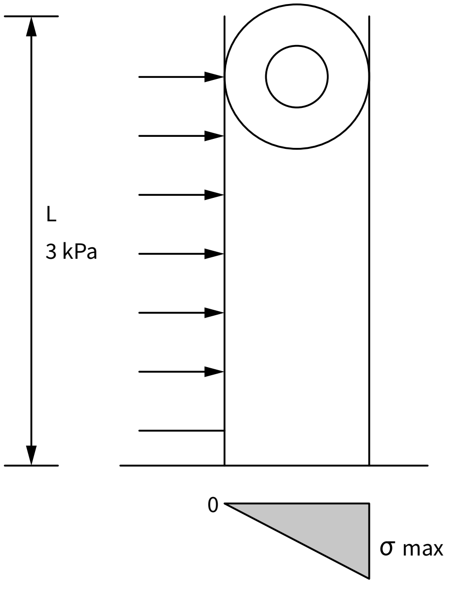
7.2 $\sigma_{comp} = -\sigma_D - \sigma_B$

$= -4\ 000 \times 1\ 071 + 107\ 998,618 \times 1\ 071$

$= -4,284M + 115,667M$

$= 111,39$ MPa

8. Consider column as a cantilever



$D = 2{,}6$ m

$d = 2{,}6 - (2 \times 0{,}06)$

$= 2{,}48$ m

$q_{steel} = 7\,800$ kg/m^3

A effect = 0,6 A projected

Effective area = 0,6 (D × L)

$= 0{,}6 \times 2{,}6 \times L$

$= 1{,}56\ L\ m^2$

Total force in column:

$F_T = P_{res} \times \text{area}$

$= 3 \times 10^3 \times 1{,}56L$

$= 4\,680\ L\ N$

$\therefore \text{Load/m} = \frac{4\,680\ L}{L} = 4\,680\ Nm$

$\therefore M = \frac{wL^2}{2} = \frac{4\,680\ L^2}{2} = 2\,340\ L^2$

$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(2{,}6^4 - 2{,}48^4) = 0{,}38633$

$\therefore \sigma_B = \frac{My}{I} = \frac{2\,340L^2 \times 1{,}3}{0{,}38633}$

$\sigma_B = 7\,874{,}097\ L^2$

Weight/m = Volqg

$= \frac{\pi}{4}(2{,}6^2 - 2{,}48^2)1 \times 7\,800 \times 9{,}81$

$= 36{,}635$ kNm

Total weight of column = $36{,}635k \times$ L

$= 36\,635$ L

$\sigma_D = \frac{F}{A}$

$= \frac{36\,635\text{ L}}{\frac{\pi}{4}(2{,}6^2 - 2{,}48^2)}$

$\sigma_D = 76\,517{,}6$ L

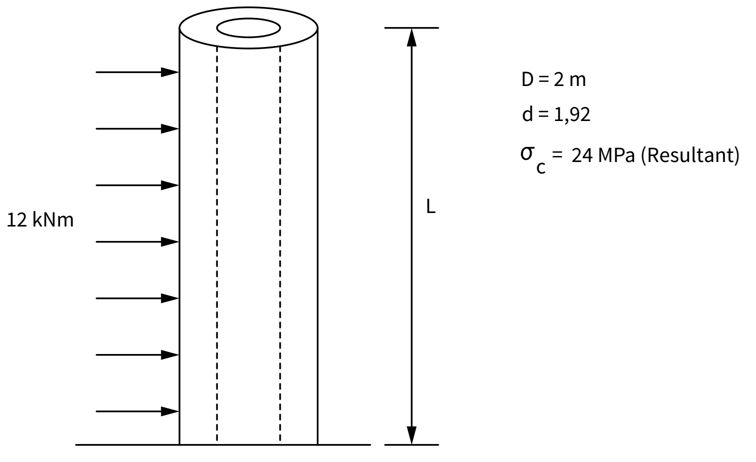
Stress zero at side of column

$\therefore$ Result stress $= 0 = \sigma_B = \sigma_D$

$\therefore 7\,874{,}097\text{ L}^2 = 76\,517{,}6$ L

$\div$ L $\therefore$ L = 9,72 m height of column

9.



9.1 Consider chimney as a cantilever

$\therefore$ Bending moment $= \frac{wL^2}{2} = \frac{0{,}2k \times L^2}{2}$

$= 600\text{ L}^2$

$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(2^4 - 1{,}92^4) = 0{,}11832\text{ m}^3$

$y = 1$

$\therefore \sigma_D = \frac{My}{I} = \frac{600\text{ L}^2 \times 1}{0{,}11832}$

$= 5\,070{,}994\text{ L}^2$

Weight/m = Volqg

$$= \frac{\pi}{4}(2^2 - 1{,}92^2)1 \times 7\,500 \times 9{,}80$$

$$= 18\,121{,}586 \text{ Nm}$$

$\therefore$ Resultant = $\sigma_R = \sigma_D + \sigma_B$

$$\therefore \text{Direct stress} = \frac{F}{A} = \frac{18\,121{,}586\text{ L}}{\frac{\pi}{4}(2^2 - 1{,}92^2)}$$

$$= 73\,575 \text{ L}$$

$$\therefore 24M = 73\,575 \text{ L} + 5\,070{,}994 \text{ L}^2$$

$$\div 5\,070{,}994 \therefore \text{L}^2 + 14{,}509 \text{ L} - 4\,732{,}8 = 0$$

$$\therefore \text{L} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= -14{,}509 \pm \frac{\sqrt{14{,}509^2 - (1)(-4{,}732{,}8)}}{2 \times 1}$$

$$\therefore \text{L} = \frac{-14{,}509 \pm 138{,}35}{2}$$

= 61,93 m length of the chimney

9.2 $\sigma_D = 73\,575 \times 61{,}93$

$= 4{,}556$ MPa

$\sigma_B = 5\,070{,}994 \text{ L}^2$

$= 5\,070{,}994 \times (61{,}93)^2$

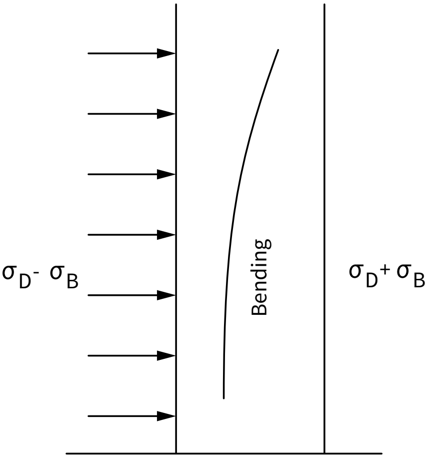
$= 19{,}449$ MPa

$\therefore$ Maximum tensile stress

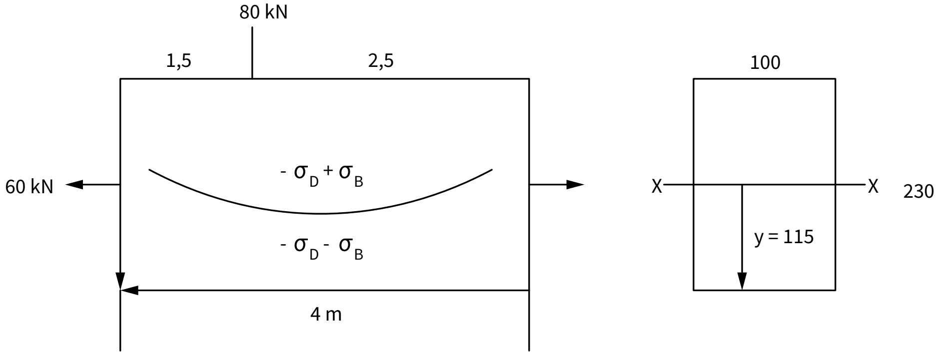
$\sigma_t = \sigma_D - \sigma_B$

$= 4{,}556 - 19{,}449$ M

$= -14{,}89$ MPa (Tensile)



10.



10.1 Bending moment $= M = \frac{Wab}{L}$

$$= \frac{80k \times 1{,}5 \times 2{,}5}{4}$$

$$= 75 \text{ kNm}$$

$$Ixx = \frac{BD^3}{12} = \frac{0{,}1 \times 0{,}23^3}{12}$$

$$= 1{,}0139 \times 10^{-4} \text{ m}^4$$

$$\therefore \sigma_B = \frac{My}{I} = \frac{75k \times 0{,}115}{1{,}0139 \times 10^{-4}}$$

$$= 85{,}07 \text{ MPa}$$

$$\sigma_D = \frac{F}{A} = \frac{60k}{0{,}1 \times 0{,}23} = 2{,}61 \text{ MPa (Tensile)}$$

$$\therefore \sigma_{max} = -\sigma_D - \sigma_B$$

$$= -2{,}61 - 85{,}07$$

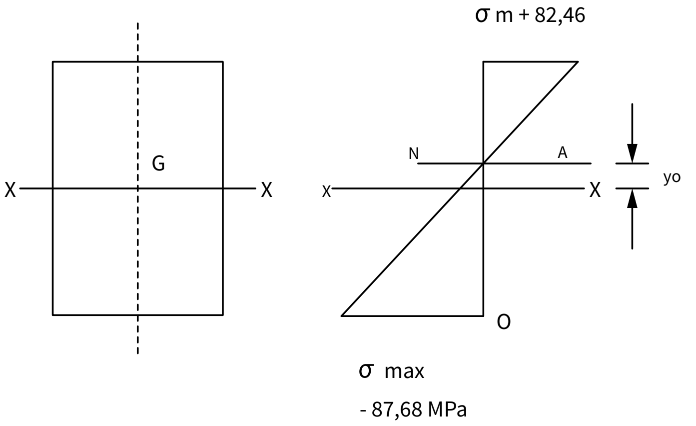
$$= 87{,}68 \text{ MPa (Tensile)}$$

$$\therefore \sigma_{min} = -\sigma_D + \sigma_B$$

$$= -2{,}61 + 85{,}07$$

$$= 82{,}46 \text{ MPa (Compressive)}$$

10.2 Position of the NA



At the NA, stress = 0

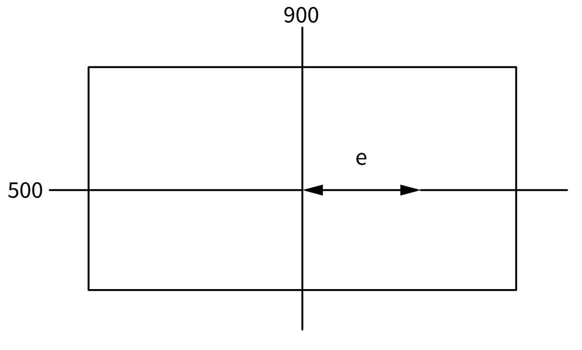
$\therefore \sigma_B = \sigma_D$

$\frac{Myo}{I} = \sigma_D$

$= \frac{75k\ yo}{1{,}0139 \times 10^{-4}}$

yp = 3,53 mm from G or xx-axis

11.1 Position of load



Direct stress = $\sigma_D = \frac{1{,}2\ M}{0{,}5 \times 0{,}9} = 2{,}667$ MPa

Bending stress = $\sigma_B = \frac{Fey}{I} = \frac{1{,}2M \times e \times 0{,}45 \times 12}{0{,}5 \times 0{,}9^3} = 17{,}778 \times 10^6 e$

Minimum resultant stress = $889k = \sigma_D - \sigma_B$

$= 889k = 2{,}667M - 17{,}778 \times 10^6 e$

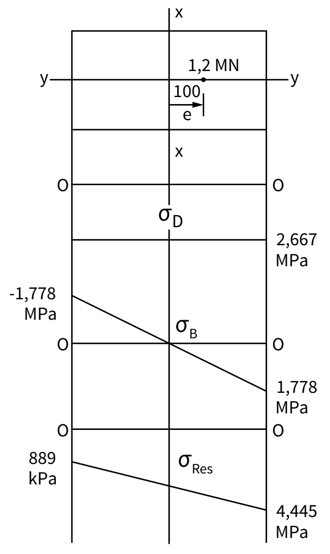
$17{,}778 \times 10^6 e = 2{,}667M - 889k$

Force from the centroid = e = 100 mm

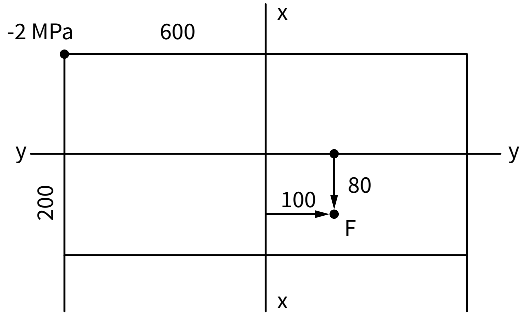
11.2 Draw stress diagram

Bending stress = $17{,}778 \times 10^6 e = 17{,}778 \times 10^6 \times 0{,}1 = 1{,}778$ MPa

Maximum resultant stress = 2,667M + 1,778M = 4,445 MPa



12.1 Magnitude of force



$$\text{Direct stress} = \sigma_D = \frac{F}{0{,}2 \times 0{,}6} = 8{,}333F$$

$$\text{Bending stress} = \sigma_{Bxx} = \frac{Fey}{I_{xx}} = \frac{F \times 0{,}1 \times 0{,}3 \times 12}{0{,}2 \times 0{,}6^3} = 8{,}333F$$

$$= \sigma_{Byy} = \frac{Fex}{I_{yy}} = \frac{F \times 0{,}08 \times 0{,}1 \times 12}{0{,}6 \times 0{,}2^3} = 20F$$

$$\text{Resultant stress} = -2M = \sigma_D - \sigma_{Bxx} - \sigma_{Byy}$$

$$= -2M = 8{,}333F - 8{,}333F - 20F$$

Force = F = 100 kN

12.2 Maximum stress in the column

Maximum resultant stress $= \sigma_D + \sigma_{Bxx} + \sigma_{Byy}$

$= (8{,}333 + 8{,}333 + 20)F$

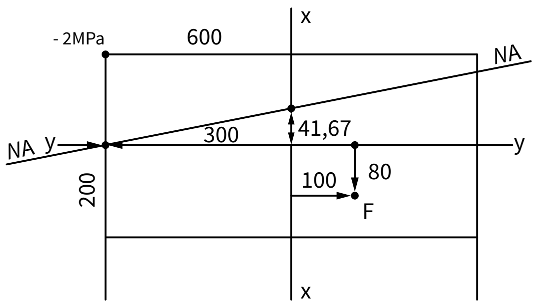
$= 36{,}666 \times 100k = 3{,}667$ MPa

12.3 Co-ordinates for NA

$$x_0 = \frac{I_{yy}}{Ae_y} = \frac{0{,}6 \times 0{,}2^3}{12 \times 0{,}2 \times 0{,}6 \times 0{,}08} = 41{,}67 \text{ mm}$$

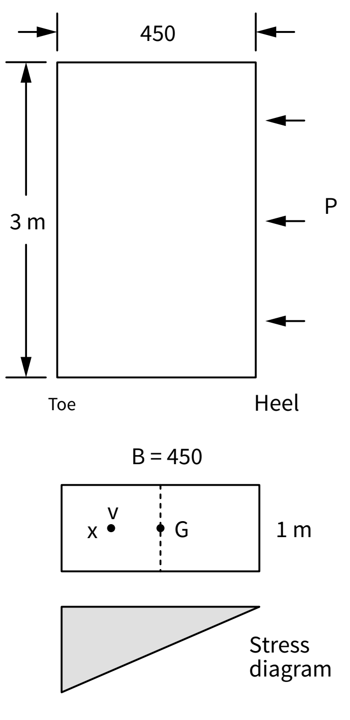
$$y_0 = \frac{I_{xx}}{Ae_x} = \frac{0{,}2 \times 0{,}6^3}{12 \times 0{,}2 \times 0{,}6 \times 0{,}1} = 300 \text{ mm}$$

12.4 Sketch



13. There is no tension in the column

$\therefore e = \frac{B}{6} = \frac{450}{6} = 75$ mm



Direct stress $= \frac{F}{A} = \frac{\text{Vol}\rho g}{0{,}45 \times 1}$

$= \frac{0{,}45 \times 1 \times 3 \times 2\,100 \times 9{,}81}{0{,}45 \times 1}$

$= 61\,803$ Pa

Bending stress

Bending moment due to the wind pressure $= M = \frac{wL^2}{2} = \frac{w3^2}{2} = 4{,}5w$

Bending stress $= \frac{My}{I} = \frac{4{,}5w \times 0{,}225 \times 12}{1 \times 0{,}45^3}$

$= 133{,}332$w

NA in outer fibre $\therefore \sigma_B = \sigma_D$

$133{,}332w = 61\,803$

$\therefore$ Wind pressure $= w = 463{,}52$ Pa/m

MODULE

6 Shear stress in beams

By the end of this module, students should be able to:

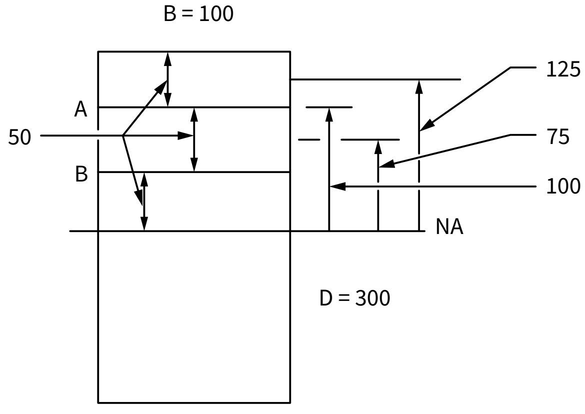
- explain the general formula for horizontal shear stress in a beam;
- calculate:
  - the maximum shear stress in a beam with a rectangular cross-section
  - the maximum shear stress in a beam with a circular cross-section
  - the shear force per unit length of a beam
  - the pitch of bolts required to fasten different layers of a beam
  - the shear stress in rectangular beams at various distances from the centroid
  - the shear stress in I-section beams at various distances from the centroid
  - the percentage of shear force carried by the web; and
- sketch the distribution of the shear stress across the beam section, showing the values at key points.

Introduction

This module focuses on the calculation of shear stresses in beams. Students will become familiar with problems related to horizontal shear stress in a beams at different levels. This module also focuses on the shear stress in beams when shear forces are applied.

Exercise 6.1 SB page 219

1.



1.1 Transverse shear stress at 50-mm intervals

$I_{xx} = \frac{BD^3}{12} = \frac{0,1 \times 0,3^3}{12} = 2,25 \times 10^{-4}\ \text{m}^4$

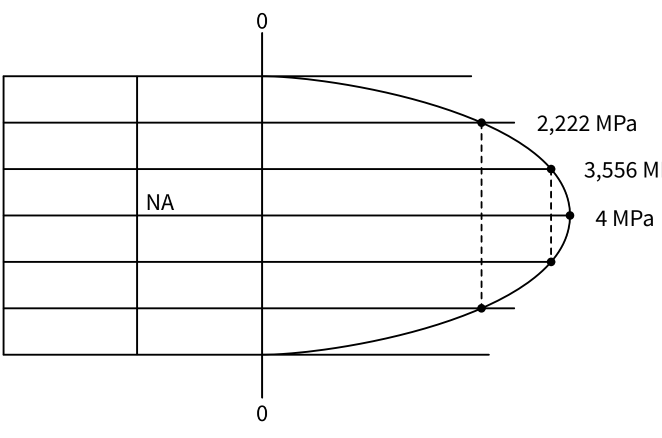
The stress at the top and bottom fibre is zero

Shear stress at A; $\tau_A = \frac{VQ}{IB} = \frac{80k \times (0,05 \times 0,1 \times 0,125)}{2,25 \times 10^{-4} \times 0,1} = 2,222$ MPa

Shear stress at B; $\tau_A = \frac{VQ}{IB} = \frac{80k \times (0,1 \times 0,1 \times 0,1)}{2,25 \times 10^{-4} \times 0,1} = 3,556$ MPa

Shear stress at NA; $\tau_A = \frac{VQ}{IB} = \frac{80k \times (0,1 \times 0,15 \times 0,0,75)}{2,25 \times 10^{-4} \times 0,1} = 4$ MPa

1.2 Shear stress diagram



1.3 Shear flow 50 mm from the top

Shear flow $= q = \frac{VQ}{I} = \frac{80k(0,1 \times 0,05 \times 0,125)}{2,25 \times 10^{-4}} = 222,22$ kN/m

2. 2.1 Maximum transverse shear stress

$$I_{xx} = \frac{\pi 0{,}08^4}{64} = 2{,}011 \times 10^{-6}\ \text{m}^4$$

Total load = 385 × 1 = 385 N

$$\text{Shear force} = V = \frac{385}{2} = 192{,}5\ \text{N}$$

$$\text{First area moment} = Q = \frac{2R^3}{3} = \frac{2 \times 0{,}04^3}{3} = 4{,}267 \times 10^{-5}\ \ \text{m}^3$$

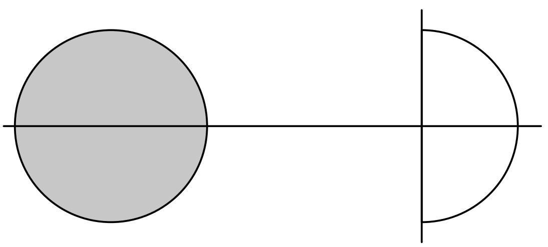
$$\text{Maximum shear stress} = \frac{VQ}{ID} = \frac{192{,}5 \times 4{,}267 \times 10^{-5}}{2{,}011 \times 10^{-6} \times 0{,}08} = 51{,}06\ \text{kPa}$$

$$\text{OR maximum shear stress} = \frac{16V}{3\pi D^3} = \frac{16 \times 192{,}5}{3\pi 0{,}08^3} = 51{,}06\ \text{kPa}$$

2.2 Average stress

$$\text{Avg}_\tau = \frac{V}{A} = \frac{192{,}5 \times 4}{\pi 0{,}08^2} = 38{,}297\ \text{kPa}$$

2.3 Stress diagram



3. 3.1 3.1.1 Shear stress in middle of flange

$$I_{xx} = \left(\frac{0{,}13 \times 0{,}22^3}{12}\right) - \left(\frac{0{,}11 \times 0{,}18^3}{12}\right) = 6{,}294 \times 10^{-5}\ \text{m}^4$$

$$\text{Middle flange;}\tau = \frac{VQ}{IB} = \frac{150k \times (0{,}01 \times 0{,}13 \times 0{,}105)}{6{,}294 \times 10^{-5} \times 0{,}13} = 2{,}502\ \text{MPa}$$

3.1.2 Shear stress in the bottom of flange

$$\text{Bottom flange;}\tau = \frac{VQ}{IB} = \frac{150k \times (0{,}02 \times 0{,}13 \times 0{,}1)}{6{,}294 \times 10^{-5} \times 0{,}13} = 4{,}766\ \text{MPa}$$

3.1.3 Shear stress at the top of the web

$$\text{Top of web; } \tau = \frac{VQ}{It} = \frac{150k \times (0{,}02 \times 0{,}13 \times 0{,}1)}{6{,}294 \times 10^{-5} \times 0{,}02} = 30{,}982\ \text{MPa}$$

3.1.4 Shear stress halfway between the NA and bottom of web

Halfway NA and web;

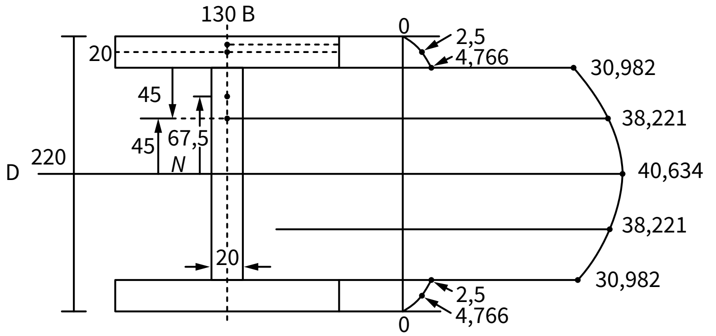
$$\tau = \frac{VQ}{It} = \frac{150k \times [(0{,}02 \times 0{,}13 \times 0{,}1) + (0{,}02 \times 0{,}045 \times 0{,}0675)]}{6{,}294 \times 10^{-5} \times 0{,}02} = 38{,}221\ \text{MPa}$$

3.1.5 At NA

$$\text{At NA; } \tau = \frac{VQ}{It} = \frac{150k \times [(0{,}02 \times 0{,}13 \times 0{,}1) + (0{,}02 \times 0{,}09 \times 0{,}045)]}{6{,}294 \times 10^{-5} \times 0{,}02}$$

$$= 40{,}634 \text{ MPa}$$

3.2 Draw shear stress diagram



3.3 The shear force carried by web

$$\tau_{avg} = \tau_{min} + \frac{2}{3}[\tau_{max} - \tau_{min}]$$

$$\tau_{AVG} = 30{,}982\text{M} + \frac{2}{3}[40{,}634\text{M} - 30{,}982\text{ M}] = 37{,}417 \text{ MPa}$$

$$V_{web} = \tau_{avg} A_{web} = 37{,}417\text{M} \times 0{,}18 \times 0{,}02 = 134{,}701 \text{ kN}$$

3.4 Percentage of shear force carried by web

$$\%\text{V web} = \frac{V_{web}}{V} \times 100 = \frac{134{,}701k}{150k} \times 100 = 89{,}9\%$$

4. 4.1 Average shear stress in the beam

$$I_{xx} = \left(\frac{0{,}14 \times 0{,}26^3}{12}\right) - \left(\frac{0{,}12 \times 0{,}22^3}{12}\right) = 9{,}86 \times 10^{-5} \text{ m}^4$$

Minimum stress at the top of web;

$$\tau_{min} = \frac{VQ}{It} = \frac{12k(0{,}14 \times 0{,}02 \times 0{,}12)}{9{,}86 \times 10^{-5} \times 0{,}02} = 20{,}446 \text{ MPa}$$

4.2 Shear force carried by the web

Maximum stress at the NA;

$$\tau_{max} = \frac{VQ}{It} = \frac{12k(0{,}14 \times 0{,}02 \times 0{,}12)}{9{,}86 \times 10^{-5} \times 0{,}02}$$

$$= \frac{12k[(0{,}14 \times 0{,}02 \times 0{,}12) + (0{,}11 \times 0{,}02 \times 0{,}055)]}{9{,}86 \times 10^{-5} \times 0{,}02} = 27{,}81 \text{ MPa}$$

$$\tau_{AVG} = \tau_{min} + \frac{2}{3}[\tau_{max} - \tau_{min}]$$

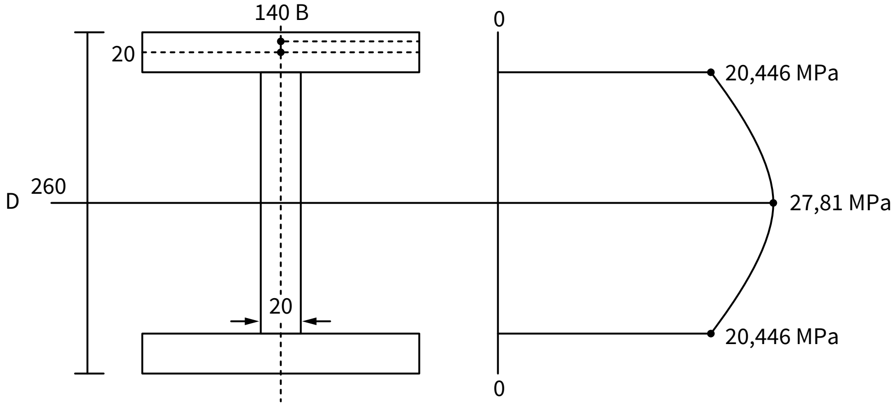
$$\tau_{AVG} = 20{,}446M + \frac{2}{3}[27{,}81M - 20{,}446\ M] = 25{,}355\ \text{MPa}$$

$$V_{web} = \tau_{avg}A_{web} = 25{,}355M \times 0{,}22 \times 0{,}02 = 111{,}562\ \text{kN}$$

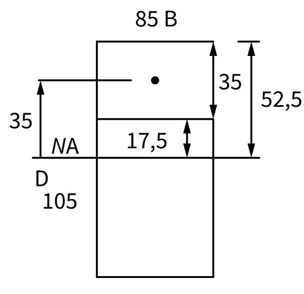
4.3 Percentage carried by web

$$\%V\ \text{web} = \frac{V_{web}}{V} \times 100 = \frac{111{,}562k}{120k} \times 100 = 96{,}35\%$$

4.4 Stress diagram (web)



5.1 Shear force in the beam



$$I_{xx} = \frac{0{,}85 \times 0{,}105^3}{12} = 8{,}2 \times 10^{-6}$$

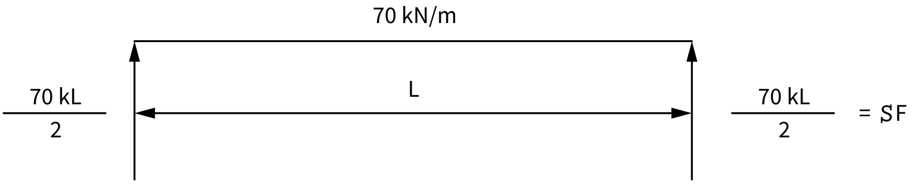
$$\tau = \frac{VQ}{IB}$$

$$\therefore V = \frac{\tau IB}{Q} = \frac{1M \times 8{,}2 \times 10^{-6} \times 0{,}085}{0{,}085 \times 0{,}035 \times 0{,}035} = 6{,}7\ \text{kN}$$

5.2 Maximum transverse shear stress

$$\text{Max shear stress} = \tau = \frac{VQ}{IB} = \frac{6{,}7k[0{,}85 \times 0{,}0525 \times 0{,}02625]}{8{,}2 \times 10^{-6} \times 0{,}085} = 1{,}13\ \text{MPa}$$

6. Consider shear stress:



$\tau = \frac{VQ}{IB}$

$\therefore$ Shear force at a support $= V = \frac{\tau IB}{Q} = \frac{30M \times 0{,}06 \times 0{,}12^3 \times 0{,}06}{12(0{,}06 \times 0{,}06 \times 0{,}03)} = 144$ kN

Shear force at a support $= \frac{wL}{2} = V$

$\frac{wL}{2} = V = 144k = \frac{70kL}{2}$

L = 4,114M

Consider bending stress:

$M = \frac{\sigma I}{y} = \frac{120M \times 0{,}06 \times 0{,}12^3}{0{,}06 \times 12} = 17\ 280$ Nm

$M = \frac{wL^2}{8} = 17\ 280 = \frac{70k \times L^2}{8}$

$L = \sqrt{\frac{8 \times 17\ 280}{70k}} = 1{,}405$ m

Use a beam with length 1,405 m

7. Consider shear stress:

$\tau = \frac{VQ}{ID}$

$\therefore$ Shear force at a support $= V = \frac{\tau ID}{Q} = \frac{5M \times \pi \times 0{,}15^4 \times 0{,}15}{64\left(\frac{2}{3}0{,}075^3\right)} = 66{,}268$ kN

$\frac{wL}{2} = V = 66{,}268k = \frac{32kL}{2}$

L = 4,142M

Consider bending stress

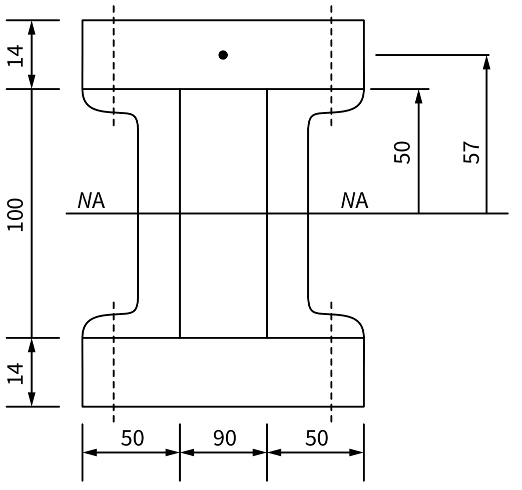
$M = \frac{\sigma I}{y} = \frac{125M \times \pi \times 0{,}15^4}{64 \times 0{,}075} = 41\ 417$ Nm

$M = \frac{wL^2}{8} = 41\ 417 = \frac{32k \times L^2}{8}$

$L = \sqrt{\frac{8 \times 41\ 417}{32k}} = 3{,}218$ m

Use a beam length of 3,218 m.

8.



$$I_{xx} = 2(2{,}053 \times 10^{-6}) + 2\left[\frac{0{,}19 \times 0{,}014^3}{12} + (0{,}19 \times 0{,}014 \times 0{,}057^2)\right]$$

$$= 2{,}1478 \times 10^{-5} m^4$$

First moment area = $Q = 0{,}19 \times 0{,}014 \times 0{,}057 = 1{,}516 \times 10^{-4}\ m^3$

Shear flow = $q = \frac{VQ}{I} = \frac{50k \times 1{,}516 \times 10^{-4}}{2{,}1478 \times 10^{-5}} = 35{,}297$ kNm

Spacing = $S = \frac{Rn}{q} = \frac{\pi \times 0{,}015^2 \times 200M \times 2}{4 \times 2{,}5 \times 35\ 297} = 80{,}11$ mm

9 9.1 Spacing

$$I_{yy} = 2\left[1{,}135 \times 10^{-6} + (2{,}797 \times 10^{-3} \times 0{,}0192^2)\right] = 4{,}332 \times 10^{-6} m^4$$

$$Q = 2{,}797 \times 10^{-3} \times 0{,}0192 = 5{,}37 \times 10^{-5}\ m^3$$

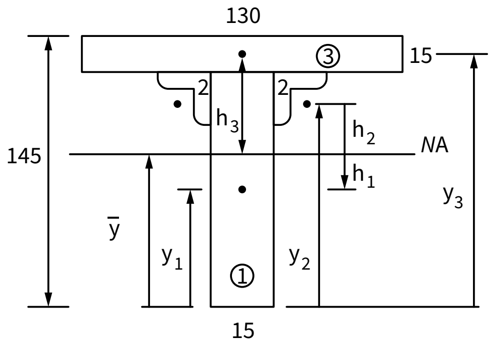
$$q = \frac{VQ}{I} = \frac{80k \times 5{,}37 \times 10^{-5}}{4{,}332 \times 10^{-6}} = 991{,}69 \text{ kNm}$$

$$S = \frac{Rn}{q} = \frac{\pi 0{,}03^2 \times 69M \times 2}{4 \times 991\ 690} = 98{,}36 \text{ mm}$$

9.2 Shear stress

$$\tau = \frac{VQ}{Ih} = \frac{80k \times 5{,}37 \times 10^{-5}}{4{,}332 \times 10^{-6} \times 0{,}18} = 5{,}51 \text{ MPa}$$

10. 10.1 Spacing of bolts A



Centroid:

No.	Area	y-distance	Ay
1	$0{,}015 \times 0{,}13 = 1{,}95 \times 10^{-3}$	0,075	$1{,}4625 \times 10^{-4}$
2	$2(0{,}7413 \times 10^{-3}) = 1{,}4826 \times 10^{-3}$	0,1148	$1{,}702 \times 10^{-4}$
3	$0{,}015 \times 0{,}13 = 1{,}95 \times 10^{-3}$	0,1375	$2{,}6813 \times 10^{-4}$
Total A	$5{,}3826 \times 10^{-3}$	ΣArea moment	$5{,}8458 \times 10^{-4}$

$$\bar{y} = \frac{5{,}8458 \times 10^{-4}}{5{,}3826 \times 10^{-3}} = 108{,}61 \text{ mm}$$

$$h_1 = \bar{y} - y_1 = 108{,}61 - 75 = 33{,}61 \text{ mm}$$

$$h_2 = y_2 - \bar{y} = 114{,}8 - 108{,}61 = 6{,}19 \text{ mm}$$

$$h_3 = y_3 - \bar{y} = 137{,}5 - 108{,}61 = 28{,}89 \text{ mm}$$

$$I_{xx} = I_1 + 2I_2 + I_3$$

$$I_1 = \frac{0{,}015 \times 0{,}13^3}{12} + \left(1{,}95 \times 10^{-3} \times 0{,}03361^2\right) = 4{,}9491 \times 10^{-6}$$

$$I_2 = 2\left[0{,}1628 \times 10^{-6} + \left(0{,}7413 \times 10^{-3} \times 0{,}00619^2\right)\right] = 3{,}8241 \times 10^{-7}$$

$$I_3 = \frac{0{,}13 \times 0{,}015^3}{12} + \left(1{,}95 \times 10^{-3} \times 0{,}02889^2\right) = 1{,}6641 \times 10^{-6}$$

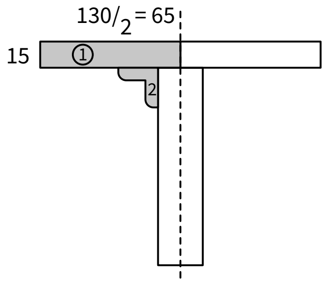
$$I_{xx} = 6{,}9956 \times 10^{-6} \text{ m}^4$$

$$Q_A = 0{,}13 \times 0{,}015 \times 0{,}02889 = 5{,}63355 \times 10^{-5} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{16k \times 5{,}63355 \times 10^{-5}}{6{,}9956 \times 10^{-6}} = 128{,}848 \text{ kNm}$$

$$S = \frac{Rn}{q} = \frac{\pi \times 0{,}012^2 \times 60M \times 2}{128\,848} = 105{,}33 \text{ mm}$$

10.2 Spacing of bolts B



$Q_3 = 0{,}065 \times 0{,}015 \times 0{,}02889 = 2{,}8168 \times 10^{-5}$

$Q_2 = 0{,}7413 \times 10^{-3} \times 0{,}00619 = 4{,}5886 \times 10^{-6}$

$Q_{\text{T for B}} = 3{,}27566 \times 10^{-5}\ \text{m}^3$

$q = \dfrac{VQ}{I} = \dfrac{16k \times 3{,}27566 \times 10^{-5}}{6{,}9956 \times 10^{-6}} = 74{,}919\ \text{kNm}$

$S = \dfrac{Rn}{q} = \dfrac{\pi \times 0{,}012^2 \times 60M \times 1}{74{,}919k} = 90{,}58\ \text{mm}$

11. $I_{yy} = 2\left[4{,}931 \times 10^{-6} + \left(5{,}876 \times 10^{-3} \times 0{,}027^2\right)\right] = 1{,}8429 \times 10^{-5}$

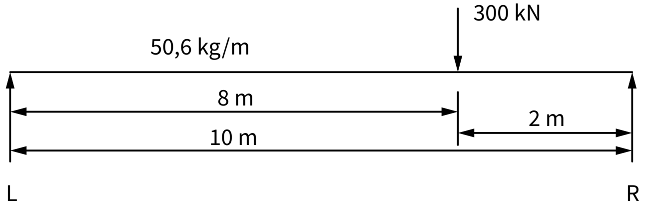
$Q = 5{,}876 \times 10^{-3} \times 0{,}027 = 1{,}58652 \times 10^{-4}$

$Rn = \frac{\pi}{4} 0{,}02^2 \times 100M \times 2 = 62{,}832\ \text{kN}$

$S = \dfrac{Rn}{q} = \dfrac{RnI}{VQ}$

$\therefore V = \dfrac{RnI}{SQ} = \dfrac{62{,}832k \times 1{,}8429 \times 10^{-5}}{0{,}14 \times 1{,}58652 \times 10^{-4}} = 52{,}133\ \text{kN}$

12. 12.1 Maximum shear force



Moments about L; $10R = 300k \times 8 + (50{,}6 \times 9{,}81 \times 10 \times 5)$

$R = 242{,}482\ \text{kN}$

Moments about R; $10L = 300k \times 2 + (50{,}6 \times 9{,}81 \times 10 \times 5)$

$L = 62{,}482\ \text{kN}$

Maximum shear force $= 242{,}482\ \text{kN} = V$

12.2 Bolt diameter

$$I_{xx} = 2\left[22{,}97 \times 10^{-6} + \left(3{,}226 \times 10^{-3} \times 0{,}1018^2\right)\right] = 1{,}128 \times 10^{-4}$$

$$Q = 3{,}226 \times 10^{-3} \times 0{,}1018 = 3{,}284 \times 10^{-4}$$

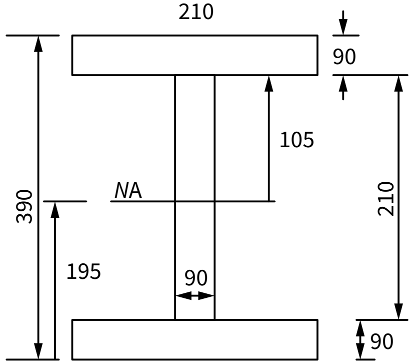
$$S = \frac{Rn}{q} = \frac{RnI}{VQ}$$

$$Rn = \frac{SVQ}{I} = \frac{0{,}15 \times 242\ 482 \times 3{,}284 \times 10^{-4}}{1{,}128 \times 10^{-4}} = 105{,}892 \text{ kN}$$

$$Rn = \frac{\pi}{4}d^2 \times 80M \times 2 = 105\ 892$$

$$d = \sqrt{8{,}42662 \times 10^{-4}} = 29{,}03 \text{ mm}$$

13. 13.1 Shear force



$$I_{xx} = \left(\frac{0{,}21 \times 0{,}39^3}{12}\right) - \left(\frac{0{,}12 \times 0{,}21^3}{12}\right) = 9{,}4549 \times 10^{-4}$$

$$Q_{flange} = 0{,}21 \times 0{,}09 \times 0{,}15 = 2{,}835 \times 10^{-3}$$

$$Q_{web} = 0{,}09 \times 0{,}105 \times 0{,}0525 = 4{,}9623 \times 10^{-4}$$

$$Q_{total} = 3{,}33113 \times 10^{-3}$$

$$\tau = \frac{VQ}{It}$$

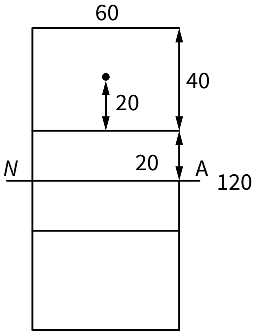
$$V = \frac{\tau It}{Q} = \frac{2M \times 9{,}4549 \times 10^{-4} \times 0{,}09}{3{,}33113 \times 10^{-3}} = 51{,}09 \text{ kN}$$

13.2 Spacing of bolts

$$q = \frac{VQ}{I} = \frac{51{,}09k \times 3{,}33113 \times 10^{-3}}{9{,}4549 \times 10^{-4}} = 179{,}999 \text{ kNm}$$

$$S = \frac{Rn}{q} = \frac{9k}{179\ 999} = 50 \text{ mm}$$

14.1 Magnitude of the point load at mid-point



$$I_{xx} = \frac{0{,}06 \times 0{,}12^3}{12} = 8{,}64 \times 10^{-6}$$

$$Q = 0{,}06 \times 0{,}04 \times 0{,}04 = 9{,}6 \times 10^{-5}$$

$$\tau = \frac{VQ}{It}$$

$$V = \frac{\tau It}{Q} = \frac{10M \times 8{,}64 \times 10^{-6} \times 0{,}06}{9{,}6 \times 10^{-5}} = 54 \text{ kN}$$

14.2 Bolt diameter

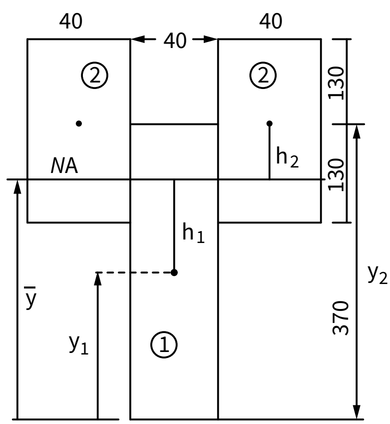
$$q = \frac{VQ}{I} = \frac{54k \times 9{,}6 \times 10^{-5}}{8{,}64 \times 10^{-6}} = 600 \text{ kN}$$

$$S = \frac{Rn}{q} \qquad \therefore Rn = Sq = 0{,}12 \times 600k = 72\ 000 \text{ N}$$

$$Rn = \frac{\pi}{4}d^2 \times 40M \times 1 = 72\ 000$$

$$d = \sqrt{2{,}292 \times 10^{-3}} = 47{,}87 \text{ mm}$$

15. 15.1 Shear stress in the bolt



No.	Area	y-distance	Ay
1	$0{,}04 \times 0{,}37 = 0{,}0148$	0,185	$2{,}738 \times 10^{-3}$
2	$2(,04 \times 0{,}26) = 0{,}0208$	0,370	$7{,}696 \times 10^{-3}$
A-total	0,0356	ΣA-mom	$0{,}010434\bar{y} = \frac{0{,}010434}{0{,}0356}$ $= 293{,}09$ mm
Total A	$5{,}3826 \times 10^{-3}$	ΣArea moment	$5{,}8458 \times 10^{-4}$

$h_1 = \bar{y} - y_1 = 293{,}09 - 185 = 108{,}09$ mm

$h_2 = y_2 - \bar{y} = 370 - 293{,}09 = 76{,}91$ mm

$I_{xx} = I_1 + I_2$

$$I_1 = \frac{0{,}04 \times 0{,}37^3}{12} + (0{,}0148 \times 0{,}10809^2) = 3{,}4176 \times 10^{-4}$$

$$I_2 = 2\left[\frac{0{,}04 \times 0{,}26^3}{12} + (0{,}0104 \times 0{,}07691^2)\right] = 2{,}5267 \times 10^{-4}$$

$I_{xx} = 5{,}9443 \times 10^{-4}\ \text{m}^4$

Consider only one side

$Q = 0{,}04 \times 0{,}26 \times 0{,}07691 = 7{,}99864 \times 10^{-4}\ \text{m}^3$

$$q = \frac{VQ}{I} = \frac{8k \times 7{,}99864 \times 10^{-4}}{5{,}9443 \times 10^{-4}} = 10{,}765 \text{ kNm}$$

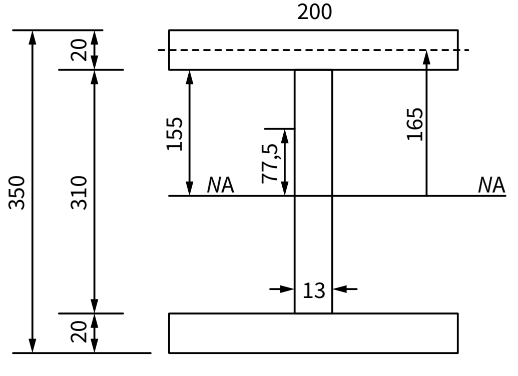
$$S = \frac{Rn}{q} \quad \therefore Rn = Sq = 0{,}35 \times 10\ 765 = 3{,}768 \text{ kN}$$

$$\tau_{avg} = \frac{Rn}{A_{bolt}} = \frac{3\ 768 \times 4}{\pi \times 0{,}012^2} = 33{,}316 \text{ MPa}$$

15.2 Transverse shear stress between the side beam and the web

$$\tau = \frac{VQ}{It} = \frac{q}{t} = \frac{10\ 765}{0{,}13} = 82{,}807 \text{ kPa}$$

16.1 Maximum shear stress



$$I_{xx} = \left(\frac{0{,}2 \times 0{,}35^3}{12}\right) - \left(\frac{0{,}187 \times 0{,}31^3}{12}\right) = 2{,}5034 \times 10^{-4}$$

$$Q_{total} = (0{,}155 \times 0{,}013 \times 0{,}0775) + (0{,}2 \times 0{,}02 \times 0{,}165) = 8{,}161625 \times 10^{-4}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{160k \times 8{,}161625 \times 10^{-4}}{2{,}5034 \times 10^{-4} \times 0{,}013} = 40{,}126 \text{ MPa}$$

16.2 Average (also referred to as the mean) shear stress in the cross-section (CS) of the beam

$$\tau_{avg\ CS} = \frac{V}{csa} = \frac{160k}{(0{,}35 \times 0{,}2) - (0{,}31 \times 0{,}187)} = 13{,}3 \text{ MPa}$$

16.3 Ratio between the maximum shear stress and the average cross-sectional shear

Stress:

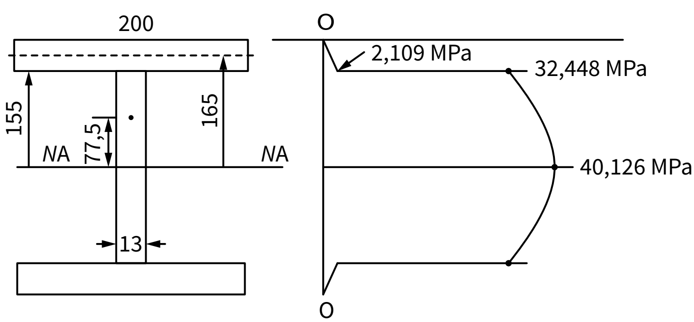
$$\text{Ratio} = \frac{\tau_{max}}{\tau_{Avg\ CS}} = \frac{40{,}126k}{13{,}3k} = 3{,}02$$

16.4 Draw the shear distribution

Shear stress at the bottom of the web

$$\tau_{flange\ bottom} = \frac{VQ}{IB} = \frac{160k \times (0{,}2 \times 0{,}02 \times 0{,}165)}{2{,}5034 \times 10^{-4} \times 0{,}2} = 2{,}109 \text{ MPa}$$

$$\tau_{top\ of\ web} = \frac{VQ}{It} = \frac{160k \times (0{,}2 \times 0{,}02 \times 0{,}165)}{2{,}5034 \times 10^{-4} \times 0{,}013} = 32{,}448 \text{ MPa}$$



16.5 Shear force taken by flanges

Shear force for one flange = $V_{flange} = \tau_{flange\ average} \times A_{flange}$

$$V_{flange} = \frac{2{,}109 \text{ M}}{2} \times (0{,}2 \times 0{,}02) = 4{,}218 \text{ kN}$$

Total shear force in both flanges = $2 \times 4{,}218k = 8{,}436$ kN

16.6 Shear force in the web

$$V_{web} = V_{total} - V_{flanges} = 160k - 8{,}436k = 151{,}564 \text{ kN}$$

16.7 Percentage shear force carried by the web

$$\% = \frac{151{,}564}{160} \times 100 = 94{,}37\%$$

MODULE

7 Closed-coiled helical springs

By the end of this module, students should be able to:

- explain the assumptions made in the design of closed-coiled helical springs;
- calculate:
  - the maximum shear stress in the spring
  - the deflection of the spring
  - the strain energy stored in the spring
  - the stiffness of the spring
  - the wire diameter
  - the mean coil diameter
  - the number of coils
  - the work done on the spring; and
- apply the same calculations of single springs to:
  - springs joined in series
  - springs joined in parallel.

Introduction

This module will familiarise students with single closed-coiled helical springs and compound springs. Some assumptions that can be made about this module are as follows.

The simple torque equation used for shafts in Module 3 is applicable in this module. It must be understood that the equations used in this module will be approximate, because the torque equation was developed for straight shafts. These equations can only be used for springs manufactured from round wire.

We assume that these springs are closed coils so that the plane of each coil of the spring is perpendicular to the line of action of the applied load. We also assume that the spring is subjected to pure torsion and that the effect of bending and shear force is neglected in the wire.

Exercise 7.1 SB page 243

1. 1.1 Maximum shear stress in the wire

$$\tau_{max} = \frac{8WD}{\pi d^3} = \frac{8 \times 2k \times 0,18}{\pi \times 0,02^3} = 114,591 \text{ MPa}$$

1.2 Maximum deflection

$$\delta = \frac{8WD^3 n}{Gd^4} = \frac{8 \times 2k \times 0,18^3 \times 18}{80 \times 10^9 \times 0,02^4} = 131,22 \text{ mm}$$

2. 2.1 Number of springs

$$\delta = \frac{8WD^3 n}{Gd^4}$$

$$W = \frac{\delta G d^4}{8D^3 n} = \frac{0,25 \times 83 \times 10^9 \times 0,03^4}{8 \times 0,5^3 \times 20} = 840,375 \text{ N}$$

U/spring = U = 0,5Wδ = 0,5 × 840,375 × 0,25 = 105,047 J

$$\text{Number of springs} = \frac{U_{total}}{U_{per\ spring}} = \frac{1,529 \times 10^6}{105,047} = 14,555$$

15 springs are required.

2.2 Shear stress in springs

$$\tau_{max} = \frac{8WD}{\pi d^3} = \frac{8 \times 840,375 \times 0,5}{\pi 0,03^3} = 39,63 \text{ MPa}$$

2.3 Stiffness of each spring

$$S = \frac{W}{\delta} = \frac{840,375}{0,25} = 3,362 \text{ kNM}$$

$$\text{Or } S = \frac{Gd^4}{8D^3} = 3,362 \text{ kNM}$$

3. 3.1 Shear stress coil wire

Torque = 0,5WD

$$W = \frac{T}{0,5D} = \frac{1,4}{0,5 \times 0,11} = 25,455 \text{ N}$$

$$\tau_{max} = \frac{8WD}{\pi d^3} = \frac{8 \times 25,455 \times 0,11}{\pi \times 0,12^3} = 4,126 \text{ MPa}$$

3.2 Angle of twist

$$\theta = \frac{16WD^2 n}{Gd^4} = \frac{16 \times 25,455 \times 0,11^2 \times 16}{82 \times 10^9 \times 0,012^4} = 0,0464 \text{ rad}$$

$$\text{Radians to degrees} = \frac{0,0464 \times 180}{\pi} = 2,659°$$

4. 4.1 Stiffness of springs

$$S = \frac{Gd^4}{8D^3} = \frac{80 \times 10^9 \times 0,008^4}{8 \times 0,052^3 \times 20} = 14,565 \text{ kNM}$$

4.2 Strain energy

$$U = \frac{16T^2Dn}{Gd^4} = \frac{16 \times 2^2 \times 0{,}052 \times 20}{80 \times 10^9 \times 0{,}008^4} = 0{,}203 \text{ J}$$

4.3 Shear stress in coils

Torque = 0,5WD

$$W = \frac{2}{0{,}5 \times 0{,}052} = 7{,}692 \text{ N}$$

$$\tau_{max} = \frac{8WD}{\pi d^3} = \frac{8 \times 7{,}692 \times 0{,}052}{\pi \times 0{,}008^3} = 1{,}989 \text{ MPa}$$

4.4 Deflection

$$\delta = \frac{8WD^3n}{Gd^4} = \frac{8 \times 7{,}692 \times 0{,}052^3 \times 20}{80 \times 10^9 \times 0{,}008^4} = 0{,}528 \text{ mm}$$

5. 5.1 Mean diameter of coil

$$\text{Number of coils} = n = \frac{L_{solid}}{d} = \frac{80}{8} = 10 \text{ coils}$$

$$\delta = \frac{8WD^3n}{Gd^4}$$

$$D^3 = \frac{82 \times 10^9 \times 0{,}008^4}{8 \times 8k \times 10}$$

$$D = \sqrt[3]{5{,}248 \times 10^{-4}} = 80{,}66 \text{ mm}$$

5.2 Load on spring

$$\delta = \frac{8WD^3n}{Gd^4}$$

$$W = \frac{\delta G d^4}{8D^3n} = \frac{0{,}02 \times 82 \times 10^9 \times 0.008^4}{8 \times 0{,}08^3 \times 10} = 160 \text{ N}$$

5.3 Shear stress in spring

$$\tau_{max} = \frac{8WD}{\pi d^3} = \frac{8 \times 160 \times 0{,}08066}{\pi \times 0{,}008^3} = 64{,}187 \text{ MPa}$$

5.4 Work done

$$U = 0{,}5W\delta = 0{,}5 \times 160 \times 0{,}02 = 1{,}6 \text{ J}$$

6. 6.1 Wire diameter

$$C = \frac{D}{d} \quad \therefore D = 8d$$

$$S = \frac{Gd^4}{8D^3n}$$

$$10k = \frac{81 \times 10^9 \times d^4}{8 \times (8d)^3 \times 10}$$

$d = 5{,}06$ mm

$D = 8 \times 5{,}06 = 40{,}48$ mm

6.2 Load on spring

$$\tau_{max} = \frac{8WD}{\pi d^3}$$

$$W = \frac{400 \times 10^6 \times \pi \times 0{,}00506^3}{8 \times 0{,}04048} = 502{,}726\ \text{N}$$

6.3 Deflection

$$\delta = \frac{8WD^3 n}{Gd^4} = \frac{8 \times 502{,}726 \times (0{,}04048)^3 \times 10}{81 \times 10^9 \times 0{,}00506^4} = 50{,}24\ \text{mm}$$

6.4 Strain energy

$$U = 0{,}5W\delta = 0{,}5 \times 502{,}726 \times 0{,}05024 = 12{,}628\ \text{J}$$

6.5 Volume of wire needed

$$U = \frac{\tau^2}{4G} \times \text{Vol}$$

$$\text{Volume wire} = \frac{U4G}{\tau^2} = \frac{12{,}628 \times 4 \times 81 \times 10^9}{(400\ \text{M})^2} = 2{,}557 \times 10^{-5}\ \text{m}^3$$

6.6 Length of wire

$$l = \pi Dn = \pi \times 40{,}48 \times 10 = 1{,}272\ \text{m}$$

7. 7.1 Wire diameter and mean diameter of coils

$$U = 0{,}5W\delta$$

$$W = \frac{1{,}2k}{0{,}5 \times 0{,}096} = 25\ \text{kN}$$

$$C = \frac{D}{d} \quad \therefore D = 6d$$

$$\tau_{max} = \frac{8WD}{\pi d^3}$$

$$300\text{M} = \frac{8 \times 25k \times 6d}{\pi \times d^3}$$

$$d = \sqrt{\frac{8 \times 25k \times 6}{\pi \times 300\ \text{M}}} = 35{,}682\ \text{mm}$$

$$D = 6 \times 35{,}682 = 214{,}095\ \text{mm}$$

7.2 Number of coils

$$U = \frac{4W^2D^3n}{Gd^4}$$

$$n = \frac{1{,}2k \times 79 \times 10^9 \times 0{,}035682^4}{4 \times (25\text{k})^2 \times 0{,}214095^3} = 6{,}264\ \text{coils}$$

7.3 Stiffness

$$S = \frac{Gd^4}{8D^3n} = \frac{79 \times 10^9 \times 0{,}035682^4}{8 \times 0{,}214095^3 \times 6{,}264} = 260{,}413\ \text{kNm}$$

8. 8.1 Total stiffness of springs

$$S_1 = \frac{Gd^4}{8D^3n} = \frac{80 \times 10^9 \times 0{,}007^4}{8 \times 0{,}032^3 \times 13} = 56{,}364 \text{ kNm}$$

$$S_2 = \frac{Gd^4}{8D^3n} = \frac{80 \times 10^9 \times 0{,}009^4}{8 \times 0{,}042^3 \times 17} = 52{,}092 \text{ kNm}$$

$$S_T = \frac{S_1S_2}{S_1 + S_2} = \frac{56\,364 \times 52\,092}{56\,464 + 52\,092} = 27{,}071 \text{ kNm}$$

8.2 Shear stress in each spring

$$\tau_{max} = \frac{8WD}{\pi d^3} = \frac{8 \times 1{,}5k \times 0{,}032}{\pi 0{,}007^3} = 356{,}359 \text{ MPa}$$

$$\tau_{max} = \frac{8WD}{\pi d^3} = \frac{8 \times 1{,}5k \times 0{,}042}{\pi 0{,}009^3} = 220{,}066 \text{ MPa}$$

9. 9.1 Stress in each spring

$$\tau_{maxS} = \frac{8WD}{\pi d^3} = \frac{8 \times 130 \times 0{,}04}{\pi 0{,}006^3} = 61{,}304 \text{ MPa}$$

$$\tau_{maxB} = \frac{8WD}{\pi d^3} = \frac{8 \times 130 \times 0{,}04}{\pi 0{,}008^3} = 25{,}863 \text{ MPa}$$

9.2 Number of coils bronze

$$\delta_s = \frac{8WD^3n}{Gd^4} = \frac{8 \times 130 \times 0{,}04^3 \times 12}{82 \times 10^9 \times 0{,}006^4} = 7{,}516 \text{ mm}$$

$$\delta_B = \delta_T - \delta_S = 10 - 7{,}516 = 2{,}484 \text{ mm}$$

$$U_{bronze} = 0{,}5W\delta_B = 0{,}5 \times 130 \times 0{,}002484 = 0{,}16146 \text{ J}$$

$$U_B = \frac{\tau_B^2}{4G_B}\pi Dn \times \frac{\pi d^2}{4}$$

$$0{,}16146 = \frac{(25{,}864 \text{ M})^2}{4 \times 42 \times 10^9}\pi \times 0{,}04n \times \frac{\pi 0{,}008^2}{4}$$

$n = 6{,}42$ coils

9.3 Stiffness of springs

$$S_S = \frac{W}{\delta_s} = \frac{130}{0{,}007516} = 17{,}269 \text{ kNm}$$

$$S_B = \frac{W}{\delta_B} = \frac{130}{0{,}002484} = 52{,}335 \text{ kNm}$$

$$S_T = \frac{S_sS_B}{S_S + S_B} = \frac{17\,269 \times 52\,335}{17\,269 + 52\,335} = 12{,}984 \text{ kNm}$$

10. 10.1 Allowable load for series spring

$$W_s = \frac{\tau\pi d^3}{8D} = \frac{145M \times \pi \times 0{,}004^3}{8 \times 0{,}025} = 145{,}77 \text{ N}$$

$$W_{SS} = \frac{\tau\pi d^3}{8D} = \frac{152M \times \pi \times 0{,}003^3}{8 \times 0{,}03} = 53{,}72 \text{ N}$$

Max load on spring = 53,72 N

10.2 Total work done

$$U_s = \frac{4W^2D^3n}{Gd^4} = \frac{4 \times 53{,}72^2 \times 0{,}025^3 \times 14}{79 \times 10^9 \times 0{,}004^4} = 0{,}125 \text{ J}$$

$$U_s = \frac{4W^2D^3n}{Gd^4} = \frac{4 \times 53{,}72^2 \times 0{,}03^3 \times 11}{73 \times 10^9 \times 0{,}003^4} = 0{,}58 \text{ J}$$

$$U_T = U_s + U_{ss} = 0{,}125 + 0{,}58 = 0{,}705 \text{ J}$$

10.3 Total deflection

$$\delta_s = \frac{8WD^3n}{Gd^4} = \frac{8 \times 53{,}72 \times (0{,}025)^3 \times 14}{79 \times 10^9 \times 0{,}004^4} = 4{,}648 \text{ mm}$$

$$\delta_{ss} = \frac{8WD^3n}{Gd^4} = \frac{8 \times 53{,}72 \times (0{,}03)^3 \times 11}{73 \times 10^9 \times 0{,}003^4} = 21{,}586 \text{ mm}$$

$$\delta_T = 26{,}234 \text{ mm}$$

10.4 Stiffness of spring

$$S_S = \frac{Gd^4}{8D^3n} = \frac{79 \times 10^9 \times 0{,}004^4}{8 \times 0{,}025^3 \times 14} = 11{,}557$$

$$S_{SS} = \frac{Gd^4}{8D^3n} = \frac{73 \times 10^9 \times 0{,}003^4}{8 \times 0{,}03^3 \times 11} = 2{,}489$$

$$S_T = \frac{S_sS_{ss}}{S_S + S_{ss}} = \frac{11\ 557 \times 2\ 489}{11\ 557 + 2\ 489} = 2{,}048 \text{ kNm}$$

11. 11.1 Number of coils outer spring

$$\delta_{outer} = \delta_{inner}$$

$$\left(\frac{8WD^3n}{Gd^4}\right)_{outer} = \left(\frac{8WD^3n}{Gd^4}\right)_{inner}$$

$$\left(\frac{W_O 0{,}1^3 n}{0{,}01^4}\right)_{Outer} = \left(\frac{W_I 0{,}08^3 \times 13}{0{,}008^4}\right)_{inner}$$

$$100\ 000W_On = 1\ 625\ 000W_I$$

$$n_{outer} = \frac{16{,}25W_I}{W_O} \ldots ①$$

But $\tau_{outer} = \tau_{inner}$

$$\left(\frac{8WD}{\pi d^3}\right)_{outer} = \left(\frac{8WD}{\pi d^3}\right)_{inner}$$

$$\frac{W_O \times 0{,}1}{0{,}01^3} = \frac{W_I \times 0{,}08}{0{,}008^3}$$

$$\therefore W_O = 1{,}563W_I \ldots ②$$

Substitute ② into ①: $n_{outer} = \frac{16{,}25}{1{,}563} = 10{,}4$ coils

11.2 Load on spring

$U_T = U_O + U_I$

$100 = \left(\frac{4W_O^2D^3n}{Gd^4}\right)_{outer} + \left(\frac{4W_I^2D^3n}{Gd^4}\right)_{inner}$

$100 = \left(\frac{4W_O^2 0{,}1^3 \times 10{,}4}{82 \times 10^9 \times 0{,}01^4}\right)_{outer} + \left(\frac{4W_I^2 0{,}08^3 \times 13}{82 \times 10^9 \times 0{,}008^4}\right)_{inner}$

$100 = 5{,}073 \times 10^{-5}W_O^2 + 7{,}927 \times 10^{-5}W_I^2 \ldots$ ③

Substitute ② into ③: $\therefore 100 = 5{,}073 \times 10^{-5}(1{,}563W_I)^2 + 7{,}927 \times 10^{-5}W_I^2$

$100 = 2{,}032 \times 10^{-4}W_I^2$

$\therefore$ Load on inner $= W_I = 701{,}517$ N

Load on outer $= W_O = 1\,096{,}471$ N

Total load on spring = 1,798 kN

11.3 Stress in springs

The stress in the springs is equal.

$\tau_O = \frac{8WD}{\pi d^3} = \frac{8 \times 1\,096{,}471 \times 0{,}1}{\pi \times 0{,}01^3} = 279{,}214$ MPa

11.4 Total deflection

$\delta_O = \delta_I$

But $U_T = 0{,}5W_T\delta_T$

$\delta_T = \frac{100}{0{,}5 \times 1\,798} = 111{,}235$ mm

12. 12.1 Load in each spring

$\delta_{inner} = 15 - 6 = 9$ mm

$\delta_{Inner} = \frac{8WD^3n}{Gd^4}$

$0{,}009 = \frac{8 \times W \times 0{,}035^3 \times 12}{80 \times 10^9 \times 0{,}004^4}$

Load on inner coil $= W_I = 44{,}78$ N

Load on outer spring 115 – 44,78 = 70,22 N

12.2 Total stiffness

$S_{inner} = \frac{W}{\delta_{inner}} = \frac{44{,}78}{0{,}009} = 4{,}976$ kNm

$S_{outer} = \frac{W}{\delta_{outer}} = \frac{70{,}22}{0{,}015} = 4{,}681$ kNm

Total stiffness = 9,757 kNm

12.3 Stiffness outer spring before connecting inner spring

$$S_{outer} = \frac{W}{\delta_{outer}} = \frac{70,22}{0,015} = 4,681 \text{ kNm}$$

12.4 Number of coils outer spring

$$\delta_{outer} = \frac{8WD^3 n}{Gd^4}$$

$$n_{outer} = \frac{0,015 \times 80 \times 10^9 \times 0,005^4}{8 \times 70,22 \times 0,045^3} = 14,65 \text{ coils}$$

12.5 Stress in each spring

$$\tau_O = \left(\frac{8WD}{\pi d^3}\right)_{outer} = \frac{8 \times 70,22 \times 0,045}{\pi \times 0,005^3} = 64,373 \text{ MPa}$$

$$\tau_I = \left(\frac{8WD}{\pi d^3}\right)_{inner} = \frac{8 \times 44,78 \times 0,035}{\pi \times 0,004^3} = 62,361 \text{ MPa}$$

12.6 Total strain energy

$$U_T = U_I + U_O$$

$$U_T = 0,5W_I\delta_I + 0,5W_O\delta_O$$

$$U_T = 0,5 \times 44,78 \times 0,009 + 0,5 \times 70,22 \times 0,015 = 0,729 \text{ J}$$

13. 13.1 Total load on spring

$$\tau_O = \left(\frac{8WD}{\pi d^3}\right)_{outer} = 130M = \frac{8 \times W_O \times 0,04}{\pi \times 0,004^3}$$

$$W_{outer} = 81,68 \text{ N}$$

$$\tau_I = \left(\frac{8WD}{\pi d^3}\right)_{inner} = 150M = \frac{8 \times W_I \times 0,03}{\pi \times 0,003^3}$$

$$W_{inner} = 53,014 \text{ N}$$

Total load on springs = 134,694 N

13.2 Deflection

Deflection outer = deflection inner

$$\therefore \text{inner deflection} = \delta = \frac{8 \times 53,04 \times 0,03^3 \times 10}{79 \times 10^9 \times 0,003^4} = 17,9 \text{ mm}$$

13.3 Coils outer spring

$$\delta_{outer} = 0,0179 = \frac{8 \times 81,68 \times 0,04^3 n}{79 \times 10^9 \times 0,004^4}$$

$$\therefore\ n_{outer} = 8,66 \text{ coils}$$

13.4 Stiffness of spring

$$S_{inner} = \frac{W}{\delta_{inner}} = \frac{53{,}014}{0{,}0179} = 2{,}962$$

$$S_{outer} = \frac{W}{\delta_{outer}} = \frac{81{,}68}{0{,}0179} = 4{,}563$$

Total stiffness = 7,525 kNm

14. $L_{closed} = nd = 16 \times 15 = 240$ mm

$\delta = 400 - 240 = 160$ mm

$$\delta = \frac{8WD^3 n}{Gd^4} = 0{,}16 = \frac{8W \times 0{,}15^3 \times 16}{79 \times 10^9 \times 0{,}018^4}$$

W = 3,072 kN

Force in the spring = force in bolt

Length of bolt = 240 + 30 = 270 mm

$$\text{Extension bolt} = x = \frac{FL}{AE} = \frac{3\,072 \times 0{,}27 \times 4}{\pi \times 0{,}017^2 \times 180 \times 10^9} = 0{,}02 \text{ mm}$$

15. 15.1 Load on the springs

The outer spring deflection is 6+12 = 18 mm

The inner spring deflection is 9+12 = 21 mm

$$\delta_{outer} = \frac{8WD^3 n}{Gd^4} = 0{,}018 = \frac{8W \times 0{,}035^3 \times 6}{79 \times 10^9 \times 0{,}004^4}$$

$W_{outer} = 176{,}886$ N

$$\delta\ inner = \frac{8WD^3 n}{Gd^4} = 0{,}021 = \frac{8W \times 0{,}025^3 \times 6}{79 \times 10^9 \times 0{,}003^4}$$

$W_{inner} = 179{,}172$ N

Total load on springs = 356,058 N

15.2 Stiffness of springs

$$S_{inner} = \frac{W}{\delta_{inner}} = \frac{179\,172}{0{,}021} = 8{,}532 \text{ kNm}$$

$$S_{outer} = \frac{W}{\delta_{outer}} = \frac{176\,886}{0{,}018} = 9{,}827 \text{ kNm}$$

Total stiffness = 18,368 kNm

15.3 Stress in each spring

$$\tau_O = \left(\frac{8WD}{\pi d^3}\right)_{outer} = \frac{8 \times 176{,}886 \times 0{,}035}{\pi \times 0{,}004^3} = 246{,}332 \text{ MPa}$$

$$\tau_I = \left(\frac{8WD}{\pi d^3}\right)_{inner} = \frac{8 \times 179{,}172 \times 0{,}025}{\pi \times 0{,}003^3} = 422{,}46 \text{ MPa}$$

16.1 Mean diameter of the second spring

$$\delta_1 = \delta_2$$

$$\left(\frac{8WD^3 n}{Gd^4}\right)_1 = \left(\frac{8WD^3 n}{Gd^4}\right)_2$$

$$\left(\frac{60^3 \times 16}{4^4}\right)_1 = \left(\frac{D^3 \times 12}{3^4}\right)_2$$

$$13\ 500 = 0{,}148D^3$$

$$D = \sqrt[3]{\frac{13\ 500}{0{,}148}} = 45 \text{ mm}$$

16.2 Allowable load on the springs

Allowable stress means that the stress in each must be used to see what force each spring can take, and the smallest will be the applied force.

Consider spring 1:

$$W_1 = \frac{\tau\pi d^3}{8D} = \frac{130M \times \pi \times 0{,}004^3}{8 \times 0{,}06} = 54{,}45 \text{ N}$$

Consider spring 2:

$$W_2 = \frac{\tau\pi d^3}{8D} = \frac{130M \times \pi \times 0{,}003^3}{8 \times 0{,}045} = 30{,}631 \text{ N}$$

Allowable force on springs is 30,631 N

16.3 Stress in each spring

Stress spring 2 =130 MPa

Stress spring 1:

$$\tau_1 = \frac{8WD}{\pi d^3} = \frac{8 \times 30{,}631 \times 0{,}06}{\pi \times 0{,}004^3} = 73{,}126 \text{ MPa}$$

16.4 Total strain energy

$$U_1 = \frac{4W^2D^3n}{Gd^4} = \frac{4 \times 30{,}631^2 \times 0{,}06^3 \times 16}{80 \times 10^9 \times 0{,}004^4} = 0{,}633 \text{ J}$$

$$U_2 = \frac{4W^2D^3n}{Gd^4} = \frac{4 \times 30{,}631^2 \times 0{,}045^3 \times 12}{80 \times 10^9 \times 0{,}003^4} = 0{,}633 \text{ J}$$

Total strain energy = 1,266 J

16.5 Total deflection

Deflection the same in both springs

$$\delta_T = 2\left(\frac{8WD^3n}{Gd^4}\right) = 2\left(\frac{8 \times 30{,}631 \times 0{,}06^3 \times 16}{80 \times 10^9 \times 0{,}004^4}\right) = 82{,}7 \text{ mm}$$

MODULE

8 Transformation of stress

By the end of this module, students should be able to:

- sketch an element to indicate the normal and shear stresses acting on a point in the material;
- calculate:
  - the resultant stress at any oblique plane if the normal and shear stresses are known
  - the angle where the normal stresses will be maximum and minimum (principal stresses)
  - the values of the principal stresses
  - the value of the maximum shear stress;
- sketch Mohr's circle to scale from given elements with normal and shear stresses; and
- determine:
  - the resultant stress at any oblique plane using Mohr's circle
  - the principal stresses and the angle where they will be using Mohr's circle
  - the value of the maximum shear stress using Mohr's circle.

Introduction

We have studied different types of stress before; compressive stress, tensile stress, shear stress in a beam and torsional stress in a shaft.

Each one was dealt with separately, but these stresses can act at a point in a member.

It is possible that the combined effects of these stresses in a particular direction are more than that of an individual stress.

Exercise 8.1 SB page 272

1. 1.1 Normal stresses and shear stress on plain PP

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x1} = \frac{-30 + 40}{2} + \frac{-30 - 40}{2}\cos(-60) + 20 \sin(-60)$$

$$\sigma_{x1} = 5 - 17{,}5 - 17{,}32 = 29{,}82 \text{ MPa (C)}$$

$$\sigma_{y1} = 5 + 17{,}5 + 17{,}32 = 39{,}82 \text{ MPa (T)}$$

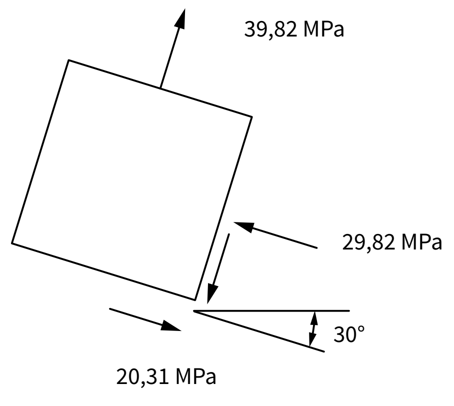
$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau \cos 2\theta$$

$$\tau_{xy} = -\left(\frac{-30 - 40}{2}\right)\sin(-60) + 20 \cos(-60)$$

$$\tau_{xy} = -30{,}31 + 10 = -20{,}31 \text{ MPa}$$

1.2 Resultant stress

$$\sigma_R = \sqrt{\sigma_{x1}^2 + \tau_{xy}^2} = \sqrt{29{,}82^2 + 20{,}31^2} = 36{,}08 \text{ MPa}$$



2. 2.1 Normal stresses

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x1} = \frac{50 - 35}{2} + \frac{50 - (-35)}{2}\cos 50 + (-25)\sin 50$$

$$\sigma_{x1} = 7{,}5 + 27{,}32 - 19{,}15 = 15{,}67 \text{ MPa (T)}$$

$$\sigma_{y1} = 7{,}5 - 27{,}32 + 19{,}15 = -0{,}67 \text{ MPa (C)}$$

2.2 Shear stress

$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau \cos 2\theta$$

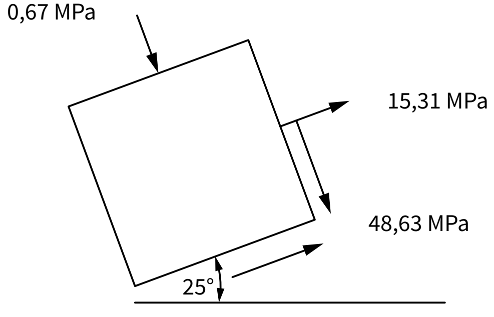
$$\tau_{xy} = -\left(\frac{50 - (-35)}{2}\right)\sin 50 + (-25)\cos 50$$

$$\tau_{xy} = -32{,}56 - 16{,}07 = -48{,}63 \text{ MPa}$$

2.3 Resultant stress

$$\sigma_R = \sqrt{\sigma_{x1}^2 + \tau_{xy}^2} = \sqrt{15{,}31^2 + 48{,}63^2} = 50{,}98 \text{ MPa}$$

2.4 Element



3. 3.1 Normal stress on the surface

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x1} = \frac{-55 - 80}{2} + \frac{-55 + 80}{2}\cos 56 + (-30)\sin 56$$

$$\sigma_{X1} = 67{,}5 - 6{,}99 - 24{,}87 = 85{,}38 \text{ MPa (C)}$$

$$\sigma_{y1} = -67{,}5 - 6{,}99 + 24{,}87 = 49{,}62 \text{ MPa (C)}$$

3.2 Shear stress

$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau \cos 2\theta$$

$$\tau_{xy} = -\left(\frac{-55 + 80}{2}\right)\sin 56 + (-30)\cos 56$$

$$\tau_{xy} = 10{,}36 - 16{,}78 = 27{,}14 \text{ MPa}$$

4. 4.1 Normal stress to plane

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x1} = \frac{30 - 25}{2} + \frac{30 + 25}{2}\cos(-140) + 18\sin(-140)$$

$$\sigma_{X1} = 2{,}5 - 21{,}07 - 11{,}57 = 30{,}14 \text{ MPa (C)}$$

$$\sigma_{y1} = 2{,}5 + 21{,}07 + 11{,}57 = 35{,}14 \text{ MPa (T)}$$

4.2 Shear stress

$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau \cos 2\theta$$

$$\tau_{xy} = -\left(\frac{30 - 25}{2}\right)\sin(-140) + 18\cos(-140)$$

$$\tau_{xy} = 1{,}61 - 13{,}79 = -12{,}18 \text{ MPa}$$

4.3 Maximum shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{30 - (-25)}{2}\right)^2 + 18_{xy}^2} = 32{,}87 \text{ MPa}$$

4.4 Element maximum shear stress

Angle $\therefore \tan 2\theta = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) = -\left(\frac{30 - (-25)}{2 \times 18_{xy}}\right) = -1{,}53$

$2\theta_S = -56{,}79° \quad \therefore \theta_S = -28{,}4°$

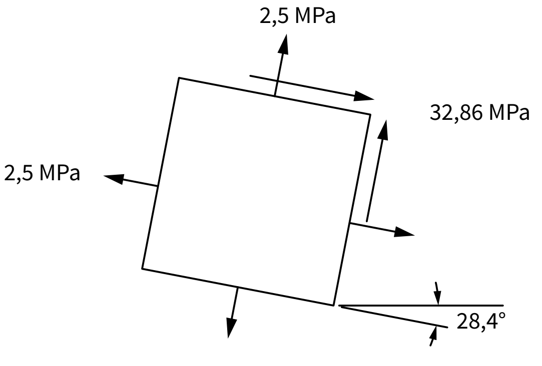
Check direction of shear stress

$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau \cos 2\theta$$

$$\tau_{xy} = -\left(\frac{30 - (-25)}{2}\right)\sin(-56{,}79) + 18\cos(-56{,}79)$$

$\tau_{xy} = 23 + 9{,}86 = 32{,}86$ MPa with x-axis

Average normal $= \frac{\sigma_x + \sigma_y}{2} = \frac{30 - 25}{2} = 2{,}5$ MPa



5.1 Normal stress: x- and y-planes

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x1} = \frac{-90 + 60}{2} + \frac{-90 - 60}{2}\cos(-60) + (-25)\sin(-60)$$

$\sigma_{X1} = -15 - 37{,}5 + 21{,}65 = 30{,}85$ MPa (C)

$\sigma_{X1} = -15 + 37{,}5 - 21{,}65 = 0{,}85$ MPa (T)

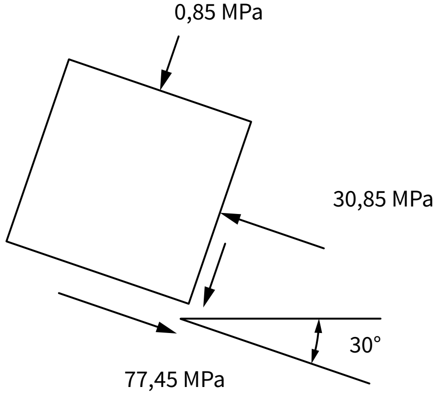
5.2 Shear stress

$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau \cos 2\theta$$

$$\tau_{xy} = -\left(\frac{-90 - 60}{2}\right)\sin(-60) + (-25)\cos(-60)$$

$$\tau_{xy} = -64{,}95 - 12{,}5 = -77{,}45 \text{ MPa}$$

5.3 Sketch element



5.4 Maximum and minimum principal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-90 + 60}{2} + \sqrt{\left(\frac{-90 - 60}{2}\right)^2 + (-25)_{xy}^2}$$

$$\Sigma_1 = -15 + 79{,}06 = 64{,}06 \text{ MPa (T)}$$

$$\sigma_2 = -15 - 79{,}06 = -94{,}06 \text{ MPa (C)}$$

5.5 Stress element

Angle of plane

$$\tan \theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times -25}{-90 - 60} = 0{,}3333$$

$$2\theta_p = 18{,}43° \quad \therefore \theta_p = 9{,}22°$$

Check which stress with which plane

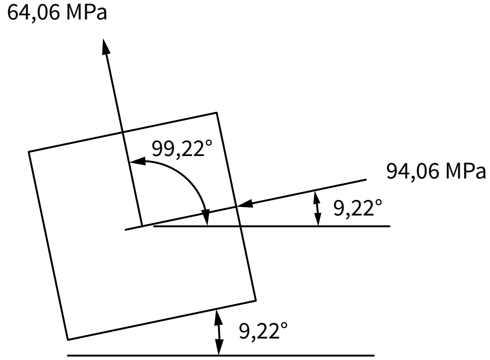
$$\sigma_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_x = \frac{-90 + 60}{2} + \frac{-90 - 60}{2}\cos(18{,}43) + (-25)\sin(18{,}43)$$

$$\sigma_X = -15 - 71{,}15 - 7{,}9 = -94{,}06 \text{ MPa (C)} = \sigma_{X1}$$

$$\therefore \sigma_{y1} = 64{,}06 \text{ MPa (T) @ } \theta_{p2} = 90 + 9{,}22 = 99{,}22°$$

$$\sigma_1 - 94{,}06 \text{ MPa (C) at } \theta_{p1} = 9{,}22°$$



5.6 Maximum shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{-90 - 60}{2}\right)^2 + (-25)_{xy}^2} = 79{,}06 \text{ MPa}$$

5.7 Element maximum shear stress

Angle $\therefore \tan 2\theta = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) = = -\left(\frac{-90 - 60}{2\times -25_{xy}}\right) = -3$

$2\theta_S = -71{,}57°$ $\therefore \theta_S = -35{,}78°$

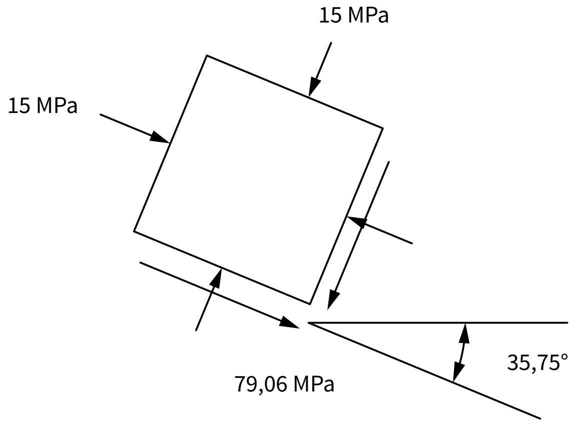
Check direction of shear stress

$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau\cos 2\theta$$

$$\tau_{xy} = -\left(\frac{-90 - 60}{2}\right)\sin(-71{,}57) + (-25)\cos(-71{,}57)$$

$\tau_{xy} = -71{,}15 - 7{,}9 = -79{,}06$ MPa with x-axis

Average normal $= \frac{\sigma_x + \sigma_y}{2} = \frac{-90 + 60}{2} = -15$ MPa



6.1 Maximum and minimum principal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{90 - 60}{2} + \sqrt{\left(\frac{90 + 60}{2}\right)^2 + (25)_{xy}^2}$$

$$\sigma_1 = 15 + 79{,}06 = 94{,}06 \text{ MPa (T)}$$

$$\sigma_2 = 15 - 79{,}06 = -64{,}06 \text{ MPa (C)}$$

6.2 Element sketch

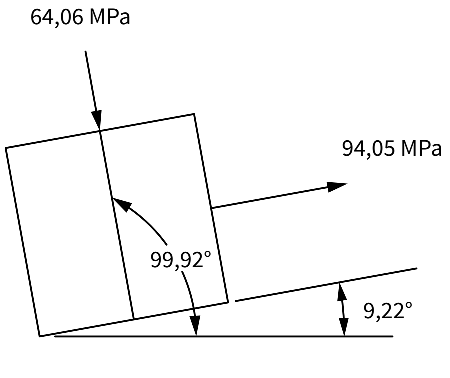
Angle $\therefore \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 25}{90 + 60} = 0{,}3333$

$2\theta_p = 18{,}44°$ and $\theta_p = 9{,}22°$

$$\sigma_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_x = \frac{90 - 60}{2} + \frac{90 + 60}{2}\cos(18{,}43) + (25)\sin(18{,}43)$$

$$\sigma_X = 15 + 71{,}15 + 7{,}9 = 94{,}06 \text{ MPa T} = \sigma_{X1} \text{ @ } 9{,}22°$$



7.1 Maximum and minimum principal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-140}{2} + \sqrt{\left(\frac{140}{2}\right)^2 + (-180)_{xy}^2}$$

$$\sigma_1 = -70 + 193{,}13 = 123{,}13 \text{ MPa } (\text{T})$$

$$\sigma_2 = -70 - 193{,}13 = -263{,}13 \text{ MPa } (\text{C})$$

7.2 Maximum shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{140}{2}\right)^2 + (-180)_{xy}^2} = 193{,}13 \text{ MPa}$$

8.1 Maximum and minimum principal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{55 + 80}{2} + \sqrt{\left(\frac{55 - 80}{2}\right)^2 + (-60)_{xy}^2}$$

$$\sigma_1 = 67{,}5 + 61{,}29 = 128{,}79 \text{ MPa } (\text{T})$$

$$\sigma_2 = 67{,}5 - 61{,}29 = -6{,}21 \text{ MPa } (\text{T})$$

8.2 Maximum shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{55 - 80}{2}\right)^2 + (-60)_{xy}^2} = 61{,}29 \text{ MPa}$$

8.3 Sketch shear stress element

$$\text{Average stress} = \left(\frac{\sigma_x + \sigma_y}{2}\right) = \frac{55 + 80}{2} = 67{,}5 \text{ MPa}$$

Angle of plane

$$\tan 2\theta_S = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) = -\left(\frac{55 - 80}{2 \times -60}\right) = -0{,}2083$$

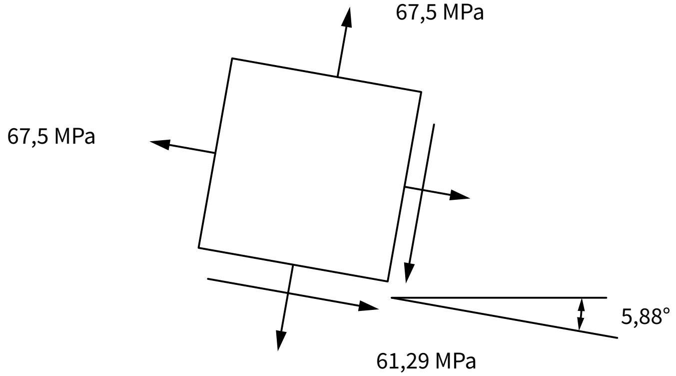
$$2\theta_S = -11.77° \quad \therefore \theta_s = -5{,}88°$$

Check direction of shear stress

$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau \cos 2\theta$$

$$\tau_{xy} = -\left(\frac{55 - 80}{2}\right)\sin(-11{,}77) + (-60)\cos(-11{,}77)$$

$\tau_{xy} = -2{,}55 - 58{,}74 = -61{,}29$ MPa with x-axis



9.1 Maximum and minimum principal stress

$T_{max} = \dfrac{80k \times 60}{2\pi 500} = 1{,}528$ kNm

$\sigma_{thrust} = \dfrac{26k \times 4}{\pi \times 0{,}06^2} = 9{,}196$ MPa

From the simple bending equation, the BM is due to thrust.

$M = \dfrac{\pi \times 0{,}06^4 \times 9{,}196 \times 10^6 \times 2}{64 \times 0{,}06} = 195$ Nm

Maximum BM = M = 1,2k + 195 = 1,395 kNm

$M_e = \frac{1}{2}\left[1\ 395 \pm \sqrt{1\ 395^2 + 1\ 528^2}\right]$

$\therefore M_{e\,max} = 1{,}731$ kNm and $M_{e\,min} = -336$ Nm

Maximum principal stress = $\sigma_1 = \dfrac{M_{e\,max}y}{I} = \dfrac{1\ 731 \times 0{,}06 \times 64}{2 \times \pi \times 0{,}06^4} = 81{,}582$ MPa

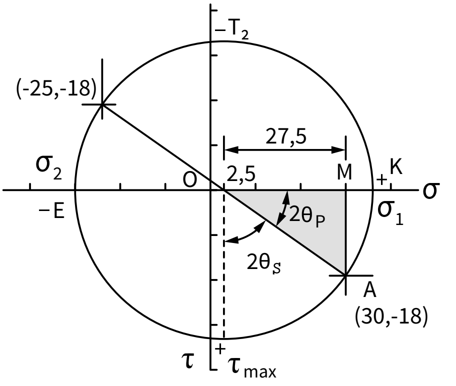
Minimum principal stress = $\sigma_2 = \dfrac{M_{e\,max}y}{I} = \dfrac{-336 \times 0{,}06 \times 64}{2 \times \pi \times 0{,}06^4} = -15{,}845$ MPa

9.2 Maximum shear stress

$\tau_{max} = \dfrac{\sigma_2 - \sigma_1}{2} = \dfrac{-15{,}845 - 81{,}582}{2} = -48{,}714$ MPa

Exercise 8.2 SB page 291

1. 1.1 Principal stresses



$C = \frac{\sigma_x + \sigma_y}{2} = \frac{30 - 25}{2} = 2{,}5$ MPa

σ_1 = OK = 3,5 cm × 10 = 35 MPa (T)±

σ_2 = OE = 3 cm × 10 = 30 MPa (C)±

τ_{max} = CT_1 = 3,3 cm × 10 = 33 MPa (C)±

Angle principal plane

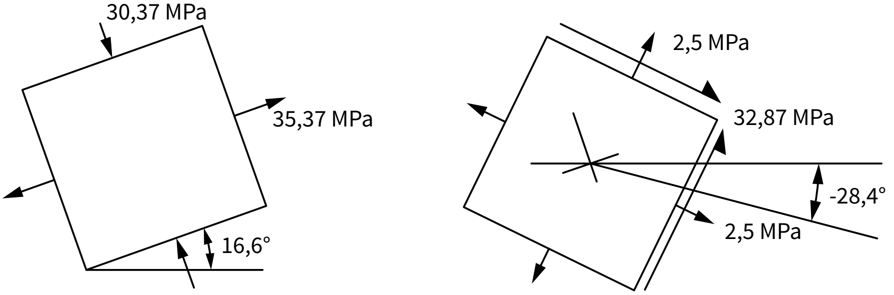
$\angle MCA = 2\theta_p = 33°$ $\therefore \theta_p = 16{,}5°$ on element

Angle shear stress plane

$\angle ACT_1 = 2\theta_S = 90 - 33 = 57°$

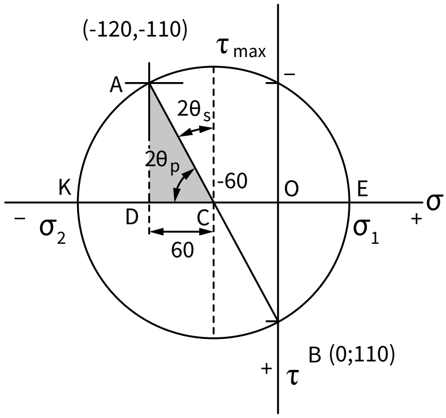
$\therefore \theta_S = 28{,}5°$

Average stress = C = 2,5 MPa tensile



2. Scale 1 cm = 10 MPa

$C = \frac{\sigma_x + \sigma_y}{2} = \frac{-120}{2} = -60$ MPa



σ_1 = OE = 6,5 cm × 10 = 65 MPa (T)±

σ_2 = OK = 18,5 cm × 10 = 185 MPa (C)±

τ_{max} = CA = CT_1 = −125,3 MPa±

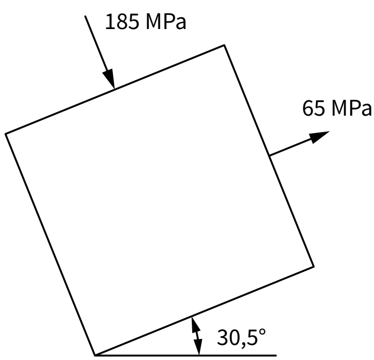
Angle principal plane

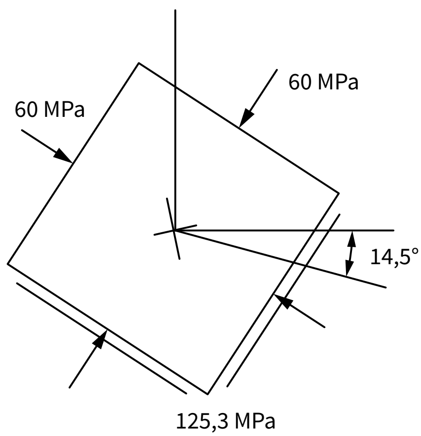
$\angle KCA = 2\theta_p = 61° \quad \therefore \theta_p = 30{,}5°$ CCW

Angle shear stress plane

$\angle ACT_1 = 2\theta_S = 90 - 61 = 29°$

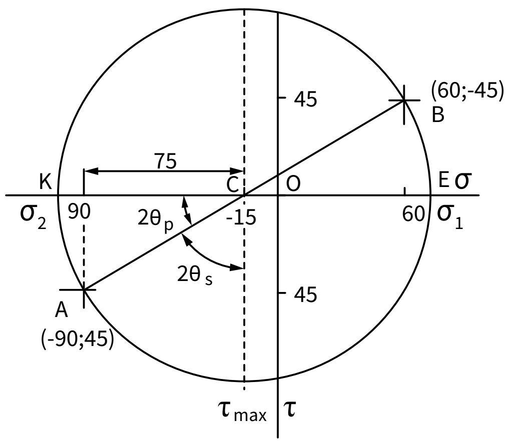
$\therefore \theta_S = 14{,}5°$ CW





3. $C = \frac{\sigma_x + \sigma_y}{2} = \frac{-90 + 60}{2} = -15$ MPa (C)

Scale 1 cm = 10 MPa



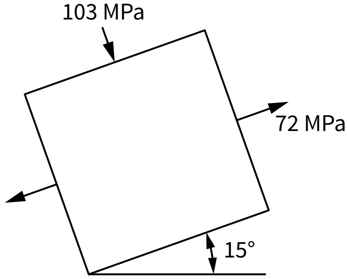
σ_1 = OE = 7,2 cm = 72 MPa (T)±

σ_2 = OK = 10,3 cm = 103 MPa (C)±

τ_{max} = CT_2 = 8,75 = 87,5 MPa

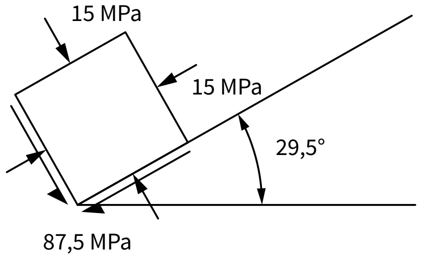
Angle principal plane

$\angle ACK = 2\theta_p = 30^o \quad \therefore \theta_p = 15°$ CCW



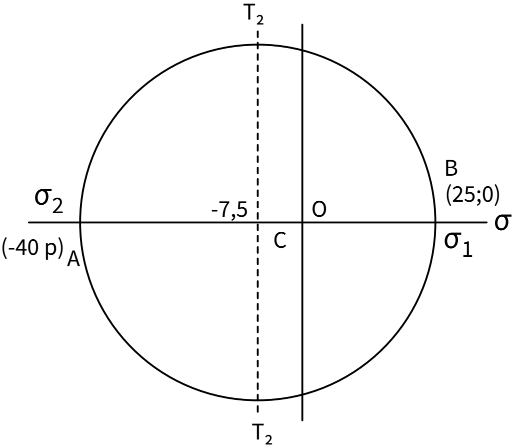
Angle shear stress plane

$\therefore \angle ACT_2 = 2\theta_S = 59° \quad \therefore \quad \theta_S = 29{,}5^o$ CCW



4. $C = \frac{\sigma_x + \sigma_y}{2} = \frac{-40 + 25}{2} = -7{,}5$ MPa

Scale 1 cm = 10 MPa



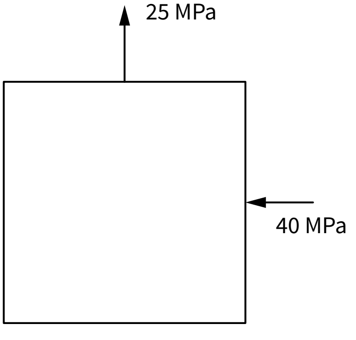
σ_1 = OB = 2,5 cm = 25 MPa (T)

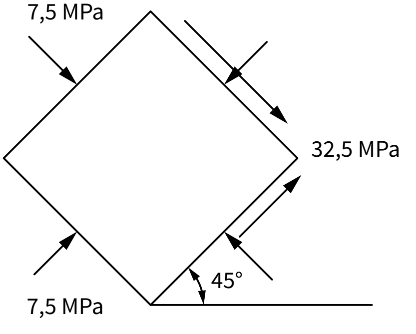
σ_2 = OA = 4 cm = 40 MPa (C)

$\tau_{max} = CT_1$ = 3,25 cm = 32,5 MPa

$2\theta_p = 0$

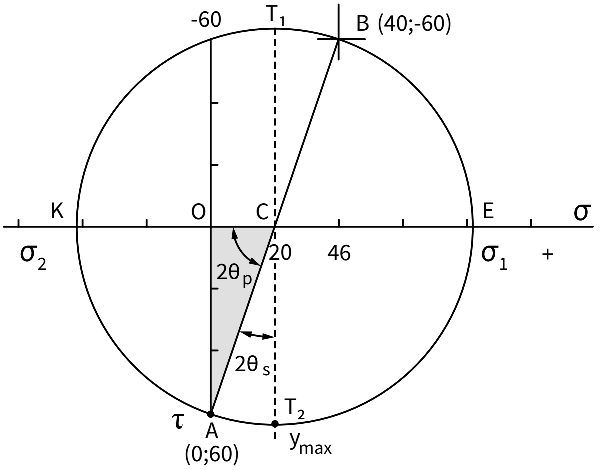
$\angle ACT_1 = 2\theta_S = 90° \quad \therefore \theta_S = 45°$





5. $C = \frac{\sigma_x + \sigma_y}{2} = \frac{40}{2} = 20$ MPa

Scale 1 cm = 10 MPa

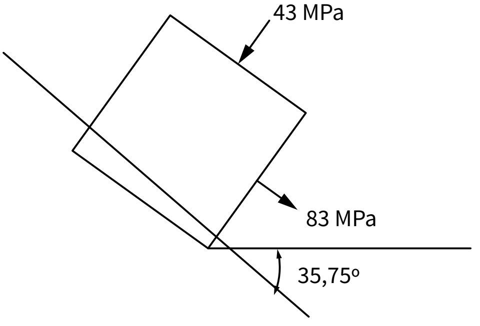


σ_1 = OE = 8,3 cm = 83 MPa (T)

σ_2 = OK = 4,3 cm = 43 MPa (C)

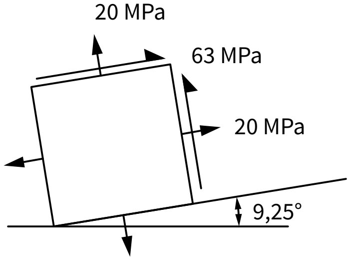
CT_2 = 6,3 cm = τ_{max} = 63 MPa

$2\theta_P = \angle OCA = 71{,}5°$ $\therefore \theta_P = 35{,}75°$ CW



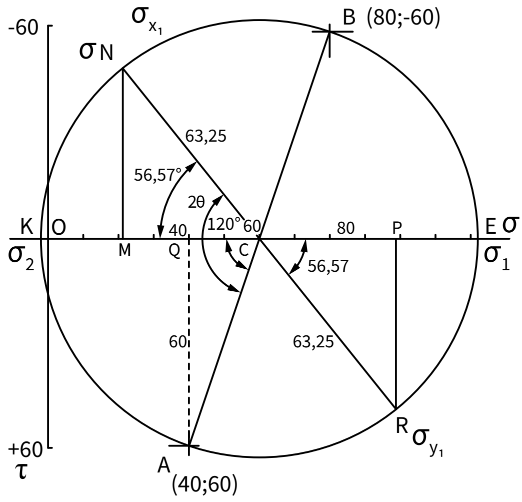
$2\theta_S = \angle ACT_2 = 90 - 71{,}5 = 18{,}5$ $\therefore \theta_S = 9{,}25°$ CCW

Average normal stress = 20 MPa (T)



6. $C = \frac{\sigma_x + \sigma_y}{2} = \frac{80 + 40}{2} = 60$ MPa

Scale 1 cm = 10 MPa

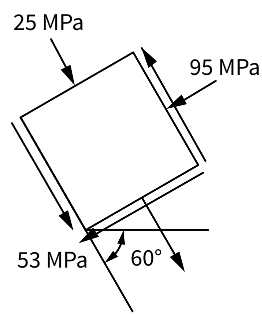


∴ σ_N = OM = 2,5 cm = 25 MPa = σ_{x1}(C)

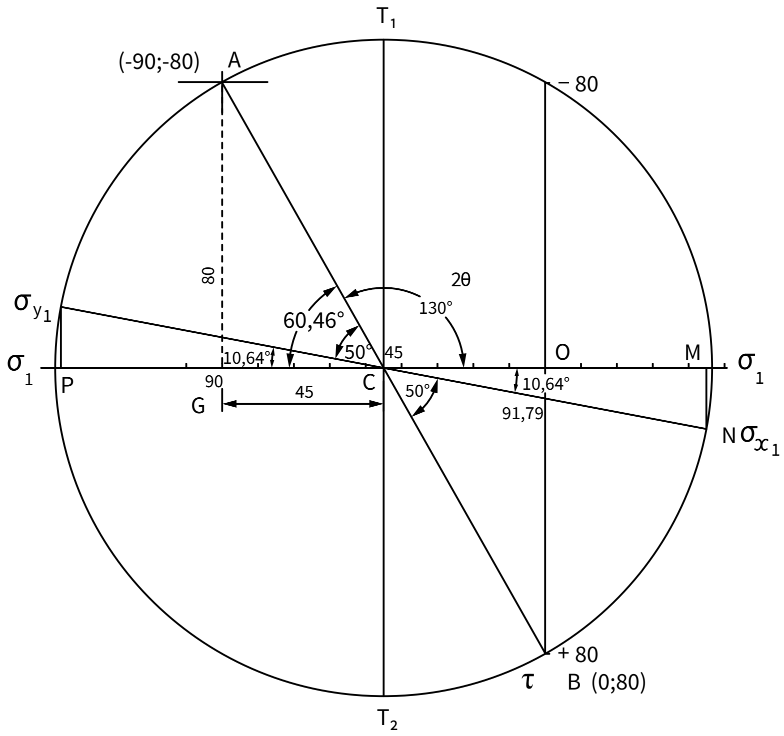
∴ σ_{y1} = OP = 9,5 cm = 95 MPa (C)

Shear stress

∴ MN = 5,3 cm = τ = 53 MPa (T)



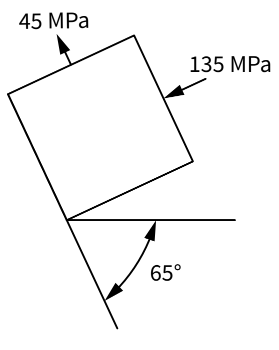
7. $C = \frac{\sigma_x + \sigma_y}{2} = \frac{-90}{2} = -45$ MPa



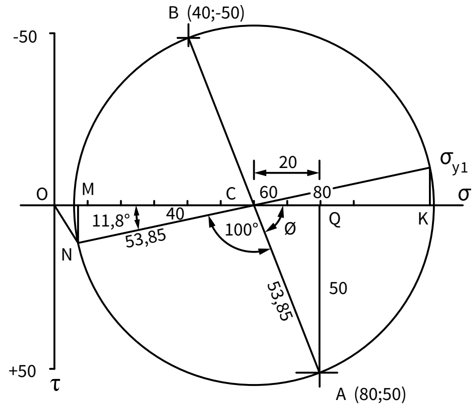
$\therefore \sigma_{N-x1}$ = OM = 4,5 cm = 45 MPa (T)

MC = CQ $\quad \therefore \sigma_{y1}$ = OP = 13,5 cm = 135 MPa (C)

Shear stress = NM = 1,7 cm = τ = 17 MPa



8. 8.1 $C = \frac{\sigma_x + \sigma_y}{2} = \frac{80 + 40}{2} = 60$ MPa

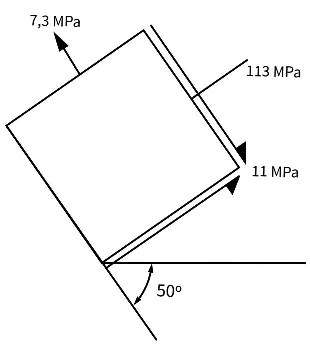


1 cm = 10 MPa

$\sigma_{N\,x1}$ = OM = 0,73 cm = 7,3 MPa (T)

σ_{y1} = OK = 11,3 cm = 113 MPa (T)

$\therefore$ Shear stress = τ_{xy} = MN = 1,1 cm = 11 MPa (C)



8.2 Resultant stress

Resultant stress = ON

σ_R = ON = 1,3 cm = 13 MPa

8.3 Calculate normal and shear stress on plane PP

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x1} = \frac{80 + 40}{2} + \frac{80 - 40}{2}\cos - 100 + 50 \sin - 100$$

$\sigma_{x1} = 60 - 3{,}47 - 49{,}24 = 7{,}29$ MPa (T)

$\sigma_{y1} = 60 + 4{,}47 + 49{,}24 = 112{,}71$ MPa (T)

Shear stress $\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$

$\tau_{xy} = -\left(\frac{80 - 40}{2}\right)\sin - 100 + 50 \cos - 100$

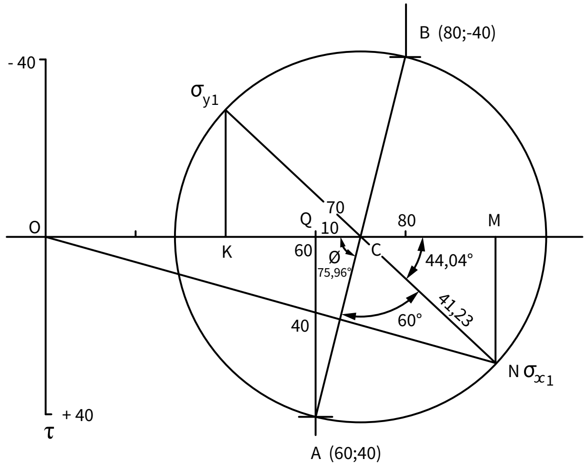
$\tau_{xy} = +19{,}7 - 8{,}7 = 11$ MPa

8.4 Resultant stress

$\sigma_{Resultant} = \sqrt{\sigma_1^2 + \tau_{xy}^2} = \sqrt{7{,}29^2 + 11^2} = 13{,}77$ MPa

9. 9.1 $C = \frac{\sigma_x + \sigma_y}{2} = \frac{60 + 80}{2} = 70$ MPa

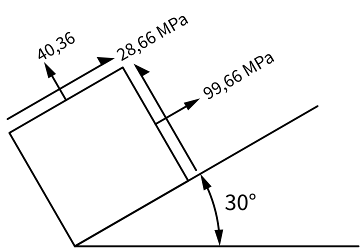
Scale 1 cm = 10 MPa



$\sigma_{N\,x1}$ = OM = 9,94 cm = 99,6 MPa (T)

σ_{y1} = OK = 4 cm = 40 MPa

$\therefore \tau$ = MN = 2,86 cm = 28,6 MPa



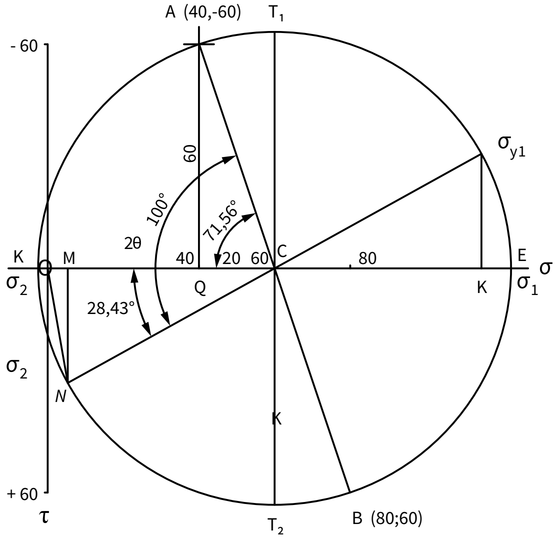
9.2 Resultant stress

Resultant stress = ON

σ_R = ON = 10,4 cm = 104 MPa

10. 10.1 $C = \frac{\sigma_x + \sigma_y}{2} = \frac{40 + 80}{2} = 60$ MPa

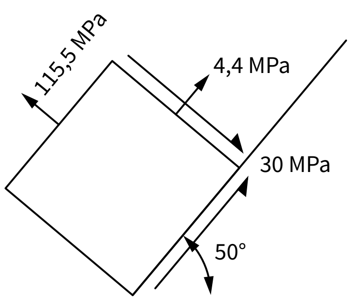
Scale 1 cm = 10 MPa



$\sigma_{N\,x1}$ = OM = 0,44 cm = 4,4 MPa (T)

σ_{y1} = OK = 11,55 cm = 115,5 MPa (T)

∴ τ = MN = 3 cm = 30 MPa



10.2 Resultant stress

Resultant stress = ON

σ_R = ON = 40,5 cm = 40,5 MPa

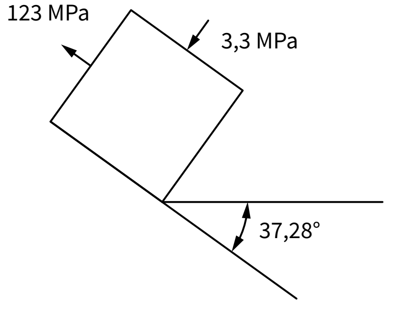
10.3 Principal stresses

σ_1 = OE = 12,3 cm = 123 MPa (T)

σ_2 = OK = 0,33 cm = 3,3 MPa (C)

Angle of plane

$\therefore \angle ACK = 2\theta_p = 74{,}56°$ $\quad \therefore \theta_p = 37{,}28°$



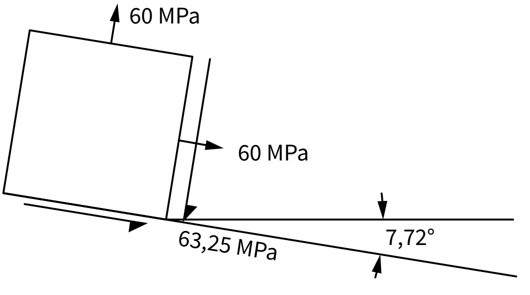
10.4 Maximum shear stress

τ_{max} = R = CA = 63,25 MPa

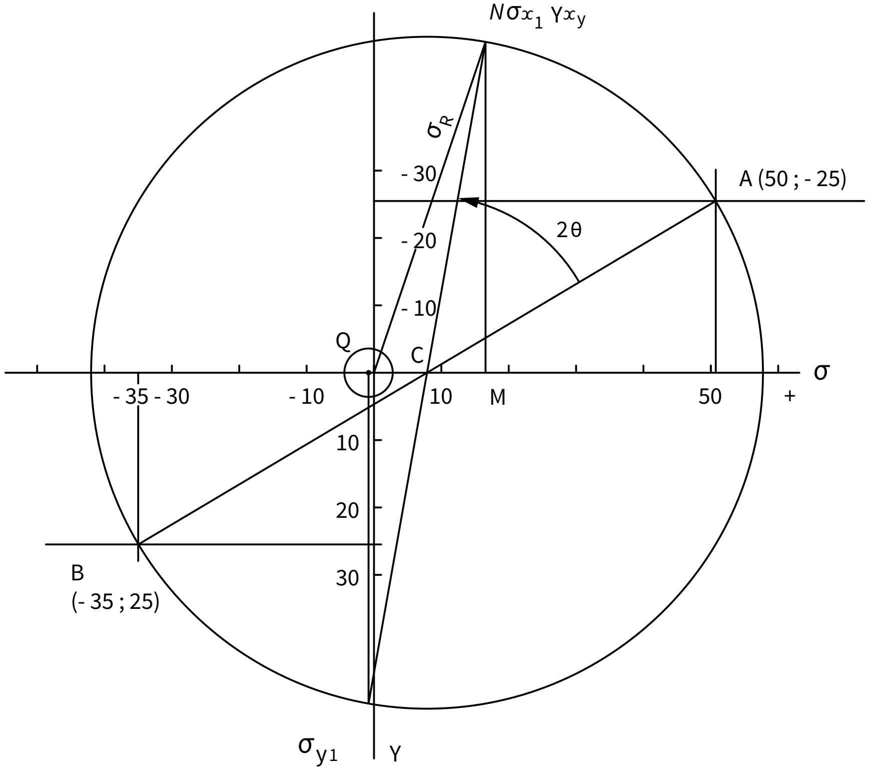
Angle of plane

$2\theta_S = \angle ACT_1 = 90 - 74{,}56 = 15{,}44$ $\quad \therefore \theta_S = 7{,}72°$

Average normal stress = C = 60 MPa (T)



11. 11.1 *This question incorporates Question 2 of Exercise 8.1.*
Scale 1 cm = 10 MPa



$\sigma_{N\,x1}$ = OM = 1,5 cm × 10 = 15 MPa (T)

σ_{y1} = OQ = 0,06 cm × 10 = 0,6 MPa (C)

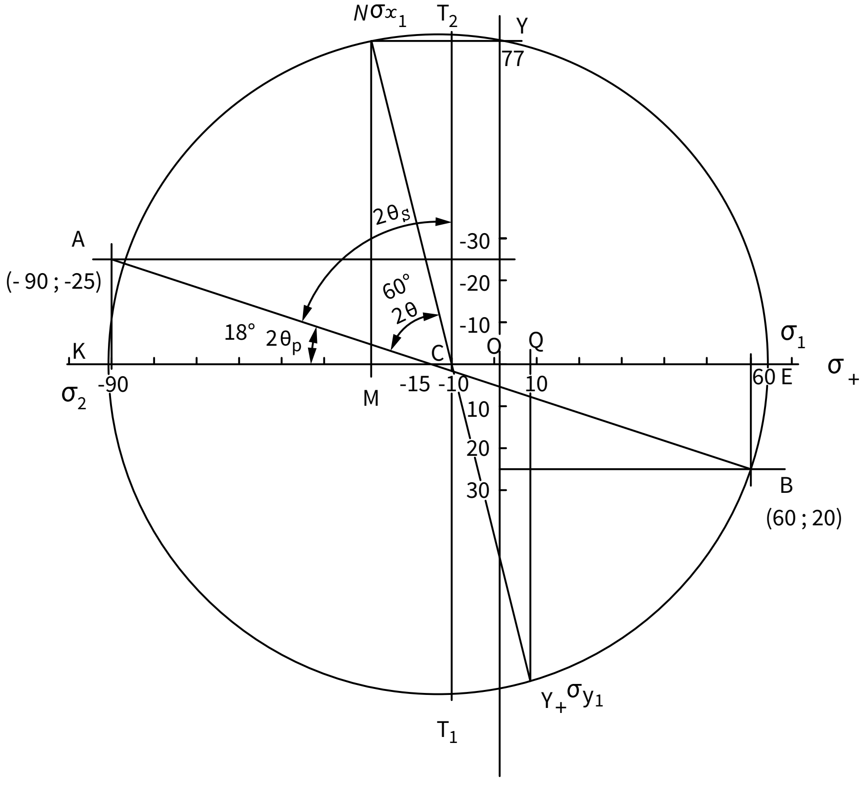
τ_{xy} = NM = 4,9 cm × 10 = 49 MPa

σ_R = ON = 5,2 cm × 10 = 52 MPa

The element sketch is the same as that in Question 2 of Exercise 8.1.

11.2 *This question incorporates Question 5 of Exercise 8.1.*

Scale 1 cm = 10 MPa

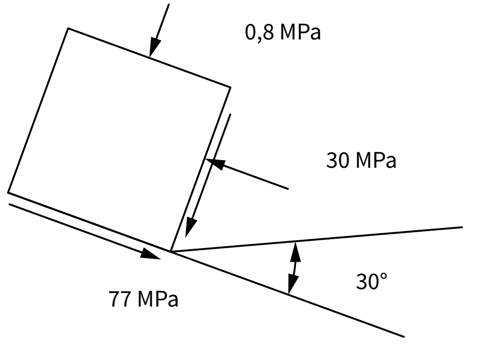


Normal stress

$\sigma_{N\,x1}$ = OM = 3 cm × 10 = 30 MPa (T)

σ_{y1} = OQ = 0,08 cm × 10 = 0,8 MPa (C)

Shear stress: τ_{xy} = NM = 7,7 cm × 10 = 77 MPa



Element sketch

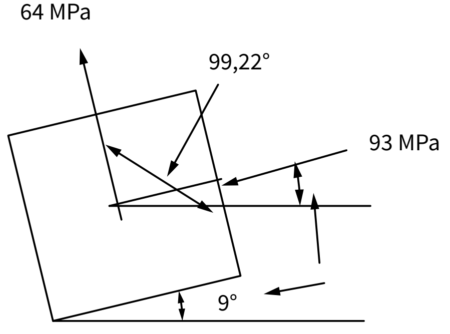
Principal stress

σ_1 = OE = 64 MPa (T)

σ_2 = OK = 93 MPa (C)

Stress element

From circle $2\sigma_p = \angle ACK = 18°$ $\therefore \theta_p = 9°$



Maximum shear stress

τ_{max} = ON = radius = 77 MPa (T)

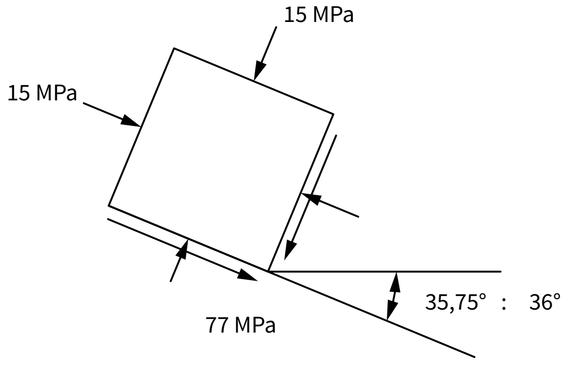
Maximum shear stress element

Average normal stress = C = 15 MPa (C)

Angle: $\angle ACT_1 = 2\theta_S = 90° - 18° = 72°$ CW $\therefore \theta_S = 36°$ CW

Note

Values that are calculated from Mohr's circle will always differ a little bit.



MODULE

9 *Forces in structural frameworks*

By the end of this module, students should be able to:

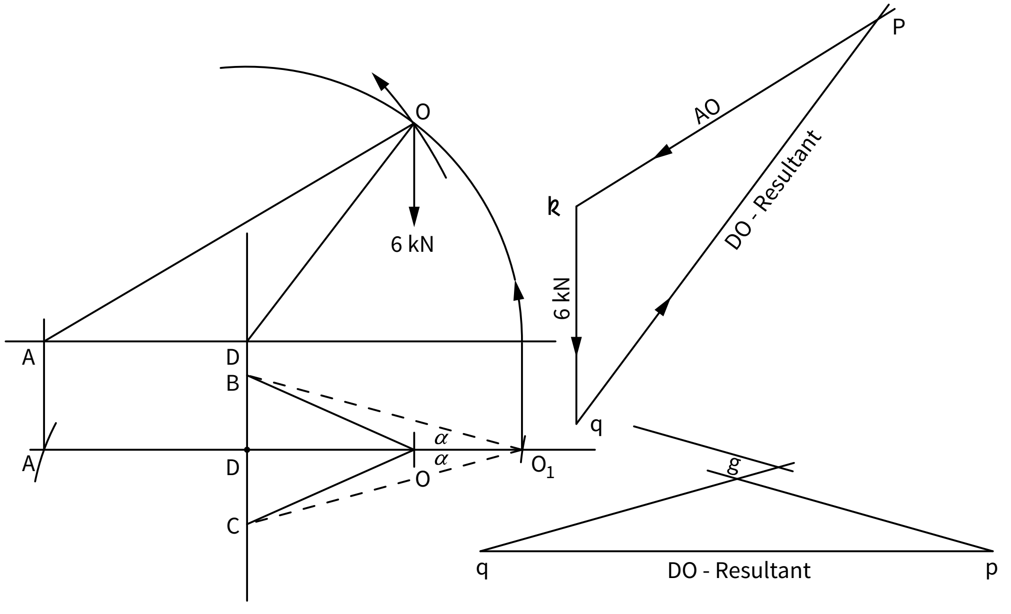
- draw the top- and side view of a sheerleg or tripod according to the appropriate scale;
- draw separate vector diagrams to graphically determine the forces in all members when a load is lifted by the sheerleg or tripod;
- set up a table to indicate the magnitude of all members (including reactions) or struts and ties;
- use a suitable scale to draw top views and side views of cranes; and
- draw separate vector diagrams to graphically determine the forces in all members when a load is lifted by a crane.

Introduction

This module deals with sheerlegs, tripods and derrick cranes. Students learn how to draw top- and side views of these structures and to determine the forces in members by use of vector diagrams.

Exercise 9.1 SB page 301

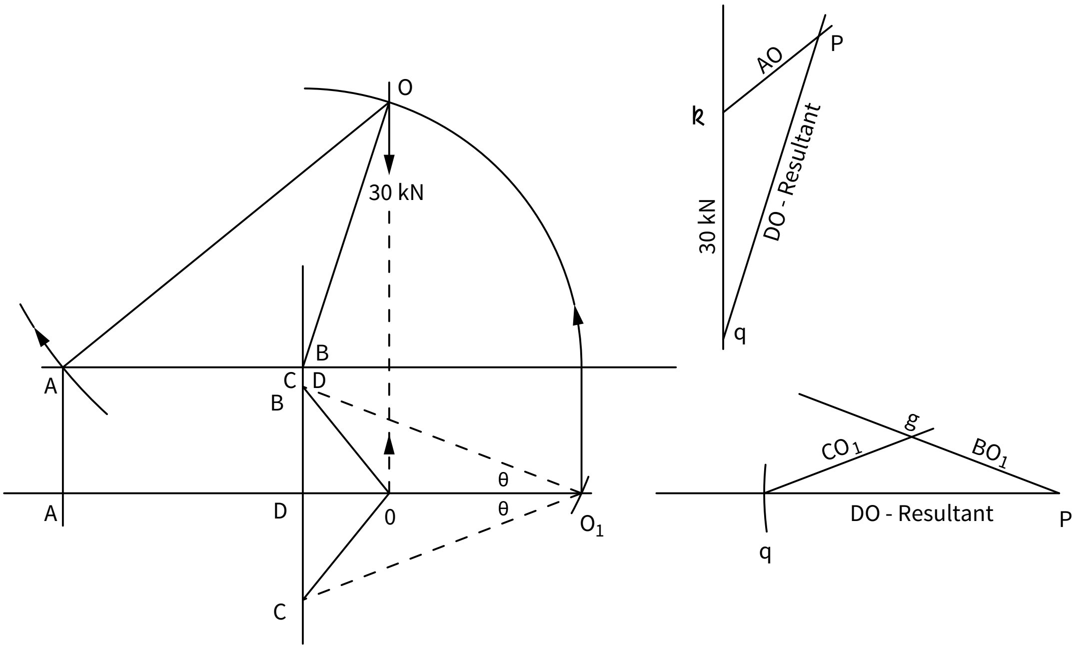
1.



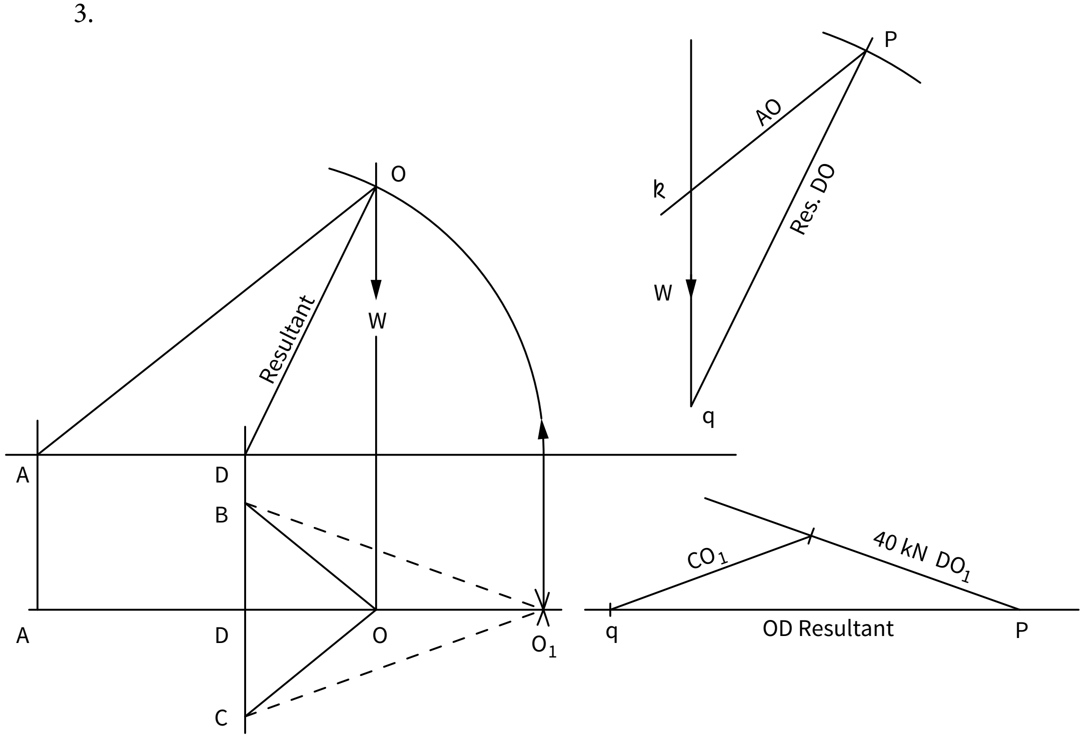
Member	Vector	Force (kN)	Nature
AO	*kp*	10	Tie
BO	*pg*	7,2	Strut
CO	*qg*	7,2	Strut

2.

Member	Vector	Force (kN)	Nature
AO	*kp*	16	Tie
BO	*pg*	23	Strut
CO	*qg*	23	Strut



3.



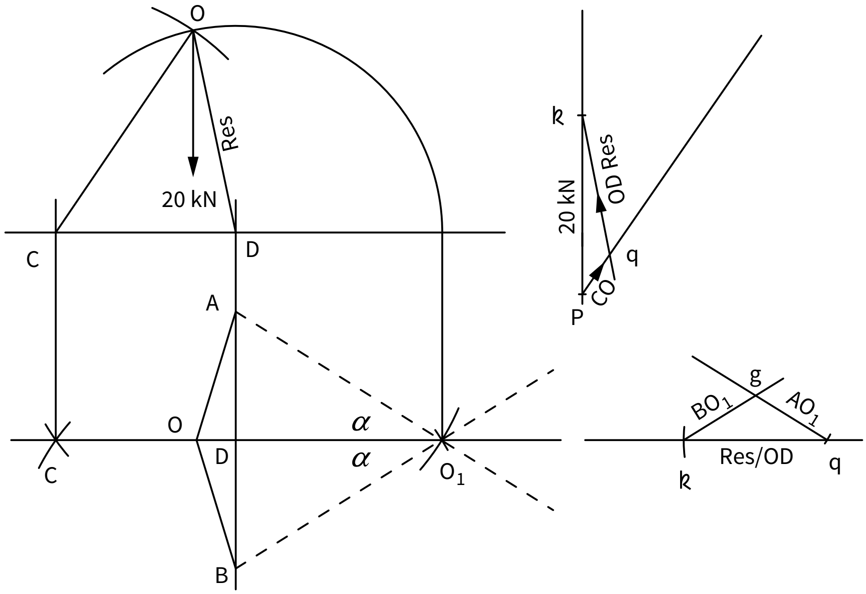
3.1 Load = 42 kN

3.2 Force in backstay 40 kN tie.

Exercise 9.2

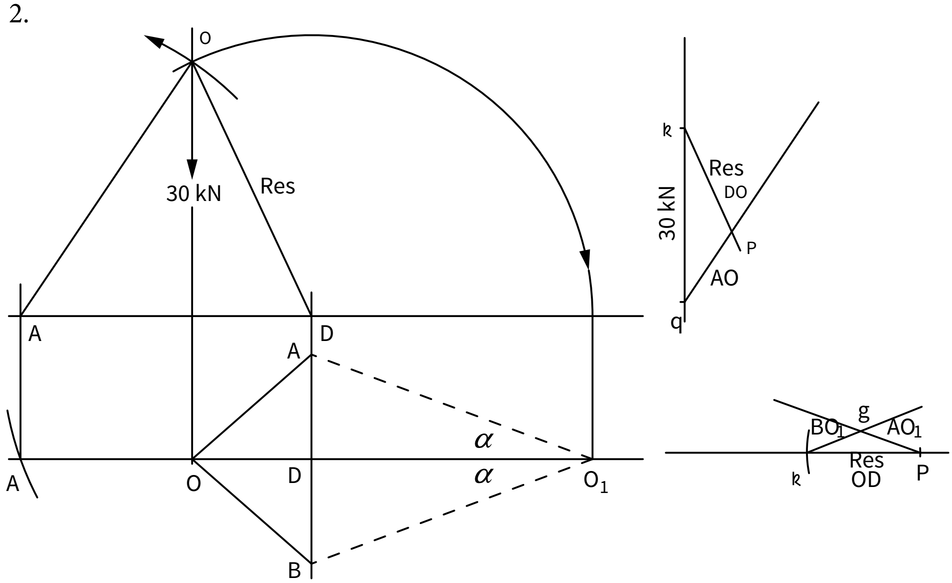
SB page 305

1.



Member	Vector	Force k(N)	Nature
CO	*pq*	5,5	Strut
AO	*qg*	9,75	Strut
BA	*kg*	9,75	Strut

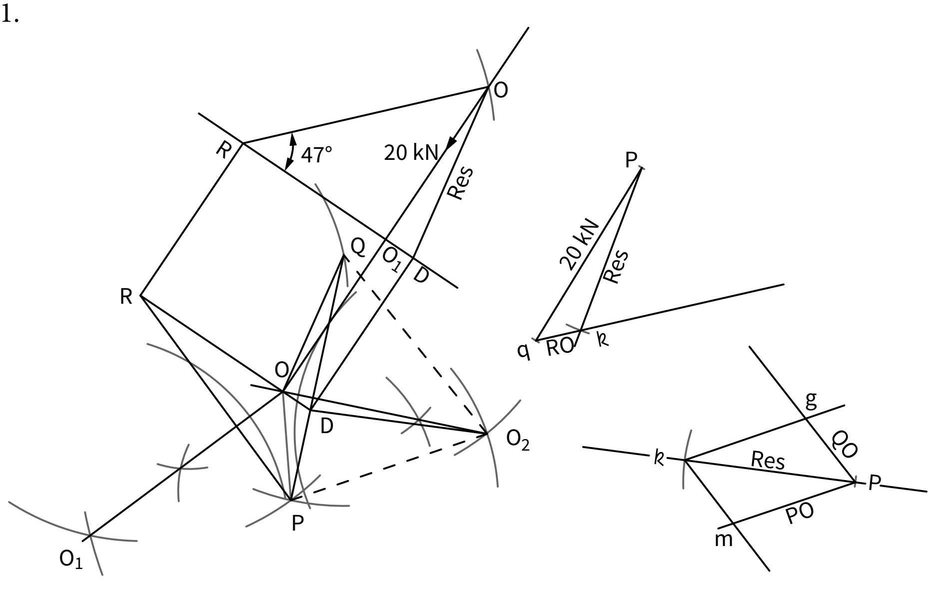
2.



Member	Vector	Force (kN)	Nature
AO	qp	15	Strut
BO	kg	10	Strut
CO	pg	10	strut

Exercise 9.3 SB page 310

1.

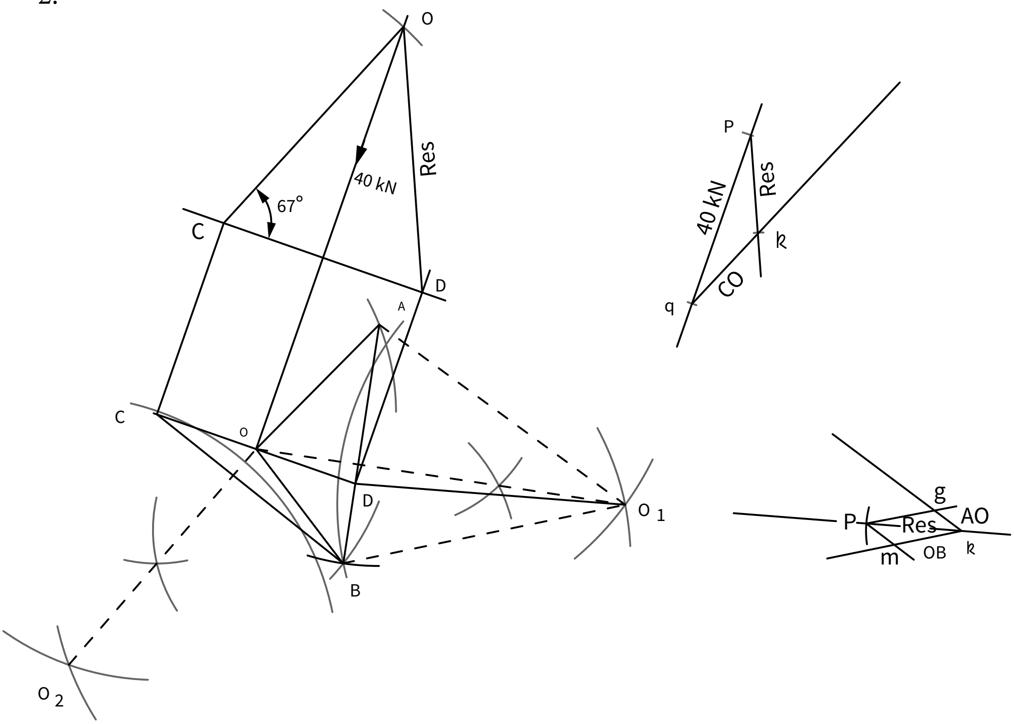


1.1

Member	Vector	Force (kN)	Nature
RO	*qk*	4,4	Strut
QO	*pg*	8	Strut
PO	*mp*	12,8	Strut

1.2 Coefficient of friction $\mu = \frac{\cos 47}{s47} = 0{,}93$

2.

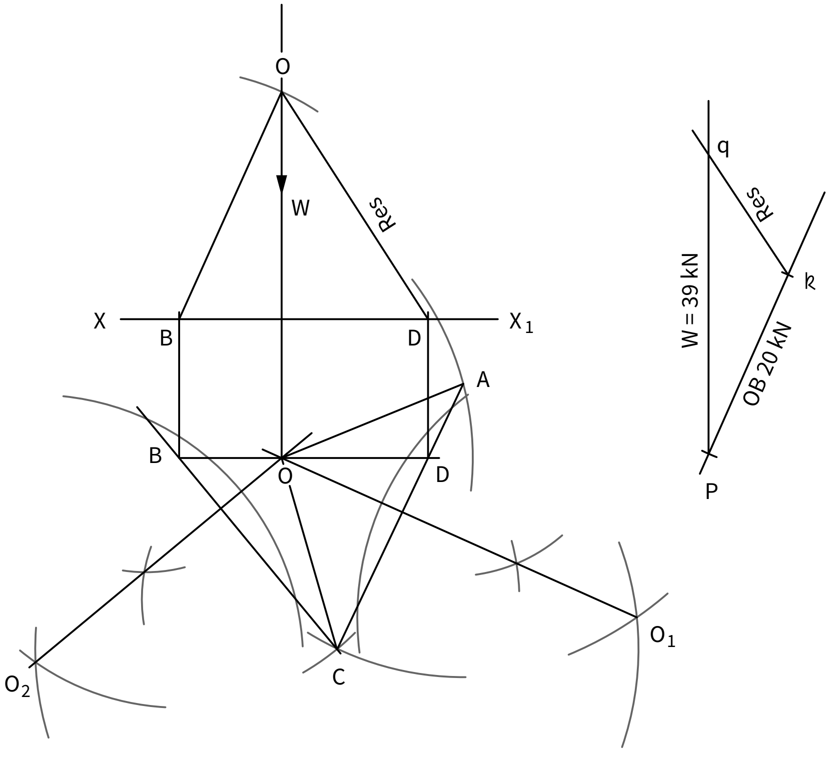


2.1

Member	Vector	Force (kN)	Nature
CO	*qk*	23,5	S
OB	*mk*	16	S
OA	*kg*	7,5	S

2.2 Coefficient of friction $\mu = \frac{\cos 67}{\sin 67} = 0{,}42$

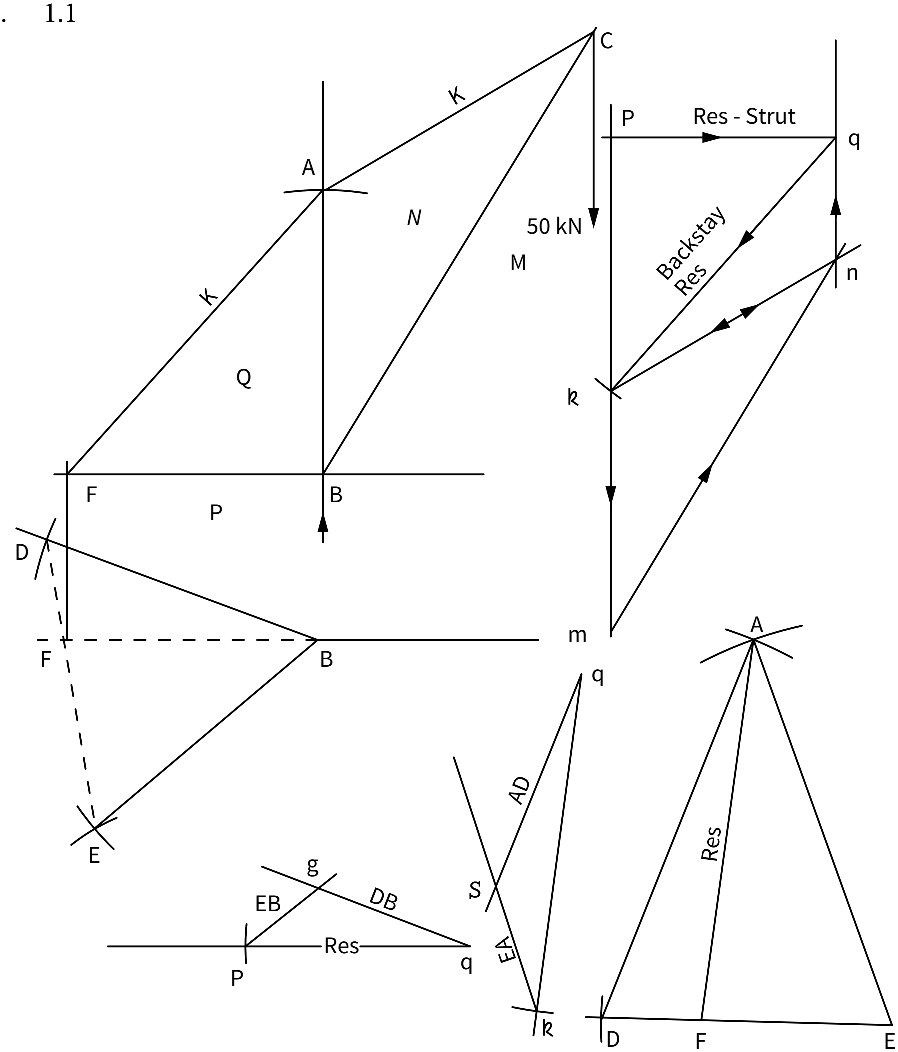
3.



Load = 32 kN

Exercise 9.4 SB page 321

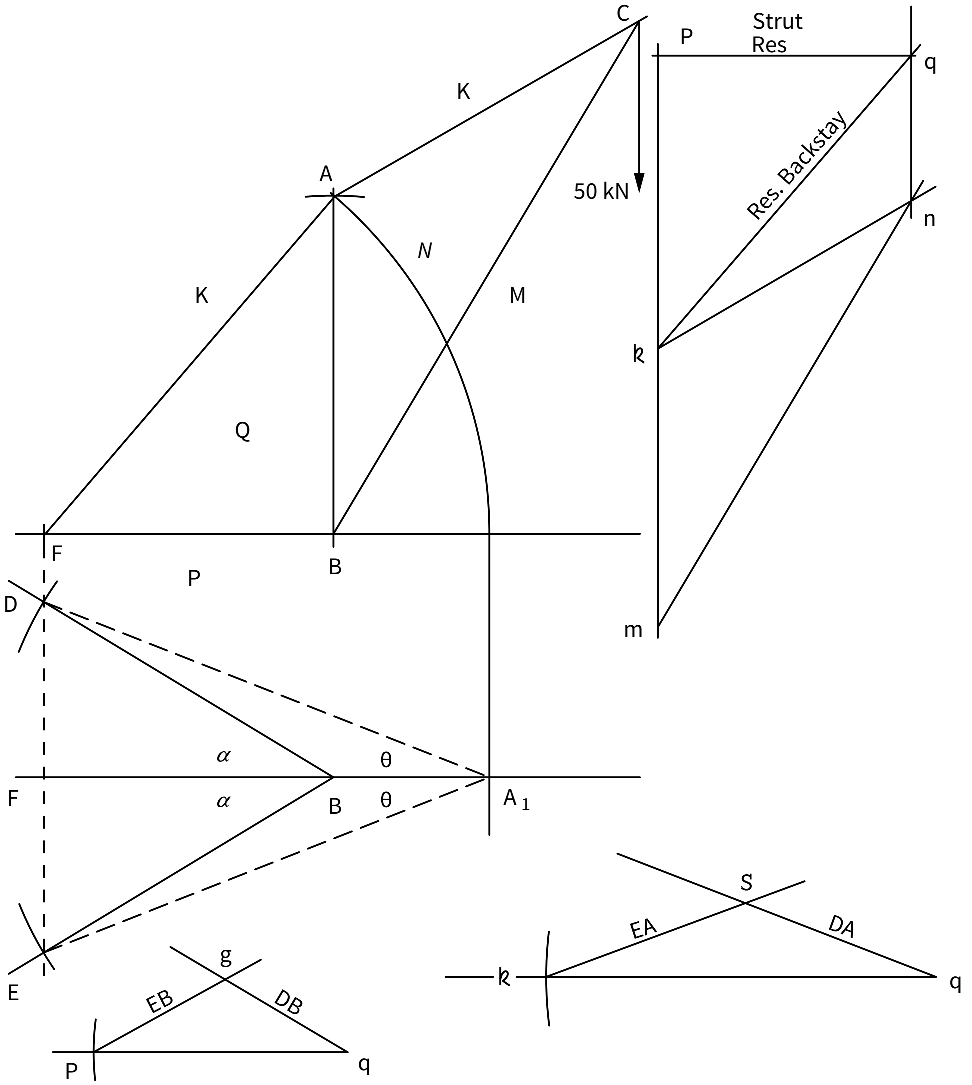
1. 1.1



EA = DA = $\sqrt{6^2 + 6^2}$ = 8,49 m

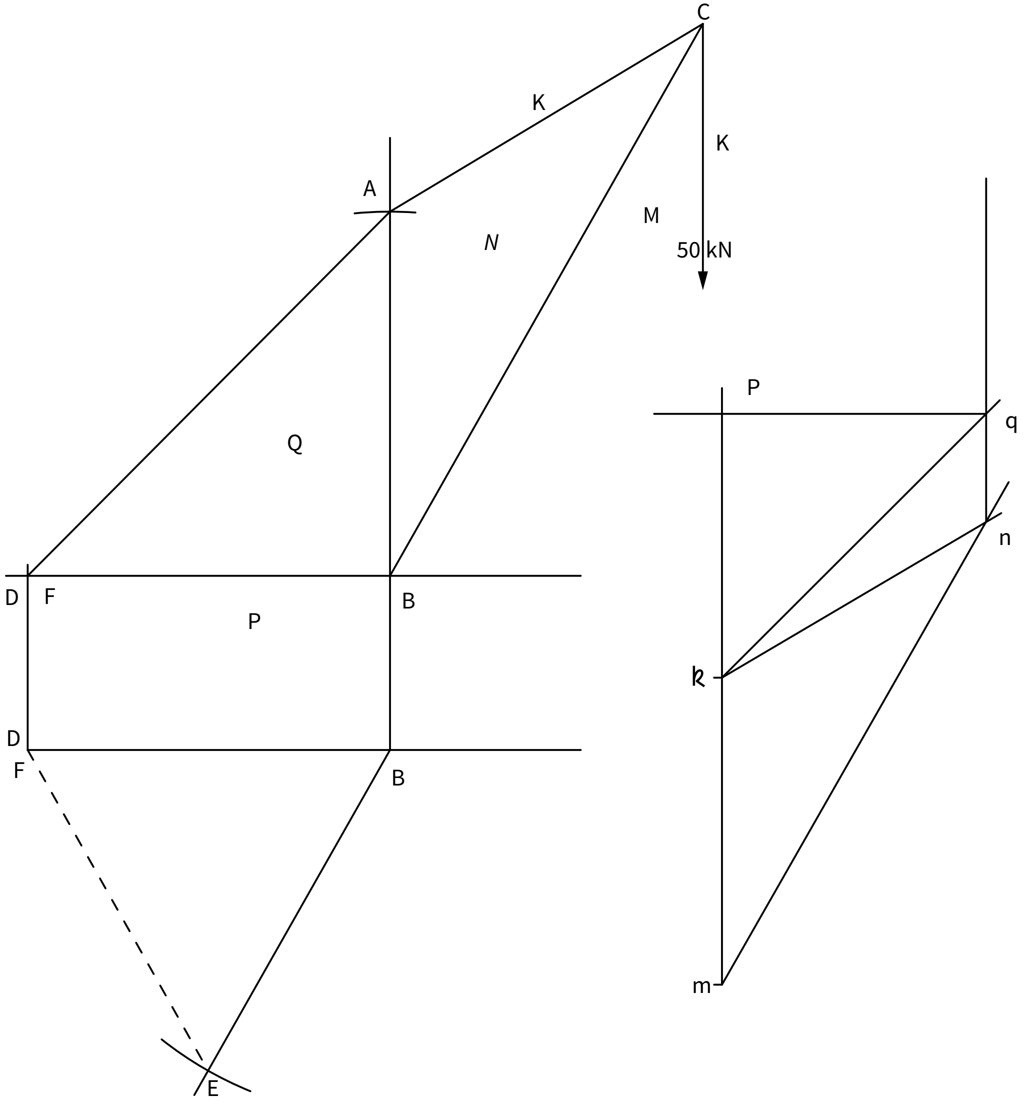
Member	Vector	Force (kN)	Nature
AC	*nk*	54	T
BC	*mn*	92	S
AB	*nq*	24	S
AD	*qs*	42	T
AE	*ks*	26	T
BD	*qg*	35	S
BE	*pq*	19	S

1.2



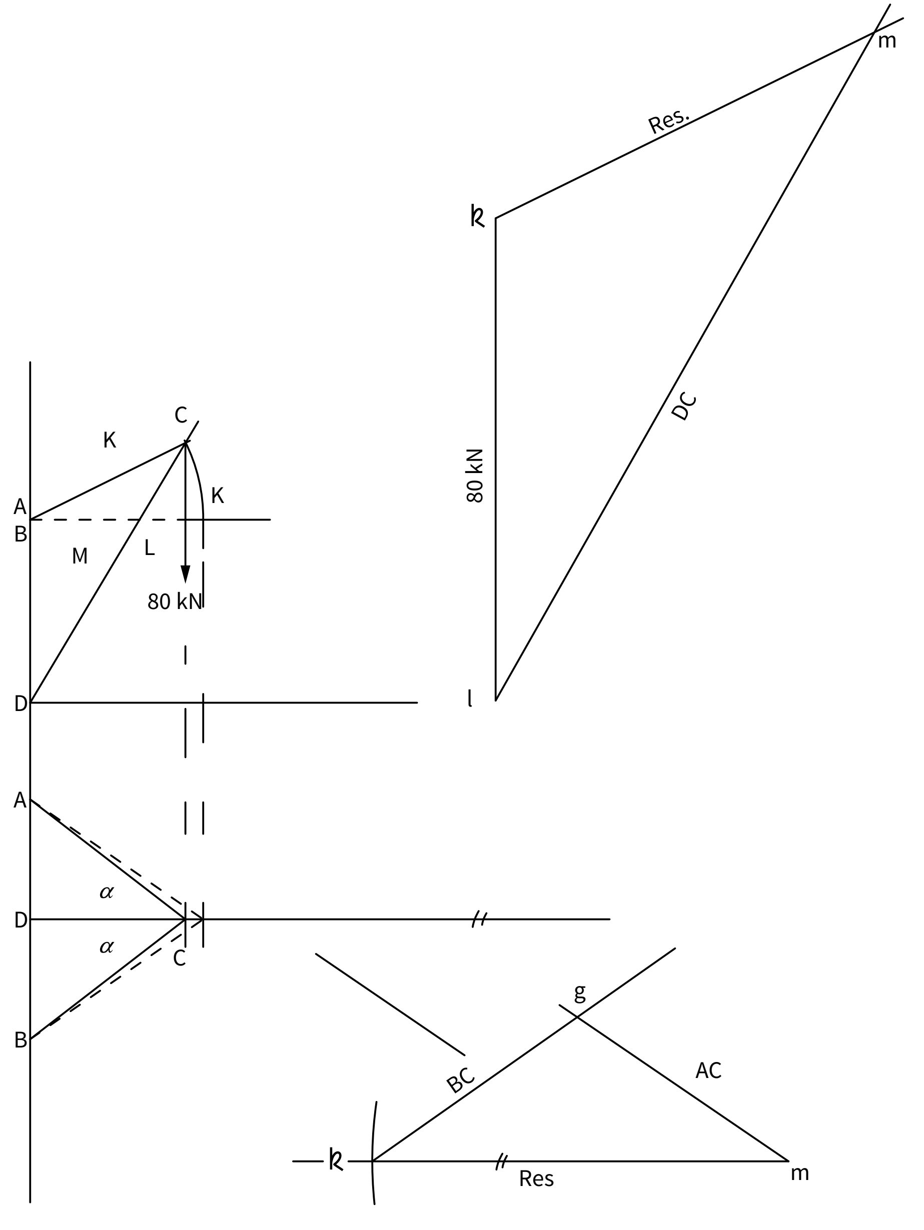
Member	Vector	Force k(N)	Nature
AC	*nk*	54	T
BC	*mn*	92	S
AB	*nq*	24	S
DA	*qs*	37	T
EA	*ks*	37	T
DB	*qg*	26	S
EB	*pg*	26	S

1.3



Member	Vector	Force (kN)	Nature
AC	*nk*	54	T
BC	*mn*	92	S
AB	*nq*	24	S
AD	*qk*	61	S
EA		0	
DB	*pq*	44	S
EB		0	

2.



Member	Vector	Force (kN)	Nature
DC	lm	128	S
AC	mg	42	T
BC	kg	42	T

Exemplar examination paper

Question 1: Thick cylinders

Two hollow cylinders were shrunk together to form a compound cylinder with an inner diameter of 100 mm and an outer diameter of 400 mm. This caused an intermediate pressure at the common diameter of 200 mm. After the cylinders were shrunk together, an internal pressure of 30 MPa was applied to the compound cylinder, causing the resultant hoop stress at the inner diameter to reach 46 MPa (compressive).

Calculate the following:

1.1 The resultant stresses in the inner cylinder at 200 mm (6)

1.2 The resultant stresses in the outer cylinder at 200 mm and 400 mm respectively (6)

1.3 Sketch a stress distribution diagram to indicate the magnitude and nature of the resultant stresses through the compound cylinder walls. (4)

Final answers for solutions:
M = 10^6 and *k* = 10^3 and G = 10^9, where M, *k* and G are not part of an equation.

[16]

Question 2: Tension in cables

The supports of a suspension bridge are 36 m apart and the on the same level. The sag of the cables is 3 m. The roadway has a total weight of 3 024 kN.

Calculate the following:

2.1 The weight per metre carried by each of the two cables (2)

2.2 The minimum and maximum tensions in each cable (2)

2.3 The diameter required for the cable if the ultimate tensile stress for the cable material is limited to 320 MPa (use a safety factor of 8) (3)

2.4 The tension in the cable 10 m from the support, measured horizontally. (3)

[10]

Question 3: Combined bending and twisting of shafts

A solid shaft with a diameter of 80 mm is subjected to a maximum torque of 4 kNm as well as a bending moment. The shear stress in the shaft is limited to 50 MPa and the principal stress is limited to 75 MPa.

Calculate the following:

3.1 The maximum bending moment by considering the shear stress (2)

3.2 The maximum bending moment by considering the principal stress (5)

3.3 The maximum bending moment allowed, and provide a reason (1)

3.4 The actual shear stress in the shaft. (2)

[10]

Question 4: Bending and deflection of beams

A steel pipe, having an inside diameter equal to half of the outside diameter, is used as a cantilever with a length of 2,5 m. It carries a uniformly distributed load of 20 kNm over the first 1,25 m from the fixed end, as well as a concentrated load at the free end. The deflection at the free end is limited to 7 mm. The modulus of elasticity for the material is 200 GPa.

Calculate the following:

4.1 The required dimensions for the pipe (4)

4.2 Choose the lightest taper flange I-profile that can replace the pipe for the same deflection limit. (1)

4.3 The maximum bending stress if the selected I-profile is used. (3)

[8]

Question 5: Combined bending and direct stresses

A crane hook has a circular cross section with a diameter of 30 mm as shown below. The distance that the load is applied from the centroid is 90 mm. The tensile stress in the material of the crane hook may not exceed 90 MPa.

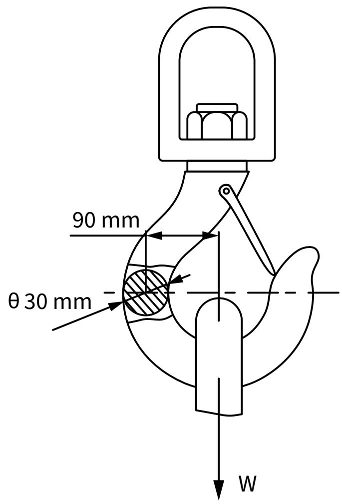


Figure 1: Crane hook

Calculate the following:

5.1 The maximum mass that may be lifted (4)

5.2 The minimum stress in the hook in magnitude and nature. (2)

[6]

Question 6: Shear stress in beams

A built-up beam consists of a hot-rolled parallel flange I-beam with dimensions of 305 × 165 × 53,6 kg/m, and a channel bolted to the I-beam by two rows of bolts. The height of the channel is 250 mm and the breadth is 80 mm, with a thickness of 20 mm right through. The build-up beam is subjected to a shear force of 100 kN. The web is connected to the flange by means of two 17-mm diameter bolts and the shear stress in the bolts is 80 MPa.

Calculate the spacing of the bolts. **[14]**

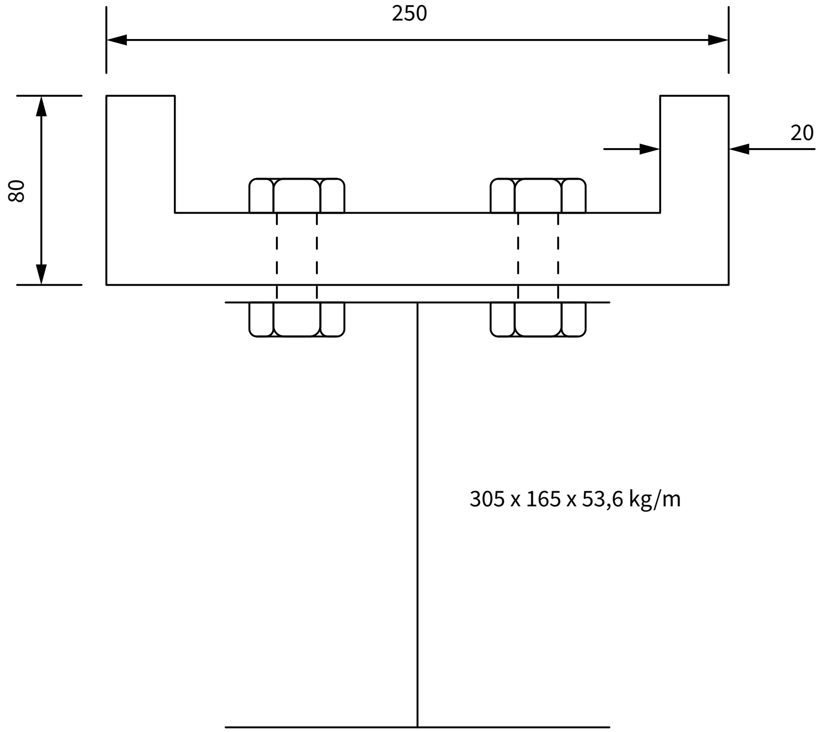


Figure 2: Built-up beam

Question 7: Close-coiled helical springs

A maximum load of 45 N compresses a coil spring to a solid length of 45 mm. The stiffness of the spring is 900 Nm and the maximum shear stress in the wire is 120 MPa. G = 40 GPa.

Calculate:

7.1 The wire diameter of the spring (8)

7.2 The diameter of the spring (1)

7.3 The number of coils. (1)

[10]

Question 8: Transformation of stress

At a point in a material, the principal stresses are 40 MPa and 100 MPa respectively, and both stresses are tensile. Turn the element 15° anti-clockwise to the plane on which the principal stresses act.

Calculate:

8.1 The normal stresses on this plane for the x-face (3)

8.2 The shear stress on this plane (2)

8.3 Use Mohr's circle to verify the answers on the stress circle. (8)

8.4 Use the circle and determine the maximum shear stress as well as the normal stress on the X-face if the element is turned 90° in the same direction to the plane above. (1)

[14]

Question 9: Forces in structural frameworks

The legs of a tripod are each 6 m long and are placed in such a way as to form an equilateral triangle ABC with sides 5 m on the ground. The tripod supports a load of 6 kN from the apex.

Use a scale of 1 cm = 1 m for the space diagram.
Use a scale of 1 cm = 1 kN for the vector diagram.

9.1 Draw the side- and top views of the tripod to the given scale to determine the apex. (6)

9.2 Draw the vector diagrams to the given scale and determine the force in each leg. Redraw the following table in your answer book and tabulate the answers. (3)

Member	Magnitude

9.3 Calculate the minimum coefficient of friction required between the legs and the ground to prevent slipping. (3)

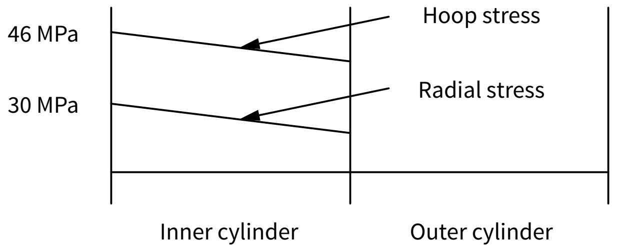
[12]

Total: 100 marks

Exemplar examination paper memorandum

Question 1: Thick cylinders

1. 1.1 Resultant stresses in the inner cylinder at 200 mm



At $d = 100$; $\sigma_R = 30\text{M} = a + \frac{b}{0{,}1^2}$

$\therefore 30\text{M} = a + 100b \ldots$ ①

and at $d = 1i = a - \frac{b}{0{,}1^2}$

$\therefore 46\text{M} = a - 100b \ldots$ ②

① + ②: $\therefore 76\text{M} = 2a$

$\therefore a = 38\text{M} \ldots$ ③

Substitute ③ into ①: $\therefore 30\text{M} = 38\text{M} + 100b$

$\therefore b = -0{,}08\text{M}$

Stresses at D = 200 mm for inner cylinder

Hoop stress: Dc = 200 $\sigma_H = 38\text{M} - \frac{-0{,}08\text{ M}}{0{,}2^2} = 40\text{ MPa}$

and Radial stress: Dc = 200 $\sigma_R = 38\text{M} + \frac{-0{,}08\text{ M}}{0{,}2^2} = 36\text{ MPa}$ (6)

1.2 The resultant stresses in the outer cylinder at 200 mm and 400 mm

At D = 400; $\sigma_R = 0 = a + \frac{b}{0{,}4^2}$

$\therefore a = -6{,}25b \ldots$ ④

At Dc = 200: $\sigma_R = 36\text{M} = a + \frac{b}{0{,}2^2} \ldots$ ⑤

$\therefore$ Substitute (4) into (5) $\therefore 36\text{M} = -6{,}25b + 25b$

$\therefore b = 1{,}92\text{M}$

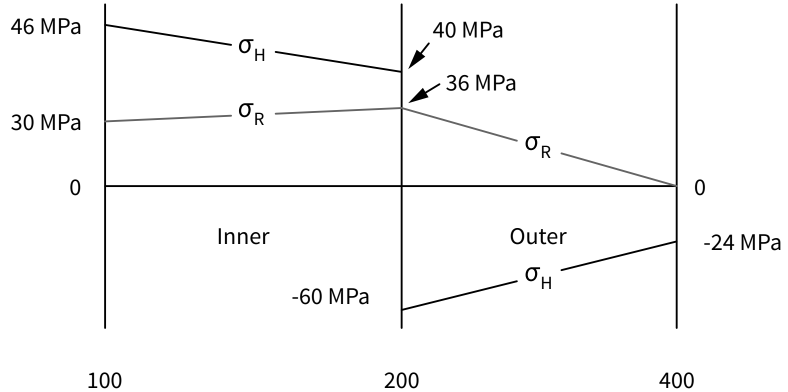
$\therefore a = -12\text{M}$

at $d = 200$; $\sigma_H = -12\text{M} - \frac{1{,}92\text{ M}}{0{,}2^2} = -60\text{ MPa (T)}$

at $d = 400$; $\sigma_H = -12M - \frac{1{,}92\ M}{0{,}4^2} = -24$ MPa (T)

Radial stress at 200 = 36 MPa
Radial stress a tD = 400 = 0 MPa (6)

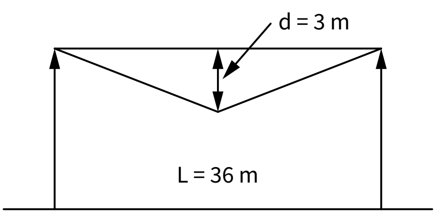
1.3 Stress distribution diagram for compound cylinder



(4)

Question 2: Tension in cables

2.



2.1 Weight per metre carried by each of the two main cables

Load per metre on bridge $= \frac{\text{Load on bridge}}{\text{Length}} = \frac{3\ 024k}{36} = 84$ kNm

Load per metre per cable $= \frac{\text{Load per metre on bridge}}{2} = \frac{84\text{k}}{2} = 42$ kNm (2)

2.2 Minimum and maximum tension in each cable

$\therefore F_H = F_{min} = \frac{wL^2}{8d} = \frac{42k\tau \times 36^2}{8 \times 3} = 2{,}268$ MN

$\therefore F_{tmax} = \sqrt{F_H^2 + \left(w\frac{L}{2}\right)^2} = \sqrt{(2{,}268\ M)^2 + \left(42k \times \frac{36}{2}\right)^2} = 2{,}391$ MN (2)

2.3 Diameter of the cable with an ultimate stress of material of 320 MPa and a factor of safety of 8

Working safe stress $= \frac{\sigma_{Ult}}{FOS} = \frac{320\ M}{8} = 40$ MPa

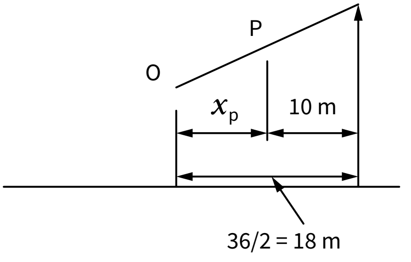
$\therefore \sigma_{safe} = \frac{F_{Tmax}}{A_{cable}}$

$\therefore A_{cable} = \frac{F_{Tmax}}{\sigma_{safe}} = \frac{2{,}391\ M}{40\ M} = 0{,}059775\ m^2$

$\therefore A = \frac{\pi}{4}D^2$

$\therefore D = \sqrt{\frac{0{,}059775 \times 4}{\pi}} = 275{,}877\ mm$ (3)

2.4 Tension in cable 10 m from the support measured horizontally



Tension in the cable at point P: From turning point O = x_p = 18 – 10 = 8 m

$\therefore$ Tension at P = $F_{tp} = \sqrt{F_H^2 + (wx_p)^2}$

$\therefore F_{tp} = \sqrt{(2{,}268\ M)^2 + (42k \times 8)^2} = 2{,}293\ MN$ (3)

Question 3: Combined bending and twisting of shafts

3. 3.1 $\therefore T_e = \frac{\pi D^3}{16}\tau = \frac{\pi 0{,}08^3}{16} \times 50 \times 10^6 = 5{,}027\ kNm$

$\therefore T_e = \sqrt{M^2 + T^2}$

$\therefore M = \sqrt{5\ 027^2 - 4\ 000^2} = 3{,}045\ kNm$ (2)

3.2 Maximum BM principal stress:

$\therefore M_e = \frac{\pi D^3}{32}\sigma = \frac{\pi \times 0{,}08^3}{32} \times 75 \times 10^6 = 3{,}77\ kNm$

$\therefore M_e = 0{,}5\left[M + \sqrt{M^2 + T^2}\right]$

$\therefore 2\ M_e - M = \sqrt{M^2 + T^2}$

$\therefore (2 \times 3\ 770 - M)^2 = M^2 + 4\ 000^2$

$7\ 540^2 - 7\ 540M - 7\ 540M + M^2 = M^2 + 4\ 000^2$

$\therefore 15\ 080M = 7\ 540^2 - 4\ 000^2$

$\therefore M = \frac{40{,}852 \times 10^6}{15\ 080} = 2{,}709\ kNm$ (5)

3.3 Maximum bending moment for shaft

$M_{max} = 2{,}709$ kNm

A bending moment of 3,045 kNm will cause a principal stress higher than 75 MPa and the shaft will fail. (1)

3.4 Actual shear stress:

$$\therefore T_e = \sqrt{M^2 + T^2} = \sqrt{2\,709^2 + 4\,000^2} = 4{,}831 \text{ kNm}$$

$$\therefore T_e = \frac{\pi D^3}{16}\tau$$

$$\therefore \tau = \frac{16 \times 4\,831}{\pi \times 0{,}08^3} = 48{,}055 \text{ MPa} \quad (2)$$

Question 4: Bending and deflection of beams

4. 4.1 Dimensions for the pipe:

$$\Delta_{max@A} = \Delta_{UDL@A} + \Delta_{pl@A}$$

$$\therefore 0{,}007 = \left[\frac{wa^4}{8EI} + \left(\frac{wa^3}{6EI} \times b\right)\right] + \frac{WL^3}{3EI}$$

$$\therefore 0{,}007I = \left[\frac{20k \times 1{,}25^4}{8 \times 200G} + \left(\frac{20k \times 1{,}25^3}{6 \times 200G} \times 1{,}25\right)\right] + \frac{2k \times 2{,}5^3}{3 \times 200G}$$

$$\therefore 0{,}007I = 3{,}052 \times 10^{-8} + 4{,}069 \times 10^{-8} + 5{,}208 \times 10^{-08}$$

$$\therefore I = 17{,}613 \times 10^{-6} \text{ m}^4$$

$D = 2d$ given

$$\therefore I = 17{,}613 \times 10^{-6} = \frac{\pi}{64}\left([2d]^4 - d^4\right)$$

$$\therefore 15d^4 = 3{,}588 \times 10^{-4}$$

$$\therefore d = \sqrt[4]{2{,}392 \times 10^{-5}} = 69{,}934 \text{ mm} \quad (4)$$

$D = 139{,}869$ mm

4.2 Select the lightest I-section taper flange

$I = 17{,}613 \times 10^{-6}$

$203 \times 102 \times 25{,}3$ kg/m

$I = 22{,}97 \times 10^{-6}$ m^4 (1)

4.3 Maximum bending stress in the selected I-section

$$\therefore M_{max} = M_{UDL} + M_{pl} + M_{weight}$$

$$M_{max} = \frac{wa^2}{2} + wL + \frac{w_{weight}L^2}{2}$$

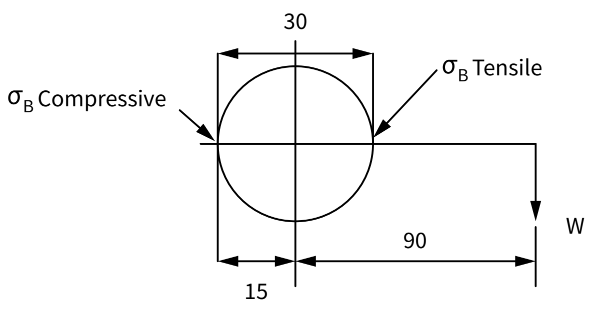
$$M_{max} = \frac{20k \times 1{,}25^2}{2} + 2k \times 2{,}5 + \frac{25{,}3 \times 9{,}81 \times 2{,}5^2}{2}$$

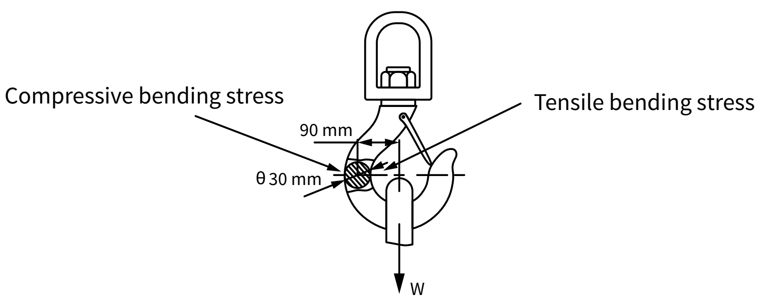
$$M_{max} = 21{,}401 \text{ kNm}$$

$$\therefore \text{Maximum stress} = \frac{My}{I} = \frac{21401 \times 0{,}2032}{22{,}97 \times 10^{-6} \times 2} = 94{,}66 \text{ MPa}$$ (3)

Question 5: Combined bending and direct stress

5. 5.1 The mass that can be lifted





$$\text{Direct stress} = \sigma_D = \frac{W}{\frac{\pi}{4}0{,}03^2} = 1\ 414{,}711W \text{ Pa (Tensile)}$$

$$\text{Bending stress} = \sigma_B = \frac{My}{I} = \frac{(W \times 0{,}09) \times 0{,}015}{\frac{\pi}{64}0{,}03^4 = 33\ 953{,}055W}$$

$$\therefore \sigma_{max} = \sigma_D + \sigma_B = -90M = -1\ 414{,}711W - 33\ 953{,}055W$$

$$\therefore W = 2{,}545 \text{ kN}$$

$$\therefore \text{Mass} = \frac{2{,}545k}{9{,}81} = 259{,}398 \text{ kg}$$ (4)

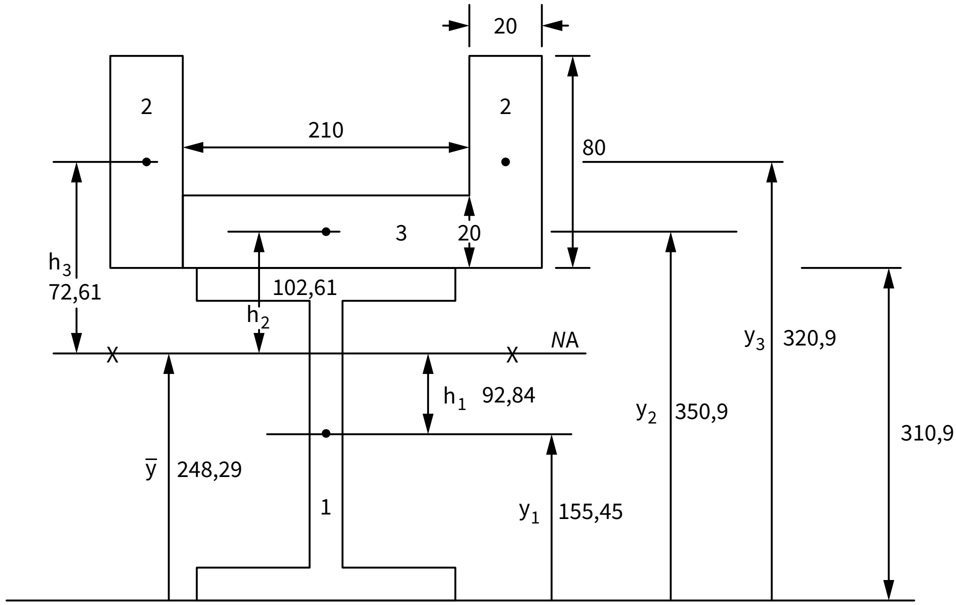
5.2 The minimum stress in the hook, magnitude and nature

$$\sigma_{min} = -\sigma_D + \sigma_B$$

$$\sigma_{min} = -(1\ 414{,}711 \times 2{,}545k) + (33\ 953{,}055 \times 2{,}545k)$$

$$= 82{,}81 \text{ MPa (Compressive)}$$ (2)

Question 6: Shear stress in beams



No	Area	Y-distance	Ay
1	$6{,}821 \times 10^{-3}$	0,15545	$1{,}0603 \times 10^{-3}$
2	$2 \times 0{,}08 \times 0{,}02 = 3{,}2 \times 10^{-3}$	0,3509	$1{,}1229 \times 10^{-3}$
3	$0{,}21 \times 0{,}02 = 4{,}2 \times 10^{-3}$	0,3209	$1{,}3478 \times 10^{-3}$
A total	0,014221	ΣA – moments	$3{,}531 \times 10^{-3}$

$\bar{y}A_T = \Sigma A - \text{moments}$

$\bar{y} = \frac{3{,}531 \times 10^{-3}}{0{,}014221} = 248{,}29 \text{ mm}$

$h_1 = \bar{y} - y_1 = 248{,}29 - 155{,}45 = 92{,}84 \text{ mm}$

$h_2 = y_2 - \bar{y} = 350{,}9 - 248{,}29 = 102{,}61 \text{ mm}$

$h_3 = y_3 - \bar{y} = 320{,}9 - 248{,}29 = 72{,}61 \text{ mm}$

$I_{xx\,\text{Total}} = I_1 + I_2 + I_3$

$I_1 = 116{,}9 \times 10^{-6} + (6{,}821 \times 10^{-3} \times 0{,}09284^2) = 1{,}7569 \times 10^{-4} \text{ m}^4$

$I_2 = 2\left[\frac{0{,}02 \times 0{,}08^3}{12} + (3{,}2 \times 10^{-3} \times 0{,}10261^2)\right] = 3{,}4546 \times 10^{-5} \text{ m}^4$

$I_3 = \left[\frac{0{,}21 \times 0{,}02^3}{12} + (4{,}2 \times 10^{-3} \times 0{,}07261^2)\right] = 2{,}2283 \times 10^{-5} \text{ m}^4$

$\therefore I_{xx} = 2{,}3252 \times 10^{-4} \text{ m}^4$

First moment of area $= Q = 2A_2 y_2' + A_3 y_3'$

$Q = 2 \times 32, \times 10^{-3} \times 0{,}10261 + (4{,}2 \times 10^{-3} \times 0{,}07261) = 9{,}617 \times 10^{-4}\ m^3$

Shear force at joint $= q = \frac{VQ}{I} = \frac{100k \times 9{,}617 \times 10^{-4}}{2{,}3252 \times 10^{-4}} = 413{,}599$ kN/m

Spacing of bolts over 1 m: $R = A_{bolt} \tau_{bolt} n1$

$R = \frac{\pi \times 0{,}017^2}{4} \times 80M \times n \times 1 = 18{,}158n$ kN

Force in bolt = Force in joint

$R = q \quad \therefore 18\ 158\ n = 413\ 599$

Number of bolt $= n = 22{,}77$ use 23 bolts

Spacing $= S = \frac{\text{Unit length} \times \text{number of rows}}{\text{Number of bolts}} = \frac{1\ 000 \times 2}{23} = 86{,}96$ mm (14)

Question 7: Closed-coiled helical springs

7. 7.1 Wire diameter

Deflection $= \delta = \frac{W}{S} = \frac{45}{900} = 0{,}05$ m

Strain energy $= U = \frac{1}{2} W\delta = 0{,}5 \times 45 \times 0{,}05 = 1{,}125$ J

But $U = \frac{\tau^2 V}{4G} \quad \therefore$ Volume $= \frac{1{,}125 \times 4 \times 40 \times 10^9}{(120 \times 10^6)^2} = 1{,}563 \times 10^{-5}\ m^3$

Also volume of wire $= \pi D n \times \frac{\pi d^2}{4} = 2{,}467\ Dnd^2 = 1{,}563 \times 10^{-5}$ … ①

Number of coils $= n = \frac{L_{solid}}{d} = \frac{0{,}045}{d}$ … ②

Shear stress wire $= \tau = \frac{8WD}{\pi d^3} \quad \therefore D = \frac{120 \times 10^6 \times \pi \times d^3}{8 \times 45}$

$D = 1{,}047 \times 10^6 d^3$ … ③

Substitute ② and ③ into ①:

$\therefore 2{,}467 \times 1{,}047 \times 10^6 d^3 \times \frac{0{,}045}{d} d^2 = 1{,}563 \times 10^{-5}$

$\therefore d^4 = 1{,}344 \times 10^{-10}$

$\therefore d = 3{,}41$ mm (8)

7.2 Spring diameter

$\therefore D = 1{,}047 \times 10^6 \times 0{,}00341^3 = 41\ 52$ mm (1)

7.3 Number of coils

$n = \frac{45}{3{,}24} = 13{,}89$ coils (1)

Question 8: Transformation of stress

8. 8.1 Normal stress x-face

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x1} = \frac{100 + 40}{2} + \frac{100 - 40}{2}\cos - 30 + 0$$

$$\sigma_{x1} = 95{,}98 \text{ MPa} \qquad (3)$$

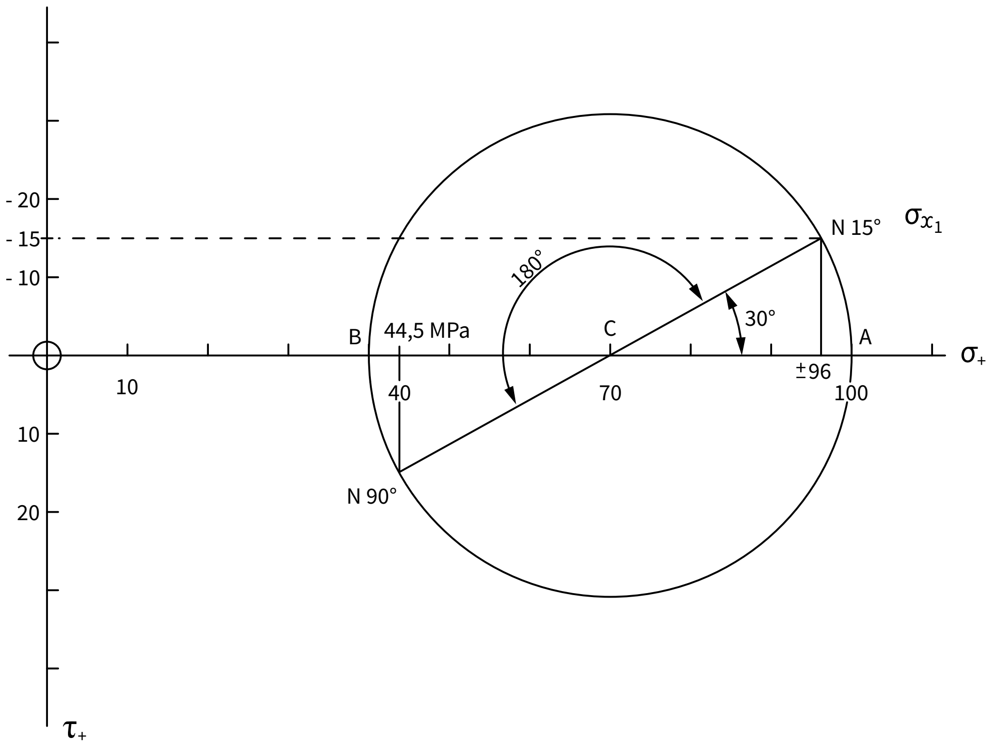
8.2 Shear stress

$$\tau_{xy} = -\left[\frac{\sigma_x - \sigma_y}{2}\right]\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{xy} = -\left[\frac{100 - 40}{2}\right]\sin - 30 + 0$$

$$\tau_{xy} = -(-15) = 15 \text{ MPa} \qquad (2)$$

8.3 Mohr's circle 15°



$$\sigma_{x1} = \pm 96 \text{ MPa}$$

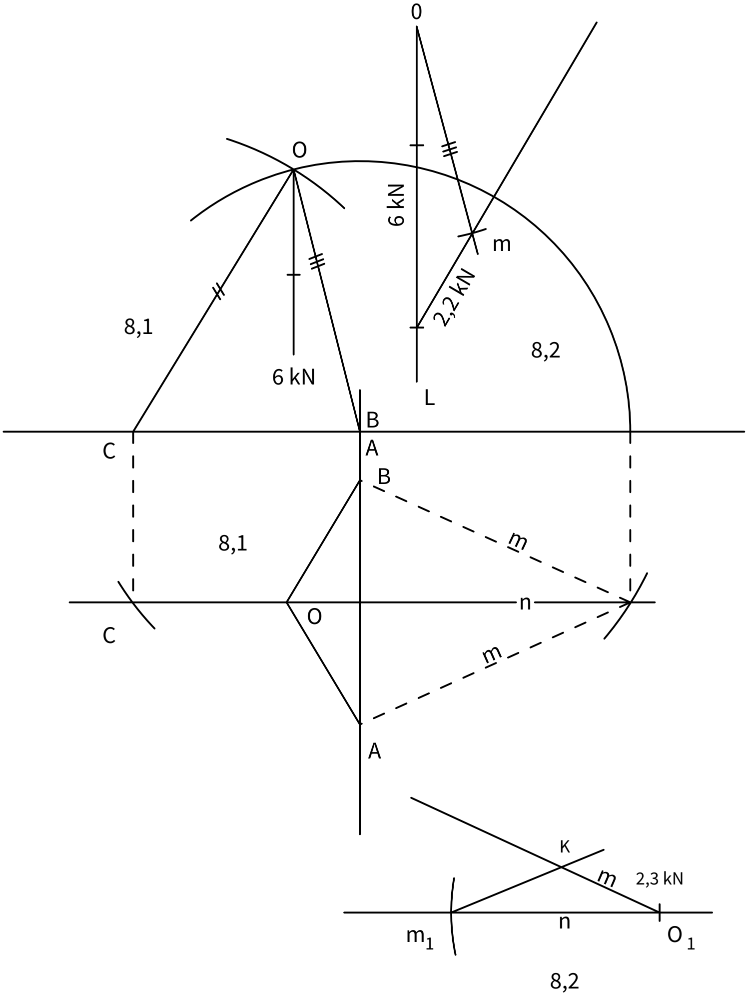
$$\tau_{xy} = 15 \text{ MPa} \qquad (8)$$

8.4 90°

$$\sigma_{x1} = \pm 44{,}5 \text{ MPa} \qquad (1)$$

Question 9: Forces in structural frameworks

9. 9.1



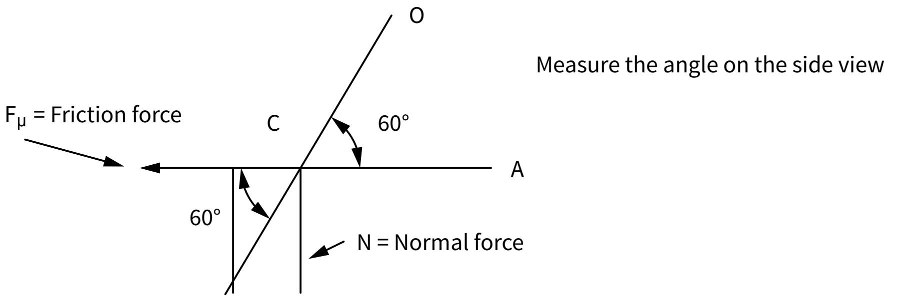
(6)

9.2 Tabulated answers

Member	Magnitude
OC	2,2 kN
OA	2,3 kN
OB	2,3 kN

(3)

9.3 Coefficient of friction between legs and the ground



From the side view angle ACO = 60° and the force in leg CO = 2,2 kN

$\therefore$ Coefficient of friction = $\mu = \frac{F_\mu}{N} = \frac{2\ 200 \cos 60°}{2\ 200 \sin 60°} = 0{,}577$ (3)

Total: 100 marks

Glossary

B

Bending stress – stress caused by the bending moment; also known as *flexural stress*

Bow's notation – a method of lettering the cells and outside spaces formed by the directions of the loads on a framed structure so that these loads can be traced

Bridge deck – the surface of a bridge

C

Catenary – the U-shaped curve formed by a heavy inextensible cable, chain or rope of uniform density suspended from its endpoints

Coefficient of friction (μ) – the ratio of the frictional force between two bodies and the force pressing them together

Common diameter (d_C) – the shared diameter of a shaft and sleeve that has undergone shrinkage

Constant horizontal force – when forces are balanced, ie. equal in magnitude with opposite directions

D

Deflection – the moving of a beam from its original position

E

Elastic stress limit – also called the *elastic limit*, is the point at which an object no longer returns to its original form

Equivalent twisting moment – the twisting moment which, when acting alone, will produce the same maximum shear stress as the applied twisting and bending moment

F

First area moment – or the first moment of area; a measure of a shapes spatial distribution in relation to its axis

Flexural stiffness – also known as *flexural rigidity*, it is the measure of deformability or the stiffness of a material subjected to bending

Flue – a duct for smoke or waste gases produced by fires

Form factor – a ratio indicating the efficiency of a building's shape

Frictionless rollers – roller supports that do not impede or create a frictional force

H

Hooke's law – strain is directly proportional to the stress that causes it

Hub – the central part of something, such as a sleeve which can be placed inside a cylinder

I

Intermediate pressure – the radial stress at the contact diameter

L

Lamé's theory – a theorem to find the solution to thick-cylinder problems

Lateral strain – the ratio of the change in diameter of a circular bar of a material to its diameter due to deformation in the longitudinal direction

O

Overhang – the distance by which the apex overhangs the base of the legs

P

Parabolic – like a parabola

Parabolic catenary – a catenary which forms a parabolic shape

Parallel axis theorem – the theorem states that the moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis passing through its centre of mass, and the product of its mass and the square of the perpendicular distance between the two parallel axes

Poisson's ratio – a constant that relates longitudinal strain in the direction of the load to lateral strain perpendicular to the load; it defines the ratio of how a material will increase lengthwise and contract widthwise when stretched

Press fit – a form of fastening between two tight-fitting parts that produces a joint held together by the friction of the parts pushing together

Proof load – the greatest load which a spring can carry without exceeding the elastic limit of the material

Proof resilience – energy stored in a spring

Proof stress – the stress at which a specified permanent elongation has taken place in the tensile test

Proportional limit – the point on a stress-strain curve where the linear, elastic deformation region transitions into a non-linear, plastic deformation region; the proportional limit determines the greatest stress that is directly proportional to strain; also known as limit of proportionality

R

Radius of curvature – the ratio of bending and moment that acts in the beam cross section; is denoted by the Greek symbol rho (ρ)

S

Section modulus – a geometric property used when designing beams or flexural members

Sheerleg – a crane consisting of two legs of equal length and a backstay

Shrinkage allowance – the difference between the change in diameter of the inner cylinder and the change in diameter of the outer cylinder

Sleeve – a tube of material which can be place inside a cylinder

Stiffness – the resistance of an elastic body to deflection or deformation by an applied force

T

Tangential force – a force acting on a moving body in the direction of a tangent to the curved path of the body

Transverse shear stress – the resistance force developed per unit in a cross-sectional area by an object to avoid deformation

Y

Young's modulus – the property of a material that indicates how easily it can stretch and deform; it is defined as a constant given the ratio of tensile stress (σ) to tensile strain (ε)