# N5

# *Strength of Materials and Structures*

## *Lecturer Guide*

**Henry T. Wickens**

Additional resource material for this title includes:

- Electronic Lecturer Guide
- Exemplar examination paper and memorandum
- PowerPoint presentation
- Past exam papers

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Tel (021) 462 3572
Fax (021) 462 3681
E-mail: info@futuremanagers.com
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# Lecturer guidance

## 1. General aims

This subject builds on the basic knowledge attained in N4 Engineering Science. This subject involves knowledge of various systems and components, hence when presenting modules for the subject, it should be ensured that the students understand each basic scientific principle in such a way that they will be able to integrate this knowledge into their applied subjects.

## 2. Specific aims

On completion of all the modules in N5 Strength of Materials and Structures, students should be able to apply the scientific principles mastered to their specific trade theory. Students should be able to apply SI units and derived units correctly. Students should be able to demonstrate understanding of subject content through the application of acquired knowledge. Students should also be able to solve problems by using subject content.

Students should be able to acquire in-depth knowledge of the following content:
1. Stress, strain and tensile testing of materials
2. Strain energy
3. Temperature-induced stresses
4. Thin cylinders and Mohr's circle
5. Simple bending of beams
6. Columns and struts
7. Shafts
8. Structural frameworks

## 3. Prerequisites

Students must have a passed N4 Engineering Science.

## 4. Duration

Full-time: 7,5 hours per week. This instructional offering may also be offered part-time or in distance-learning mode.

## 5. Evaluation

Candidates must be evaluated continually as follows:

### 5.1 ICASS trimester mark

5.1.1 Two formal class tests must be written for full-time and part-time students (or two assignments for distance-learning students only)

5.1.2 Students must obtain a minimum of 40% in order to qualify to write the final examination.

5.1.3 Assessment marks are valid for a period of one year and are referred to as ICASS trimester marks.

5.1.4 Calculation of trimester mark:
Weight of test or assignment 1 = 30% of the syllabus
Weight of test or assignment 2 = 70% of the syllabus.

## 5.2 Examination

5.2.1 The examination shall consist of 100% of the syllabus.

5.2.2 The duration shall be three hours.

5.2.3 The minimum pass percentage shall be 40%.

5.2.4 This is a closed-book examination.

5.2.5 Knowledge, understanding, application and evaluation are important aspects of the subject and should be weighted as follows:

| Knowledge | Understanding | Application | Evaluation |
|---|---|---|---|
| 60% | 20% | 15% | 5% |

## 5.3 Promotion mark

The promotion mark, consisting of the combination of the trimester and examination marks, shall be a minimum of 40%.

# 6. Weighted values of modules

| Modules | Weighting (%) |
|---|---|
| 1. Stress, strain and tensile testing of materials | 8 |
| 2. Strain energy | 8 |
| 3. Temperature-induced stresses | 14 |
| 4. Thin cylinders and Mohr's circle | 14 |
| 5. Simple bending of beams | 14 |
| 6. Columns and struts | 14 |
| 7. Shafts | 14 |
| 8. Structural frameworks | 14 |
| **Total** | **100** |

# 7. Work schedule

| Week | Topic | Content | Hours |
|---|---|---|---|
| 1 | **Module 1**<br>Stress, strain and tensile testing of materials | 1.1 Stress and strain and tensile testing of materials<br>1.2 Compound bars | 8 hours |
| 2 | **Module 2**<br>Strain energy | 2.1 Strain energy<br>2.2 Gradually applied load<br>2.3 Suddenly applied load<br>2.4 Shock load | 8 hours |
| 3–4 | **Module 3**<br>Temperature-induced stresses | 3.1 Temperature-induced stresses<br>3.2 Resultant stresses | 14 hours |
| 4–5 | **Module 4**<br>Thin cylinders and Mohr's circle | 4.1 Stresses in thin cylinders<br>4.2 Strain in thin cylinders<br>4.3 Mohr's stress circle | 14 hours |
| 5–6 | **Module 5**<br>Simple bending of beams | 5.1 The theory of simple bending<br>5.2 Hot-rolled structural steel sections | 14 hours |
| 7–8 | **Module 6**<br>Columns and struts | 6.1 Introduction to columns and struts<br>6.2 Fixing of ends and calculations | 14 hours |
| 9 | **Module 7**<br>Shafts | 7.1 Introduction to limits in shaft design<br>7.2 Circular and compound shaft calculations | 14 hours |
| 10 | **Module 8**<br>Structural frameworks | 8.1 Introduction to structural frameworks<br>8.2 Calculations on structural frameworks | 14 hours |
| **TOTAL** | | | **100 hours** |

## 8. Lesson plan template

| LESSON | Content/Outcomes to be covered this week | List of examples to be done in class by the lecturer to explain the outcome/ concept | Facilitation method (Please tick) | | Teaching resources/ aids (Please tick) | | Student activity (exercise in textbook/additional supporting task) to be done this week |
|---|---|---|---|---|---|---|---|
| WEEK 1 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | Introduction to lessons | | | | |
| | | | | | | | |
| | | | Recapping/Reinforcement | | | | |
| | | | | | | | |

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<tr><td rowspan="8">WEEK 2</td><td rowspan="8"></td><td rowspan="8"></td><td>Lecture</td><td></td><td>White board/ OHP</td><td></td><td rowspan="8"></td></tr>
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|---|---|---|---|---|---|---|---|
| WEEK 3 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | Introduction to lessons | | | | |
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|---|---|---|---|---|---|---|---|
| WEEK 4 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | Introduction to lessons | | | | |
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|---|---|---|---|---|---|---|---|
| WEEK 5 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
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|---|---|---|---|---|---|---|---|
| WEEK 6 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | Introduction to lessons | | | | |
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|---|---|---|---|---|---|---|---|
| WEEK 7 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
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|---|---|---|---|---|---|---|---|
| WEEK 8 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
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|---|---|---|---|---|---|---|---|
| WEEK 9 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | Introduction to lessons | | | | |
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MODULE

# 1 Stress, strain and tensile testing of materials

**By the end of this module, students should be able to**:

- name the three different types of stresses;
- calculate direct and shear stresses;
- explain what strain is and calculate it;
- explain what modulus of elasticity is and calculate it;
- calculate the change in length, final length and percentage change in length of the bar;
- draw a stress-strain graph and use it to obtain information about a material;
- draw a force-extension graph or a stress-strain graph and use it to calculate Young's modulus for a material;
- calculate stresses for different materials connected in parallel, including pipe with threaded bar and nut;
- calculate stresses when different materials are connected in series;
- calculate change in length of each material; and
- calculate the final length of a compound bar.

Machine parts and members of structures are always subjected to loads, which cause stress in the parts or members. Due to the load on these parts and members there will be a change in length, which will cause strain in the parts and members.

## Exercise 1.1 SB page 29

1. Note that it is very important to sketch the problem.

1.1 Working stress $= \frac{\text{maximum stress}}{\text{FoS}}$

$= \frac{120}{3}$

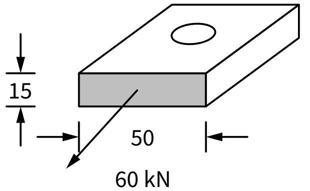
$= 40 \text{ MPa}$

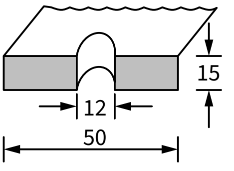
1.2 Safe load = σ.A

$= 40 \times \frac{\pi}{4}(0{,}009456^2)$

$= 2{,}809$ kN

2.



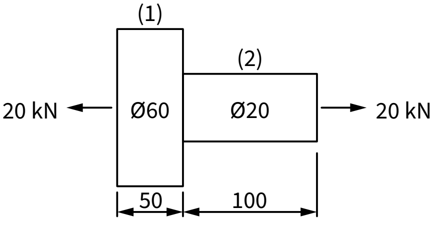




Effective area = (50 – 12) × 15

$= 570 \text{ mm}^2$

$\therefore \sigma = \frac{F}{A} = \frac{60k}{570 \times 10^{-6}} = 105{,}26$ MPa

3.




Basic equation: $E = \frac{\sigma}{\varepsilon} = \frac{\frac{F}{A}}{\frac{x}{L}} = \frac{FL}{Ax} = \frac{\sigma_L}{x}$

3.1 Maximum stress smallest area $= \sigma = \frac{F}{A} = \frac{20k}{\frac{\pi}{4}902^2} = 63{,}662$ MPa

3.2 $x_T = x_1 + x_2$

$= \frac{FL_1}{A_1E} + \frac{FL_2}{A_2E} = \frac{20k}{200G}\left[\frac{0{,}05}{\frac{\pi}{4}0{,}06^2} + \frac{0{,}1}{\frac{\pi}{4}0{,}02^2}\right] = 0{,}0336$ mm

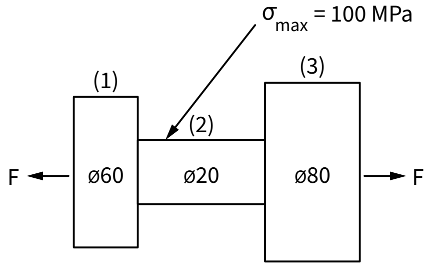
3.3 $\varepsilon_T = \varepsilon_1 + \varepsilon_2 \therefore \varepsilon = \frac{\sigma_1}{E} = \frac{F}{A_1E} = \frac{20k}{\frac{\pi}{4}0{,}06^2} \times 200G = 3{,}537 \times 10^{-5}$

$\varepsilon_2 = \frac{\sigma_2}{E} = \frac{F}{A_2E} = \frac{20k}{\frac{\pi}{4}0{,}02^2} = 200G = 3{,}183 \times 10^{-4}$

$\therefore \varepsilon_T = \varepsilon_1 + \varepsilon_2 = 3{,}537 \times 10^{-5} + 3{,}183 \times 10^{-4} = 3{,}537 \times 10^{-4}$

3.4 Percentage elongation $= \frac{x}{L_{original}} \times \frac{100}{1} = \frac{0{,}0336}{150} \times \frac{100}{1} = 0{,}022\%$

4.




E = 200 GPa

4.1 Maximum stress smallest area $\therefore F = \sigma_{max} A_2$

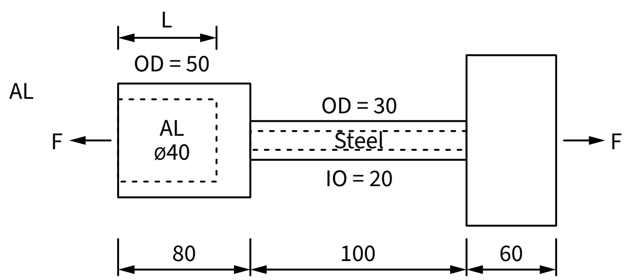
$= 100\text{ M} \times \frac{\pi}{4} 0{,}02^2 = 31{,}416\text{ kN}$

This is the maximum force that can be applied due to the weakest section.

4.2 $\sigma_1 = \frac{F}{A_1} = \frac{31{,}416k}{\frac{\pi}{4}0{,}06^2} = 11{,}11\text{ MPa}$

$\sigma_3 = \frac{F}{A_3} = \frac{31{,}416k}{\frac{\pi}{4}0{,}08^2} = 6{,}25\text{ MPa}$

5.




$\sigma_{AL} = 155$ MPa

$E_{al} = 69$ GPa

$\sigma_s = 465$ MPa

$E_s = 207$ GPa

$\sigma_c = 247$ MPa

$E_C = 110$ GPa

5.1 Aluminium maximum allowable on working stress

$= \frac{\sigma_{max}}{FoS} = \frac{155}{3} = 51{,}67\text{ MPa}$

Steel maximum allowable on working stress $= \frac{\sigma_{max}}{FoS} = \frac{465}{3} = 155\text{ MPa}$

Copper maximum allowable on working stress

$= \frac{\sigma_{max}}{FoS} = \frac{247}{3} = 82{,}33\text{ MPa}$

5.2 $F_{AL} = \sigma_A A_A = 51{,}67M \times \frac{\pi}{4}(0{,}05^2 - 0{,}04^2) = 36{,}523$ kN

$F_S = \sigma_S A_S = 155M \times \frac{\pi}{4}(0{,}03^2 - 0{,}02^2) = 60{,}868$ kN

$F_C = \sigma_c A_c = 82{,}33M \times \frac{\pi}{4}(0{,}09^2) = 523{,}76$ kN

$\therefore$ Maximum force = 36,523 kN

The smallest load is the maximum, because the other loads will damage the aluminium. The strain in aluminium will be higher, as the maximum allowable for aluminium is 51,67 MPa.

5.3 $\sigma_a = 51{,}67$ MPa acting force

$$\sigma_s = \frac{F}{A} = \frac{36{,}523k}{\frac{\pi}{4}\left(0{,}03^2 - 0{,}02^2\right)} = 93{,}01 \text{ MPa}$$

$$\sigma_c = \frac{F}{A} = \frac{36{,}523k}{\frac{\pi}{4}0{,}09^2} = 5{,}74 \text{ MPa}$$

This will be the stresses in the other materials when the maximum load is applied.

5.4 Total extension: $x_T = x_a + x_s + x_c$ ①

| Basic equation: $E = \frac{\sigma}{\varepsilon} = \frac{F/A}{X/L} = \frac{FL}{Ax} = \frac{\sigma L}{x}$ (refer to Student Book) |
|---|

$$x_s = \frac{\sigma_s L_s}{E_s} = \frac{93{,}01M \times 0{,}1}{207G} = 0{,}0449 \text{ mm}$$

$$x_c = \frac{\sigma_c L_c}{E_c} = \frac{5{,}74M \times 0{,}06}{110G} = 3{,}131 \times 10^{-3} \text{ mm}$$

From ①: $x_a = x_T - (x_s + x_c)$

$= 0{,}0981 - (0{,}0449 + 3{,}131 \times 10^{-3})$

$= 0{,}050069$

But $x_a = x_{hollow} + x_{solid}$ due to the two different areas.

$$= \frac{FL_H}{A_H E_a} + \frac{FL_s}{A_s E_a}$$

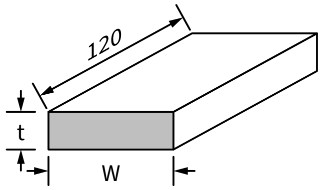
$$\therefore 0{,}050069 \times 10^{-3} = \frac{36\,523}{69G}\left[\frac{L_H}{\frac{\pi}{4}\left(0{,}05^2 - 0{,}04^2\right)} + \frac{\left(0{,}08 - L_H\right)}{\frac{\pi}{4}0{,}05^2}\right] \quad (\therefore L_S = (80 - L_H))$$

$\therefore 94{,}5914 = 1\,414{,}711\, L_H + 509{,}296(0{,}08 - L_H)$

$53{,}848 = 905{,}415\, L_H$

$L_H = 59{,}473$ mm

6.




$\sigma = 80$ MPa

$\varepsilon = 1{,}054 \times 10^{-3}$

L = 120 mm

6.1 $E = \frac{\sigma}{\varepsilon} = \frac{80\text{ M}}{1{,}054 \times 10^{-3}} = 75{,}9$ GPa

6.2 $\varepsilon = \frac{x}{L}$ $\quad \therefore x = L\varepsilon = 120 \times 1{,}054 \times 10^{-3} = 0{,}1265$ mm

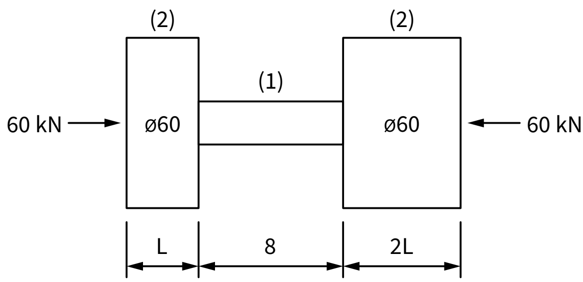
6.3 $\sigma = \frac{F}{A}$ $\quad \therefore A = \frac{F}{\sigma} = \frac{24k}{80M} = 3 \times 10^{-4}\text{ m}^2$

$A = wt$ $\quad \therefore 3 \times 10^{-4} = 0{,}05 \times t$

$\therefore t = 6$ mm

6.4 Percentage elongation $= \frac{x}{L_{origin}} \times \frac{100}{1} = \frac{0{,}1265}{120} \times \frac{100}{1} = 0{,}105\%$

7.




$\sigma_{max} = 100$ MPa

$x_T = 0{,}0718$ mm

E = 200 GPa

7.1 $\sigma = \frac{F}{A}$ $\quad \therefore A = \frac{F}{\sigma} = \frac{60k}{100M} = 6 \times 10^{-4} = \frac{\pi}{4}D^2$

D = 27,64 mm

7.2 $x_T = x_1 + x_2$

$x_2 = x_T - x_1$

$= 0{,}0718 - \left(\frac{\sigma L}{E}\right)$

$= 0{,}0718 - \left(\frac{100M \times 0{,}08}{200G}\right)$

$= 0{,}0718 - 0{,}04$

$= 0{,}0318$ mm

$x_2 = \frac{FL}{AE} = \frac{60k \times 3L}{\frac{\pi}{4}0{,}06^2 \times 200G} = 0{,}0318 \times 10^{-3}$

$\therefore$ L = 100 mm

$\therefore$ 2L = 200 mm

8. 8.1 Allowable stress $= \frac{84}{4} = 21$ MPa

Refer to the Student Book. Point C is the point that the particles in the material separate before setting and the safe stress must be less.

$\therefore$ Use FoS

8.2 $F = \sigma.A = 21M \times \frac{\pi}{4}0{,}02^2 = 6{,}597$ kN

8.3 $x = \frac{\sigma L}{E} = \frac{21M \times 0{,}25}{200G} = 0{,}026$ mm

8.4 Final length $= L_o - x = 250 - 0{,}026 = 249{,}974$ mm

9. $F = \sigma.A = 300\ M \times \pi D \times t$ (use the shear area $\pi Dt$)

$= 300\ M \times \pi \times 0{,}01 \times 0{,}02$

$= 188{,}496$ kN

10 10.1

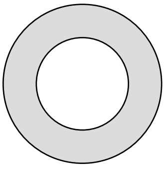


*Figure 1*

OD = 110

ID = 90

$F = \sigma.A = 60\ M \times \pi \times 0{,}09 \times 0{,}012$ (shear area in collar $= \pi dt$)

$= 203{,}575$ kN

10.2 $\sigma = \frac{F}{A} = \frac{203{,}575}{\frac{\pi}{4}0{,}09^2} = 32$ MPa (shaft diameter)

10.3 $\sigma = \frac{F}{A} = \frac{203{,}575k}{\frac{\pi}{4}\left(0{,}11^2 - 0{,}09^2\right)}$ (contact area of collar – see Figure 1)

$= 64{,}8$ MPa

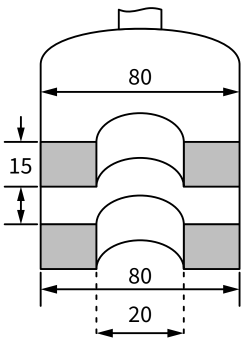
11. 11.1 Area: $s = \frac{F}{A} \therefore \frac{F}{\sigma} = \frac{100k}{100M} = 1 \times 10^{-3} = \frac{\pi}{4}D^2$

$D = 35{,}68$ mm

11.2 $t = \frac{F}{A} = \frac{100k}{2 \times \frac{\pi}{4}0{,}02^2} = 159{,}155$ MPa (pin shear in two areas

$\therefore 2 \times$ area of pin [cross-sectional area])

11.3




**Important**

Remember, the answers for graph questions will always be ± values (Questions 9–11).

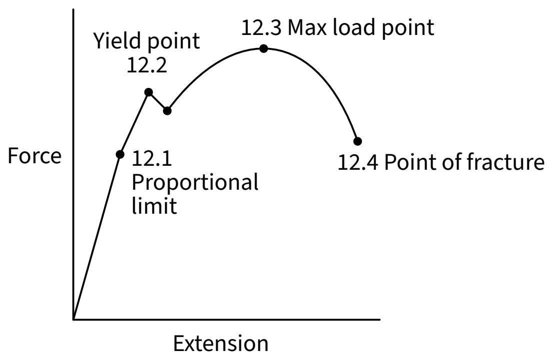
In practice, more tests are done to get the correct answers. We only do one calculation.

Two legs of fork

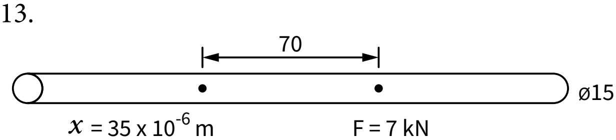
$\therefore$ Area = 2(80 – 20)15 = 1 800 mm$^2$

$$\sigma t = \frac{F}{A} = \frac{100k}{1\ 800 \times 10^{-6}} = 55{,}56 \text{ MPa}$$

12.




13.




Basic equation:

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{X/L} = \frac{FL}{Ax} = \frac{\sigma L}{x}$$

13.1 $\sigma = \frac{F}{A} = \frac{7k}{\frac{\pi}{4}0{,}015^2} = 39{,}612$ MPa (see 3.1 in the Student Book)

13.2 $E = \frac{\sigma L}{x} = \frac{39{,}612\ M \times 0{,}07}{35 \times 10^{-6}} = 79{,}224$ GPa (see 3.2 in the Student Book)

14. 14.1 $\sigma = \frac{F}{A} = \frac{28k}{\frac{\pi}{4}0{,}012^2} = 247{,}57$ MPa

14.2 $E = \frac{\sigma L}{x} = \frac{247{,}57\ M \times 0{,}06}{0{,}19 \times 10^{-3}} = 78{,}18$ MPa

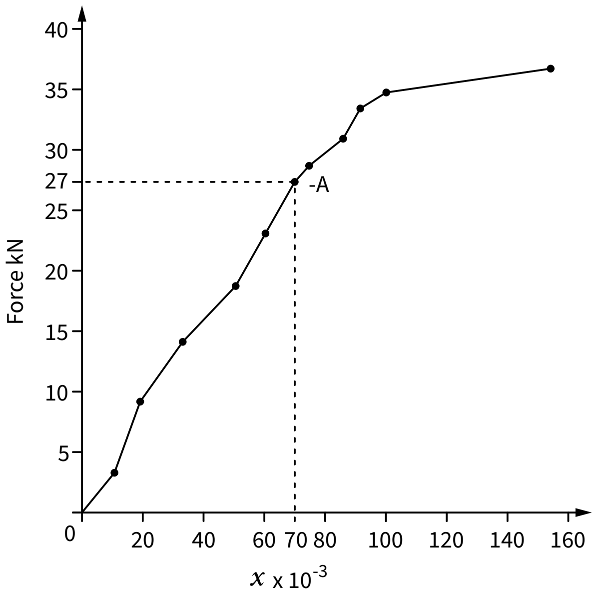
14.3 $\sigma_{max} = \frac{F_{max}}{\text{area}} = \frac{52k}{\frac{\pi}{4}0{,}012^2} = 459{,}78$ MPa

14.4 Actual stress $= \frac{40k}{\frac{\pi}{4}0{,}009^2} = 628{,}76$ MPa

14.5 % elongation $= \frac{x}{L} \times \frac{100}{1} = \frac{8}{60} \times \frac{100}{1} = 13{,}33\%$

14.6 % reduction in area $= \left(\frac{12^2 - 9^2}{12^2}\right) \times \frac{100}{1} = 43{,}75\%$

15.




Change units of extension to metres.

| F(kN) | 3,5 | 8,5 | 13,5 | 18,8 | 23,5 | 28,5 | 31 | 33,5 | 35 | 36 |
|---|---|---|---|---|---|---|---|---|---|---|
| $x \times 10^{-3}$ | 9 | 22 | 35 | 48 | 61 | 74 | 87 | 93 | 104 | 157 |

15. 15.1 $E = \frac{FL}{Ax} = \frac{28{,}5k \times 0{,}060}{\frac{\pi}{4}0{,}012^2 \times 74 \times 10^{-6}} = 204{,}32$ GPa

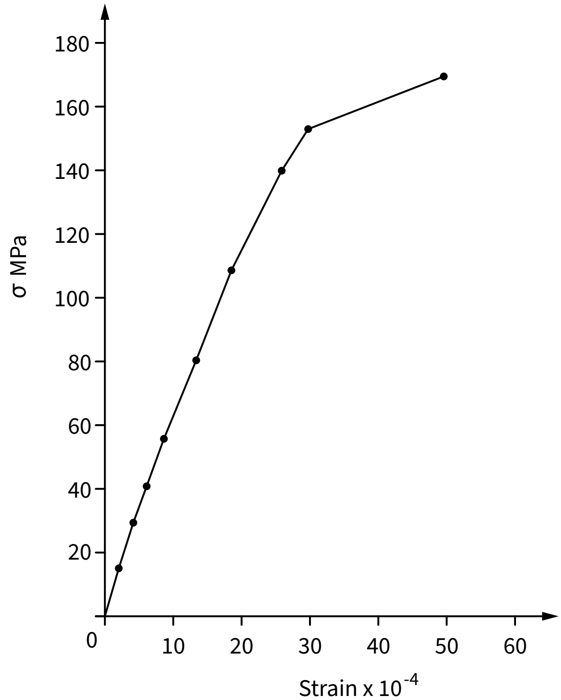
15.2 $s = \frac{F}{A} = \frac{27k}{\frac{\pi}{4}0{,}012^2} = 238{,}73$ MPa

15.3 $s = \frac{F}{A} = \frac{46k}{\frac{\pi}{4}0{,}012^2} = 406{,}73$ MPa

15.4 Percentage $x = \frac{x}{L} \times \frac{100}{1} = \frac{9}{60} \times \frac{100}{1} = 15\%$

15.5 Percentage $A = \frac{12^2 - 8^2}{12^2} \times \frac{100}{1} = 55{,}56\%$

16. 16.1

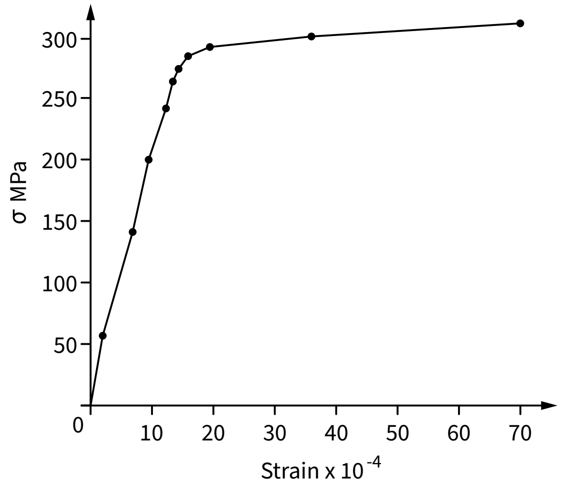



16.2 $E = \frac{FL}{Ax} = \frac{42 \times 10^3}{6 \times 10^{-4}} = 70$ GPa

Study the graph and select any value on the straight line of the graph. Take values from the table given, which is on the straight line (refer to the Student Book).

16.3 $s = \frac{F}{A} = \frac{21k}{100 \times 10^{-6}} = 210$ MPa (refer to the Student Book)

17.




| **F** | 25 | 55 | 80 | 95 | 104 | 109 | 114 | 117 | 118 | 120 |
|---|---|---|---|---|---|---|---|---|---|---|
| ***x* mm** | 0,08 | 0,176 | 0,255 | 0,303 | 0,332 | 0,35 | 0,41 | 0,52 | 0,88 | 1,75 |
| **MPa σ** | 62,87 | 138,33 | 201,2 | 238,9 | 261,56 | 274,14 | 286,7 | 294,3 | 296,8 | 301,8 |
| $\varepsilon = \frac{x}{L}$ | $3,2 \times 10^{-4}$ | $7,04 \times 10^{-4}$ | $1,02 \times 10^{-3}$ | $1,212 \times 10^{-3}$ | $1,328 \times 10^{-3}$ | $1,4 \times 10^{-3}$ | $1,64 \times 10^{-3}$ | $2,08 \times 10^{-3}$ | $3,52 \times 10^{-3}$ | $7 \times 10^{-3}$ |
| $\times 10^{-4}$ | 3,2 | 7,04 | 10,2 | 12,12 | 13,28 | 14 | 16,4 | 20,8 | 35,2 | 70 |

17.1 $E = \frac{\sigma}{\varepsilon} = \frac{201,2 \text{ M}}{1,02 \times 10^{-3}} = \pm 197,3 \text{ GPa}$

80 kN force on straight line ∴ use 201,2-MPa stress. If the stress for the 55 kN or 95 kN was used, the answer would be close to 197,3 GPa.

17.2 $s = \frac{F}{A} = \pm 261,56 \text{ MPa}$

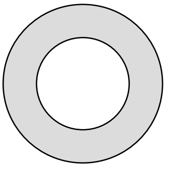
Read from graph where straight line stops.

17.3 ± 286,7 MPa

Read from graph just after straight line stops.

18. Safe stress $= \frac{465}{5} = 93 \text{ MPa}$

Refer to the Student Book.



OD = 100

ID = 220

Force in wall = σ.A

$= 93\text{M} \times \frac{\pi}{4}\left(0,22^2 - 0,1^2\right)$

$= 2\ 804,814 \text{ kN}$

∴ Internal pressure $= \frac{F}{A} = \frac{2\ 804,814k}{\frac{\pi}{4}011^2}$

$= 357,12 \text{ MPa}$

- Calculate the area and store in memory of calculator.

  $\therefore \sigma = \frac{F}{A}$

  Divide each force by the area to obtain stress for each force.
- To calculate the strain, use calculator and divide the extension by the gauge length for each extension (both can be in mm, not to change to m – keep the units the same, m or mm).

19.1 Proportional limit is the point on a stress-strain curve where the linear, elastic deformation region transitions into a non-linear, plastic deformation region.

19.2 Tensile strength is the maximum stress caused by the maximum load before the material fractures.

19.3 The modulus of elasticity is the property of a material that indicates how easily it can stretch and deform.

20. Any three of the following:
    - Compressive
    - Tensile
    - Torsion
    - Shear force

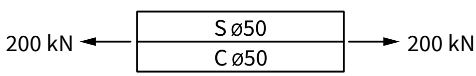
21. 21.1 Stress is the ability of an object to resist the effects of an external force.

    21.2 Strain is the deformation of an object due to an internal state of stress.

22. Hooke's law states that strain is directly proportional to the stress that causes it.

## Exercise 1.2 — SB page 45

1.




$E_s = 207$ GPa

$E_c = 100$ GPa

1.1

$$F_T = F_C + F_S$$
$$\therefore 20k = \sigma_c A_c + \sigma_s A_s$$
$$= \frac{\pi}{4} 0{,}0^2(\sigma_c - \sigma_s)$$
$$\therefore 101{,}859 \times 10^6 = \sigma_c + \sigma_s \qquad ①$$
$$x_c = x_s$$
$$\therefore \frac{\sigma_c L_c}{E_c} = \frac{\sigma_s L_s}{E_s}$$
$$\therefore \sigma_c = \frac{\sigma_c E_s}{E_c} = \frac{\sigma_c 207}{100} = 2{,}07\sigma c \qquad ②$$

Substitute ② in ① $\therefore 101{,}859 \times 10^6 = 2{,}07\sigma_c$

$\therefore \sigma_c = 33{,}179$ MPa

$\sigma_c = 68{,}68$ MPa

1.2 Final length: $L_F = L_O + x$

$= 100 + x_c$

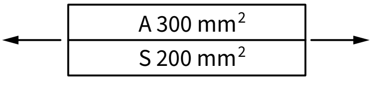
$\therefore 0{,}100 + \frac{\sigma_c L_c}{E_c} = 0{,}1 + \frac{33{,}179 \times 10^6 \times 0{,}1}{100G}$

$= 0{,}1 + 3{,}318 \times 10^{-5}$

$= 100{,}033$ mm

1.3 Percentage elongation $= \frac{x}{L_{origin}} \times \frac{100}{1} = \frac{0{,}033}{100} \times \frac{100}{1} = 0{,}033\%$

2.




$\varepsilon = 0{,}0005$

$E_S = 209$ GPa

$E_A = 104{,}5$ GPa

$F_T = F_A + F_S$

2.1 $x_s = x_c$

But $x = \frac{\sigma L}{E}$

$\therefore \sigma = \frac{Ex}{L}$

$= E\varepsilon \qquad \left(\frac{x}{L} = \varepsilon\right)$

$\therefore \sigma_s = E_s\varepsilon$ and $\sigma_A = E_c\varepsilon$

$\therefore \sigma_s = 209G \times 0{,}0005 = 103{,}5$ MPa

$\therefore \sigma_A = 104{,}5G \times 0{,}0005 = 52{,}25$ MPa

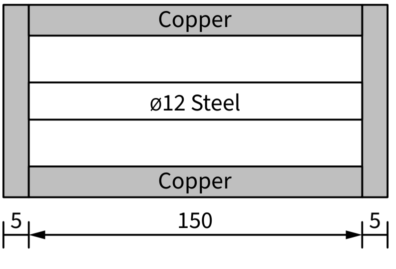
2.2 $F_s = \sigma_s A_s$

$= 103{,}5 \times 10^6 \times 200 \times 10^{-6} = 20{,}7$ kN

$F_A = \sigma_A A_A$

$= 52{,}25M \times 300 \times 10^{-6} = 15{,}675$ kN

3.



OD = 30

I$d$ = 25

$p$ = 1,5 mm pitch

$E_S$ = 200 GPa

$E_C$ = 100 GPa

3.1 $$F_s = F_c$$

$$\therefore \sigma_c A_s = \sigma_c A_c$$

$$\therefore \sigma_s \frac{\pi}{4} 0{,}012^2 = \sigma_s \frac{\pi}{4}\left(0{,}03^2 - 0{,}025^2\right)$$

$$\therefore \sigma_s = 1{,}91\sigma_c \quad ①$$

$$x_T = x_s + x_c$$

$$x_T = \frac{2}{8} \times 1{,}5 = 0{,}375 \text{ mm}$$

$$\therefore 0{,}375 \times 10^{-3} = \frac{\sigma_s L_s}{E_s} = \frac{\sigma_c L_c}{E_c}$$

$$= \frac{\sigma_s(150 + 10)}{200G} + \frac{\sigma_c(0{,}15)}{100G}$$

$$\times 200G: \sigma\, 75 \times 10^6 = 0{,}16\sigma_s + 0{,}3\sigma_c \quad ②$$

Substitute ① in ②: $75 \times 10^6 = 0{,}16\,(1{,}91\sigma_c) + 0{,}3\sigma_c$

$$= 0{,}6056\, \sigma_c$$

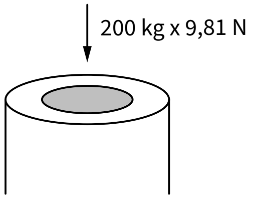
$$\therefore \sigma_c = 123{,}84 \text{ MPa}$$

$$\therefore \sigma_s = 236{,}54 \text{ MPa}$$

3.2 $$x_s = \frac{\sigma_s L_s}{E_s} = \frac{236{,}54M \times 0{,}16}{200G} = 0{,}189 \text{ mm}$$

$$x_c = \frac{\sigma_c L_c}{E_c} = \frac{123{,}84M \times 0{,}15}{100G} = 0{,}186 \text{ mm}$$

4.




OD = 500

$E_S$ = 140G

I$d$ = 420

$E_C$ = 14G

L = 2 m

4.1 $F_T = F_1 + F_C$

$$200 \times 9{,}81 = \sigma_1 A_1 + \sigma_c A_c$$

$$= \sigma_1 \tfrac{\pi}{4}\left(0{,}5^2 - 0{,}42^2\right) + \sigma_c\left(\tfrac{\pi}{4}0{,}42^2\right)$$

$$1\ 962 = 0{,}058\sigma_{c1} + 0{,}139\sigma_c \quad ①$$

$$x_s = x_c$$

$$\therefore \frac{\sigma_s L_s}{E_s} = \frac{\sigma_c L_c}{E_c}$$

$$\therefore \sigma_s = \frac{\sigma_c 140}{14}$$

$$= 10\ \sigma_c \quad ②$$

Substitute ② in ①: $1\ 962 = 0{,}058(10\sigma_c) + 0{,}139\sigma_c$

$$= 0{,}719\sigma_c$$

$$\sigma\sigma_c = 2{,}729 \text{ kPa}$$

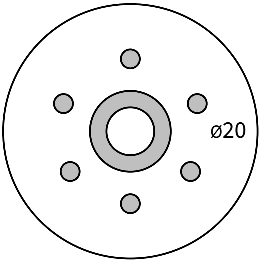
From ②: $\sigma_S = 27{,}29$ kPa

4.2 $x_s = x_c = \frac{\sigma_c L_c}{E_c} = \frac{2\ 729 \times 2}{14G} = 3{,}9 \times 10^{-4}$ mm

4.3 $U = \frac{1}{2}Fx_T = \frac{1}{2}(200 \times 9{,}81) \times 3{,}9 \times 10^{-7}$

$$= 3{,}826 \times 10^{-4} \text{ J}$$

5.




$D = 100$

$D = 300$

$d = 80$

$$A_T = \tfrac{\pi}{4}0{,}3^2 = 0{,}07069 \text{ m}^2$$

$$A_S = A_P + A_R$$

$$= \tfrac{\pi}{4}\left(0{,}1^2 - 0{,}3^2\right) + \left(\tfrac{\pi}{4} \times 0{,}02^2 \times 6\right)$$

$$= 2{,}827 \times 10^{-3} + 1{,}885 \times 10^{-3}$$

$$A_S = 4{,}712 \times 10^{-3} \text{ m}^3$$

$$\therefore A_C = 0{,}07069 - 4{,}712 \times 10^{-3} - \tfrac{\pi}{4}0{,}08^2$$

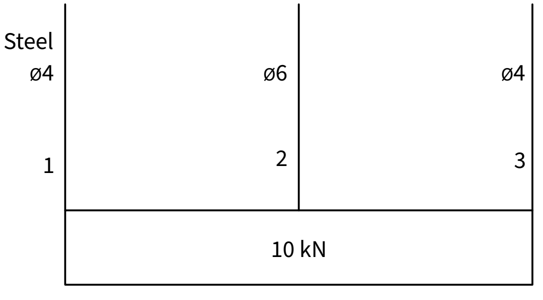
$$A_C = 0{,}061\ m^2$$
$$F_T = F_S + F_C$$
$$300k = \sigma_s A_s + \sigma_c A_c$$
$$= \sigma_c 4{,}712 \times 10^{-3} + \sigma_c 0{,}061 \quad ①$$
$$x_s = x_c$$
$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{G_c}$$
$$\therefore \sigma_s = 15\sigma_c \quad ②$$

Substitute ② in ①: $300k = 15\ \sigma_c \times 4{,}712 \times 10^{-3} + \sigma_c 0{,}061$

$$= 0{,}13168\sigma_c$$
$$\sigma_c = 2{,}278\ MPa$$

From ②: $\sigma_s = 34{,}17$ MPa

6.




$$E_S = 200G$$
$$E_C = 100G$$
$$x_1 = x_2 = x_3$$
$$L_1 = L_2 = L_3 = 3\ m$$
$$No_3 = d = 4$$
$$F = 30\ kN$$
$$x = 60$$
$$L = 3$$

6.1 $E_3 = \frac{FL}{Ax}$

$$= \frac{30k \times 3}{\frac{\pi}{4}0{,}004^2 \times 0{,}06} \qquad \therefore F_T = F_1 + F_2 + F_3$$

$$E_3 = 119{,}37\ GPa \qquad 10k = F_1 + F_2 + F_3 \quad ①$$

$$x_1 = x_2 = x_3$$
$$\therefore x_1 = x_2$$
$$\frac{F_1 L_1}{A_1 E_1} = \frac{F_2 L_2}{A_2 E_2}$$

$$\therefore \frac{F_1}{\frac{\pi}{4}0{,}004^2 \times 200G} = \frac{F_2}{\frac{\pi}{4}0{,}006^2 \times 100G}$$

$$\therefore F_1 = \frac{F_2 \frac{\pi}{4}0{,}006^2 \times 200G}{\frac{\pi}{4}0{,}006^2 \times 100G}$$

$$F_1 = 0{,}889\ F_2 \qquad ②$$

$$x_3 = x_2$$

$$\therefore \frac{F_3L_3}{A_3E_3} = \frac{F_2L_2}{A_2E_2}$$

$$\therefore \frac{F_3}{\frac{\pi}{4}0{,}004^2 \times 119{,}37G} = \frac{F_2}{\frac{\pi}{4}0{,}006^2 \times 100G}$$

$$\therefore F_3 = \frac{F_2 0{,}006^2 \times 119{,}37G}{0{,}006^2 \times 100G}$$

$$= 0{,}531\ F_2 \qquad ③$$

Substitute ② and ③ in ①: $10k = 0{,}889\ F_2 + F_2 + 0{,}531\ F_2$

$\therefore F_2 = 4{,}132$ kN

From ②: $F_1 = 3{,}674$ kN

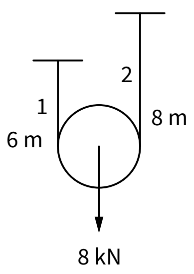
From ③: $F_3 = 2{,}194$ kN

6.2 $x_T = x_1 = \frac{\sigma_1 L_1}{E_1}$

$$= \frac{F_1L_1}{A_1E_1} = \frac{3{,}674k \times 3}{\frac{\pi}{4}0{,}004^2\ 200G}$$

$$= 4{,}386 \text{ mm}$$

7.




A = 900 m$^2$

$x_1 = x_2$

E = 1 GPa

7.1 Load in each rope $= \frac{8}{2} = 4$ kN

$$\therefore \sigma = \frac{F}{A} = \frac{4k}{900 \times 10^{-6}}$$

$$= 4{,}44 \text{ MPa}$$

7.2 Because it is a continuous rope.

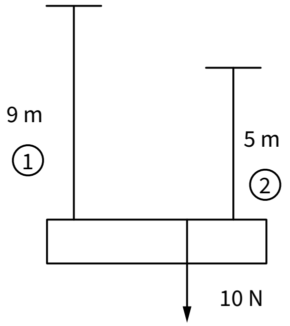
$\therefore$ Average strength of rope $= \frac{6+8}{2} = 7$ m

$$\therefore\ x = \frac{\sigma L_{AV}}{E}$$

$$= \frac{4{,}44 \text{ m} \times 7}{1G}$$

$$= 31{,}08 \text{ mm}$$

8.




8.1 $\therefore F_T = F_1 + F_2$ ①

$x_1 = x_2$

$$\therefore \frac{F_1L_1}{A_1G_1} = \frac{F_2L_2}{A_2G_2} \qquad \begin{bmatrix} A_1 & = & A_2 \\ G_1 & = & G_2 \end{bmatrix}$$

$\therefore \times$ AG: $F_1L_1 = F_2L_2$

$$\therefore F_2 = \frac{F_1 9}{5}$$

$$= 1{,}8\ F_1 \qquad ②$$

Substitute ② in ①: $10k = F_1 + 1{,}8\ F_1$

$\therefore F_1 = 3{,}571$ kN

$\therefore F_2 = 6{,}429$ kN

$$\sigma_1 = \frac{F_1}{A} = \frac{3{,}571k}{\frac{\pi}{4}0{,}06^2} = 2{,}84 \text{ MPa}$$

$$\sigma_2 = \frac{F_2}{A} = \frac{6{,}429k}{\frac{\pi}{4}0{,}04^2} = 5{,}12 \text{ MPa}$$

8.2 $\sigma_1 = \frac{\sigma_1}{E} = \frac{2{,}84M}{1{,}1G} = 2{,}582 \times 10^{-3}$

$$\sigma_2 = \frac{\sigma_2}{E} = \frac{5{,}12M}{1{,}1G} = 4{,}655 \times 10^{-3}$$

8.3 $x_1 = x_2$ and $\epsilon_2 = \frac{x_2}{L_2}$

$\therefore x_1 = \epsilon_1 L_1$ $\therefore x_2 = \epsilon_2 L_2$

$x_1 = 2{,}582 \times 10^{-3} \times 9$ $x_2 = 4{,}655 \times 10^{-3} \times 5$

$= 23{,}24$ mm $= 23{,}28$ mm

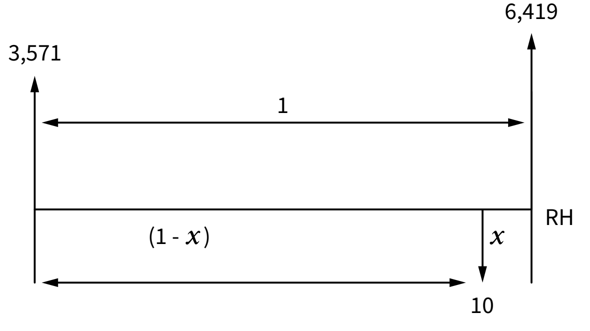
$\therefore U_1 = \frac{1}{2} F_1 x_1$ $\frac{1}{2} 3{,}571k \times 23{,}28 \times 10^{-3}$

41,57 J

$U_2 = \frac{1}{2} F_1 x_1 = \frac{1}{2} \times 6{,}429k \times 23{,}28 \times 10^{-3}$

$= 74{,}83$ J

8.4

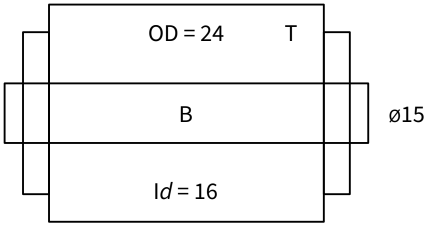



Moments about right $\therefore 10x = 3{,}571 \times 1$

$x = 0{,}3571$ m

10 kN load is 357,1 mm from right

9.




$p = 1{,}2$ mm

$\sigma_B = 20$ MPa

$G_S = 210$ GPa

$E_B = 110$ GPa

$L_B = 1{,}2$ m

$L_T = 1{,}188$ m

9.1

$$F_B = F_T$$
$$x_T = x_B + x_T$$
$$\sigma_B A_B = \sigma_T A_T$$
$$\therefore 20 \times \frac{\pi}{4} 0{,}015^2 = \sigma_T \frac{\pi}{4}\left(0{,}024^2 - 0{,}016^2\right)$$
$$\sigma_T = 14{,}063 \text{ MPa}$$

9.2

$$F_B = F_T$$
$$\sigma_B A_B = \sigma_T A_T$$
$$\sigma_B = \sigma_T \left(\frac{0{,}024^2 - 0{,}016^2}{0{,}015^2}\right)$$
$$\sigma_B = 1{,}42\ \sigma_T \quad ①$$
$$x_T = x_B + x_{TU}\ (x_T = \text{length} \times \text{pitch})$$
$$(1{,}2 \times 1{,}5)10^{-3} = \frac{\sigma_B L_B}{E_B} + \frac{\sigma_T L_T}{E_T}$$
$$1{,}8 \times 10^{-3} = \frac{\sigma_B 1{,}2}{210\text{G}} + \frac{\sigma_T 1{,}188}{110\text{G}}$$
$$\times\ 210\text{G}: \therefore 378 \times 10^6 = 1{,}2\sigma_B + 2{,}268\sigma_T \quad ②$$

Substitute ① in ② $\therefore 378 \times 10^6 = 1{,}2(1{,}42\sigma_T) + 2{,}268\sigma$

$$\therefore \sigma_{\text{brass}} = \frac{378\text{M}}{3072} = 95{,}17 \text{ MPa}$$
$$\sigma_{\text{bolt}} = 135{,}14 \text{ MPa}$$

Resultant stress = stress tight nut + initial stress

$$\therefore \sigma_{\text{brass}} = 95{,}17 + 14{,}06 = 109{,}23 \text{ MPa}$$
$$\sigma_{steel} = 135{,}14 + 20 = 155{,}14 \text{ MPa}$$

MODULE

# 2 Strain energy

**By the end of this module, students should be able to**:

- explain what strain energy is and how to calculate it;
- express strain energy in terms of force and stress;
- explain what gradually applied load is;
- calculate strain energy when load is gradually applied (single and compound bars);
- explain what suddenly applied load is;
- calculate strain energy when a sudden load is applied (single and compound bars);
- explain what a shock load is;
- calculate strain energy when a shock load is applied (single and compound bars); and
- calculate the height a load may fall in order not to exceed the stress limit.

When a member is subjected to an axial force, the dimensions will change and this means that the member will extend or shorten. That means that the force had moved a distance and an amount of work was done. Energy is then stored in the member, which is called strain energy.

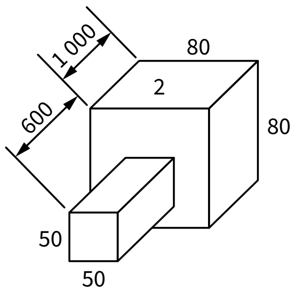
## Exercise 2.1 SB page 56

1. $U = \frac{1}{2}Fx = \frac{1}{2}F\left(\frac{FL}{AG}\right) = \frac{F^2L}{2AG}$

$= \frac{(60k)^2 \times 0{,}5}{2 \times \frac{\pi}{4}0{,}04 \times 206G}$

$= 3{,}477\ \text{J}$

2. 2.1




$$U_T = U_1 + U_2 = \tfrac{1}{2}Fx_1 + \tfrac{1}{2}Fx_2$$

$$= \tfrac{1}{2}F(x_1 + x_2)$$

$$= \tfrac{1}{2}F\left[\frac{FL_1}{A_1E} + \frac{FL_2}{A_2E}\right]$$

$$= \frac{FL}{2E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2}\right]$$

$$= \frac{(100k)^2}{2 \times 206G}\left[\frac{0,6}{0,05^2} + \frac{1}{0,08^2}\right]$$

$$= 9,618 \text{ J}$$

2.2 $x_T = x_1 + x_2$

$$= \frac{FL_1}{A_1E} + \frac{FL_2}{A_2E}$$

$$= \frac{100k}{206G}\left[\frac{0,6}{0,05^2} + \frac{1}{0,08^2}\right]$$

$$= 0,192 \text{ mm}$$

2.3 $\varepsilon_T = \varepsilon_1 + \varepsilon_2$

$$= \frac{x_1}{L_1} + \frac{x_2}{L_2}$$

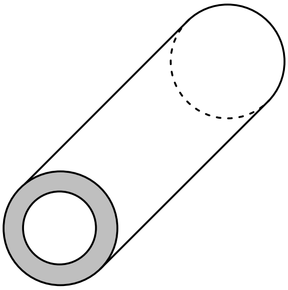
$$= \frac{FL_1}{A_1EL_1} + \frac{FL_2}{A_2EL_2}$$

$$= \frac{100k}{206G}\left[\frac{1}{0,05^2} + \frac{1}{0,08^2}\right]$$

$$= 2,7 \times 10^{-4}$$

2.4 $\sigma_{max} = \frac{F}{A_1} = \frac{100k}{0,05^2} = 40$ MPa (maximum stress in smallest area)

3.



$ID = 50$

$OD = 80$

$E = 200\ GPa$

$U = 30\ J$

3.1 $U = \frac{1}{2}Fx$

$= \frac{1}{2}F\left(\frac{FL}{AE}\right)$

$\therefore 30 = \frac{F^2L}{2AE} = \frac{F^2 \times 0,7}{2 \times \frac{\pi}{4}\left(0,08^2 - 0,05^2\right)200G}$

$\therefore F^2 = \frac{3,676 \times 10^{10}}{0,7}$

$F = \sqrt{5,251 \times 10^{10}}$

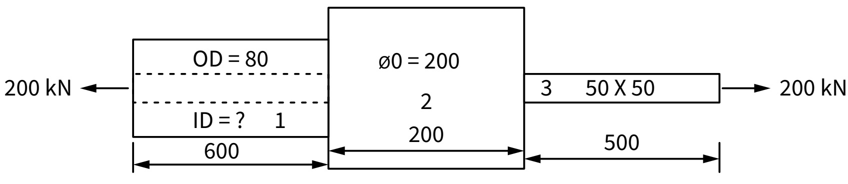
$= 229,149\ kN$

3.2 $\sigma = \frac{F}{A} = \frac{229,149k}{\frac{\pi}{4}\left(0,08^2 - 0,05^2\right)} = 74,811\ MPa$

3.3 $x = \frac{\sigma L}{E} = \frac{74,811M \times 0,7}{200G} = 0,262\ mm$

3.4 $\varepsilon = \frac{\sigma}{E} = \frac{74,811M}{200G} = 3,741 \times 10^{-4}$

4.




$E = 210\ GPa$

4.1 $$\varepsilon_T = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

$$= \frac{\sigma_1}{E} + \frac{\sigma_2}{E} + \frac{\sigma_3}{E}$$

$$= \frac{1}{E}\left[\frac{F}{A_1} + \frac{F}{A_2} + \frac{F}{A_3}\right]$$

$$= \frac{F}{E}\left[\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3}\right]$$

$$\therefore 7{,}548 \times 10^{-4} = \frac{200k}{210G}\left[\frac{1}{A_1} + \frac{1}{\frac{\pi}{4}0{,}1^2} + \frac{1}{0{,}05^2}\right]$$

$$\therefore \frac{1}{A_1} = 265{,}216$$

$$A_1 = 3{,}771 \times 10^{-3}$$

$$\therefore \frac{\pi}{4}\left(0{,}08^2 - d^2\right) = 3{,}771 \times 10^{-3}$$

$$d = 40 \text{ mm}$$

4.2 $$x_T = x_1 + x_2 + x_3$$

$$= \frac{FL_1}{A_1E} + \frac{FL_2}{A_2E} + \frac{FL_3}{A_3E}$$

$$= \frac{F}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$

$$= \frac{200k}{210G}\left[\frac{0{,}06}{\frac{\pi}{4}\left(0{,}08^2 - 0{,}04^2\right)} + \frac{0{,}2}{\frac{\pi}{4}0{,}1^2} + \frac{0{,}5}{0{,}05^2}\right]$$

$$= 0{,}366 \text{ mm}$$

4.3 Check for smallest area

$$A_1 = \frac{\pi}{4}(0{,}08^2 - 0{,}04^2) = 3{,}77 \times 10^{-3} \text{ m}^2$$

$$A_3 = 0{,}05^2 = 2{,}5 \times 10^{-3} \text{ m}^2$$

$$\sigma_{max} = \frac{F}{A_{min}}$$

$$= \frac{200k}{2{,}5 \times 10^{-3}}$$

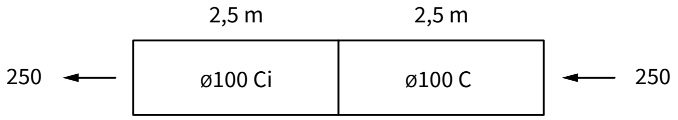
$$= 80 \text{ MPa}$$

We must check for the smallest area to calculate, because the hollow area can be smaller, depending on the diameters.

4.4 $$U_T = \frac{1}{2}Fx_T = \frac{1}{2} \times 200k \times 0{,}366 \times 10^{-3}$$

$$= 36{,}6 \text{ J}$$

5.




$E_{Ci} = 80$ GPa
$E_C = 100$ GPa

5.1 $U_T = U_1 + U_2$

$= \frac{1}{2}Fx_1 + \frac{1}{2}Fx_2 = \frac{1}{2}F[x_1 + x_2]$

$\therefore = \frac{F}{2}\left[\frac{FL_1}{A_1E_1} + \frac{FL_2}{A_2E_2}\right]$

$= \frac{F^2}{2}\left[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right]$

$= \frac{(250k)^2}{2}\left[\frac{2,5}{\frac{\pi}{4}0,1^2 \times 80G} + \frac{2,5}{\frac{\pi}{4}0,1^2 \times 100G}\right]$

$= 223,812$ J

5.2 $U_T = \frac{1}{2}Fx_T \qquad d = ?$

$= \frac{1}{2}F\left[\frac{FL}{AE}\right]$

$\therefore 223,812 = \frac{F^2L}{2AE} = \frac{(250k)^2 \times 5}{2A \times 210G}$

$\therefore A = 3,324 \times 10^{-3}$

$\therefore \frac{\pi}{4}d^2 = 3,324 \times 10^{-3}$

$d = 65,06$ mm

> **Important**
> Students must show more calculation steps.

## Exercise 2.2 — SB page 62

1. 1.1 Maximum instantaneous stress

$F = 2W = 2 \times 5,0k = 10$ kN

$\therefore \text{Stress} = \frac{F}{A} = \frac{10k \times 4}{\pi 0,026^2} = 18,345$ MPa

1.2 Total extension $x_T = \frac{\sigma L}{E} = \frac{18,345M \times 3}{200G} = 0,275$ mm

2. 2.1 Stress in rod gradually applied

$\sigma = \frac{F}{A} = \frac{12k \times 4}{\pi 0,014^2} = 77,953$ MPa

2.2 Diameter of rod 12 $k$ suddenly applied

$W = 2F = 2 \times 12k = 24$ kN

$$\sigma = \frac{F}{A} = 105M = \frac{24k \times 4}{\pi d^2}$$

$$\therefore d = \sqrt{\frac{24k \times 4}{\pi \times 105M}} = 17{,}1 \text{ mm}$$

3. 3.1 Magnitude of load

$$\sigma = \frac{F}{A}$$

$$\therefore F = \sigma A = 210M \times 400 \times 10^{-6} = 84 \text{ kN}$$

But $F = 2W \therefore$ Load applied $= \frac{84}{2} = 42$ kN

3.2 Energy

$$U_T = U_1 + U_2$$

$$U_T = \frac{2W^2 L_1}{A_1 E} + \frac{2W^2 L_2}{A_2 E}$$

$$U_T = \frac{2 \times (42k)^2 \times 0{,}15}{200G}\left(\frac{1}{700 \times 10^{-6}} + \frac{1}{400 \times 10^{-6}}\right) = 10{,}359 \text{ J}$$

**Note**

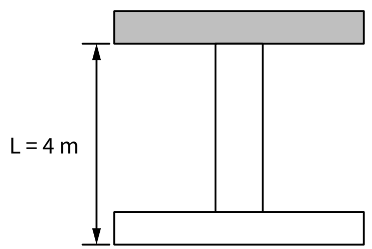
Note that the formula provides that F = 2W, and therefore we use the load applied.

3.3 Extension $x_{Total} = \frac{2WL_1}{A_1 E} + \frac{2WL_2}{A_2 E} = \frac{2 \times 42k \times 0{,}15}{200G}\left(\frac{1}{700 \times 10^{-6}} + \frac{1}{400 \times 10^{-6}}\right)$

$= 0{,}248$ mm

## Exercise 2.3 — SB page 68

1. $E = \frac{\sigma}{\varepsilon} = \frac{\frac{F}{A}}{\frac{x}{L}} = \frac{FL}{Ax} = \frac{\sigma L}{x}$ (basic equation still applies)



$d = 80$

$E = 206$ GPa

$W = 4$ kN

$h = 200$ mm

1.1 PE = U

$W(h + x_T) = \frac{1}{2}Fx_T \qquad \left(x_T = \frac{FL}{AE}\right)$

$\therefore 4k\left(0{,}2 + \frac{F \times 4}{\frac{\pi}{4}0{,}08^2 \times 200G}\right) = \frac{F^2 \times 4}{2 \times \frac{\pi}{4}0{,}08^2 \times 206G}$

$\therefore 4k(0{,}2 + 3{,}863 \times 10^{-9}\ F) = 1{,}931 \times 10^{-9}\ F^2$

$\therefore 800 + 1{,}545 \times 10^{-5}\ F = 1{,}931 \times 10^{-9}\ F^2$

$\div\ 1{,}931 \times 10^{-9}$: $F^2 - 8\ 002{,}072\ F - 4{,}143 \times 10^{11} = 0$

$$F = \frac{-(-8\ 002{,}072 \pm \sqrt{\left(-8\ 002{,}072\right)^2 - 4(1)\left(-4{,}143 \times 10^{11}\right)})}{2 \times 1}$$

$$F = \frac{8\ 002{,}072 \pm 1\ 287\ 337}{2}$$

$= 647{,}67$ kN

$\therefore \sigma = \frac{F}{A} = \frac{647{,}67k}{\frac{\pi}{4}0{,}08^2} = 128{,}85$ MPa

1.2 PE = U

$W(h + x) = \frac{1}{2}Fx$

$\therefore 40k(0{,}02 + 3{,}863 \times 10^{-9}\ F) = 1{,}931 \times 10^{-9}\ F^2$

$\therefore 800 + 1{,}545 \times 10^{-4}\ F = 1{,}931 \times 10^{-9}\ F^2$

$\div\ 1{,}931 \times 10^{-9}$: $F^2 - 80\ 010{,}357\ F - 4{,}143 \times 10^{11}$

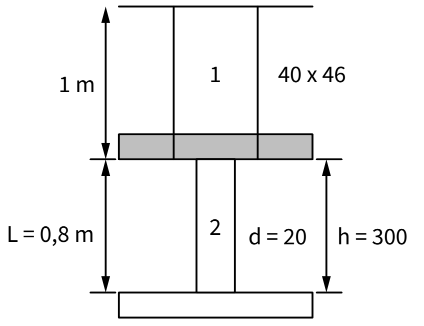
$$F = \frac{80\ 010{,}357 \pm \sqrt{b^2 - 4ac}}{2(1)}$$

$$F = \frac{80\ 010{,}357 \pm 1\ 289\ 796{,}1}{2}$$

$= 684{,}9$ kN

$\sigma = \frac{F}{A} = \frac{684{,}9}{\frac{\pi}{4}0{,}08^2} = 136{,}2$ MPa

2.




E = 210 GPa

W = 200 × 9,81 = 1 962 N

2.1 PE = U

$$W(h + x_T) = \tfrac{1}{2}Fx_T \quad ①$$

$$x_T = x_1 + x_2$$

$$= \frac{FL_1}{A_1E} + \frac{FL_2}{A_2E}$$

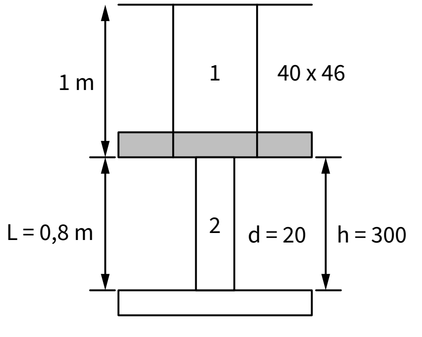
$$= \frac{F}{210G}\left[\frac{1}{0{,}04^2} + \frac{O_18}{\frac{\pi}{4}0{,}02^2}\right]$$

$$x_T = 1{,}51 \times 10^{-8}\ F \quad ②$$

Substitute ② in ①:

$$\therefore 1\ 962(0{,}3 + 1{,}51 \times 10^{-8}\ F) = \tfrac{1}{2}F \times 1{,}51 \times 10^{-8}\ F$$

$$\therefore 588{,}6 + 2{,}963 \times 10^{-5}\ F = 7{,}55 \times 10^{-9}\ F^2$$




$$\div\ 7{,}55 \times 10^{-9}:\ F^2 - 3\ 924{,}503\ F - 7{,}796 \times 10^{-10} = 0$$

$$F = \frac{-\left(-3\,924{,}503\right) \pm \sqrt{b^2 - 4ac}}{2(1)}$$

$$F = \frac{3\,924{,}503 \pm 558\,441{,}1}{2}$$

$$= 281{,}183 \text{ kN}$$

$$A_1 = 0{,}06^2 = 1{,}6 \times 10^{-3} \text{ m}$$

$$A_2 = \tfrac{\pi}{4}0{,}02^2 = 3{,}162 \times 10^{-4}$$

$$\therefore \sigma_{max} = \frac{281{,}183k}{3{,}142 \times 10^{-4}} = 894{,}92 \text{ MPa}$$

2.2 From ②: $x_T = 1{,}51 \times 10^{-8}$ F

$$= 1{,}51 \times 10^{-8} \times 281{,}183k$$

$$= 4{,}246 \text{ mm}$$

2.3 $U_T = \frac{1}{2}Fx_T = \frac{1}{2} \times 281{,}183 \times 4{,}246 \times 10^{-3}$

$$= 596{,}932 \text{ J}$$

2.4 $\sigma_1 = \frac{W}{A_1} = \frac{1\,962}{0{,}04^2} = 1{,}226$ MPa

$$\sigma_2 = \frac{W}{A_2} = \frac{1\,962}{3{,}14^2 \times 10^{-4}} = 6{,}244 \text{ MPa}$$

2.5 $U_1 = \frac{1}{2}Fx_1 = \frac{F^2L}{2AE} = \frac{1\,962^2 \times 1}{2 \times 0{,}04^2 \times 210G}$

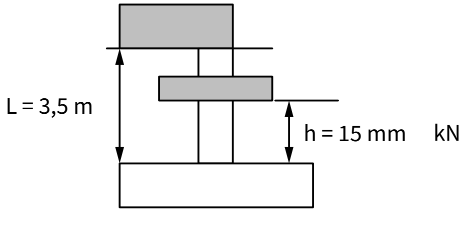
$$= 5{,}728 \times 10^{-3} \text{ J}$$

$$U_2 = \frac{F^2L}{2AE} = \frac{1\,962^2 \times 0{,}8}{2 \times \frac{\pi}{4}0{,}02^2 \times 210G} = 0{,}0233 \text{ J}$$

$$U_T = U_1 + U_2 = 0{,}0291 \text{ J}$$

Final strain is the strain under load gradually applied.

3.




$A = 700 \text{ mm}^2$

$x_T = 2{,}5$ mm

$E = 206$ GPa

$PE = U$

3.1 $E = \frac{\sigma L}{x}$

$\therefore 206G = \frac{\sigma 3{,}5}{2{,}5 \times 10^{-3}}$

$\sigma = 147{,}43$ MPa

3.2 $PE = U$

$\therefore W(h + x_T) = \frac{1}{2}Fx_T$

$\therefore W(0{,}015 + 2{,}5 \times 10^{-3}) = \frac{1}{2}\sigma.A \times 2{,}5 \times 10^{-3}$

$0{,}0175W = \frac{1}{2} \times 147{,}143M \times 700 \times 10^{-6} \times 2{,}5 \times 10^{-3}$

$W = 7{,}357$ kN

3.3 $\text{Mass} = \frac{\text{weight}}{g} = \frac{7{,}357k}{9{,}81}$

$= 749{,}96$ kg

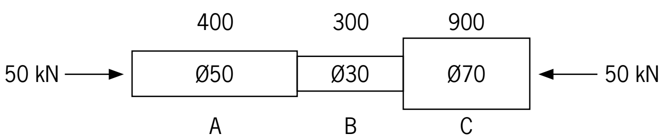
4.1 $F = 2W$

$\therefore F = 2 \times 50 = 100k$

$\therefore \sigma = \frac{F}{A} = \frac{100k}{\frac{\pi}{4}0{,}026^2} = 188{,}349$ MPa

4.2 $x_T = \frac{\sigma L}{E} = \frac{188{,}349M \times 3}{200G} = 2{,}825$ mm

5.




5.1 $\sigma_A = \frac{50k}{\frac{\pi}{4}0{,}05^2} = 25{,}46$ MPa

$\sigma_B = \frac{50k}{\frac{\pi}{4}0{,}03^2} = 70{,}74$ MPa

$\sigma_C = \frac{F}{A} = \frac{50k}{\frac{\pi}{4}0{,}07^2} = 12{,}99$ MPa

5.2 $x_A = \frac{\sigma_A L_A}{E} = \frac{25{,}46M \times 0{,}4}{200G} = 0{,}051$ mm

$x_B = \frac{\sigma_B L_B}{E} = \frac{71{,}74M \times 0{,}03}{200G} = 0{,}106$ mm

$x_C = \frac{\sigma_C L_C}{E} = \frac{12{,}99M \times 10^6 \times 0{,}9}{200G} = 0{,}058$ mm

5.3 $U_A = \frac{1}{2}Fx_A = \frac{1}{2} \times 50k \times 0{,}051 \times 10^{-3} = 1{,}275\ J$

$U_B = \frac{1}{2}Fx_B = \frac{1}{2} \times 50k \times 0{,}106 \times 10^{-3} = 2{,}65\ J$

$U_C = \frac{1}{2}Fx_C = \frac{1}{2} \times 50k \times 0{,}058 \times 10^{-3} = 1{,}45\ J$

5.4 Strain

$\varepsilon_A = \frac{\sigma_A}{E} = \frac{25{,}46M}{200G} = 1{,}273 \times 10^{-4}$

$\varepsilon_B = \frac{\sigma_B}{E} = \frac{x_B}{L_B} = \frac{0{,}106 \times 10^{-3}}{0{,}3} = 3{,}53 \times 10^{-4}$

$\varepsilon_C = \frac{x_C}{L_C} = \frac{0{,}058 \times 10^{-3}}{0{,}9} = 6{,}44 \times 10^{-5}$

5.5 $U_T = U_1 + U_2 + U_3 = 5{,}375\ J$ (add the values in 5.3)

OR

$U_T = \frac{1}{2}Fx_T$

$= \frac{1}{2} \times 50k\ (0{,}051 + 0{,}106 + 0{,}058)10^{-3}$

$= 5{,}375\ J$

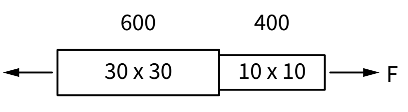
6.1 $U = \frac{1}{2}Fx = \frac{1}{2}F\frac{FL}{AE}$

$\therefore 3 = \frac{F^2 \times 0{,}6}{2 \times \frac{\pi}{4}0{,}05^2 \times 200G}$

$F = 62{,}666\ kN$

6.2 $\sigma = \frac{F}{A} = \frac{62{,}666k}{\frac{\pi}{4}0{,}05^2} = 31{,}92\ MPa$

7.




7.1 $x_T = x_1 + x_2$

$= \frac{FL_1}{A_1E} + \frac{FL_2}{A_2E}$

$\therefore 0{,}15 \times 10^{-3} = \frac{F}{209G}\left[\frac{0{,}6}{0{,}03^2} + \frac{0{,}4}{0{,}01^2}\right]$

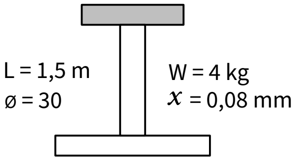
$0{,}15 \times 10^{-3} = 2{,}2329 \times 10^{-8}\ F$

$F = 6{,}718\ kN$

7.2 $\sigma_{max} = \frac{F}{A_{min}} = \frac{6{,}718k}{0{,}01^2}$ (maximum stress in smallest area)

$= 67{,}18\ MPa$

8.




$E = \frac{FL}{Ax}$

$= \frac{4 \times 9,81 \times 1,5}{\frac{\pi}{4}0,03^2 \times 0,08 \times 10^{-3}}$

$= 1,041$ GPa

$\therefore$ PE = U

$W(h + x_T) = \frac{1}{2}Fx_T$

$4 \times 9,81 \left(0,003 + \frac{FL}{AE}\right) = \frac{1}{2}F\left(\frac{FL}{AE}\right)$

$4 \times 9,81 \left(0,003 + \frac{F \times 1,5}{\frac{\pi}{4}0,03^2 \times 1,014G}\right) = \frac{F^2 \times 1,5}{2AE}$

$4 \times 9,81(0,00^3 + 2,0385 \times 10^{-6}\ F) = 1,019 \times 10^{-6}\ F^2$

$0,11772 + 7,999 \times 10^{-5}\ F = 1,019 \times 10^{-6}\ F^2$

$\div\ 1,019 \times 10^{-6}$: $F^2 - 78,5\ F - 115\,525,02 = 0$

$F = \frac{78,5 \pm \sqrt{b^2 - 4ac}}{2(1)}$

$= \frac{78,5 \pm 684,297}{2}$

$= 381,4$ N

$\therefore \sigma = \frac{F}{A} = \frac{381,4}{\frac{\pi}{4}0,03^2} = 539,569$ kPa

9. PE = U

$\therefore W(h + x) = \frac{1}{2}Fx$

$25k\left(h + \frac{\sigma L}{E}\right) = \frac{1}{2}\sigma.A \times \frac{\sigma L}{E}$

$25k\left(h + \frac{102M \times 0,8}{205G}\right) = \frac{(102M)^2}{2} \times \frac{\pi}{4}0,12^2 \times \frac{0,8}{205G}$

$\therefore 25k\ (h + 3,98 \times 10^{-4}) = 229,593$

$h = 8,786$ mm

10. 10.1 $U = \frac{1}{2}Fx_T = \frac{1}{2}\sigma.A.\frac{\sigma L}{E} = \frac{\sigma^2 AL}{2E}$

$8 = \frac{\sigma^2 \times 0{,}03^2 \times 2}{2 \times 198G}$

$\sigma^2 = 1{,}76 \times 10^{15}$

$\sigma = 41{,}95$ MPa

10.2.1 $U = \frac{1}{2}Fx \therefore 8 = \frac{F^2 L}{2AE} = \frac{F^2 \times 2}{2 \times 0{,}03^2 \times 198G}$

$\therefore F = 37{,}757$ kN

10.2.2 $U = Wx = \frac{W(2W)L}{EA} = \frac{2WL}{EA}$ (F = 2W for suddenly applied)

$8 = \frac{2 \times W^2 \times 2}{0{,}03^2 \times 198G}$

$W = 18{,}879$ kN

10.2.3 $W(h \times x) = U$

$W(0{,}02 + x_T) = 8$ ①

But $x_T \therefore 8 = \frac{1}{2}Fx_T$

$8 = \frac{1}{2} \times 37{,}757\ kx_T$

$\therefore x_T = 4{,}238 \times 10^{-4}$ ②

**Important**

For U to be constant, the force F must also be constant.
$\therefore$ From 10.2.1 F = 37,757 $kx_T$

Substitute ② in ①: $W(0{,}02 \times 4{,}238 \times 10^{-4}) = 8$

$W = 391{,}7$ N

11.1 $U = \frac{1}{2}Fx$

$= \frac{1}{2} \times 2W \times \frac{2WL}{EA}$ (F = 2W)

$\therefore 8 = \frac{2W^2 L}{EA}$

$= \frac{2W^2 \times 1{,}5}{320 \times 10^{-6} \times 200G}$

$\therefore W = 13{,}064$ kN

$\therefore$ Force in rod = F = 2W = 26,128 kN

11.2 $U = \frac{1}{2}Fx \qquad (F = W)$

$$8 = \frac{F^2L}{2AE}$$

$$= \frac{F^2 \times 1{,}5}{2 \times 320 \times 10^{-6}E} \qquad (E = 200G)$$

$$\therefore F = 26{,}128 \text{ kN}$$

**Important**

Students must show all values.

11.3 $PE = U$

$$\therefore W(h + x) = \frac{1}{2}Fx$$

$$W(h + x) = \frac{F^2L}{2AE} = 8$$

$$\therefore \frac{F^2 \times 1{,}5}{2AE} = 8$$

$$F = 26{,}128 \text{ kN}$$

**Important**

Students must show all values. This is only showing the method of calculation.

11.4.1 $F = 2W$

$$x = \frac{FL}{AE}$$

$$\therefore = \frac{26{,}128k \times 1{,}5}{AE}$$

$$= 0{,}612 \text{ mm}$$

11.4.2 $x = \frac{FL}{AE} = \frac{26{,}128k \times 1{,}5}{AE} = 0{,}612 \text{ mm}$

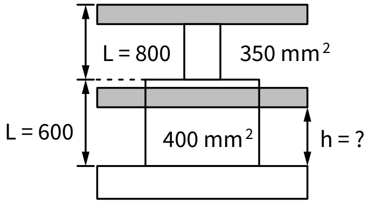
11.4.3 $x = W(h + x) = \frac{1}{2}Fx$

$$W(h + x) = 8 = \frac{1}{2}Fx$$

$$= \frac{1}{2} \times 26{,}128k$$

$$\therefore x = 0{,}612 \text{ mm}$$

12.




$x_T = 1$ mm

$W = 300 \times 9{,}81$

$E = 210$ GPa

$PE = U$

$W(h + x_T) = \frac{1}{2}Fx_T$ ①

$x_T = x_1 + x_2$

$= \frac{F}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2}\right]$

$1 \times 10^{-3} = \frac{F}{210G}\left[\frac{0{,}8}{350 \times 10^{-6}} + \frac{0{,}6}{400 \times 10^{-6}}\right]$

$\therefore F = 55{,}472$ kN ②

Substitute ② in ①: $300 \times 9{,}87(h + 1 \times 10^{-3}) = \frac{1}{2}55{,}472k \times 1 \times 10^{-3}$

$h = 8{,}424$ mm

13.1 $U = \frac{1}{2}Fx$

$4 = \frac{1}{2}F \times \frac{FL}{AE}s$

$= \frac{F^2 \times 2}{2 \times 2\,500 \times 10^{-6} \times 200G}$

$\therefore F = 44{,}721$ kN

13.2.1 $F = 2W$

$= 2 \times 44{,}721$

$= 89{,}442$ kN

13.2.1 $PE = U$

$W(h + x_T) = \frac{1}{2}Fx$

$44{,}721\left(5 \times 10^{-3} + \frac{F \times 2}{2\,500 \times 10^{-6}E}\right) = \frac{F^L 2}{2AE}$

$\therefore 223{,}605 + 1{,}789 \times 10^{-4}\, F = 2 \times 10^{-9}\, F^2$

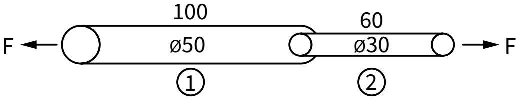
$\div\, 2 \times 10^{-9}: \therefore F^2 - 89\,450F - 1{,}118 \times 10^{11} = 0$

$$\therefore F = \frac{89\,450 \pm \sqrt{b^2 - 4ac}}{2(1)}$$

$$= \frac{89\,450 \pm 674\,693{,}49}{2}$$

$$= 382{,}072 \text{ kN}$$

14.1




$U = 10$ J

$U_T = U_1 + U_2$

$$\therefore 10 = \tfrac{1}{2}F(x_1 + x_2)$$

$$x = \frac{FL}{AE}$$

$$\therefore 10 = \frac{F^2}{2E}\left[\frac{0{,}1}{\frac{\pi}{4}0{,}05^2} + \frac{0{,}06}{\frac{\pi}{4}0{,}03^2}\right]$$

$$\therefore 10 \times 2 \times 200G = F^2\,[50{,}93 + 84{,}883]$$

$$F = 171{,}617 \text{ kN}$$

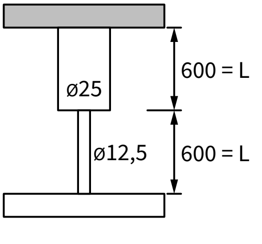
14.2 $\sigma_1 = \dfrac{171{,}617k}{\frac{\pi}{4}0{,}05^2} = 87{,}4$ MPa

$\sigma_1 = \dfrac{171{,}617k}{\frac{\pi}{4}0{,}03^2} = 242{,}79$ MPa

14.3 $x_1 = \dfrac{\sigma_1 L_1}{E} = \dfrac{87{,}4M \times 0{,}1}{200G} = 0{,}044$ mm

$x_2 = \dfrac{\sigma_2 L_2}{E} = \dfrac{242{,}79M \times 0{,}06}{200G} = 0{,}0728$ mm

15.




If stress = 220 MPa when load falls *h* metres, stress is maximum in smallest area.

∴ Force in rod will be:

$F = \sigma.A = 220\text{ M} \times \frac{\pi}{4}0{,}0125^2$

$= 26{,}998$ kN

$$\therefore x_T = x_1 + x_2 = \frac{FL_1}{A_1E} + \frac{FL_2}{A_2E}$$

$$\therefore x_T = \frac{26\,998 \times 0{,}6}{\frac{\pi}{4}0{,}025^2 \times 207G} + \frac{26\,998 \times 0{,}6}{\frac{\pi}{4}0{,}0125^2 \times E} \text{ (E = 207G)}$$

$$= 7{,}971 \times 10^{-4} \text{ m}$$

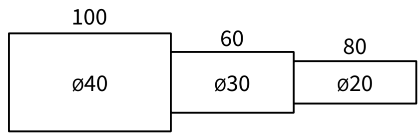
But: PE = U

$$W(h + x_T) = \tfrac{1}{2}Fx_T$$

$$\therefore 12 \times 9{,}81(h + 7{,}971 \times 10^{-4}) = \frac{1}{2} \times 26\,998 \times 7{,}971 \times 10^{-4}$$

$$h = 90{,}6 \text{ mm}$$

16.1




E = 210 GPa

$$x_T = x_1 + x_2 + x_3 \left(x = \frac{FL}{AE}\right)$$

$$\therefore 0{,}177 \times 10^{-3} = \frac{F}{E}\left[\frac{0{,}1}{\frac{\pi}{4}0{,}04^2} + \frac{0{,}06}{\frac{\pi}{4}0{,}03^2} + \frac{0{,}08}{\frac{\pi}{4}0{,}02^2}\right]$$

$$\therefore F = 88{,}68 \text{ kN}$$

16.2 $D = 1{,}2d$ ①

$$x_T = \frac{FL}{AE}$$

$$\therefore A = \frac{70k \times 924}{0{,}177 \times 10^{-3} \times 210G}$$

$$= 451{,}9774 \text{ mm}^2$$

$$A = \frac{\pi}{4}(D^2 - d^2) = 451{,}9774$$

$D^2 - d^2 = 575{,}4755$ ②

Substitute ① in ②: $(1{,}2d)^2 - d^2 = 575{,}4755$

$$\therefore 1{,}44d^2 - d^2 = 575{,}4755$$

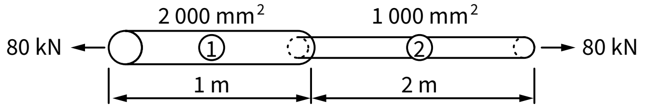
$$d = 36{,}165 \text{ mm}$$

$$D = 43{,}4 \text{ mm}$$

**Important**

Students must show all steps.

17.




$E = 200 \text{ GPa}$

$U_T = U_1 + U_2$

$= \frac{FL_1}{A_1E} + \frac{FL_2}{A_2E}$

$= \frac{80k}{200G}\left[\frac{1}{2\,000 \times 10^{-6}} + \frac{2}{1\,000 \times 10^{-6}}\right]$

$= 40 \text{ J}$

Uniform bar:

Volume of step bar $= V_1 + V_2 = A_1L_1 + A_2L_2$

$= (2\,000 \times 1\,000) + (1\,000 \times 2\,000)$

$= 4 \times 10^6 \text{ mm}^3$

Volume of step bar = volume of single bar

$\therefore 4 \times 10^6 = A_S \times 3\,000$

$A_S = 1\,333{,}33 \text{ m}^2$

$\therefore U \text{ single} = \frac{1}{2}Fx_T$

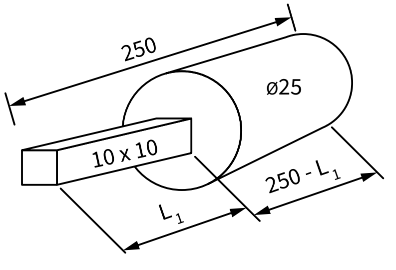
$= \frac{F_2L}{2AE}$

$= \frac{(80k)^2 \times 3}{2 \times 1\,333{,}33 \times 10^{-6} \times 200G}$

$= 36 \text{ J}$

$\therefore$ 2 sections more energy.

18.




18.1 Max stress ⇒ small area

$\therefore F = \sigma.A = 200 \text{ m} \times 0{,}01^2$

$= 20 \text{ kN}$

18.2 $x_T = x_1 + x_2$

But $U = \frac{1}{2}Fx_T$

$\therefore x_T = \frac{2U}{F}$

$= \frac{2 \times 1{,}3}{20k}$

$= 0{,}13$ mm

18.3 Stress in round bar $= \frac{F}{A_2}$

$= \frac{20k}{\frac{\pi}{4}0{,}025^2}$

$= 40{,}744$ MPa

$x_T = x_1 + x_2$

$= \frac{\sigma_1 L_1}{E} + \frac{\sigma_1 L_2}{E}$

$0{,}13 \times 10^{-3} = \frac{200\text{M } L_1}{200\text{G}} + \frac{40{,}744\text{M}\left(0{,}25 - L_1\right)}{200\text{G}}$

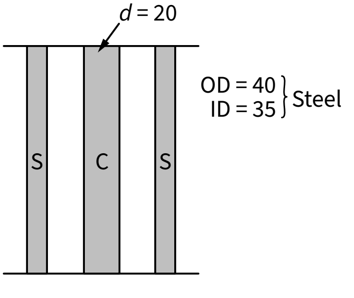
$\therefore \times 200\text{G}: 26 \times 10^6 = 200\text{M } L_1 + 40{,}744\text{M } (0{,}25 - L_1)$

$\div 10^6: 26 = 200L_1 + 10{,}186 - 40{,}744L_1$

$15{,}814 = 159{,}256L_1$

$L_1 = 99{,}3$ mm

19. 19.1




W = ?

L = 400

$h = 80$

$x_T = 0{,}16$ mm

$x_{C_T} = x_{S_T}$ ①

$\therefore$ Use values of copper: $\sigma_c \div E_C = \frac{\sigma_C L}{x_c}$ (Can also use values of steel: $L_S = L_C$)

$$\sigma_c = \frac{E_C x_c}{L}$$

$$\sigma_c = \frac{100G \times 0{,}16 \times 10^{-3}}{0{,}4}$$

$$= 40 \text{ MPa}$$

$$\therefore \sigma \text{ Steel} = \frac{E_s x_s}{L}$$

$$= \frac{198G \times 0{,}16 \times 10^{-3}}{0{,}4}$$

$$= 79{,}2 \text{ MPa}$$

19.2 $U_T = U_C + U_{steel}$

$$\therefore U_C = \tfrac{1}{2}Fx$$

$$= \tfrac{1}{2}\sigma.Ax = \frac{1}{2} \times 40M \times \tfrac{\pi}{4}0{,}02^2 \times 0{,}16 \times 10^{-3}$$

$$= 1{,}005 \text{ J}$$

$$U_S = \tfrac{1}{2}Fx$$

$$= \tfrac{1}{2}\sigma.Ax$$

$$= \frac{1}{2} \times 79{,}2M \times \frac{\pi}{4}(0{,}04^2 - 0{,}035^2) \times 0{,}16 \times 10^{-3}$$

$$= 1{,}866 \text{ J}$$

19.3 PE = U

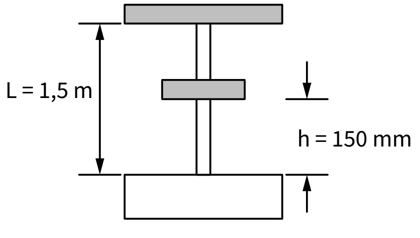
$$W(h + x) = U_1 + U_2$$

$$W(0{,}08 + 0{,}16 \times 10^{-3}) = 1{,}005 + 1{,}866$$

$$0{,}08016W = 2{,}871$$

$$W = 35{,}82 \text{ N}$$

20.




$\sigma = 80$ mPa

$d = 25$

E = 200G

$PE = U$

$W(h + x) = \frac{1}{2}Fx$ ①

$\therefore x = \frac{\sigma L}{E}$

$= \frac{80M \times 1{,}5}{200G}$

$= 6 \times 10^{-4}$ m ②

Substitute ② in ①: $W(h + x) = \frac{1}{2}\sigma.Ax$

$\therefore W(0{,}15 + 6 \times 10^{-4}) = \frac{1}{2} \times 80M \times \frac{\pi}{4}0{,}25^2 \times 6 \times 10^{-4}$

$W = 78{,}227$ N

$\text{Mass} = \frac{78{,}227}{9{,}81}$

$= 7{,}97$ kg

MODULE

# 3 Temperature-induced stresses

**By the end of this module, students should be able to**:

- calculate change in length due to temperature for different materials (free expansion or contraction);
- calculate stresses due to temperature rise or cooling for different materials connected in series and then prevented to expand or contract;
- calculate stresses due to temperature rise or cooling for different materials connected in parallel, including a pipe with a threaded bar and nut;
- calculate the final length of a compound bar;
- calculate resultant stresses due to temperature and external forces when materials are connected in parallel or series;
- calculate the resultant change in length of a compound bar; and
- calculate the final length of a compound bar.

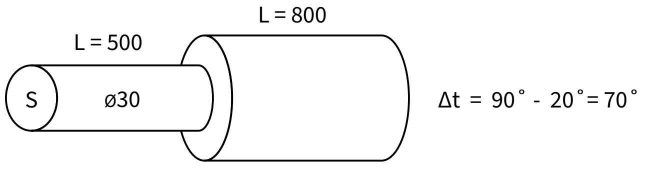
Compound bars are bars that most often consist of two different materials of which the strengths are different. These kinds of bars are found in construction and machine members, and we must be aware of the effects on these bars when they are subjected to loads and temperature changes.

## Exercise 3.1 SB page 92

**Important**

A sketch of the question must be made for the temperature question that will solve 90% of the problem.

1.




Free expansion:

$\therefore \Delta L_C = \alpha_c tL = 18 \times 10^{-6} \times 70 \times 0{,}8 = 1{,}008$ mm

$\Delta L_S = \alpha_s tL = 12 \times 10^{-6} \times 70 \times 0{,}5 = 1{,}42$ mm

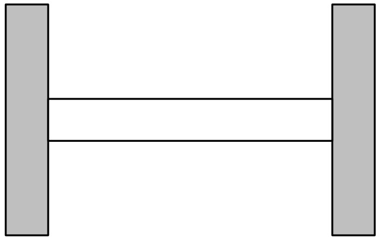
and $L_T = 1\,300$ mm,

$\therefore L_F = L_T + \Delta L_C + \Delta L_S$

$= 1\,300 + 1{,}008 + 1{,}42$

$= 1\,301{,}428$ mm

2.



$t = 50°$

$\varepsilon = 175$ GPa

$\alpha = 15 \times 10^{-6}$

2.1 With no free expansion: Stress = strain × E

$\Sigma = \varepsilon E$

$= \frac{\alpha tL}{L} \times E \left(\therefore \varepsilon = \frac{\text{change in length}}{\text{original length}} = \frac{\alpha tL}{L} = \alpha t\right)$

$= \alpha tE$

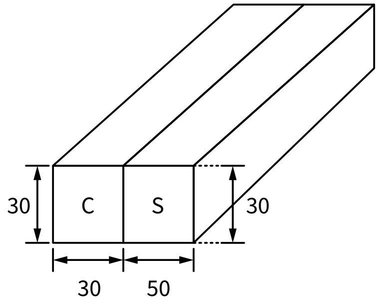
$= 15 \times 10^{-6} \times 50 \times 175G$

$= 131{,}25$ MPa

2.2 $\sigma = \frac{\alpha tL}{L} = \alpha t = 15 \times 10^{-6} \times 50$

$= 7{,}5 \times 10^{-4}$

3.

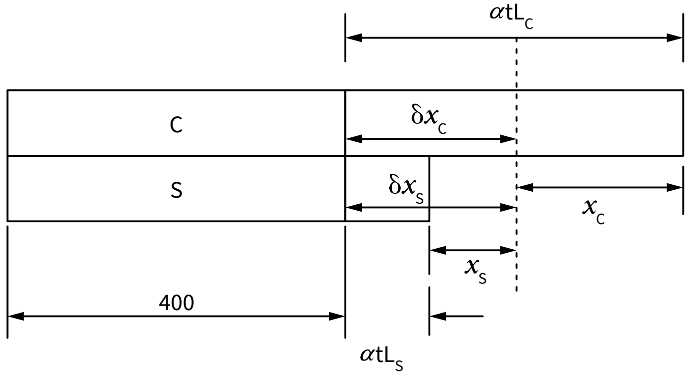



L = 400

t = 90°

Steel 30 × 50

Copper 30 × 30




3.1 $F_C = F_S$

$\delta x_c = \delta x_s$

$\therefore \alpha tL - x_c = \alpha tL + x_s$

$\alpha_c tL_c - \alpha_s tL_s = x_c + x_s$

$$= \frac{F_C L_C}{A_C E_C} + \frac{F_S L_S}{A_S E_S}$$

$$\div L{:}\ 90(17{,}5 - 13{,}5)10^{-6} = \frac{F}{0{,}03^2 \times 100G} + \frac{F}{0{,}03^2 \times 0{,}05 \times 209G}$$

$\times$ 209G: $75{,}24 \times 10^6 = 2\,322{,}22F + 666{,}66F$

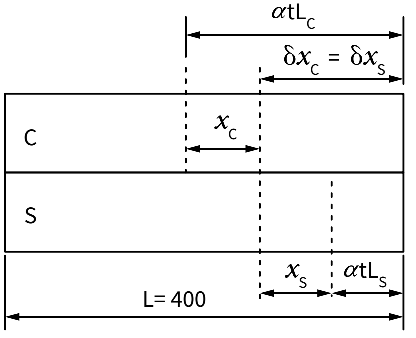
$75{,}24 \times 10^6 = 2\,988{,}89F$

$F_S = 25{,}173$ kN $= F_C$

$$\sigma_s = \frac{F_C}{A_C} = \frac{25{,}173k}{0{,}03 \times 0{,}05} = 16{,}78 \text{ (T)}$$

$$F_C = \frac{25{,}173k}{0{,}03^2} = 27{,}97 \text{ MPa (C)}$$

3.2 $t = T_1 - T_2 = 20 - (-5) = 25°C$




$F_S = F_C$ ①

$\therefore \delta x_s = dx_c$

$\alpha tL_s + x_s = \alpha tL_c + x_c$

$\alpha tL_c - \alpha tL_s = x_s + x_c$

$\div L: L_S = L_C \therefore 25(17{,}5 - 13{,}5)10^{-6} = \frac{F_C}{A_S E_S} + \frac{F_C}{A_C E_C}$

$= \frac{F}{(30 \times 50)209G} + \frac{F}{(30 \times 30)100G}$

$\times 209G: 20{,}9 \times 10^6 = 666{,}67F + 2\,322{,}22F$

$F = 6{,}993$ kN

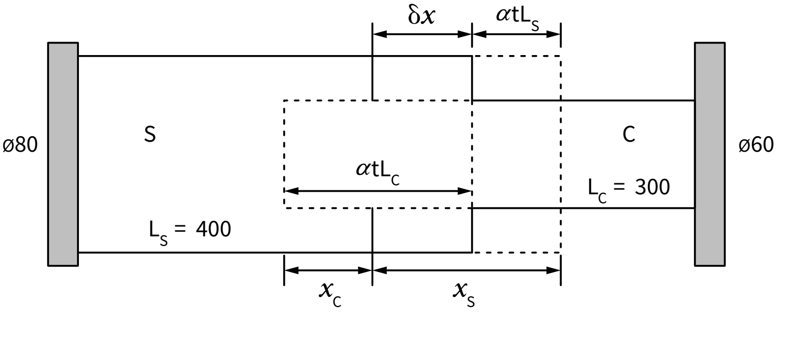
$\therefore \sigma_s = \frac{F}{A_S} = \frac{6\,993}{30 \times 50} = 4{,}66$ MPa (C)

$\therefore \sigma_s = \frac{F}{A_C} = \frac{6\,993}{(30)^2} = 7{,}77$ MPa (T)

3.3 $\varepsilon_c = \frac{\sigma_c}{E} = \frac{27{,}97M}{100G} = 2{,}797 \times 10^{-4}$

$\varepsilon_s = \frac{\sigma_s}{E} = \frac{16{,}78M}{209G} = 8{,}029 \times 10^{-5}$

4.



**Important**

Students must show all calculations.

4.1 $F_C = F_S$

$\delta x_c = \delta x_s$

$\therefore \alpha t L_c - x_c = x_s - \alpha t L_s$

$\alpha_c t L + \alpha_s t L = x_s + x_c$

$(18 \times 10^{-6} \times 80 \times 0{,}3) + (12 \times 10^{-6} \times 80 \times 0{,}4)(t = 80°)$

$$= \frac{F \times 0{,}4}{\frac{\pi}{4}0{,}08^2 \times 100G} \times \frac{F \times 0{,}3}{\frac{\pi}{4}0{,}06^2 \times 100G}$$

$\times$ 209G: 171,36M = 79,58F + 222,82F

= 302,39F

F = 566,68 kN

$$\sigma_s = \frac{F}{A_S} = \frac{566{,}68 \times 10^3}{\frac{\pi}{4}0{,}08^2} = 112{,}74 \text{ MPa}$$

$$\sigma_s = \frac{F}{A_C} = \frac{566{,}68 \times 10^3}{\frac{\pi}{4}0{,}06^2} = 200{,}42 \text{ MPa}$$

4.2 Use information for steel or use information for copper

$\therefore \delta x_s = x - \alpha t L$

$$= \frac{\sigma_s L}{E} - \alpha t L$$

$$= \frac{112{,}74M \times 0{,}4}{210G} - (12 \times 10^{-5} \times 80 \times 0{,}4)$$

= 0,169 mm (shorter)

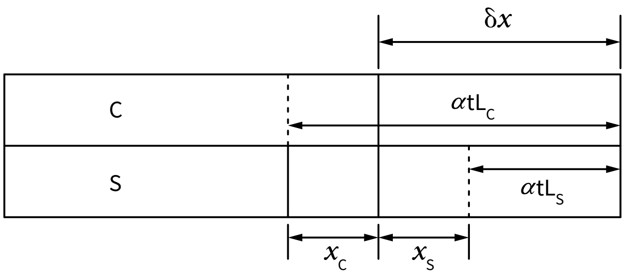
4.3 $\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{112{,}74M}{210G} = 5{,}369 \times 10^{-4}$

$\varepsilon_c = \frac{\sigma_c}{E_c} = \frac{200{,}42M}{100G} = 2{,}004 \times 10^{-3}$

4.4 $U = \frac{1}{2}Fx = \frac{1}{2} \times 566{,}68k \times 0{,}169 \times 10^{-3}$

= 47,88 J

5.



5.1 $\delta x_c = \delta x_s$

$\alpha t L_c - x_c = \alpha t L_s + x_s$

$\alpha_c t L - \alpha_s t L = x_s + x_c$

$(18 \times 10^{-6} \times 30 - 12 \times 10^{-6} \times 30) = \frac{F}{400 \times 10^{-6} \times 200G} + \frac{F}{400 \times 10^{-6} \times 100G}$

$\times$ 200G: 36M = 2 500F + 5 000F

$\therefore$ F = 4,8 kN

$\sigma_S = \frac{F}{A} = \frac{4{,}8k}{400 \times 10^{-6}} = 12$ MPa (C)

$\sigma_c = \frac{F}{A} = \frac{4{,}8k}{400 \times 10^{-6}} = 12$ MPa (T)

Compound bar: $F_T = F_C + F_S$

$100k = F_C + F_S$ ①

$x_s = x_c$

$\frac{F\not{L}}{\not{A}E} = \frac{F\not{L}}{\not{A}E}$

$\frac{F_S}{200G} = \frac{F_C}{100G}$

$F_S = 2F_C$ ②

Substitute ② in ①: $100k = F_C + 2F_C$

$F_C = 33{,}333$ kN

$F_S = 66{,}667$ kN

$\therefore \sigma_c = \frac{33{,}333k}{400} = 83{,}33$ MPa (T)

$\sigma_s = \frac{66{,}667k}{400} = 166{,}67$ MPa (T)

$\therefore R_s\sigma_c = -12 - 83{,}33 = 95{,}33$ MPa (T)

$R_s\sigma_s = +12 - 166{,}67 = 154{,}67$ MPa (T)

5.2 Use information of copper for cooling

$\delta x_c = \alpha t L - x_c$

$= 18 \times 10^{-6} \times 30 \times 0{,}6 - \frac{FL}{AE}$

$= 3{,}24 \times 10^{-4} - \frac{4{,}8k \times 0{,}6}{400 \times 10^{-6} \times 100G}$

$= 3{,}24 \times 10^{-4} - 7{,}2 \times 10^{-5}$

$= 2{,}52 \times 0^{-4}$ m

$= 0{,}252$ mm

$\therefore L_F = L_O - \delta x$

$= 600 - 0{,}252$

$= 599{,}748$ mm

**Note**

For Question 5.2, the information of steel can also be used.

For tension of 100 kN, use steel:

$x_S = x_C$

$\therefore x_S = \frac{\sigma_S L_F}{E_S}$

$= \frac{166{,}67M \times 0{,}599748}{200G}$

$= 0{,}5$ mm

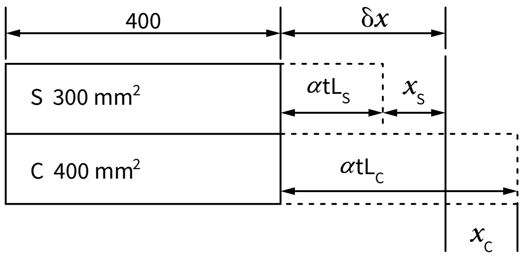
$\therefore$ Final total length after coding and load:

$L_{F_T} = F_F + x_{load}$

$= 599{,}748 + 0{,}5$

$= 600{,}248$ mm

6.




6.1 $\delta x_c = \delta x_s$

$\alpha tL - x = \alpha tL + x$

$\alpha_c tL - \alpha_s tL = x_s + x_c$

$\therefore 40(18 - 12)10^{-6} = \frac{F}{300 \times 200G} + \frac{F}{400 \times 100G}$

$\times$ 200G: $48m = 3\,333{,}3^1F + 5\,000F$

$\therefore F = 5\,760$ N

$\sigma_s = \frac{F_S}{A_S} = \frac{5\,760}{300 - m} = 19{,}2$ MPa (T)

$\sigma_s = \frac{F_S}{A_S} = \frac{5\,760}{400 - m} = 14{,}4$ MPa (C)

6.2 $F_T = F_S + F_C$ ①

$x_s = x_c$

$\frac{F_S L}{A_S E_S} = \frac{F_C L}{A_C E_C}$

$\therefore F_S = \frac{F_C 300 \times 200}{400 \times 100}$

$= 1{,}5\ F_C$ ②

Substitute ② in ① $\therefore 80 = 1{,}5\ F_C + F_C$

$F_C = 32$ kN

$F_S = 48$ kN

$$\sigma_s = \frac{F_S}{A_S} = \frac{48k}{300 \times 10^{-6}} = 160 \text{ MPa (C)}$$

$$\sigma_s = \frac{F_C}{A_C} = \frac{32k}{400 \times 10^{-6}} = 80 \text{ MPa (C)}$$

6.3 Res: $\sigma_c = +80 + 14{,}4 = 94{,}6$ MPa (C)

Res: $\sigma_s = +160 - 19{,}2 = 140{,}8$ MPa (C)

6.4 $L_F = L_O + \delta x_c - x_c$ (use information of one material)

Copper information: $400 + [\alpha tL - x] - \frac{\sigma_c L}{E_C}$

$$= 400 + \left[18 \times 10^{-6} \times 40 \times 94 - \frac{14{,}4M \times 0{,}4}{100G}\right] - \left(\frac{80M \times 0{,}4}{100G}\right)$$

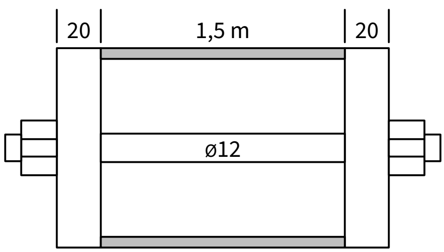
$= 400 + 0{,}23 - 0{,}32$

$= 399{,}91$ mm

**Note**

Subjected to load means it is compressed.

7.




D = 40

$d = 30$

$F_C = F_S$

$x_T = x_S + x_C$

7.1 Stress due to nuts tightening

Total change in length:

$$x_T = \frac{80}{360} \times 1{,}75 = 0{,}389 \text{ mm}$$

$\therefore x_T = x_s + x_c$

$$0{,}389 \times 10^{-3} = \frac{F_S L_S}{A_S E_S} + \frac{F_C L_C}{A_C E_C}$$

$$= \frac{F1{,}54}{\frac{\pi}{4}0{,}012^2 200G} + \frac{F1{,}5}{\frac{\pi}{4}\left(0{,}04^2 - 0{,}03^2\right)100G}$$

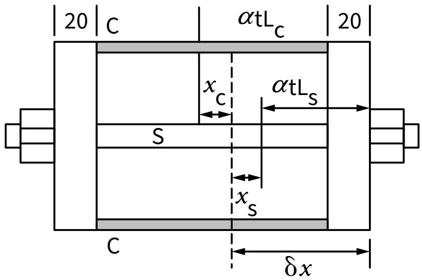
$\times$ 200G: 77,8M = 13 616,59F + 5 456,74F

F = 4,079 kN

$$\sigma_s = \frac{F}{A_S} = \frac{4{,}079k}{\frac{\pi}{4}0{,}012^2} = -36{,}07 \text{ MPa (T)}$$

$$\sigma_c = \frac{F}{A_C} = \frac{4{,}079k}{\frac{\pi}{4}\left(0{,}04^2 - 0{,}03^2\right)} = +7{,}42 \text{ MPa (C)}$$

Stress due to temperature cooling




$L_C = 1{,}5$

$L_S = 1{,}5 + 0{,}04$

$= 1{,}54$

Cooling:

$F_S = F_C \therefore \delta x_c = \delta x_s$

$\alpha t L_c - x_c = \alpha t L_s + x s_s$

$\alpha_c tL - \alpha_s tL = x_s + x_c \left[\therefore x = \frac{FL}{AE}\right]$

$\therefore (18 \times 10^{-6} \times 20 \times 1{,}5) - (12 \times 10^{-6} \times 20 \times 1{,}54)$

$$= \frac{F1{,}54}{\frac{\pi}{4}12^2 \times 200G} + \frac{F1{,}5}{\frac{\pi}{4}\left(40^2 - 30^2\right) \times 100G}$$

$\times$ 200G: 34,08M = 13 616,59F + 5 456,74F

F = 1,788 kN

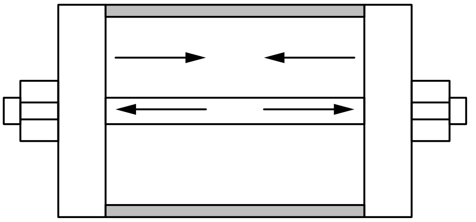
$$\sigma_s = \frac{F}{A_S} = \frac{1{,}788k}{\frac{\pi}{4}12^2} = 15{,}81 \text{ MPa (C)}$$

$$\sigma_s = \frac{F}{A_C} = \frac{1{,}788k}{\frac{\pi}{4}\left(40^2 - 30^2\right)} = 3{,}25 \text{ MPa (T)}$$

$\therefore$ Res: $\sigma_c = +7{,}42 - 3{,}25 = +4{,}17$ MPa (C)

Res: $\sigma_s = -36{,}07 + 15{,}81 = -20{,}26$ MPa (T)

7.2 Steel information



$x_F = x_{nut} - \delta x_s$

$\delta x_s = \delta tL + x$

$= (12 \times 10^{-6} \times 20 \times 1{,}54) + \frac{20{,}26M \times 1{,}54}{200G}$

$= 0{,}526$ mm (cooled)

7.3 After nut was turned 80°

$x_s = \frac{F_S L_S}{A_S E_S} = \frac{4{,}079k \times 1{,}54}{A\,E_S}$

$= 0{,}278$ mm

$\therefore x_c = x_T - x_s$

$= 0{,}389 - 0{,}278$

$= 0{,}111$ mm

$U_s = \frac{1}{2} F x_s$

$= \frac{1}{2} \times 4{,}079k \times 0{,}278 \times 10^{-3}$

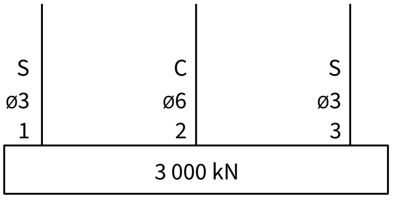
$= 0{,}567$ J

$U_c = \frac{1}{2} F x_c$

$= \frac{1}{2} \times 4{,}079k \times 0{,}111 \times 10^{-3}$

$= 0{,}226$ J

8.




8.1 $F_T = F_1 + F_2 + F_3$

$F_T = F_1 + F_2 + F_3$ ①

$\therefore 3\,000k$

$x_1 = x_2 = x_3$

$\therefore x_1 = x_2$

$\frac{F_1 \not{L}_1}{A_1 E_1} = \frac{F_2 \not{L}_2}{A_2 E_2} \quad (\div \text{ L as } L_1 = L_2)$

$\therefore F_1 = \frac{F_2 A_1 E_1}{A_2 E_2}$

$= \frac{3^2 200G}{6^2 100G} F_2$

$F_1 = 0{,}5\ F_2 \qquad ②$

$x_1 = x_3$

$\therefore \frac{F_1}{A_1 E_1} = \frac{F_3}{A_1 E_1}$

$F_1 = F_3 = 0{,}5\ F_2 \qquad ③$

Substitute ② and ③ in ①:

$\therefore 3k = 0{,}5\ F_2 + F_2 + 0{,}5\ F_2$

$= 2F_2$

$F_2 = 1\ 500$ N

$F_C = 1{,}5$ kN

$F_S = 0{,}5 \times 1{,}5 = 750$ kN/wire

8.2 $\sigma_c = \frac{1\ 500}{\frac{\pi}{4} 0{,}006^2} = -53{,}05$ MPa (T)

$\sigma_s = \frac{750}{\frac{\pi}{4} 0{,}003^2} = -106{,}1$ MPa

$F_C = F_S$

Temp. $\delta x_c = \delta x_s$

$\alpha_c tL - \alpha_s tL = x_s + x_c \left(x = \frac{FL}{AE}\right)$

$100(18 - 12)10^{-6} = \frac{F}{A_S E_S} + \frac{F}{A_C E_C}$

$\therefore 6 \times 10^{-4} = \frac{F}{2 \times \frac{\pi}{4} 3^2 200G} + \frac{F}{\frac{\pi}{4} 6^2 100G}$

$\times$ 200G: 120M = 141 471,06F

F = 848,23 N

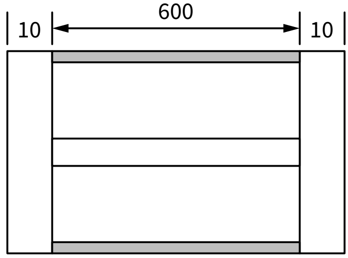
$\sigma_c = \frac{848{,}23}{2 \times \frac{\pi}{4} 3^2} = -60$ MPa

$\sigma_c = \frac{848{,}3}{\frac{\pi}{4} 6^2} = -30$ MPa

$\sigma_{res}\ \sigma_c = -53{,}05 + 30 = -23{,}05$ MPa (T)

Res $\sigma_s = -106 - 60 = -166$ MPa (T)

9.




$L_B = 620$

$d = 20$

$D = 28$

$L_{tube} = 600$ $\quad$ $L_{bolt} = 620$

$\sigma_B = 30$ MPa $\quad$ $\sigma_T = 127{,}3$ MPa

$E = 200G\ 12 \times 10^{-6}$

$E = 100G\ 12 \times 10^{-6}$

$t = (30° - t_2)$

Stress in bolt and tube will be zero when their final lengths are equal.

**Note**

Tube length = 600
Bolt length = 600 + 10 + 10
= 620

Also

Res $\sigma_B = 0 = -30_{nut} + 30_{temp}$

$\sigma_T = 0 = +\ 27{,}3_{nut} - 27{,}3_{temp}$

But $\delta x_B = \delta x_T$

$\therefore\ \alpha_T t L_T - \alpha_B t L_B = x_B + x_T$

$$t[18 \times 10^{-6} \times 0{,}6 - 12 \times 10^{-6} \times 0{,}620] = \frac{\sigma_B L_B}{E_B} + \frac{\sigma_T L_T}{E_T}$$

$$3{,}36 \times 10^{-6} t = \frac{30M \times 0{,}62}{200G} + \frac{27{,}3 \times 0{,}6}{100G}$$

$$= 9{,}3 \times 10^{-5} + 1{,}638 \times 10^{-4}$$

$$= 2{,}568 \times 10^{-4}$$

$$T = 76{,}43°$$

Tube compressive and bolt tensile when nut was tightened:

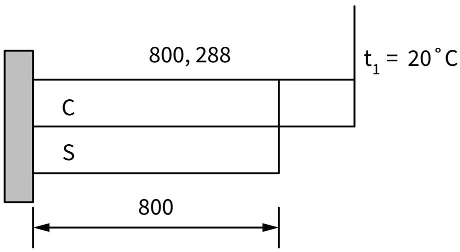
$T = t_1 - t_2$

$76{,}43 = 30 - t_2$

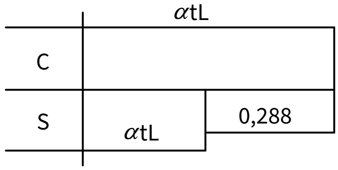
$\therefore t_2 = 30 - 76{,}43$

$= -46{,}43°C$

10.




10.1 Bar must be cooled for both to have the same length.




$\alpha_c tL = \alpha_s tL + 0{,}288 \times 10^{-3}$

$t(\alpha_c L_c = \alpha_s L_s) = 0{,}288 \times 10^{-3}$

$t(18 \times 10^{-6} \times 0{,}800288 - 12 \times 10^{-6} \times 0{,}8) = 0{,}288 \times 10^{-3}$

$t = \frac{0{,}288 \times 10^{-3}}{4{,}805184 \times 10^{-6}}$

$= 59{,}94C$

$t = t_1 - t_2$

$t_2 = t_1 - t$

$= 20 - 59{,}94$

$= -39{,}94°C$

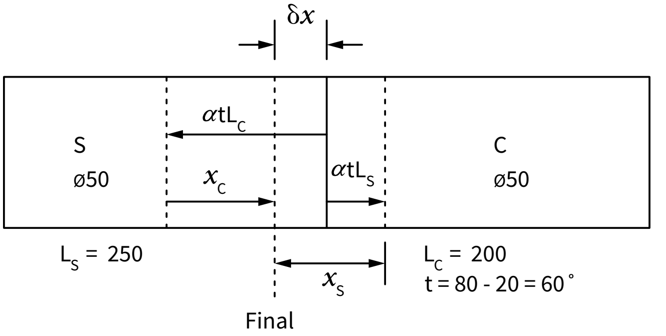
10.2 $\alpha tL_s = 12 \times 10^{-6} \times 0{,}8 \times 59{,}96$

$= 0{,}575424$ mm

$\therefore L_N = 800 - 0{,}575424$

$= 799{,}425$ mm

11.




11.1 $\delta t L_c = 20 \times 10^{-6} \times 60 \times 0{,}2 = 0{,}24$ mm

$\delta t L_s = 12 \times 10^{-6} \times 60 \times 0{,}25 = 0{,}18$ mm

11.2 $\delta x_c = \delta x_s$

$\alpha t L_c - x_c = x_s - \alpha t L_s$

$\alpha t L_c + \alpha t L_s = x_s + x_c$

(From 11.1)

$$(0{,}24 + 0{,}18)10^{-3} = \frac{\sigma_S L_S}{E_S} + \frac{\sigma_C L_C}{E_C}$$

$$= \frac{\sigma_S 0{,}25}{200G} + \frac{\sigma_C 0{,}2}{95G}$$

$\times$ 200G: $\therefore 84M = 0{,}25\ \sigma_s + 0{,}421\ \sigma_c$ ①

But:

$F_C = F_S$

$\sigma_s A_s = \sigma_c A_c \quad (A_S = A_C)$

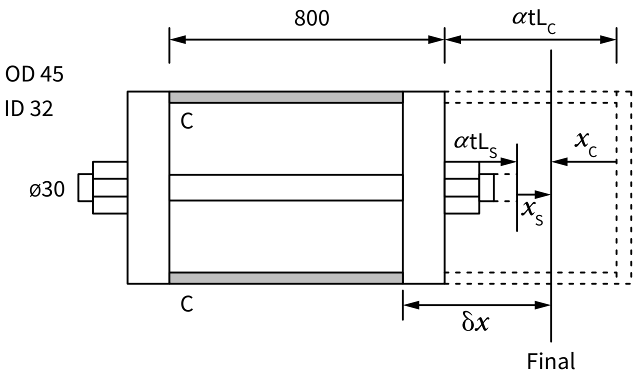
$\sigma_s = \sigma_c$ ②

Substitute ② in ①:

$84M = 0{,}25\ \sigma_c + 0{,}421\ \sigma_c$

$\sigma_c = \sigma_s = 125{,}19$ MPa

12.



12.1 $F_S = F_C$

$\delta x_c = \delta x_s$

$\alpha t L_c + x_c = \alpha t L_s + x_s$

$\alpha t L_c - \alpha t L_s = x_s + x_c \left(x = \frac{FL}{AE}\right)$

$80(18 - 12)10^{-6} = \frac{F}{\frac{\pi}{4}30^2 200G} + \frac{F}{\frac{\pi}{4}\left(45^2 - 32^2\right)100G}$

$\times$ 200G: 96M = 1 414,711F + 2 543,935F

F = 24,251 kN

$\sigma_s = \frac{F}{A_S} = \frac{24{,}251k}{\frac{\pi}{4}30^2} = 34{,}31$ MPa (T)

$\sigma_s = \frac{F}{A_C} = \frac{24{,}251k}{\frac{\pi}{4}\left(45^2 - 32^2\right)} = 30{,}85$ MPa (C)

12.2 $F_F = L_O + \delta x_s$

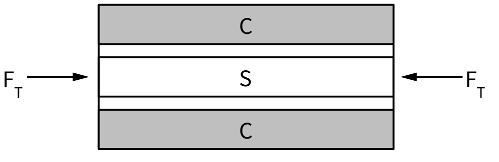
$= 800 + \alpha t L + x \qquad x = \frac{\sigma L}{E}$

$= 800 + (80 \times 12 \times 10^{-6} \times 0{,}8) + \frac{34{,}31M \times 0{,}8}{200G}$

$= 800 + 0{,}905$

$= 800{,}905$ mm

12.3




$F_T = F_C + F_S$

$x_s = x_c = 0{,}5$ mm

$\frac{F_S L}{A_S E_S} = 0{,}5 \times 10^{-3}$

$F_S = \frac{A_S E_S \times 0{,}5 \times 10^{-3}}{L}$

$= \frac{\left(\frac{\pi}{4}30^2\right)\left(200G\right)\left(0{,}5\times10^{-3}\right)}{0{,}800905}$

$= 88{,}257$ kN

$\therefore x = \frac{F_C L}{A_C E_C} = 0{,}5 \times 10^{-3}$

$F_C = \frac{\frac{\pi}{4}\left(45^2 - 32^2\right)\left(100G\right)0{,}5\times10^{-3}}{0{,}800905}$

$= 49{,}081$ kN

$\therefore F = F_C + F_S$

$= 49{,}081 + 88{,}257$

$= 137{,}338$ kN

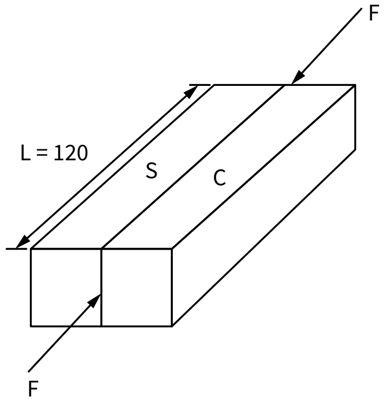
12.4 $\sigma_s = \frac{F}{A_S} = \frac{88{,}257k}{\frac{\pi}{4}30^2} = +124{,}86$ MPa

$\sigma_c = \frac{F}{A_C} = \frac{49{,}081k}{\frac{\pi}{4}\left(45^2 + 32^2\right)} = +62{,}43$ MPa

$\therefore$ Res $\sigma_c = +62{,}43 + 30{,}85 = 93{,}28$ MPa (C)

Res $\sigma_s = +124{,}86 + 34{,}31 = 90{,}55$ MPa (C)

13.




$x_T = 0{,}3$

$\Delta t = t$ °C

$F_C = 0$

$A_S = 110$ mm$^2$

$A_C = 210$ mm$^2$

13.1 No load in copper means there is NO stress in the copper. After a change in temperature, the resultant stress in the copper is zero, which is equal to free expansion (free expansion = no stress).

$\Delta x = \alpha t L$

$\Delta x = x_T = 0{,}3 = \alpha t L$

$$\Delta t = \frac{0{,}3 \times 10^{-3}}{\alpha_c L}$$

$$= \frac{0{,}3 \times 10^{-3}}{18 \times 10^{-6} \times 0{,}12}$$

$= 138{,}89$ °C

13.2 Change in length of steel due to temperature = $\alpha_s t L_s$

$\therefore \Delta_L = 12 \times 10^{-6} \times 138{,}89 \times 0{,}12$

$= 0{,}2$ mm

Distance that F must compress steel:

$= 0{,}3 - 0{,}2$

$x = 01$ mm

$\therefore$ Force required:

$$X = \frac{FL}{AE} = 0{,}1 \times 10^{-3}$$

$$F = \frac{xAE}{L}$$

$$= \frac{0{,}1 \times 10^{-3} \times 110 \times 10^{-6} \times 200G}{0{,}12}$$

$= 183{,}33$ kN

MODULE

# 4 *Thin cylinders and Mohr's circle*

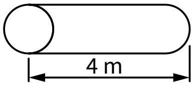
**By the end of this module, students should be able to**:

- calculate circumferential stress in a thin cylinder when it is subjected to internal pressure;
- calculate longitudinal stress in a cylinder when it is subjected to internal pressure;
- calculate theoretical thickness or diameter of a cylinder when longitudinal and circumferential efficiency is involved;
- calculate longitudinal strain in a thin cylinder when it is subjected to internal pressure;
- calculate the circumferential strain in a thin cylinder when it is subjected to internal pressure;
- calculate volumetric strain in a thin cylinder when it is subjected to internal pressure;
- construct Mohr's stress circle when given stresses in the *x*- and *y*-planes in order to determine the stresses in a different plane.

In industry, we work with vessels under pressure every day, so it is important the we know how to calculate the safety conditions for these vessels.

## Exercise 4.1 SB page 109

1.




$\varnothing d = 2$ m
P = 1,2 MPa
$t = 25$ mm

1.1 Tensile stress $\sigma_t = \frac{P_i D}{2t}$

$= \frac{1{,}2\ M \times 2}{4 \times 0{,}025}$

$= 48$ MPa

1.2 Longitudinal stress $\sigma_L = \frac{P_i D}{4t}$

$= \frac{1{,}2M \times 2}{4 \times 0{,}025}$

$= 24$ MPa

2. $P_t$4 MPa D = 2 m Yield stress = 500 MPa
FoS

Safe stress $= \frac{500}{5} = 100$ MPa

$\therefore$ Tensile stress:$\sigma t = \frac{P_d D}{2t}$

$\therefore t = \frac{4M \times 2}{2 \times 100M}$

$= 40$ mm

Longitudinal stress $\sigma_L = \frac{P_i D}{4t}$

$\therefore t = \frac{4M \times 2}{4 \times 100M}$

$= 0{,}02$ m

$= 20$ mm

Use 40-mm plate thickness.

**Note**

There are two stresses; calculate a plate thickness for each.

3. 2 MPA $\sigma_{allowable}$ 80 MPa $d = 1{,}2$ m of 72%
Spherical shell; no tensile stress, only longitudinal stress

$\therefore \sigma_L = \frac{P_i D}{4t\eta}$

$t = \frac{4 \times 1{,}2}{4 \times 80M \times 0{,}72}$

$= 10{,}42$ mm

4. 4.1 D = 1,6 m $t = 12$ mm $\eta_L = 72\%$ $\eta_C = 68\%$ 70 MPa

$\sigma_t = \frac{P_i D}{2t\eta_C}$

$\therefore P_i = \frac{70M \times 2 \times 0{,}012 \times 0{,}72}{1{,}6}$

$= 756$ kPa

$$\sigma_L = \frac{P_i D}{4t\eta_C}$$

$$\therefore P_i = \frac{70M \times 4 \times 0{,}012 \times 0{,}68}{1{,}6}$$

$$= 1{,}428 \text{ MPa}$$

Maximum allowable = 756 kPA

> **Note**
>
> The smallest pressure is the maximum, because the higher pressure will destroy the cylinder.

4.2 Change in cross-sectional area

Area original $A = \frac{\pi d^2}{4} = \frac{\pi 2^2}{4} = \pi \text{ m}^2$

Change in diagram $= \delta d = \frac{pd^2}{4tE}(2 - \gamma)$

$$\delta d = \frac{756k \times 2^2}{4 \times 0{,}012 \times 200G}(2 - 0{,}3) = 5{,}355 \times 10^{-4} \text{ m}$$

$$d_{new} = d_{org} + \delta_d = 2 + 5{,}355 \times 10^{-4} = 2{,}00054 \text{ mm}$$

$$A_{new} = \frac{\pi}{4}2{,}00045^2 = 3{,}143 \text{ m}^2$$

Change in area $= A_{new} - A_{org} = 3{,}143 - 3{,}142 = 0{,}001 \text{ m}^2$

5. D = ? Yield stress = 550 MPa $\eta = 75\%$

$t = 16$ $P_i = 1{,}8$ MPa FoS = 6

$\therefore$ Allowable stress $= \frac{550}{6} = 91{,}67$ MPa

$$\sigma_t = \frac{P_i D}{2t\eta}$$

$$\therefore D = \frac{(91{,}67M \times 2 \times 0{,}016 \times 0{,}75)}{1{,}8M}$$

$$= 1{,}22 \text{ m}$$

$$\sigma_C = \frac{P_i D}{4t\eta}$$

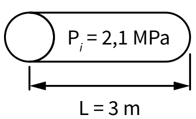
$$\therefore D = \frac{91{,}67M \times 4 \times 0{,}016 \times 0{,}75}{1{,}8M}$$

$$= 2{,}44 \text{ m}$$

Use 1,22 m.

> **Note**
>
> If a bigger diameter is used, the stress will be more than the allowable stress and destroy the cylinder.

6.




$t = 12$ mm

$d = 115$ m

6.1 $\sigma t = \frac{P_i D}{2t} = \frac{(2{,}1M \times 1{,}5)}{2 \times 0{,}012} = 131{,}25$ MPa

6.2 $\sigma_C = \frac{P_i D}{4t} = \frac{2{,}1M \times 1{,}5}{4 \times 0{,}012} = 65{,}63$ MPa

6.3 $F = P_i DL$

$= 2{,}1M \times 1{,}5 \times 3$

$= 9{,}45$ MN

6.4 $F = \frac{P_i \pi}{4} d^2$

$= 2{,}1M \times \frac{\pi}{4} 1{,}5^2$

$= 3{,}71$ MN

7. F = 14 MN

7.1 $\sigma = \frac{F}{A} = \frac{14M}{\pi D t} = \frac{14M}{\pi \times 2{,}5 \times 0{,}025}$

$= 71{,}3$ MPa

7.2 $\sigma = \frac{P_i D}{4t}$

$\therefore P_i = \frac{(71{,}3M \times 4 \times 0{,}025)}{2{,}5}$

$= 2{,}85$ MPa safe pressure

7.3 Change in internal volume

$\delta v = \frac{pdV}{4tE}(5 - 4\gamma)$

Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 1{,}25^3 = 0{,}5113 \text{ m}^3$

$\delta v = \frac{2{,}85M \times 2{,}5 \times 0{,}5113}{4 \times 0{,}025 \times 210G}(5 - 4 \times 0{,}3) = 6{,}592 \times 10^{-3} \text{ m}^3$

8. L = 2,5 m  $P_i = 1{,}2$ MPa  $t = 10$

D = 1,2 m  $\sigma_L = 36$ MPa  $\sigma_t = 72$ MPa

8.1 Force resist longitudinal

$\text{Force}_{resist} = \sigma_t 2tL$

$= 72M \times 2 \times 0{,}01 \times 2{,}5$

$= 3{,}6$ MN

8.2 Force acting longitudinal joint

$F_{acting} = P_i \times DL$

$= 1{,}2M \times 2{,}5 \times 1{,}2$

$= 3{,}6$ MN

8.3 Circumferential joint

Force resisting

$\text{Force}_{resist} = \sigma_C \times \pi D t$

$= 36M \times \pi \times 1{,}2 \times 0{,}01$

$= 1{,}357$ MN

8.4 Force acting

$F_{act} = P_i \times \frac{\pi}{4}d^2$

$= 1{,}2M \times \frac{\pi}{4}1{,}2^2$

$= 1{,}357$ MN

9.1 $d = 2$ m  $P_i = 1{,}7$ MPa  $\sigma_t = 80$ MPa  $\eta = 75\%$

$\sigma_t = \frac{P_i D}{2t\eta_C}$

$= \frac{1{,}7M \times 2}{2 \times 80M \times 0{,}75}$

$= 28{,}33$ mm

9.2.1 Circumferential strain

$\sigma_L = \frac{\sigma_H}{2} = \frac{80M}{2} = 40$ MPa

$\therefore \varepsilon_H = \frac{\sigma_H - \gamma\sigma_L}{E} = \frac{80M - (0{,}29 \times 40M)}{180G} = 3{,}8 \times 10^{-4}$

9.2.2 Longitudinal strain

$\varepsilon_L = \frac{\sigma_L - \gamma\sigma_H}{E} = \frac{40m - (0{,}29 \times 80M)}{180G} = 9{,}333 \times 10^{-5}$

9.2.3 Change area

$A_{org} = \frac{\pi}{4}2^2 = 3{,}142\ m^3$

Change diagram $\delta_d = \frac{pd^2}{4tE}(2 - \gamma) = \frac{1{,}7M \times 2^2}{4 \times 0{,}03 \times 180G}(2 - 0{,}29)$

$= 0{,}5383$ mm

$d_{new} = 2 + 5{,}383 \times 10^{-4} = 2{,}0005383$ m

$A_{new} = \frac{\pi}{4}2{,}0005383^2 = 3{,}143\ m^2$

$\delta_A = 3{,}143 - 3{,}142 = 0{,}001\ m^2$

9.2.4 Change in length

$\delta_L = \frac{pd}{4tE}(1 - 2\gamma) = \frac{1{,}7M \times 2}{4 \times 0{,}03 \times 180G}(1 - 2 \times 0{,}29) = 0{,}0661$ mm

9.2.5 New volume

$L_{new} = 900{,}0661$ m and $A_{new} = 3{,}143\ m^2$

$V_{new} = AL = 3{,}143 \times 0{,}9000661 = 2{,}829\ m^3$

10. $\sigma_y = 450$ MPa

$t = 15$  $\eta_t = 50\%$  $d = 2{,}1$  $P_i = 2$ MPa  $\eta_L = 75\%$

$\therefore \sigma t = \frac{P_i d}{2t\eta_C} = \frac{2M \times 2{,}1}{2 \times 0{,}015 \times 0{,}75}$

$= 186{,}67$ MPa

$\therefore FoS = \frac{\sigma_y}{\sigma_t} = \frac{450}{186{,}67} = 2{,}41$

Take 3

$$\sigma_C = \frac{P_i d}{4t\eta_C} = \frac{2M \times 2{,}1}{4 \times 0{,}015 \times 0{,}5}$$

$$= 140 \text{ MPa}$$

$$\therefore \text{FoS} = \frac{\sigma_y}{\sigma_L} = \frac{450}{140} = 3{,}21$$

Use 4 as the FoS.

11. $\sigma = 120$ MPa  D = 2,5  $t = 14$  $\eta_L = 80\%$  $\eta_C = 40\%$

11.1 $\sigma_t = \dfrac{P_i D}{2t\eta_C}$

$$P_i = \frac{\sigma_t \eta_L}{D}$$

$$= \frac{120M \times 4 \times 0{,}014 \times 0{,}4}{2{,}5}$$

$$= 1{,}0752 \text{ MPa}$$

$$\sigma_C = \frac{P_i D}{4t\eta_C}$$

$$P_i = \frac{120M \times 4 \times 0{,}014 \times 4}{2{,}5}$$

$$= 1{,}0752 \text{ MPa}$$

11.2 1,0752 MPa

12. 12.1 Pressure

$$V_{org} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi 0{,}6^3 = 0{,}9048 \text{ m}^3$$

$$\delta_V = \frac{3pdV}{4tE}(1 - \gamma) = \frac{3p \times 1{,}2 \times 0{,}9048}{4 \times 0{,}008 \times 210G}(1 - 0{,}33) = 85 \times 10^{-6}$$

$$p = \frac{571200}{2{,}1824} = 261{,}733 \text{ kPa}$$

12.2 Stress in material

$$\sigma_L = \frac{pd}{4t} = \frac{261\,733 \times 1{,}2}{4 \times 0{,}008} = 9{,}815 \text{ MPa}$$

12.3 Material fail

$$p = \frac{\sigma_L 4t}{d} = \frac{300M \times 4 \times 0{,}008}{1{,}2} = 8 \text{ MPa}$$

13. 13.1 Change in volume

$$V_{org} = \frac{\pi}{4}d^2 \times L$$

$$d = \sqrt{\frac{60 \times 10^{-3} \times 4}{2\pi}} = 195{,}441 \text{ mm}$$

$$\delta v = \frac{pdV}{4tE}(5 - 4\gamma)$$

$$\delta v = \frac{2M \times 0{,}195441 \times 60 \times 10^{-3}}{4 \times 0{,}008 \times 200G}(5 - 4 \times 0{,}3) = 1{,}393 \times 10^{-8} \text{ m}^3$$

13.2 New length

$$\delta_L = \frac{pdL}{4tE}(1 - 2\gamma) = \frac{2M \times 0{,}195441 \times 2}{4 \times 0{,}008 \times 200G}(1 - 2 \times 0{,}3) = 3{,}636 \times 10^{-5} \text{ m}$$

$$L_{new} = 2 + 3{,}636 \times 10^{-5} = 2{,}00003636 \text{ m}$$

13.3 New diameter

$$\delta_d = \frac{pd^2}{4tE}(2 - \gamma) = \frac{2M \times 0{,}195441^2}{4 \times 0{,}008 \times 200G}(2 - 0{,}3) = 2{,}029 \times 10^{-5} \text{ m}$$

$$d_{new} = 195{,}441 + 0{,}02029 = 195{,}461 \text{ mm}$$

14. Poisson's ratio

$$V_{org} = \frac{\pi}{4}0{,}16^2 \times 1{,}2 = 0{,}02413 \text{ m}^3$$

$$\delta v = \frac{pdV}{4tE}(5 - 4\gamma)$$

$$18{,}9 \times 10^{-6} = \frac{9M \times 0{,}16 \times 0{,}02413}{4 \times 0{,}006 \times 210G}(5 - 4\gamma)$$

$$5 - 4\gamma = \frac{18{,}9 \times 10^{-6}}{6{,}894 \times 10^{-6}}$$

$$4\gamma = 5 - 2{,}742$$

$$\gamma = 0{,}565$$

15. 15.1 Longitudinal and hoop stress

$$V_{org} = \frac{\pi}{4}2^2 \times 1{,}6 = 5{,}027 \text{ m}^3$$

$$\delta v = \frac{pdV}{4tE}(5 - 4\gamma)$$

$$16 \times 10^{-6} = \frac{p2 \times 5{,}027}{4 \times 0{,}006 \times 210G}(5 - 4 \times 0{,}33)$$

$$p = 2{,}18 \text{ kPa}$$

$$\sigma_H = \frac{pd}{2t} = \frac{2{,}18k \times 2}{2 \times 0{,}006} = 363{,}333 \text{ kPa}$$

$$\sigma_L = \frac{pd}{4t} = \frac{2{,}18k \times 2}{4 \times 0{,}006} = 181{,}667 \text{ kPa}$$

15.2 With efficiency

$$\sigma_H = \frac{\sigma_{H\text{ no effic}}}{\acute{\eta}_L} = \frac{363{,}333}{0{,}85} = 427{,}333 \text{ kPa}$$

$$\sigma_L = \frac{\sigma_{L\text{ no effic}}}{\acute{\eta}_H} = \frac{181{,}667}{0{,}45} = 403{,}704 \text{ kPa}$$

15.3 Change in volume; new pressure

Pressure is directly proportional to the pressure, which means the percentage the volume changes the pressure will change with the same percentage.

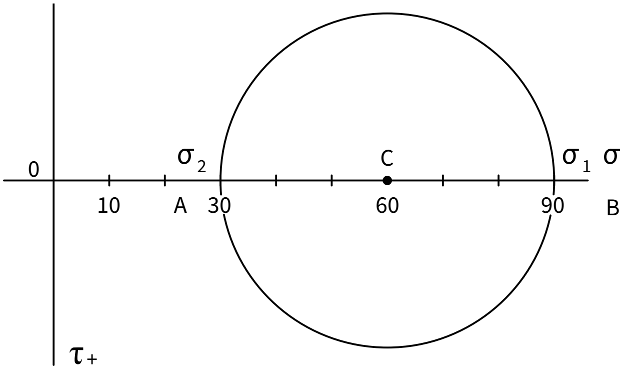
New pressure will be = % × $p$ = 1,12 × 2,18 = 2,442 kPa

## Exercise 4.2 — SB page 134

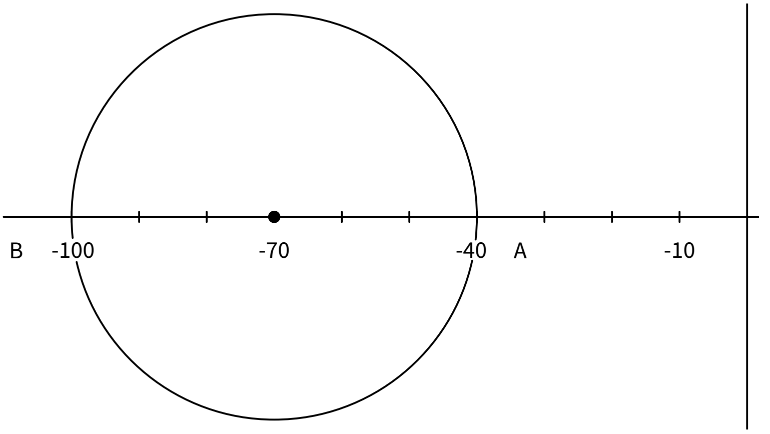
1. Positions of the principal stresses in circles (a)–(c)

Scale: 1 cm = 10 MPa

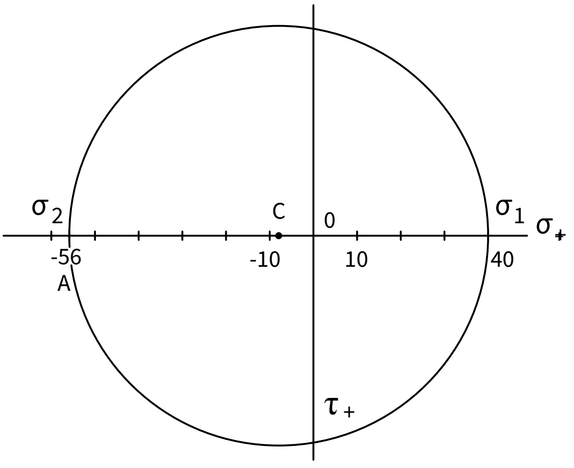
(a) $C = \frac{30 + 90}{2} = +60$




(b) $C = \frac{-40 - 100}{2} = -70$



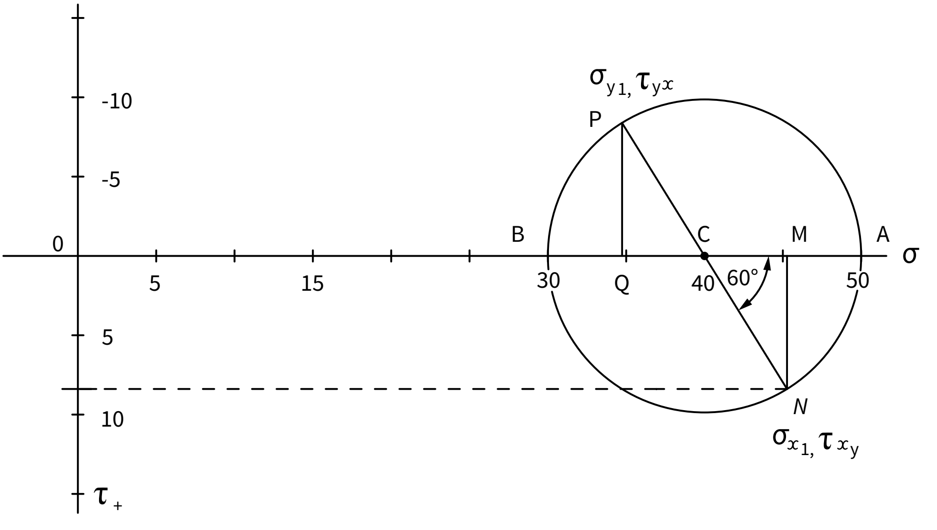
(c) $C = \frac{-56 + 40}{2} = -8$




2. Scale: 1 cm = 5 MPa

$C = \frac{50 + 30}{2} = +40$

2.1 Normal stresses




Normal stress $x$-face = OM = $\sigma_{x1}$ = 9 × 5 = 45 MPa (T)

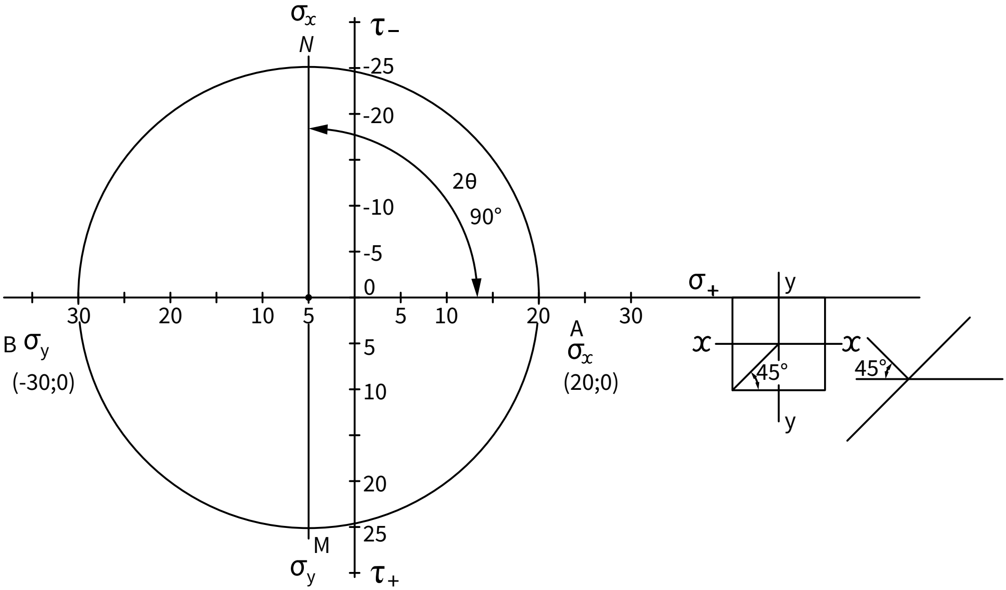
Normal stress $y$-face = OQ = $\sigma_{y1}$ = 7 × 5 = 35 MPa (T)

2.2 Shear stress

Shear stress at $x$-face = MN = $\tau_{xy}$ = 1,6 × 5 = 8 MPa (T)

3. Scale: 1 cm = 5 MPa

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{20 - 30}{2} = -5 \text{ MPa}$$

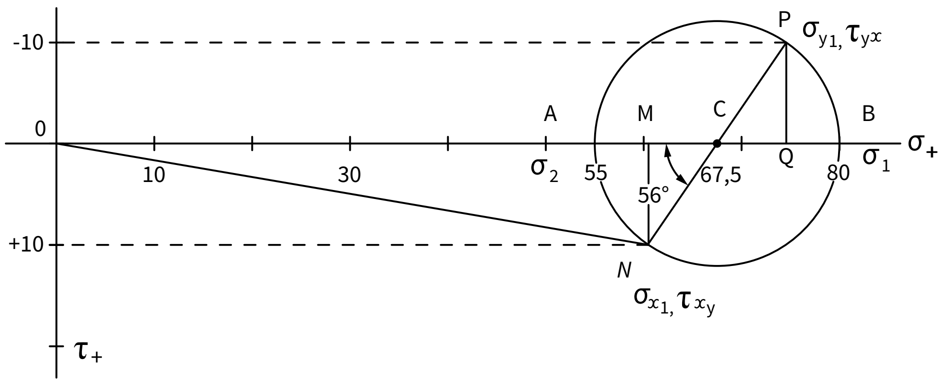



Normal stresses on both axes are zero = 0

4.1 Normal stresses

Scale: 1 cm = 10 MPa

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{55 + 80}{2} = 67{,}5 \text{ MPa}$$




∴ Normal stress on the X-face = $\sigma_{x1}$ = OM = 6,2 × 10 = ±62 MPa (T)
∴ Normal stress on the Y-face = $\sigma_{y1}$ = OQ = 7,4 × 10 = ±74 MPa (T)

4.2 Shear stresses

Shear stress $x$-face = $\tau_{xy}$ = MN = 1 × 10 = ±10 MPa (T)

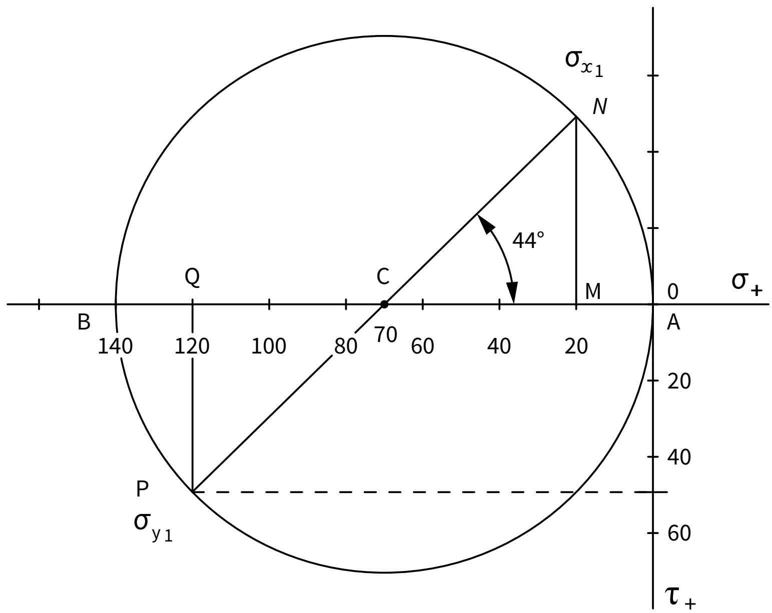
Shear stress $y$-face = $\tau_{yx}$ = PQ = 1 × 10 = ±10 MPa (T)

4.3 Resultant stress

Resultant stress = $\sigma_R$ = ON = 6,1 × 10 = 61 MPa

5. Scale: 1 cm = 20 MPa

$C = \frac{\sigma_x + \sigma_y}{2} = \frac{0 - 140}{2} = -70$ MPa



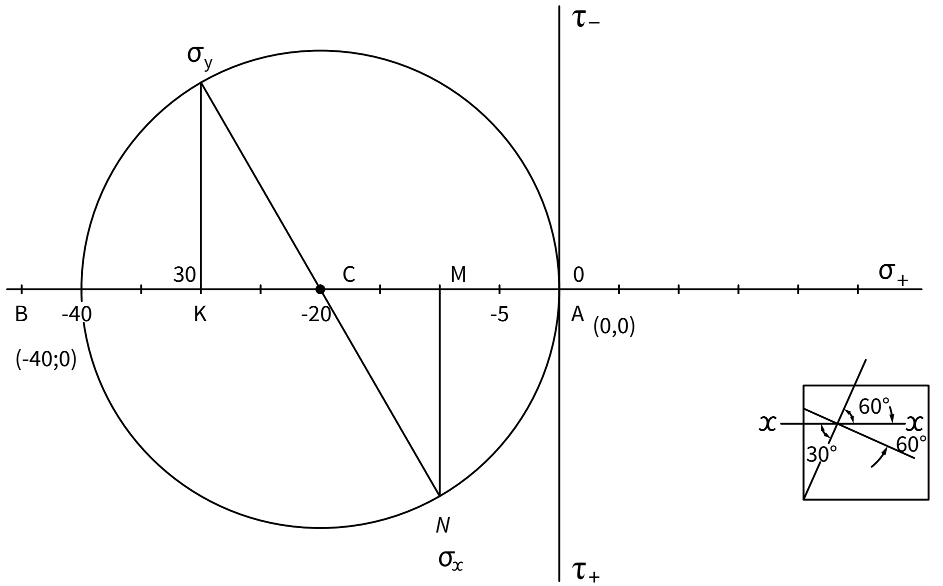

5.1 Normal stress on X-face = $\sigma_{x1}$ = OM = 1 × 20 = ±20MPa (C)

5.2 Normal stress on Y-face = $\sigma_{y1}$ = OQ = 2 × 20 = ±120 MPa (C)

5.3 Shear stress $y$-face = $\tau_{yx}$ = PQ = 2,25 × 20 = ±45 MPa (T)

6. Scale: 1 cm = 5 MPa

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{0 - 40}{2} = -20 \text{ MPa}$$




Normal stress on X-face = $\sigma_{x1}$ = OM = 2 × 5 = ±10MPa (C)

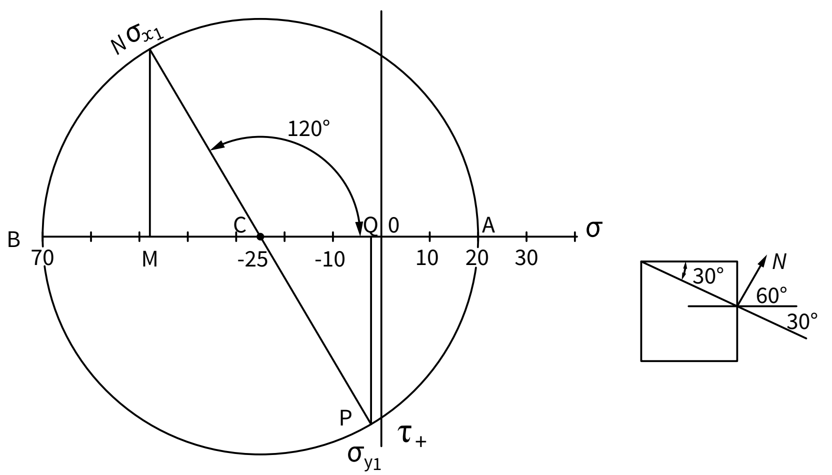
Normal stress on Y-face = $\sigma_{y1}$ = OK = 6 × 5 = ±30 MPa (C)

7. Scale 1 cm = 10 MPa

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{20 - 70}{2} = -25 \text{ MPa}$$

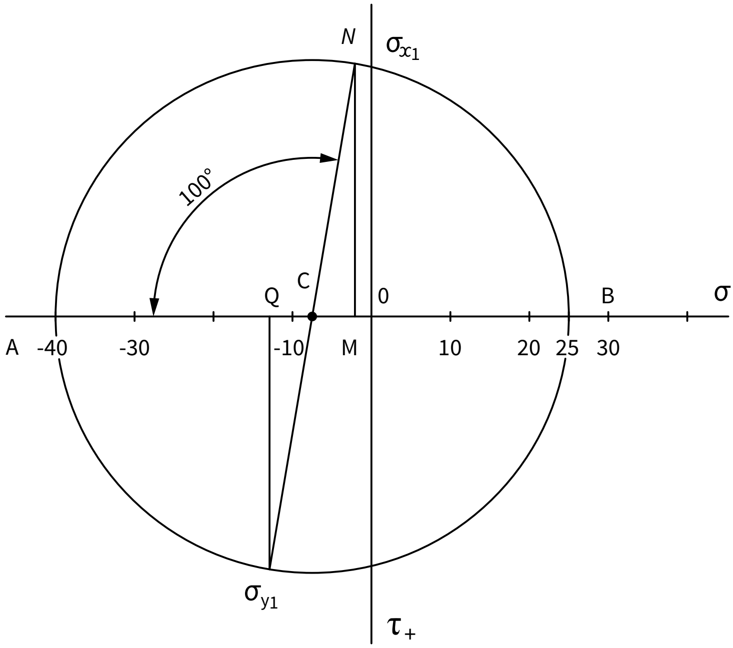
Normal stress on X-face = $\sigma_{x1}$ = OM = 4,7 × 10 = ±47MPa (C)

Normal stress on Y-face = $\sigma_{y1}$ = OQ = 0,15 × 10 = ±1,5 MPa (C)



8. Scale: 1 cm = 10 MPa

$C = \frac{\sigma_x + \sigma_y}{2} = \frac{-40 + 25}{2} = -7{,}5$ MPa




Normal stress on X-face = $\sigma_{x1}$ = OM = 0,2 × 10 = ±2 MPa (C)

Normal stress on Y-face = $\sigma_{y1}$ = OQ = 1,4 × 10 = ±14 MPa (C)

9. 9.1 Allowable diameter

Safe stress = $\frac{400M}{2{,}5}$ = 160 MPa

Hoop stress: $\sigma_H = \frac{pd}{2t\cap_L}$

Diameter $d = \frac{\sigma_H 2t\cap_L}{p} = \frac{160M \times 2 \times 0{,}016 \times 0{,}75}{3M} = 1{,}28$ m

Longitudinal stress: $\sigma_L = \frac{pd}{4t\cap_t}$

Diameter $d = \frac{\sigma_L 4t\cap_t}{p} = \frac{160M \times 4 \times 0{,}016 \times 0{,}5}{3M} = 1{,}71$ m

Allowable diameter 1,28 m due to hoop stress 160 MPa

9.2 Longitudinal and hoop stresses

Hoop stress = 160 MPa

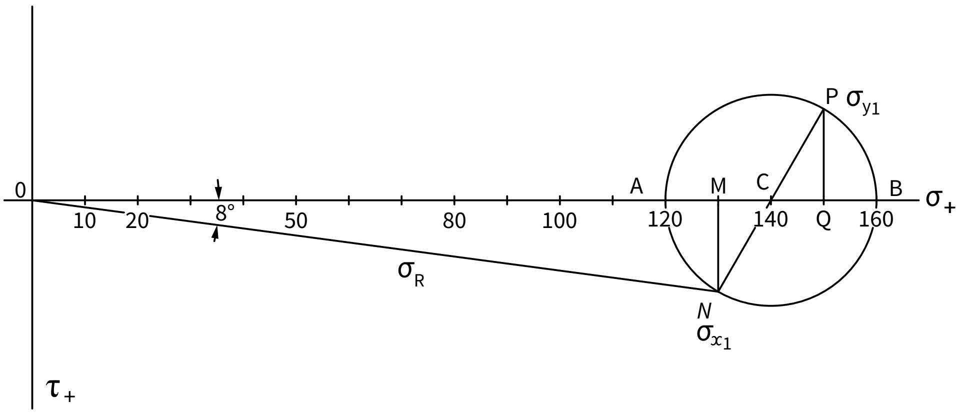
Longitudinal stress = $\sigma_L = \frac{pd}{4t\cap_t} = \frac{3M \times 1{,}28}{4 \times 0{,}016 \times 0{,}5} = 120$ MPa

9.3 Mohr's circle

Both stresses are tensile

Scale: 1 cm = 10 MPa

$C = \frac{\sigma_x + \sigma_y}{2} = \frac{-120 - 160}{2} = -140$ MPa




Normal stress on X-face = $\sigma_{x1}$ = OM = 13 × 10 = ±130MPa (T)

Normal stress on Y-face = $\sigma_{y1}$ = OQ = 14 × 10 = ±140 MPa (T)

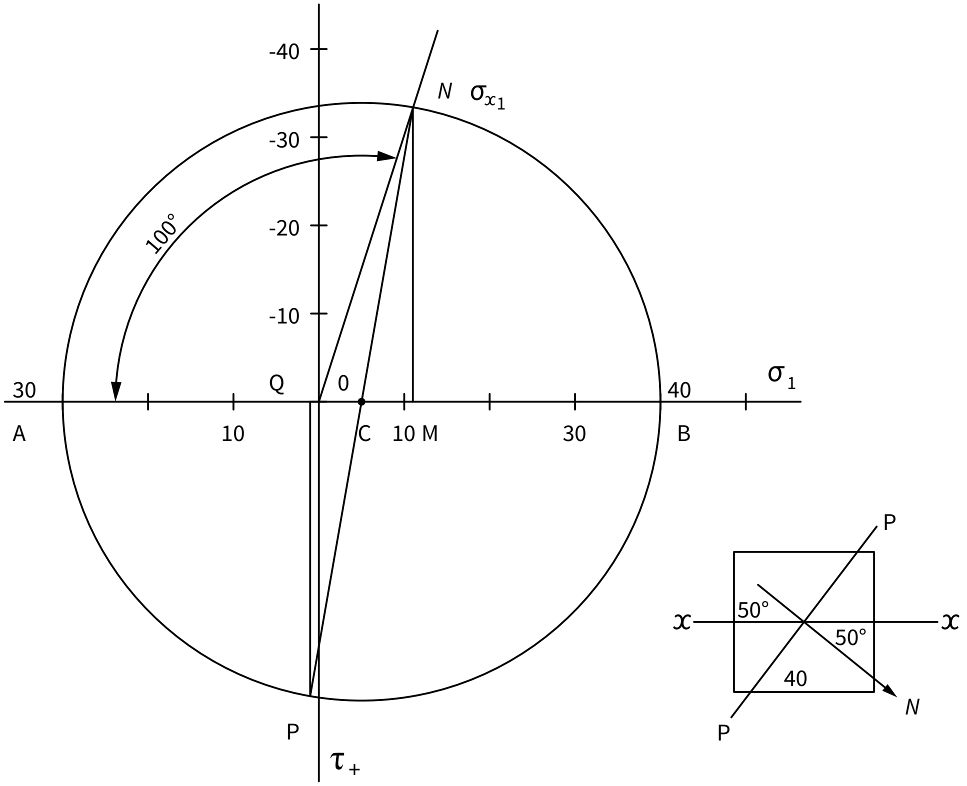
9.4 Resultant stress

Resultant stress = $\sigma_R$ = ON = 13 × 10 = 130 MPa at 8°

10. 10.1 Normal stress on $x$- and $y$-faces

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{-30 + 40}{2} = 5 \text{ MPa}$$

Scale: 1 cm = 10 MPa




Normal stress on X-face = $\sigma_{x1}$ = OM = 1,1 × 10 = ±11MPa (T)

Normal stress on Y-face = $\sigma_{y1}$ = OQ = 0,18 × 10 = ±1,8 MPa (C)

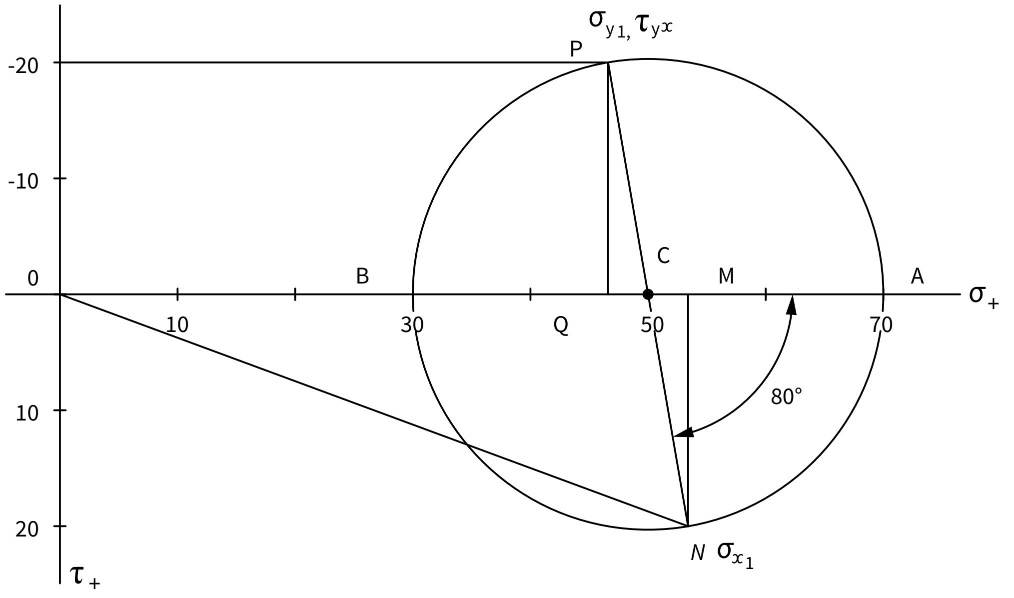
10.2 Shear stress on the $x$-face

Shear stress $x$-face = MN = 3,4 × 10 = 34 MPa (T)

10.3 Resultant stress

$\sigma_R$ = ON = 3,5 × 10 = 35 MPa

11. $C = \frac{\sigma_x + \sigma_y}{2} = \frac{-70 - 30}{2} = -50$ MPa




Scale: 1 cm = 10 MPa

11.1 Normal stress on X-face = $\sigma_{x1}$ = OM = 5,5 × 10 = ±55 MPa (T)

11.2 Shear stress on the *y*-face

$\tau_{yx}$ = QP = 2 × 10 = 20 MPa T

11.3 Resultant stress

$\sigma_R$ = ON = 5,8 × 10 = 58 MPa

MODULE

# 5 *Simple bending of beams*

**By the end of this module, students should be able to**:

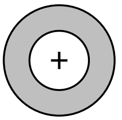
- use the simple bending equation;
- use bending moment formulae for standard beams;
- calculate the maximum moment of resistance for simply supported beams and cantilevers supporting a maximum of one uniformly distributed load (UDL) and one point load only;
- calculate position and value of maximum bending moment for simply supported beams supporting UDL over full length, plus an eccentric point load;
- calculate centroid position (neutral axis) for standard and built-up beams under pure bending;
- calculate the second moment of area about both axes for standard and built-up beams;
- calculate maximum and minimum stresses for built-up beams;
- sketch a stress distribution diagram to indicate the position of the neutral axis;
- select information for standard steel sections from section tables;
- select suitable standard profiles from section tables using a section modulus with the bending stress limit given;
- calculate maximum and minimum stresses in standard profiles from section tables;
- calculate the centroid position (neutral axis) for standard and built-up beams under pure bending;
- calculate the second moment of area about both axes for standard and built-up beams;
- calculate maximum and minimum stresses for built-up beams; and
- sketch a stress distribution diagram to indicate the position of the neutral axis.

The critical point on a beam will be the point of failure, which is at the point of maximum bending moment.

The point of maximum bending moment is the point where the stress will be at its maximum and, for the beam to be safe, the stress must be in the elastic limit of the material.

## Exercise 5.1 SB page 159

1.



OD =500 ID = 460 $\frac{M}{1} = \frac{\sigma}{y}$

$I = \frac{\pi}{64}(0{,}5^4 - 0{,}46^4) = 8{,}701 \times 10^{-4}\ m^6$

$y = \frac{500}{2} = 0{,}25$

$\therefore M = \frac{\sigma I}{y} = \frac{60M \times 8{,}701 \times 10^{-4}}{0{,}25}$

$= 208{,}823$ kNm

But $M_{max} = M_L + M_w$

$= \frac{wL^2}{8} + \frac{wL^2}{8}$

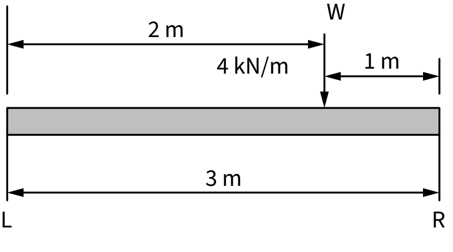
$208\ 823 = \frac{L^2}{8}[1\ 304 + 2\ 308]$

$L = \sqrt{462{,}51}$

$= 21{,}51$ m

2. Moments about L; $\therefore 3R = 2W + 4k \times 3 \times \frac{3}{2}$

$R = 0{,}667W + 6k$



Maximum BM at point load; $M_{max} = 1 \times R - \left(4k \times 1 \times \frac{1}{2}\right)$

$$= 1(0{,}667W + 6k) - 2k$$

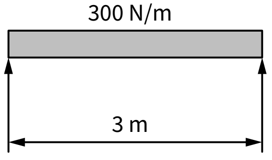
$$= 0{,}667W + 4k \ldots (1)$$

$M_{max} = \frac{\sigma I}{y} = \frac{100M \times 0{,}08 \times 0{,}4^3}{0{,}2 \times 12} = 213{,}35 \text{ kNm} \ldots (2)$

$(1) = (2) \quad \therefore 0{,}667W + 4k = 213{,}35k$

W = 313,86 kN

3.




D = 50

$\frac{M}{I} = \frac{\sigma}{y}$: $M = \frac{\sigma I}{y} = \frac{120M \times \pi 0{,}05^4}{0{,}025 \times 64} = 1\,472{,}63 \text{ Nm}$

$1\,472{,}63 = \frac{wL^2}{8} + \frac{wL^2}{8}$

$= \frac{300 \times 3^2}{8} + \frac{w3^2}{8}$

W = 1,009 kNm

4.1 $6R = (80 \times 22) + \left(12 \times 6 \times \frac{6}{3}\right)$

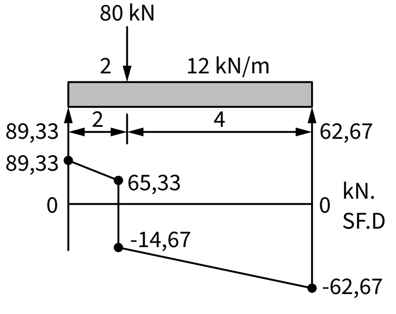
R = 62,67 kN

$6L = (80 \times 4) + \left(12 \times 6 \times \frac{6}{2}\right)$

L = 89,33 kN

4.2 PL 2 m from left shear force values: 89,33; 65,33l – 14,67; –62,67

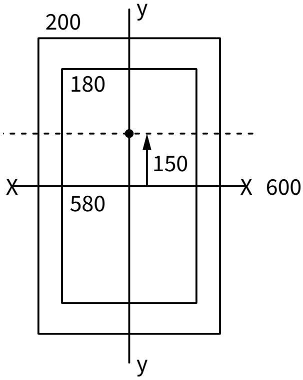
4.3




$M_{max} = (89{,}33 \times 2) - \left(12 \times 2 \times \frac{2}{2}\right)$(at 80 kN point of inflection)

$= 154{,}6 \text{ kNm}$

4.4




$$I_{xx} = \frac{1}{12}\left[0{,}2 \times 0{,}6^3 - 0{,}18 \times 0{,}58^3\right]$$

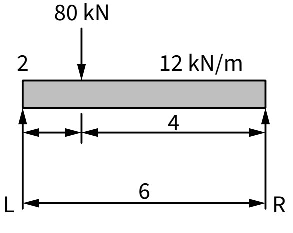
$$= 6{,}7332 \times 10^{-4}\ \text{m}^4$$

$$\frac{M}{I} = \frac{\sigma}{y} \qquad \sigma = \frac{My}{I}$$

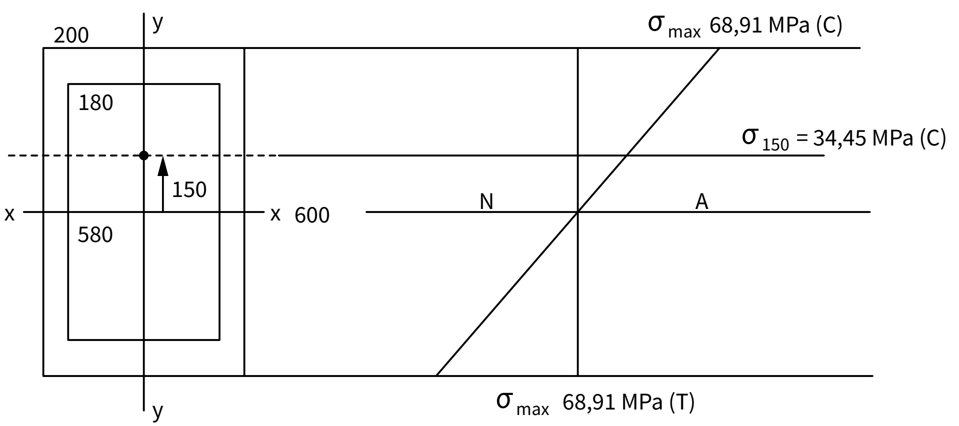
$$= \frac{154{,}66 \times 10^3 \times 0{,}3}{6{,}7332 \times 10^{-4}}$$

$$= 68{,}91\ \text{MPa}$$

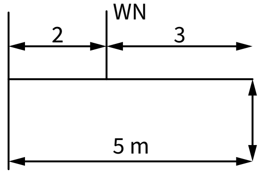
4.5



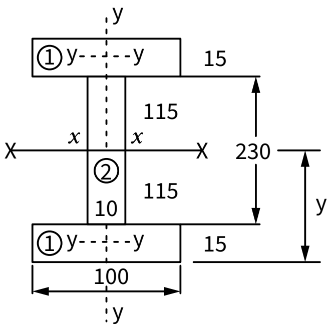

4.6 Stress diagram



5.




$\sigma = 80$ MPa




$y = 115 + 15 = 130$ mm

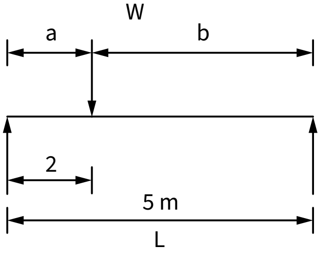
$I_{xx} = 2\left[I_{1yy} + A_1 h_1^2\right] + \left[I_{2xx} + A_2 h_2^2\right]$

$h_1 = 122{,}5$ mm $\qquad h_2 = 0$

$$\therefore I_{xx} = 2\left[\frac{0{,}01 \times 0{,}015^3}{12} + 0{,}1 \times 0{,}015 \times 0{,}1225^2\right] + \frac{0{,}01 \times 0{,}23^3}{12}$$

$$= 45{,}075 \times 10^{-6} + 10{,}139 \times 10^{-6}$$

$$= 55{,}214 \times 10^{-6} \text{ m}^4$$



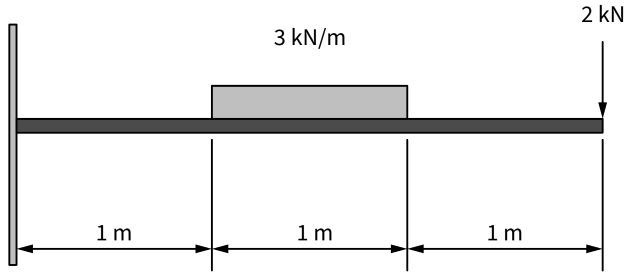

$$M = \frac{\sigma I}{y} = \frac{80M \times 55{,}214 \times 10^{-6}}{0{,}13}$$

$$= 33{,}978 \text{ kNm}$$

$$M = \frac{W_{ab}}{L}$$

$$33\ 978 = \frac{W \times 2 \times 3}{5} \times W = 28{,}315 \text{ kN}$$

6.




$$I_{xx} = \frac{0{,}03 \times D^3}{12} = 2{,}5 \times 10^{-3} D^3 \ldots (1)$$

**Important**

Students must show all steps in calculations.

6.1 BM 1 m from free end $M = 2k \times 1 = 2$ kNm

$$I = \frac{My}{\sigma} = \frac{2k \times D}{120M \times 2} = 2{,}5 \times 10^{-3} D^3$$

$$D^2 = \frac{2k}{120M \times 2 \times 2{,}5 \times 10^{-3}}$$

D = 57,74 mm

6.2 BM 2 m from free end; $M = (2k \times 2) + \left(3 \times 1 \times \frac{1}{2}\right) = 5{,}5$ kNm

$$D^2 = \frac{5{,}5k}{120M \times 2 \times 2{,}5 \times 10^{-3}}$$

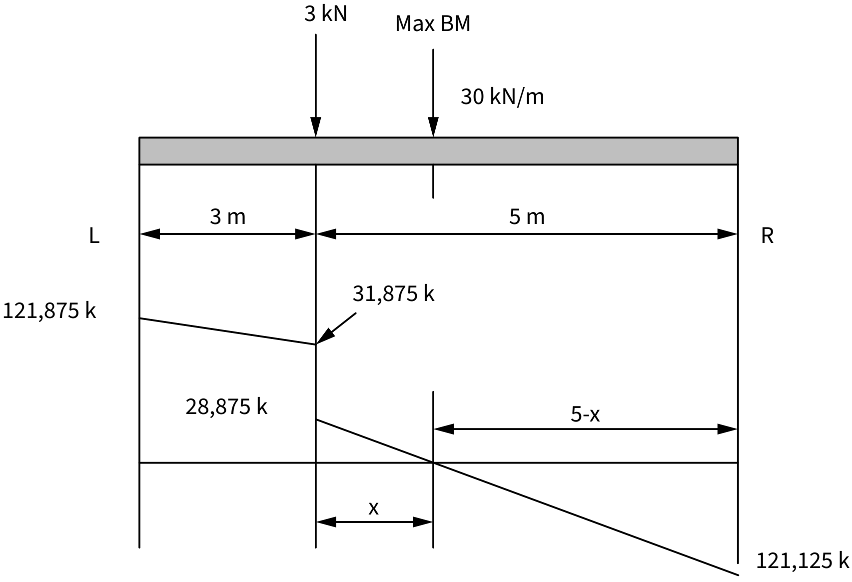
D = 95,74 mm

6.3 BM at fixed end; $M = (2k \times 3) + (3k \times 1 \times 1{,}5) = 10{,}5$ kNm

$$D^2 = \frac{10{,}5k}{120M \times 2 \times 2{,}5 \times 10^{-3}}$$

D = 132,288 mm

7.




7.1 Maximum BM

Reactions: Moments about L; 8R = $(3k \times 3) + (30k \times 8 \times 4)$

R = 121,125 kN

Moments about R; 8L = $(3k \times 5) + (30k \times 8 \times 4)$

L = 121,875 kN

Determine $x$; $\tan \propto = \tan \propto \therefore \frac{28,875k}{x} = \frac{121,125k}{(5-x)}$

$4,195x = 5 - x$

X = 962,464 mm

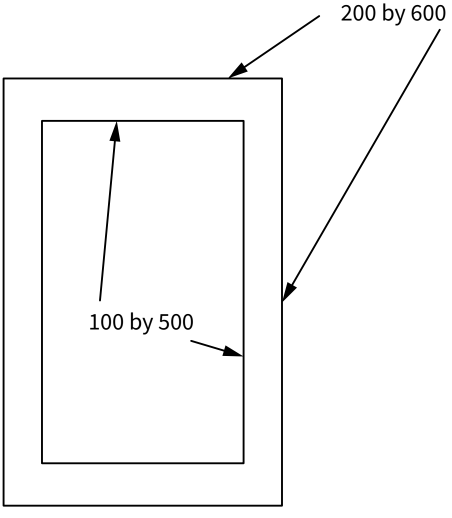
Maximum BM is (5 – 0,962464) = 4,038 m from the right support

$M_{max} = (121,125k \times 4,038) - \left(3i \times 4,038 \times \frac{4,038}{2}\right)$

$= 489,103k - 244,582k$

$M_{max} = 244,521$ kNm

7.2 Maximum stress




$$I_{xx} = \frac{1}{12}\left[\left(0,2 \times 0,6^3\right) - \left(0,1 \times 0,5^3\right)\right] = 0,01089 \text{ m}^4$$

$$\text{Stress:}\sigma = \frac{My}{I} = \frac{244,521k \times 0,3}{0,01089} = 6,736 \text{ MPa}$$

7.3 Section modulus about X-axis

$$Z_{xx} = \frac{I_{xx}}{y} = \frac{0,01089}{0,3} = 0,363 \text{ m}^3$$

7.4 Section modulus about *yy*-axis

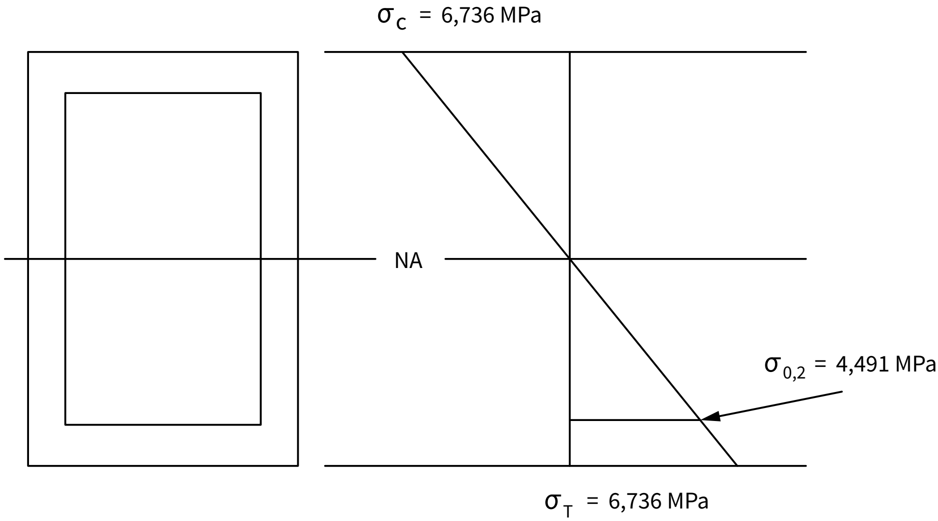
$$I_{yy} = \frac{1}{12}\left[\left(0,6 \times 0,2^3\right) - \left(0,5 \times 0,1^3\right)\right] = 3,583 \times 10^{-4} \text{ m}^4$$

$$Z_{yy} = \frac{I_{yy}}{x} = \frac{3,583 \times 10^{-4}}{0,1} = 3,583 \times 10^{-4} \text{ m}^3$$

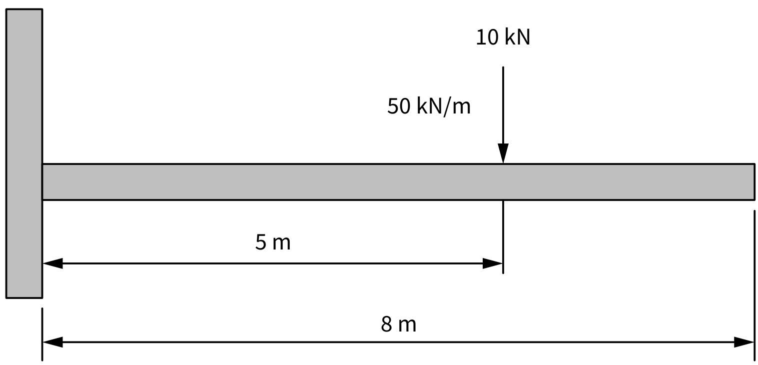
7.5 Stress 200 mm below *x*-axis

$$\sigma_{\text{at 200 mm}} = \frac{M y_{0,2}}{I_{xx}} = \frac{244,521k \times 0,2}{0,01089} = 4,491 \text{ MPa}$$

7.6




8.




8.1 Maximum BM

$M_{max} = (10k \times 5) + (50k \times 8 \times 4) = 1\ 650$ kNm

8.2 Dimensions for beam

$D = 2d \ldots (1)$

$I_{xx} = \frac{\pi}{64}[D^4 - d^4] = \frac{\pi}{64}[(2d)^4 - d^4] = \frac{\pi 15 d^4}{64} \ldots (2)$

$y = \frac{D}{2} = \frac{2d}{2} = d \ldots (3)$

$\sigma = \frac{My}{I} = 160M = \frac{1\ 650k \times d \times 64}{\pi 15 d^4}$

$d^3 = \frac{1\ 650k \times 64}{\pi 15}$

$d = 241$ mm

$D = 2 \times 241 = 482$ mm

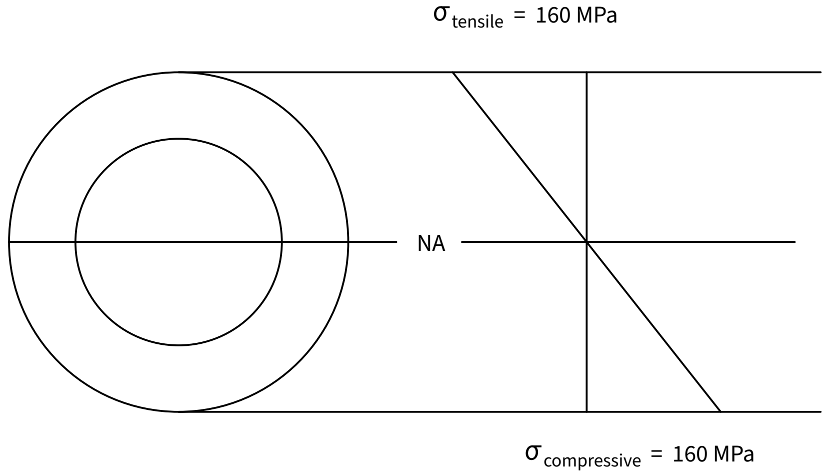
8.3 Stress at point load

BM at point load $= 50k \times 3 \times \frac{3}{2} = 225$ kNm

From (2) $I_{xx} = \frac{\pi 15 d^4}{64} = \frac{\pi 15 \times 0{,}241^4}{64} = 2{,}484 \times 10^{-3}\ \text{m}^4$

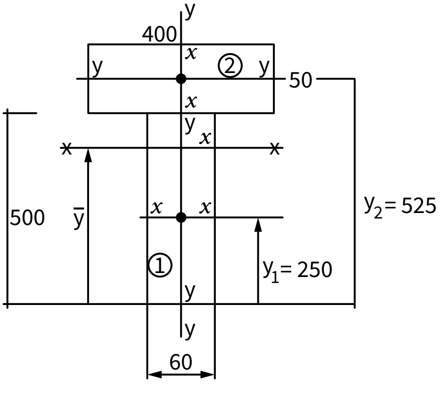
Stress $= \sigma = \frac{My}{I} = \frac{225k \times 0{,}241}{2{,}484 \times 10^{-3}} = 21{,}83$ MPa $\left(y = \frac{D}{2} = \frac{482}{2} = 241\right)$

8.4 Stress diagram at maximum stress




## Exercise 5.2 SB page 173

1.




$\bar{y} A_T = A_1 y_1 + A_2 y_2$

1.1 $y(400 \times 50 + 60 \times 500) = (500 \times 60 \times 250) + 50\ 000\ y = 18 \times 10^6$

$\bar{y} = 360$ mm

$h_1 = \bar{y} - g = 360 - 250 = 110$ m

$h_2 = y_2 - \bar{y} = 525 - 360 = 165$ mm

1.2 $I_{xx} = I_{xx} + A_1 h_1^2 + I_{2yy} + A_2 h_2^2$

$= \left(\frac{0{,}06 \times 0{,}5^3}{12} + 0{,}06 \times 0{,}4 \times 0{,}11^2\right) + \left(\frac{0{,}4 \times 0{,}05^3}{12} + 0{,}4 \times 0{,}05 \times 0{,}165^2\right)$

$= 9{,}88 \times 10^{-4} + 5{,}487 \times 10^{-4}$

$= 1\ 536{,}7 \times 10^{-6}\ m^4$

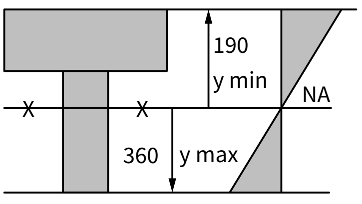
1.3 $I_{yy} = I_1 + A_1 h_1^2 + I_2 + A_2 h_2^2 \qquad h_1 = h_2 = 0$

$= I_{xx} + I_{2yy}$

$= \frac{0{,}05 \times 0{,}4^2}{12} \times \frac{0{,}5 \times 0{,}06^3}{12}$

$= 275{,}67 \times 10^{-6}\ m^4$

1.4




$\therefore \frac{\sigma M}{I} = \frac{\sigma}{y}$

$M = \frac{WL}{4} = \frac{200k \times 5}{4} = 250\ kNM$

$\therefore \sigma_{max} = \frac{M_{y_{max}}}{I_{xx}} = \frac{250k \times 0{,}36}{1\ 536{,}7 \times 10^{-6}}$

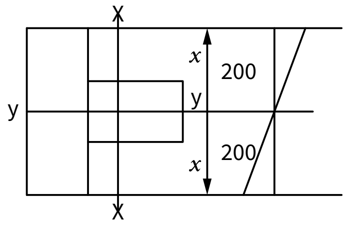
$= 58{,}57\ MPa$

$\sigma_{min} = \frac{M_{y_{min}}}{I_{xx}}$

$= \frac{250k \times 0{,}19}{1\ 536{,}7 \times 10^{-6}}$

$= 30{,}91\ MPa$

1.5




$\sigma_{max} = \sigma_{min} \qquad \therefore x_t = x_c$

$\therefore \sigma = \frac{250k \times 0{,}2}{275{,}67 \times 10^{-6}} \qquad \sigma = \frac{M_x}{I_{yy}}$

$\sigma = 181{,}38\ MPa$

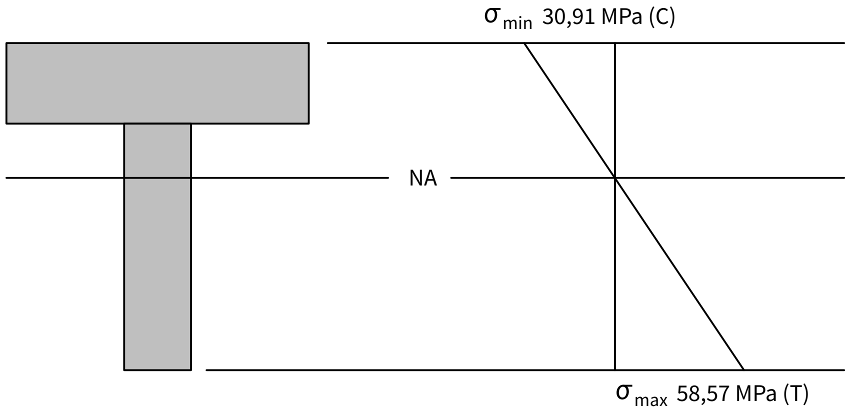
1.6 $Z_{max} = \frac{M}{\sigma_{max}} = \frac{250k}{58{,}57M}$

$= 4{,}268 \times 10^{-3}\ m^3$

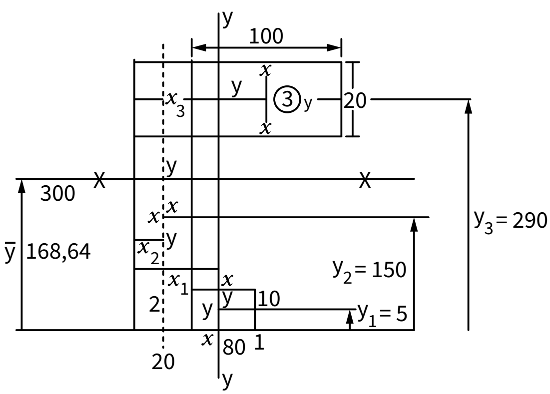
$Z_{min} = \frac{M}{\sigma_{min}} = \frac{250k}{30{,}91M}$

$= 8{,}088 \times 10^{-3}\ m^3$

1.7




2. 2.1




$\bar{y} A_T = A_1 y_1 = A_2 y_2 = A_3 y_3$

| No. | Area | | $y$ | $A_y$ |
|---|---|---|---|---|
| 1 | 10 × 80 | 800 | 5 | 4 000 |
| 2 | 20 × 300 | 6 000 | 150 | 900$k$ |
| 3 | 20 × 100 | 2 000 | 290 | 580$k$ |
| | Total | 8 800 | ΣA – max | 1 484$k$ |

$\therefore \bar{y}_{A_T} = \Sigma\text{Area} - \text{moment}$

$\bar{y} = 8{,}8k = 1\,484k$

$\bar{y} = 168{,}64 \text{ mm}$

$\bar{x}\,A_T = \Sigma\text{Area} - \text{moments}$

$\bar{x}8\,800 = \Sigma\text{Area} - \text{moments}$

| No. | Area | *x* | $A_x$ |
|---|---|---|---|
| 1 | 800 | 60 | 48*k* |
| 2 | 6 000 | 10 | 60*k* |
| 3 | 2 000 | 70 | 140*k* |
| | | ΣA – max | 248*k* |

$\therefore \bar{x}\,A_T = \Sigma A - \text{moment}$

$\bar{x}8\,800 = 248 \times 10^3$

$\bar{x} = 28{,}18 \text{ mm}$

2.2 $I_{xx} = I_{1yy} + A_1 h_1^2 + I_{2xx} + A_2 h_2^2 + I_{3yy} + A_3 h_3^2$

$h_1 = 168{,}4 - 5 = 163{,}64 \text{ mm}$

$h_2 = 168{,}64 - 150 = 18{,}64 \text{ mm}$

$h_3 = 190 - 168{,}64 = 121{,}64 \text{ m}$

$I_{1T} = I_{1yy} + A_1 h_1^2 = \frac{0{,}08 \times 0{,}01^3}{12} + 0{,}08 \times 0{,}01 \times 0{,}16364^2$

$= 2{,}143 \times 10^{-5} \text{ m}^4$

$I_{2T} = I_{2yy} + A_2 h_2^2 = \frac{(0{,}02 \times 0{,}3)^3}{12} + 0{,}02 \times 0{,}3 \times 0{,}01864^2$

$= 4{,}708 \times 10^{-5} \text{ m}^4$

$I_{3T} = I_{3yy} + A_3 h_3^2 = \frac{0{,}1 \times 0{,}02^3}{12} + 0{,}1 \times 0{,}02 \times 0{,}12164^2$

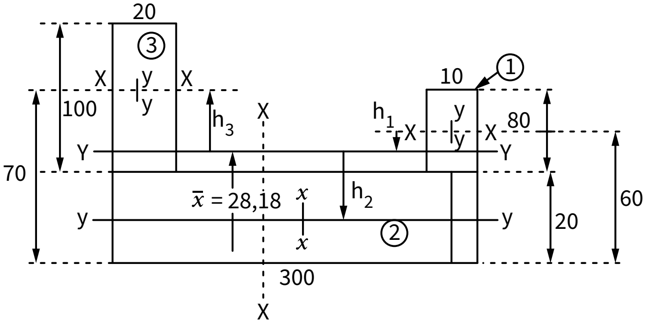
$= 2{,}966 \times 10^{-5}$

$I_{xx} = I_{1T} + I_{2T} + I_{3T}$

$= (2{,}143 + 4{,}708 + 2{,}966)10^{-5}$

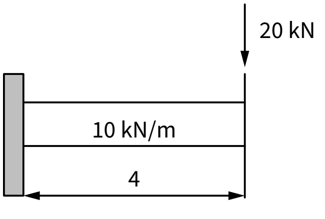
$= 9{,}817 \times 10^{-5} \text{ m}$

2.3

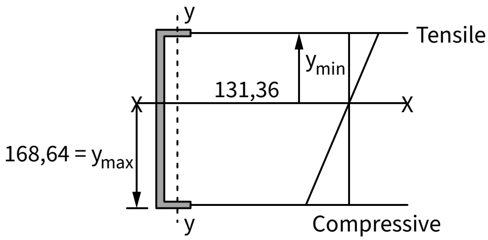



$$h_1 = 60 - 28{,}18 = 31{,}82$$
$$h_2 = 28{,}18 - 10 = 18{,}18$$
$$h_3 = 70 - 28{,}18 = 41{,}82$$
$$I_{yy} = I_{1xx} + A_1 h_1^2 + I_{2yy} + A_2 h_2^2 + I_{3xx} + A_3 h_3^2$$
$$I_{1T} = I_{xx} + A_1 h_1^2$$
$$= \frac{0{,}01 \times 0{,}08^3}{12} + [0{,}01 \times 0{,}08 \times 0{,}03182^2]$$
$$= 1{,}2367 \times 10^{-6}\ \text{m}^4$$
$$I_{2T} = I_{yy} + A_2 h_2^2$$
$$= \frac{(0{,}3 \times 0{,}02^2)}{12} + [0{,}3 \times 0{,}2 \times 0{,}01818^2]$$
$$= 2{,}183 \times 10^{-6}\ \text{m}^4$$
$$I_{3T} = I_{xx} + A_3 h_3^2$$
$$= \frac{0{,}02 \times 0{,}1^3}{12} + [0{,}02 \times 0{,}1 \times 0{,}04182^2]$$
$$= 5{,}164 \times 10^{-6}\ \text{m}^4$$
$$I_{yy} = I_{1T} + I_{2T} + I_{3T}$$
$$= (1{,}2367 + 2{,}183 + 5{,}164)10^{-6}$$
$$= 8{,}584 \times 10^{-6}\ \text{m}^4$$

2.4




$$M = WL + \frac{wL^2}{2} = (20 \times 4) - \left(10 \times 4 \times \frac{4}{2}\right)$$
$$= 160\ \text{kNm}$$




$$\frac{M}{F} = \frac{\sigma}{y}$$

$$\sigma_t = \frac{My}{I_{xx}} = \frac{160 \times 10^3 \times 0{,}13136}{98{,}17 \times 10^{-4}}$$

$$= 214{,}1 \text{ MPa}$$

2.5 $\sigma_{max} = \frac{My}{I} = 160 \times 10^3 \text{ kNm}$

$$Z_{max} = \frac{I_{xx}}{y_{max}} = \frac{98{,}17 \times 10^6}{0{,}16864}$$

$$= 5{,}821 \times 10^{-4} \text{ m}^3$$

2.6 Maximum stress

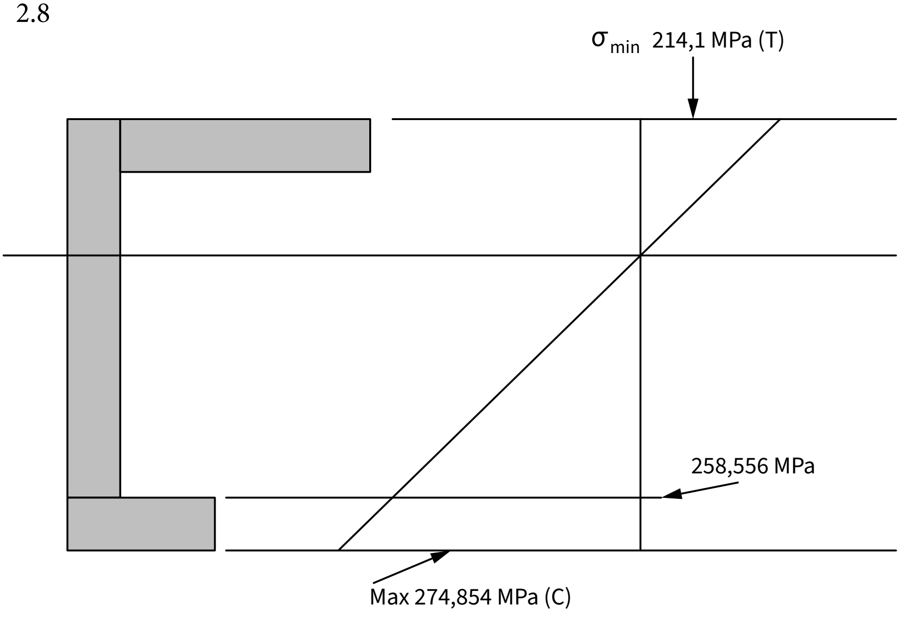
$$\sigma_{max} = \frac{My_{max}}{I} = \frac{160\text{k} \times 0{,}168{,}64}{98{,}17 \times 10^{-6}} = 274{,}854 \text{ MPa}$$

2.7 Stress at top of bottom flange

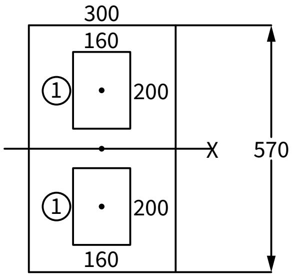
Distance from NA to top of bottom flange = 168,64 – 10 = 158,64 mm

$$\sigma_{\text{at top of flange}} = \frac{My_{\text{to top flange}}}{I} = \frac{160k \times 0{,}15864}{98{,}17 \times 10^{-6}} = 258{,}556 \text{ MPa}$$

2.8



3.1




$$I_{xx} = I_{xxT} - 2\left[I_{xx1} + A_1 h_1^2\right]$$

$$= \frac{0{,}3 \times 0{,}57^3}{12} - 2\left[\frac{0{,}16 \times 0{,}2^3}{12} + 0{,}16 \times 0{,}2 \times 0{,}125^2\right]$$

$$= 4{,}63 \times 10^{-3} - 1{,}213 \times 10^{-3}$$

$$= 3{,}4165 \times 10^{-3}\ \text{m}^4$$

$$\frac{M}{I} = \frac{\sigma}{y} \quad \therefore M = \frac{\sigma I}{y}$$

$$= \frac{130 \times 10^6 \times 3{,}4165 \times 10^{-3} \times 2}{0{,}57}$$

$$= 1{,}558\ \text{MNm}$$

3.2 $\frac{M}{I} = \frac{\sigma}{y}$

$$\therefore Z = \frac{M}{\sigma}$$

$$= \frac{1{,}558 \times 10^6}{130 \times 10^6}$$

$$= 0{,}0119876\ \text{m}^3$$

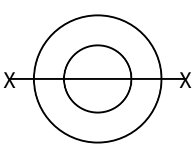
$$Z_{\text{shaft}} = \frac{I}{y} = \frac{\pi D^4 \times 2}{64D} = \frac{\pi D^3}{32}$$

$$\therefore \frac{\pi D^3}{32} = 0{,}0119876$$

$$D^3 = 0{,}1221$$

$$D = 496{,}11\ \text{mm}$$

4.



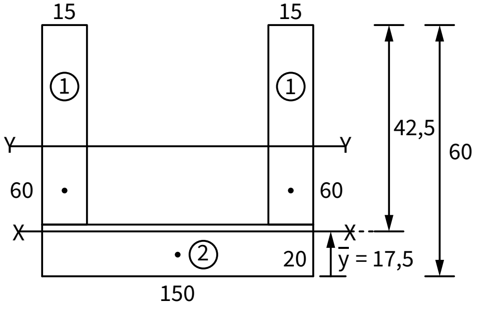

(a) $D = 80 \qquad d = 16{,}986$

$$Z_{xx} = \frac{I}{y} = \frac{\pi(D^4 - d^4) \times 2}{64D}$$

$$= \frac{\pi}{32}\left(\frac{(0{,}08^4 - 0{,}016984^4)}{0{,}08}\right)$$

$$= 50{,}163 \times 10^{-6}\ m^3$$

(b)




$$\bar{y}A_T = \Sigma A_{mom}$$

$$\bar{y}(2 \times 15 \times 60) + (150 \times 20) = 2(15 \times 60 \times 30) + (150 \times 20 \times 10)$$

$$4\ 800y = 84\ 000$$

$$\bar{y} = 17{,}5\ mm$$

$$I_{yy} = 2\left(I_{1T} + A_1 h_1^2\right) + I_2 + A_2 h_2^2$$

$$= 2\left[\frac{(0{,}015 \times 0{,}06^3)}{12} + 0{,}015 \times 0{,}06 \times 0{,}0125^2\right] + \left[\frac{0{,}15 \times 0{,}02^3}{12} + 0{,}15 \times 0{,}02 \times 0{,}0075^2\right]$$

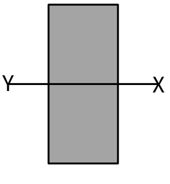
$$= 8{,}2125 \times 10^{-7} + 2{,}6875 \times 10^{-7}$$

$$= 1{,}09 \times 10^{-6}\ m^3$$

$$Z_{max} = \frac{I_{yy}}{y_{max}} = \frac{1{,}09 \times 10^{-6}}{0{,}0425}$$

$$= 25{,}647 \times 10^{-6}\ m^3$$

(c)



$$Z_{xx} = \frac{I}{y}$$

$$= \frac{BD^3 \times 2}{12D}$$

$$= \frac{BD^3}{6}$$

$$= \frac{0{,}06 \times 0{,}08^2}{6}$$

$$= 64 \times 10^{-3}\ m^3$$

(d) $I_{xx} = \frac{1}{12}[BD^3 - bd^3]$

$$= \frac{1}{12}[(0{,}06 \times 0{,}18^3) - (0{,}04 \times 0{,}15^3)]$$

$$= 1{,}791 \times 10^{-5}$$

$$= Z_{xx} = \frac{I_{xx}}{y} = \frac{1{,}791 \times 10^{-5}}{0{,}09}$$

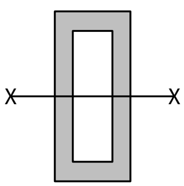
$$= 199 \times 10^{-6}\ m^3$$

Channel = $23{,}647 \times 10^{-6}\ m^3$

Pipe = $50{,}163 \times 10^{-6}\ m^3$

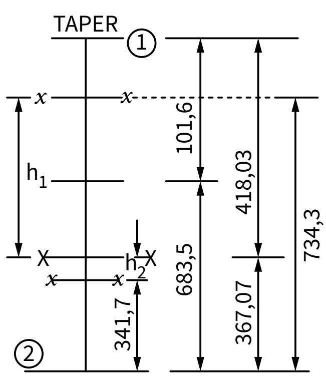
Solid rectangular = $64 \times 10^{-6}\ m^3$

Hollow rectangular = $199 \times 10^{-6}\ m^3$




## Exercise 5.3 SB page 190

1. $\bar{y}A_T = A_1y_1 + A_2y_2$



$\bar{y}(17{,}84 \times 10^{-3} + 1{,}23 \times 10^{-3})$

$= 17{,}84 \times 10^{-3} \times \frac{0{,}6835}{2} + 1{,}23 \times 10^{-3} \times 0{,}7343$

$= 7{,}000009 \times 10^{-3}$

$\bar{y} = 367{,}07 \text{ mm} \qquad (418{,}03)$

1.2 $I_{xx} = I_{xx} + A_1 h_1^2 + I_{xx} + A_2 h_2^2$

$h_1 = 734{,}3 - 367{,}07 = 367{,}23$

$h_2 = 367{,}07 - 341 = 25{,}32$

$I_{xx} = (2{,}176 \times 10^{-6} + 1{,}23 \times 10^{-3} \times 0{,}36723^2) + (1\,363 \times 10^{-6} + 17{,}34 \times 10^{-3} \times 0{,}02532^2)$

$= 1{,}681 \times 10^{-4} + 1{,}374 \times 10^{-3}$

$= 1\,542 \times 10^{-3} \text{ m}^4 \qquad 1\,542 \times 10^{-6} \text{ m}^4$

$I_{yy} = I_{yy_{1T}} + I_{yy_{2B}}$

$= 0{,}2528 \times 10^{-6} + 51{,}83 \times 10^{-6}$

$= 52{,}083 \times 10^{-6} \text{ m}^6$

1.3 $\frac{M}{I} = \frac{\sigma}{y} \quad \therefore \sigma_{max} = \frac{My_{max}}{I_{xx}} = \frac{400k \times 0{,}41803}{1\,542 \times 10^{-6}}$

$= 108{,}44 \text{ MPa}$

1.4 Stress at the XX-axis of the taper flange

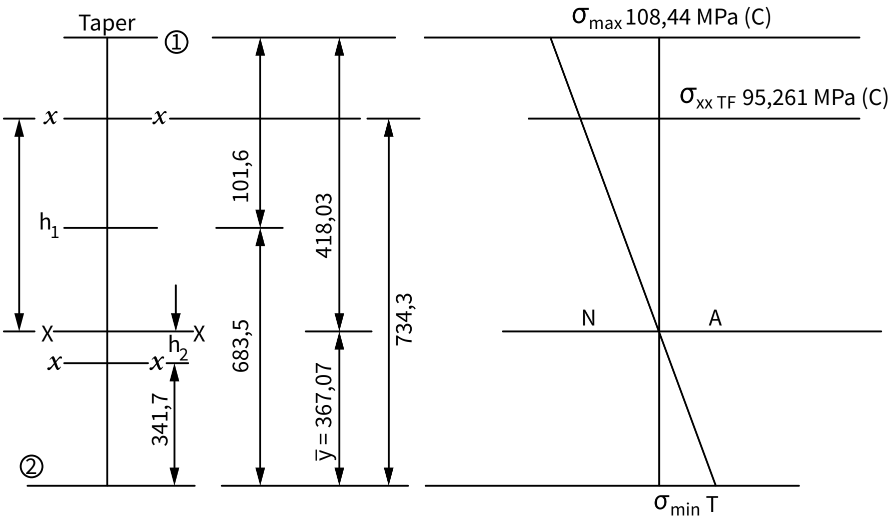
Y-distance from the common *xx* to the *xx* of the taper flange

$= h_1 = 367{,}23 \text{ mm}$

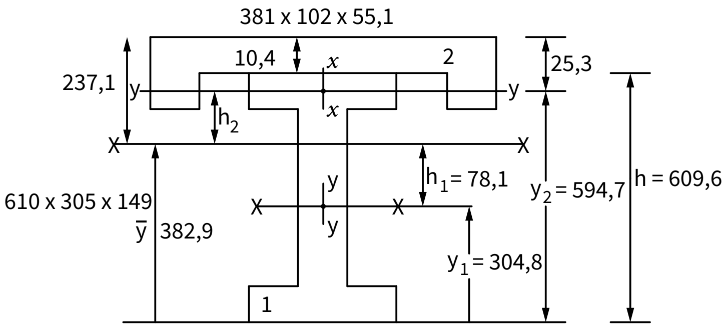
∴ Stress at XX taper flange

$= \sigma_{xxTF} = \frac{Mh_1}{I_{xx\,max}} = \frac{400k \times 0{,}36723}{1\,542 \times 10^{-6}} = 95{,}261 \text{ MPa (C)}$

1.5 Stress diagram




2.




2.1 $\bar{y}\,A_T = A_1 y_1 + A_2 y_2$

$\bar{y}(19{,}03 \times 10^{-3} + 7{,}019 \times 10^{-3} \times 0{,}3048) + (7{,}019 \times 10^{-3} \times 0{,}5947)$

$26{,}049 \times 10^{-3}\,\bar{y} = 9{,}975 \times 10^{-3}$

$\bar{y} = 382{,}9$ mm

2.2 $I_{xx} = I_1 + A_1 h_1^2 + I_2 + A_2 h_2^2$

$h_1 = 382{,}9 - 304{,}8 = 78{,}1$ mm $(\bar{y} - y_1 = h_1)$

$h_2 = 594{,}7 - 382{,}9 = 211{,}8$ mm $(y_2 - \bar{y} = h_2)$

$\therefore I_{xx} = I_{xx} + A_1 h_1^2 + I_{yy} + A_2 h_2^2$

$= (1\,247 \times 10^{-6} + 19{,}03 \times 10^{-3} \times 0{,}0781^2) + (5{,}849 \times 10^{-6} + 7{,}019$
$\times 10^{-3} \times 0{,}2118^2)$

$= 1{,}363 \times 10^{-3} + 3{,}307 \times 10^{-4}$

$= 1\,683{,}792 \times 10^{-6}$ m$^4$

2.3 $I_{yyT} = I_{xx2} + I_{yy1}$

$= 149{,}1 \times 10^{-6} + 93{,}08 \times 10^{-6}$

$= 242{,}18 \times 10^{-6}\ \text{m}^4$

2.4 $Z_{max} = \dfrac{I_{xx}}{y_{max}} = \dfrac{1\,683 \times 10^{-6}}{0{,}3829}$

$= 4\,397{,}47 \times 10^{-6}$

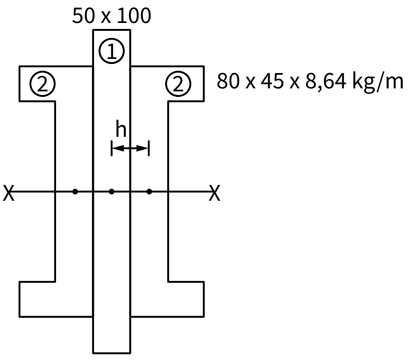
$Z_{min} = \dfrac{I_{xx}}{y_{min}} = \dfrac{1\,683 \times 10^{-6}}{0{,}2371}$

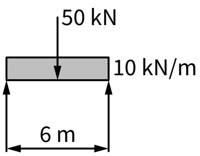
$= 7\,101{,}6 \times 10^{-6}$

2.5 $M = \dfrac{\sigma I}{y_{max}} = \dfrac{120M \times 1\,683{,}792 \times 10^{-6}}{0{,}3829}$

$=527{,}7\ \text{kNm}$

3.




$h = \dfrac{50}{2} + 14{,}5 = 39{,}5$

3.1 $I_{xxT} = 2I_{xx2} + I_{xx1}$

$= \left(2 \times 1{,}059 \times 10^{-6}\right) \times \dfrac{0{,}05 \times 0{,}1^3}{12}$

$= 6{,}285 \times 10^{-6}\ \text{m}^4$

3.2 $I_{yy1} + 2\left[I_{yy2} + A_2 h_2^2\right]$

$= \frac{0{,}1 \times 0{,}05^3}{12} + 2[0{,}1936 \times 10^{-6} + 1{,}102 \times 10^{-3} \times 0{,}0395^2]$

$= 1{,}0417 \times 10^{-6} + 3{,}826 \times 10^{-6}$

$= 4{,}8677 \times 10^{-6}\ \text{m}^4$

3.3 $Z_{xx\max} = \frac{I_{xx}}{y_{\max}} = \frac{6{,}285 \times 10^{-6}}{0{,}05}$

$= 1{,}257 \times 10^{-4}\ \text{m}^3$

3.4 $M = \frac{WL}{4} + \frac{wL^2}{8}$

$= \frac{50 \times 10^3 \times 6}{4} + \frac{10k \times 6^2}{8}$

$= 75k + 45k$

$= 120\ \text{kNm}$

3.5 $\frac{M}{I} = \frac{\sigma}{y}: \sigma_{\max} = \frac{M_{y\max}}{I_{xx}}$

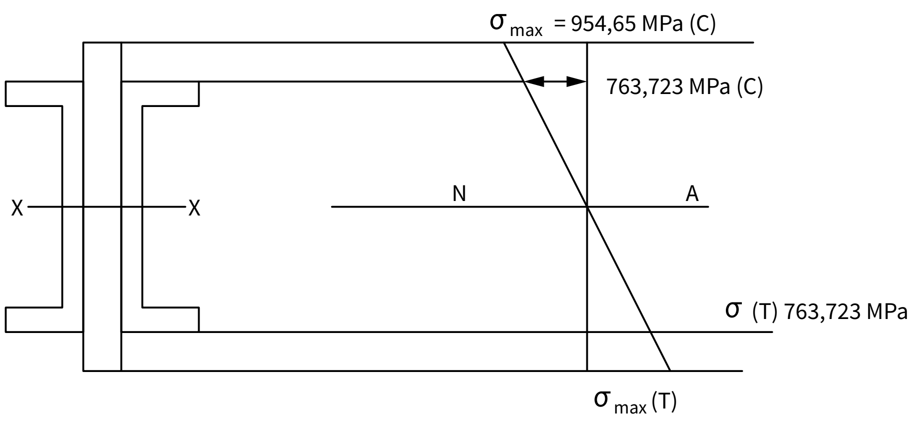
$= \frac{120 \times 10^3 \times 0{,}05}{6{,}285 \times 10^{-6}}$

$= 954{,}65\ \text{MPa}$

3.6 The stress at the top of the channel

Stress at top of channel $= \sigma_{top} = \frac{My}{I_{xx}} = \frac{120k \times 0{,}04}{6{,}285 \times 10^{-4}} = 763{,}723\ \text{MPa}$

3.7 Stress diagram



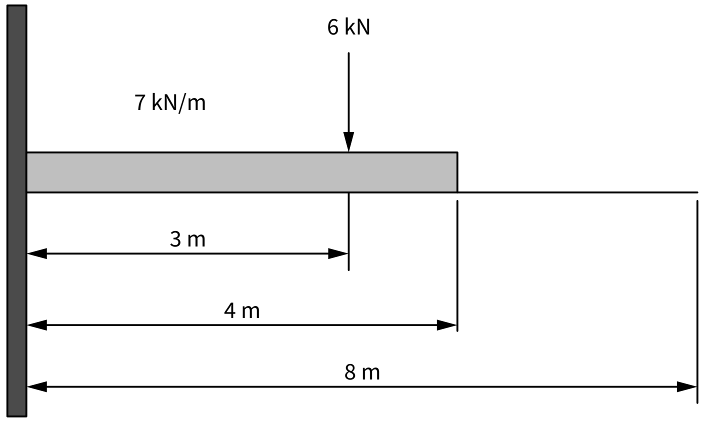
3.8 $Z_{xx} = \frac{M}{\sigma} = \frac{120k}{954{,}65 \times 10^{-6}}$

$= 125{,}7 \times 10^{-6}\ m^3$

(Taper flange) 178 × 102 × 21,5 kg/m

$Z_{exx} = 170{,}2 \times 10^{-6}\ m^3$ (nearest bigger value)

4. 4.1




Maximum BM = $(6k \times 3) + (7k \times 4 \times 4/2) = 74$ kNm

$\therefore Z_{xx} = \frac{M}{\sigma} = \frac{74k}{80M} = 924 \times 10^{-6}\ m^3$

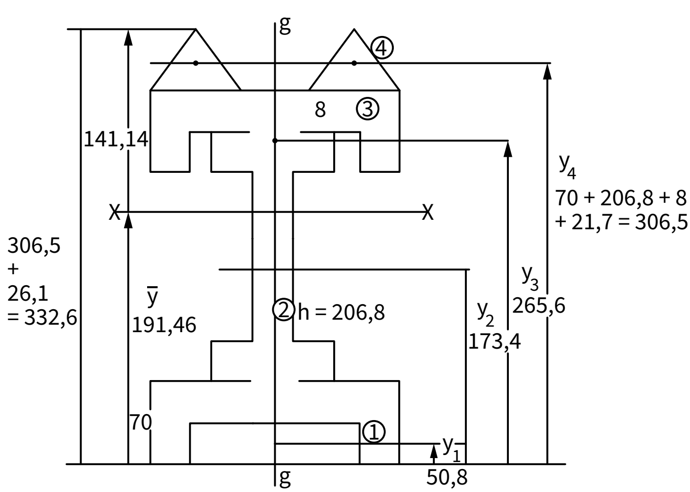
406 × 178 × 53,8 kg/m

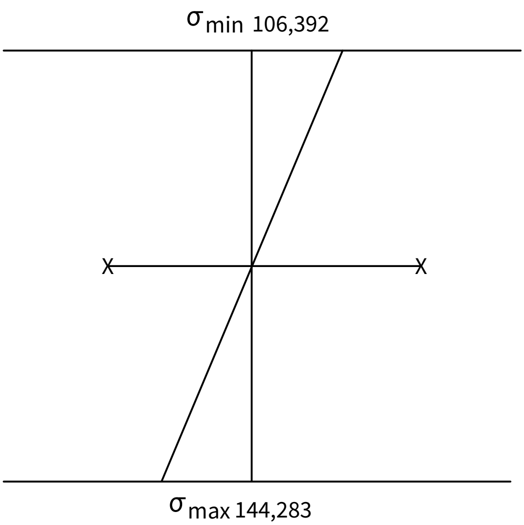
$Z_{xx} = 927{,}4 \times 10^{-6}\ m^3$

4.2 Actual stress

$\sigma = \frac{M}{Z} = \frac{74k}{927{,}4 \times 10^{-6}} = 79{,}793$ MPa

5.






5.1 $\bar{y}_{A_T} = \Sigma A - \text{moments}$

| No. | $A \times 10^{-3}$ | $y$ | $A_y$ |
|---|---|---|---|
| 1 | 2,797 | 0,0508 | $1,421 \times 10^{-4}$ |
| 2 | 3,8 | 0,1734 | $6,5892 \times 10^{-4}$ |
| 3 | 2,797 | 0,2656 | $7,429 \times 10^{-4}$ |
| 4 | $2 \times 1,107$ | 0,3065 | $6,7859 \times 10^{-4}$ |
| $A_T$ | 11,608 | ΣA – moments | $2,2224818 \times 10^{-3}$ |

$\therefore \bar{y}_{A_T} = \Sigma A - \text{mom}$

$\bar{y}\ 11,608 \times 10^{-3} = 2,2224818 \times 10^{-3}$

$\bar{y} = 191,46 \text{ mm}$

5.2 $I_{xx} = I_{T1} + I_{T2} + I_{T3} + I_{T4}$

$\therefore I_{T1} = I_{yy1} + A_1 h_1^2 \qquad (h_1 = 191,46 - 50,8 = 140,66)$

$= 1,135 \times 10^{-6} + 2,797 \times 10^{-3} \times 0,14066^2$

$= 5,6474 \times 10^{-5} \text{ m}^4$

$I_{T2} = I_{xx2} + A_2 h_2^2 \qquad (h_2 = 191,46 - 173,4 = 18,06)$

$= 28,88 \times 10^{-6} + 3,8 \times 10^{-3} \times 0,01806^2)$

$= 3,0119 \times 10^{-5}$

$I_{T3} = I_{yy3} + A_3 h_3^2 \qquad (h_3 = 265,6 - 191,46 = 74,14)$

$= 1,135 \times 10^{-6} + 2,797 \times 10^{-3} \times 0,07414^2$

$= 1,6509 \times 10^{-5}$

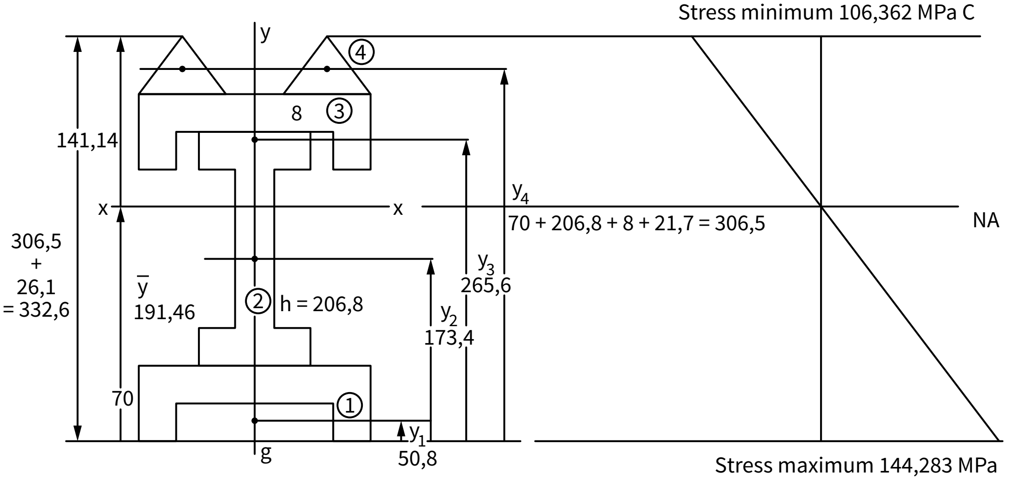
$$I_{T4} = 2\left[I_{vv4} + A_4 h_4^2\right] \qquad (h_4 = 306{,}5 - 191{,}46 = 115{,}04)$$
$$= 2\left[0{,}1479 \times 10^{-6} + 1{,}107 \times 10^{-3} \times 0{,}11504^2\right]$$
$$= 2{,}9596 \times 10^{-5}\ \text{m}^4$$
$$\therefore I_{xx} = I_{T1} + I_{T2} + I_{T3} + I_{T4}$$
$$= [5{,}6474 + 3{,}0119 + 1{,}6509 + 2{,}9596]10^{-5}$$
$$= 132{,}698 \times 10^{-6}\ \text{m}^4$$
$$= I_{yyT} = 2I_{xx\text{channel}} + I_{yy\text{I-sec}} + 2\left[I_{uu} + A_4 h_4^2\right]$$
$$I_{T4} = 2\left[I_{uu} + A_4 h_4^2\right] \qquad h = \frac{86}{2} = 43$$
$$= 2\left[0{,}5507 \times 10^{-6} + 1{,}107 \times 10^{-3} \times 0{,}043^2\right]$$
$$= 5{,}1951 \times 10^{-6}$$
$$\therefore I_{yyT} = \left(2 \times 13{,}54 \times 10^{-6}\right) + 3{,}838 \times 10^{-6} + 5{,}1951 \times 10^{-6}$$
$$= 36{,}113 \times 10^{-6}\ \text{m}^4$$

5.3 $$Z_{max} = \frac{I_{xx}}{y_{max}} = \frac{132{,}698 \times 10^{-6}}{0{,}199146}$$
$$= 693{,}08 \times 10^{-6}\ \text{m}^3$$

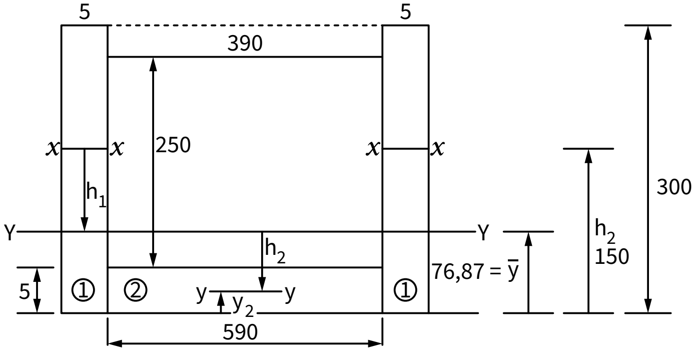
5.4 $$k_{xx} = \sqrt{\frac{I_{xx}}{A_T}}$$
$$= \sqrt{\frac{132{,}698 \times 10^{-6}}{11{,}608 \times 10^{-3}}}$$
$$= 106{,}92\ \text{mm}$$
$$k_{yy} = \sqrt{\frac{I_{yg}}{A_T}}$$
$$= \sqrt{\frac{36{,}113 \times 10^{-6}}{11{,}608 \times 10^{-3}}}$$
$$= 55{,}78\ \text{mm}$$

5.5 $$\sigma_{min} = \frac{My_{min}}{I_{xx}} \qquad \sigma_{max} = \frac{My_{max}}{I_{xx}}$$
$$= \frac{100k \times 0{,}14114}{132{,}698 \times 10^{-6}} \qquad = \frac{(100k \times 0{,}19146)}{132{,}698 \times 10^{-6}}$$
$$= 106{,}362\ \text{MPa} \qquad = 144{,}283\ \text{MPa}$$

5.6 Stress diagram




6. $\rho = 1\,100\ \text{kg/m}^3$




$\bar{y}\,A_T = \Sigma A - \text{moments}$

$\bar{y}(600 \times 300) - (590 \times 295) = (600 \times 300 \times 150) - (590 \times 295 \times 152{,}5)$

$5\,950\,\bar{y} = 457\,375$

$\bar{y} = 76{,}87\ \text{mm}$

$\overline{h_1 = y_1 - y = 73{,}13}$

$h_2 = \bar{y} - y_2 = 75{,}37$

$$\therefore I_{xx} = 2\left[I_1 + A_1 h_1^2\right] + I_2 + A_2 h_2^2$$

$$= 2\left[\frac{0{,}005 \times 0{,}3^3}{12} + 0{,}005 \times 0{,}3 \times 0{,}07313^2\right] + \left[\frac{0{,}59 \times 0{,}005^3}{12} + 0{,}59 \times 0{,}005 \times 0{,}07437^2\right]$$

$$= 3{,}8544 \times 10^{-5} + 1{,}6322 \times 10^{-5}$$

$$= 5{,}48663 \times 10^{-5}\ \text{m}^4$$

$$M = \frac{\sigma I_{xx}}{y_{max}} \qquad y_{max} = 300 - 76,87 = 223,13$$

$$= \frac{60M \times 5,48663 \times 10^{-5}}{0,22313}$$

$$= 14,754 \text{ kNm}$$

Weight of slime/m = volume.$\rho g$

$= A \times L \times \rho g$

$= (0,59 \times 0,25) \times 1 \times 1\,100 \times 9,81$

$= 1\,591,6725$ Nm

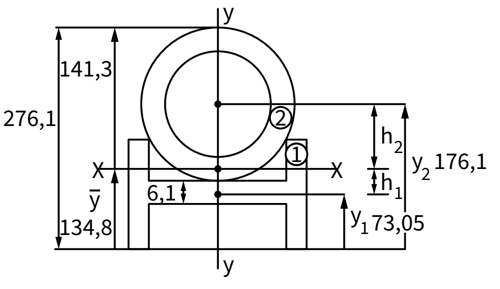
$$M = \frac{wL^2}{8}$$

$$\therefore L = \sqrt{\frac{8M}{W}}$$

$$= \sqrt{\frac{8 \times 14,754 \times 10^{-3}}{1\,591,6725}}$$

$= 8,61$ m

7.




D = 200 $\qquad$ $d$ = 180

Channel: $h$ = 251,5 $\qquad$ $b$ = 146,1

7.1 $gA_T = \Sigma A$ – moment

| | **A** | ***y*** | **$A_y$** |
|---|---|---|---|
| 1 | $3,991 \times 10^{-3}$ | 73,05 | $2,9162 \times 10^{-4}$ |
| 2 | $\frac{\pi}{4}(0,2^2 - 0,18^2)$ | | |
| | $5,969 \times 10^{-3}$ | 176,1 | $1,051 \times 10^{-3}$ |
| $A_T$ | $9,961 \times 10^{-3}$ | ΣM | $1,3428 \times 10^{-3}$ |

$\therefore yA_T = \Sigma A$ – moment

$$y = \frac{1,3428 \times 10^{-3}}{9,961 \times 10^{-3}}$$

$= 134,8$ mm

7.2 $I_{xx} = I_1 + A_1 h_1^2 + I_2 + A_2 h_2^2$

$h_1 = 134{,}8 - 73{,}05 = 61{,}75$

$h_2 = 176{,}1 - 134{,}8 = 41{,}3$

$I_{T1} = I_{1yy} + A_1 h_1^2$

$= 4{,}476 \times 10^{-6} + 3{,}992 \times 10^{-3} \times 0{,}06175^2$

$= 19{,}698 \times 10^{-6}\ \text{m}^4$

$I_{T2} = I_{2xx} + A_2 h_2^2$

$= \frac{\pi}{64}(0{,}2^4 - 0{,}18^4) + \frac{\pi}{4}(0{,}2^2 - 0{,}18^2) \times 0{,}0413^2$

$= 2{,}701 \times 10^{-5} + 1{,}018 \times 10^{-5}$

$= 37{,}19 \times 10^{-6}\ \text{m}^4$

$\therefore I_{xx} = I_{T1} + I_{T2}$

$= 19{,}698 \times 10^{-6} + 37{,}19 \times 10^{-6}$

$I_{xx} = 56{,}888 \times 10^{-6}\ \text{m}^4$

$I_{yy_2} = \frac{\pi}{64}(0{,}2^4 - 0{,}18^4)$

$= 27{,}01 \times 10^{-6}\ \text{m}^4$

$I_{yy} = I_{xx1} + I_{yy2}$

$= 44{,}28 \times 10^{-6} + 27{,}01 \times 10^{-6}$

$I_{yy} = 71{,}29 \times 10^{-6}\ \text{m}^4$

7.3 $k_{xx} = \sqrt{\frac{I_{xx}}{A_T}}$

$= \sqrt{\frac{(56{,}888 \times 10^{-6})}{(9{,}961 \times 10^{-3})}}$

$= 75{,}57$ mm

$k_{yy} = \sqrt{\frac{I_{yy}}{A_T}}$

$= \sqrt{\frac{71{,}29 \times 10^{-6}}{9{,}961 \times 10^{-3}}}$

$= 84{,}6$ mm

Smallest $k$-value = 75,57 mm

7.4 $Z_{max} = \frac{M}{\sigma_{max}}$

$= \frac{I_{xx}}{y_{max}}$

$= \frac{56,888 \times 10^{-6}}{0,1443}$

$= 402,604 \times 10^{-6}\ m^3$

7.5 $\frac{M}{I} = \frac{\sigma}{y} \quad \therefore M = \frac{\sigma I_{xx}}{y_{max}}$

$M = \frac{175 \times 10^6 \times 56,888 \times 10^{-6}}{0,1413}$

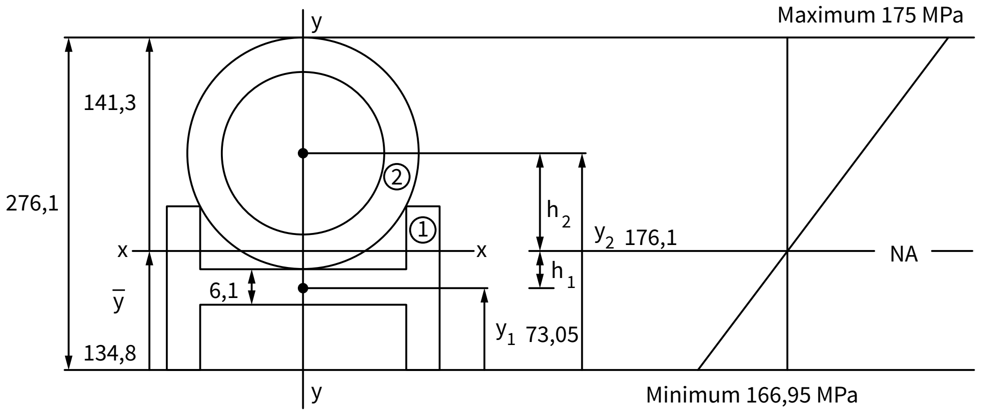
$= 70,46$ kNm

7.6 Minimum bending stress

Consider the maximum stress at the top of the beam; where $y = 141,3$ mm and the minimum stress at the bottom of beam $y = 134,8$ mm.

$\therefore$ Minimum stress $= \sigma_{min} \frac{My}{I_{xx}} = \frac{70,46k \times 0,1348}{56,888 \times 10^{-6}} = 166,95$ MPa

7.7 Stress diagram




7.8 $\frac{M}{I} = \frac{\sigma}{y}$

$\therefore I = \frac{M.y}{\sigma}$

$y = ? \quad \therefore I = ?$

$\therefore Z = \frac{M}{\sigma}$

$= \frac{70,46 \times 10^3}{175 \times 10^{-6}}$

$= 402,63 \times 10^{-6}\ m^3$

$$\therefore Z = \frac{I}{y}$$

$$\therefore \frac{\frac{\pi}{64}(D^4 - D^4)}{\frac{D}{2}} = Z$$

$$\therefore \frac{\frac{\pi}{64}(D^4 - d^4)}{\frac{D}{2}} = 402{,}63 \times 10^{-6}$$

$$\therefore \frac{D^4 - d^4}{D} = \frac{402{,}63 \times 10^{-6} \times 64}{2\pi}$$

$$\therefore \frac{0{,}4^2 - d^2}{0{,}4} = 4{,}101 \times 10^{-3}$$

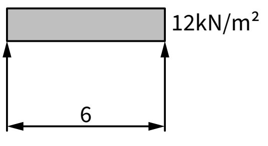
$$0{,}4^4 - d^4 = 1{,}64 \times 10^{-3}$$

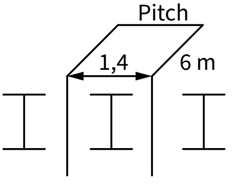
$$d^4 = 0{,}4^4 - 1{,}64 \times 10^{-3}$$

$$= 0{,}02396$$

$$\therefore d = 393{,}4 \text{ mm}$$

8.




$\sigma_{max} = 480$

Acting safe stress $= \frac{450}{4} = 120$ MPa

Per pitch $\therefore F_T = W/m^2 \times A$

$$= 12 \times 10^3 \times (1{,}4 \times 6)$$

$$= 100{,}8 \text{ kN}$$

$$\therefore \text{Load/m} = \frac{100{,}8}{6}$$

$$= 16{,}8 \text{ kNm}$$

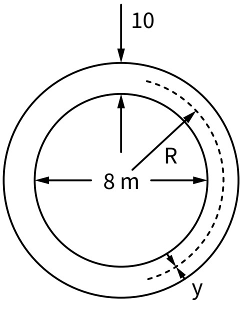
$$\therefore M = \frac{wL^2}{8} = \frac{16{,}8 \times 6^2}{8} = 75{,}6 \text{ kNm}$$

$$\therefore Z = \frac{M}{\sigma} = \frac{75{,}6 \times 10^3}{120 \times 10^6}$$

$$= 630 \times 10^{-6} \text{ m}^3$$

Nearest $305 \times 165 \times 46{,}1$ kg/m $(647 \times 10^{-6})$

9.




$\frac{\sigma}{y} = \frac{E}{R} \quad \therefore y = \frac{0,01}{2} = 0,005$ m and

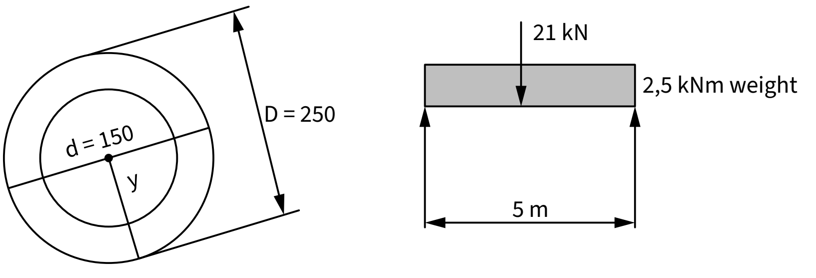
$R = \frac{8}{2} + \frac{0,01}{2} = 4,005$ m

$\therefore \sigma = E.\frac{y}{R}$

$= \frac{100 \times 10^9 \times 0,005}{4,005}$

$= 124,84$ MPa

10.




$I = \frac{\pi}{64}(0,25^4 - 0,15^4) = 1,669 \times 10^{-4}\ m^4$

$\therefore M_{max} = \frac{\sigma I}{y} = \frac{40 \times 10^6 \times 1,669 \times 10^{-4}}{0,125}$

$= 53,407$ kNm

$M_{max} = M_{PL} + M_{weight} + M_{UDL}$

$\therefore M_{max} = \frac{WL}{4} + \frac{wL^2}{8} + \frac{wL^2}{8}$

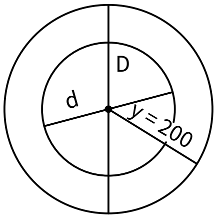
$\therefore 53\ 407 = \frac{21 \times 5}{4} + \frac{2,5 \times 5^2}{8} + \frac{w5^2}{8}$

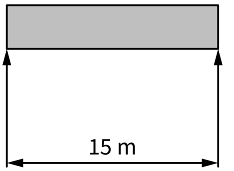
$= 26\ 250 + 7\ 812,5 + \frac{w25}{8}$

$\therefore \frac{w25}{8} = 19\ 344,575$

$w = 6,19$ kNm (safe UD load)

11.




11.1 $I = \frac{\pi}{64}(0{,}4^4 - 0{,}38^4)$

$= 233{,}098 \times 10^{-6}\ m^4$

$\therefore M = \frac{\sigma I}{y} = \frac{40 \times 10^6 \times 233{,}098 \times 10^{-6}}{0{,}2}$

$= 46\ 619{,}6$ Nm

$\therefore M_{max} = \frac{wL^2}{8} + \frac{wL^2}{8}\ (M_{max} = M_{load} + M_{weight})$

$46\ 619{,}6 = \frac{1 \times 10^3 \times 15^2}{8} = \frac{w15^2}{8}$

$46\ 619{,}6 = 28{,}125 + \frac{wL^2}{8}$

$\therefore \frac{w15^2}{8} = 18\ 494{,}6$

$\therefore$ Load/m $= w = 657{,}59$ Nm

But W = volume $\rho \times g$

$657{,}59 = A \times 1 \times 1\ 200 \times 9{,}87$

Water area is $\therefore A = 0{,}05586\ m^2$

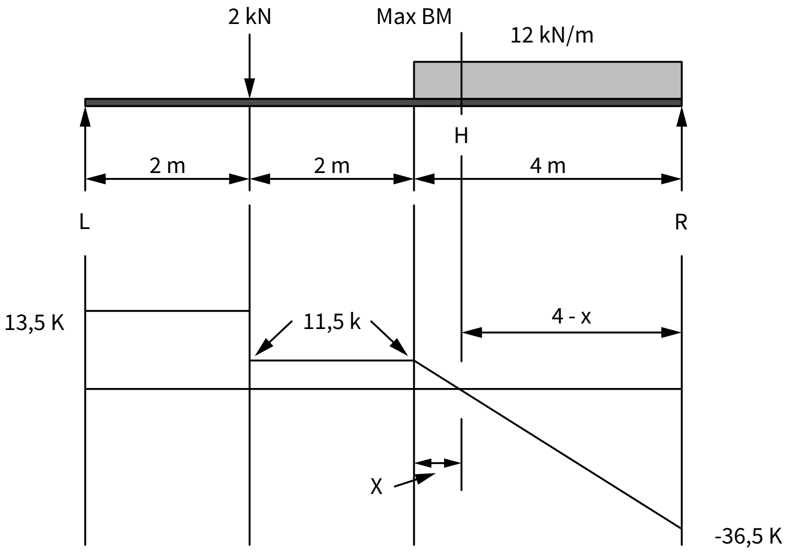
11.2 Percentage area full

$\frac{\text{Area of water}}{\text{Inner area of pipe}} \times \frac{100}{1}$

$\therefore \frac{0{,}05586}{\frac{\pi}{4} \times 0{,}38^2} \times \frac{100}{1}$

$= 49{,}25\%$

12.1 Maximum BM




Reactions; moments about L; 8R = $(2k \times 2) + (12k \times 4 \times 6)$

R = 36,5 kN

Moments about R; 8L = $(2k \times 6) + (12k \times 4 \times 2)$

L = 13, 5 kN

Maximum BM at H

$\tan \propto = \tan \propto \quad \therefore \frac{11{,}5k}{x} = \frac{36{,}5k}{4 - X}$

3,174X = 4 – X

X = 0,958 m

Distance from H to R = 4 – 0,958 = 3,042 m

Maximum BM at H = $(36{,}5k \times 3{,}042) - \left(12k \times 3{,}042 \times \frac{3{,}042}{2}\right)$

= 55,51 kNm

12.2 Maximum stress

Maximum stress = $\sigma_{max} = \frac{M}{Z} = \frac{55{,}51k}{3{,}917 \times 10^{-4}} = 141{,}716$ MPa

12.3 Select section

Built-up Z = $391{,}7 \times 10^{-6}$

Use I-section 305 × 102 × 32,8 kg/m

12.4 Maximum stress

$\sigma_{max} = \frac{M}{Z} = \frac{55{,}51k}{415 \times 10^{-6}} = 133{,}759$ MPa

MODULE

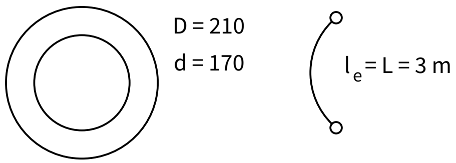
# 6 *Columns and struts*

**By the end of this module, students should be able to**:
- differentiate between columns and struts;
- define the term *effective length*;
- calculate the effective length for both pinned ends, both fixed ends, one end pinned and one end fixed, as well as one end fixed and the other end free;
- calculate the slenderness ratio; and
- calculate buckling loads using Euler's theory as well as Rankine's theory.

Columns and struts are integral components of many structures. Like all structural members that are subjected to various loads and forces, this module covers the ways in which members, and particularly columns and struts, may fail by buckling and how to incorporate buckling load and failure theory into your calculations to prevent failure.

## Exercise 6.1 SB page 208

1.




D = 210 $\quad d = 170 \quad l_e = L = 3$ m

1.1 $P_E = \frac{\pi^2 EI}{l_e} = \left(\pi^2 \times 210G \times \frac{\frac{\pi}{64}(0{,}210^4 - 0{,}17^4)}{3^2}\right)$

$= 12{,}54$ MN

1.2 $P_R = \dfrac{\sigma_c A}{1 + a\left(\frac{l_e}{R}\right)^2}$

$= 300\text{M} \times \dfrac{\frac{\pi}{4}(0{,}21^2 - 0{,}17^2)}{1 + \left(\frac{1}{7\,500}\left(\frac{3}{0{,}06755}\right)\right)}$

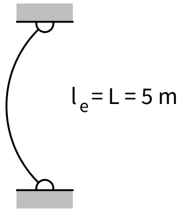
$= 2{,}836 \text{ MN}$

$\therefore l_e = \sqrt{\dfrac{I}{A}}$

$= \sqrt{\left(\dfrac{\frac{\pi}{4}(0{,}21^4 - 0{,}17^4)}{\frac{\pi}{4}(0{,}21^2 - 0{,}17^2)}\right)}$

$= 0{,}06755 \text{ m}$

2. 2.1




$P_E = \dfrac{\pi^2 EI}{le^2}$

$1{,}2 \times 10^6 = \dfrac{\pi^2 \times 200 \times 10^9 I}{5^2}$

$\therefore I_{yy} = 15{,}198 \times 10^{-6} \text{ m}^4$

$\therefore 203 \times 203 \times 46{,}6 \text{ kg/m}$

2.2 2.2.1 $S_R = \dfrac{l_e}{k} = \dfrac{2{,}5}{0{,}0512}$

$= 48{,}83:1$

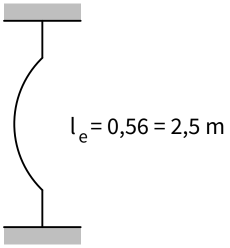
2.2.2 $P_R = \dfrac{\sigma_c A}{1 + a\left(\frac{l_e}{k}\right)^2}$

$l_e = \sqrt{\dfrac{I}{A}} = 51{,}2$

$\therefore P_R = \dfrac{300\text{M} \times 5{,}882 \times 10^{-3}}{1 + \dfrac{1}{7\,500\left(\frac{2{,}5}{0{,}0512}\right)^2}}$

$= 1{,}339 \text{ MN}$

3. 3.1




$P_R = \dfrac{\sigma_c A}{1 + a\left(\frac{l_e}{k}\right)^2}$

$$\therefore 1{,}5 \times 10^6 = \frac{80 \times 10^6\,\text{A}}{1 + \frac{1}{6\,400\left(\frac{2{,}5}{k}\right)^2}}$$

$$1{,}5 \times 10^6 = \frac{80 \times 10^6\,\text{A}}{1 + \frac{9{,}766 \times 10^{-4}}{k^2}}$$

$$\therefore 1 + \frac{9{,}766 \times 10^{-4}}{k^2} = 53{,}333\text{ A} \ldots ①$$

$$\text{But } k^2 - \frac{\text{I}}{\text{A}} \qquad \frac{\pi(\text{D}^4 - d^4)^4}{64\pi(\text{D}^2 - d^2)} = \frac{(\text{D}^2 - d^2)(\text{D}^2 + d^2)}{16(\text{D}^2 - d^2)}$$

$$\frac{\text{D}^2 + d^2}{16}$$

$$\therefore k^2 = 0{,}0625(\text{D}^2 + d^2) \ldots ②$$

$$\text{Substitute into:} \quad \therefore 1 + \frac{9{,}766 \times 10^{-4}}{0{,}0625(\text{D}^2 + d^2)} = 53{,}333 \times \frac{\pi}{4}(\text{D}^2 - d^2)$$

$$\therefore 1 + \frac{0{,}01625}{\text{D}^2 + d^2} = 41{,}888(\text{D}^2 - d^2)$$

$$\times (\text{D}^2 + d^2){:}(\text{D}^2 + d^2) + 0{,}01562 = 41{,}888\left(\text{D}^4 - d^4\right)$$

$$\boxed{(\text{D}^2 + d^2)(\text{D}^2 - d^2) = (\text{D}^4 - d^4)}$$

$$\therefore \text{D}^2 = d^2 + 0{,}01562 = 41{,}888\ \text{D}^4 - 41{,}888\ d^4$$

$$\therefore 0{,}26^2 + d^2 + 0{,}01562 = 41{,}888(0{,}26^4) - 41{,}888\ d^4$$

$$\therefore d^2 + 0{,}08322 = 0{,}1914 - 41{,}888\ d^4$$

$$\therefore 41{,}888\ d^4 + d^2 - 0{,}1082 = 0$$

$$d^4 - x^2 d = x$$

$$\therefore 41{,}888x^2 + x - 0{,}1082 = 0$$

$$\therefore x = -1 \pm \frac{\sqrt{(1^2 - 4(41{,}888)(-0{,}1082))}}{2(41{,}888)}$$

$$= \frac{-1 \pm 4{,}374}{83{,}776}$$

$$= \frac{3{,}374}{83{,}776}$$

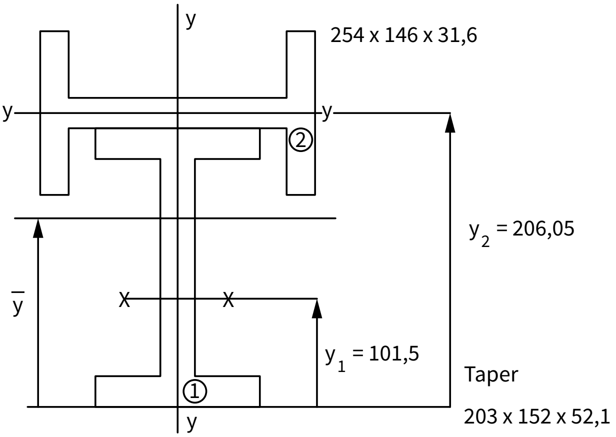
$$x = d^2 = 0{,}04027$$

$$\therefore d = 200{,}67\text{ mm}$$

3.2 $$\text{P}_\text{E} = \left(\frac{\pi^2 \text{EI}}{l_e}\right) = \frac{\left(\pi^2 85 \times 10^9\left(\frac{\pi}{64}[0{,}26^4 - 0{,}2006^4]\right)\right)}{2{,}5^2}$$

$$= 19{,}424\text{ MN}$$

4.




$\bar{y}A_Y = \Sigma A$ – moments

| No. | A | $y$ | $A_y$ |
|---|---|---|---|
| 1 | $6{,}641 \times 10^{-3}$ | 0,101,5 | $6{,}740615 \times 10^{-4}$ |
| 2 | $3{,}99 \times 10^{-3}$ | 0,20605 | $8{,}225516 \times 10^{-4}$ |
| $A_T$ | 0,010633 | ΣA – moment | $1{,}4966131 \times 10^{-3}$ |

$\bar{y}.0{,}010633 = 1{,}4966131 \times 10^{-3}$

$\bar{y} = 140{,}75$ mm

$$I_{xx} = I_{xxl} + A_1 h_1^2 + I_{yy2} + A_2 h_2^2$$

$$= \left(\frac{47}{0{,}76 \times 10^{-6} + 6{,}641 \times 10^{-3} \times 0{,}03925^2}\right) + 4{,}476 \times 10^{-6} + 3{,}992 \times 10^{-3} \times 0{,}0653^2$$

$$= 79{,}46 \times 10^{-6} \text{ m}^4$$

$$I_{yy} = I_{xx2} + I_{yy2}$$

$$= 44{,}28 \times 10^{-6} + 8{,}098 \times 10^{-6}$$

$$= 52{,}378 \times 10^{-6} \text{ m}^4$$

4.2 $\quad SR \therefore k = \sqrt{\frac{I_{yg}}{A_T}} = \frac{\sqrt{52{,}378 \times 10^{-6}}}{0{,}010633} = 0{,}07019$ m

$$SR = \frac{l_e}{k} = \frac{0{,}707 \times 4{,}5}{0{,}07019} = 45{,}33:1$$

4.3 $\quad P_R = \dfrac{\sigma A}{1 - a\left(\frac{l_e}{k}\right)^2}$

$$= \frac{280 \times 10^6 \times 0{,}010633}{1 + \frac{1}{7\,500}\left(\frac{0{,}707 \times 4{,}5}{0{,}07019}\right)^2}$$

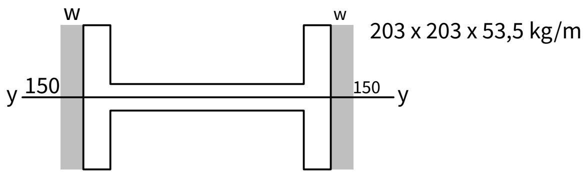
$$= 2{,}34 \text{ MN}$$

5. $l_e = 0{,}707 \times 6 = 4{,}242$

$$P_E = \frac{\pi^2 EI}{l_e^2}$$

$$3{,}5 \times 10^6 = \frac{\pi^2 \times 200G \times I}{4{,}242^2}$$

$$I = 31{,}91 \times 10^{-6}\ m^4$$




$$I_{yy} = I_{yy} \times 2\left(\frac{w \times 0{,}15^3}{12}\right)$$

$$31{,}91 \times 10^{-6} = 16{,}78 \times 10^{-6} + 2(2{,}8125 \times 10^{-4} w)$$

$$= 2(2{,}8125 \times 10^{-4} w)$$

$$w = 26{,}89\ mm$$

6. L = 4 m  D = 80  $P_E$ = 150 kN  E = 200 GPa

6.1 $P_E = \dfrac{\pi^2 EI}{l_e^2}$  $\therefore 150k = \dfrac{\pi^2 EI}{4^2}$

$\therefore I_{yy} = 1{,}216 \times 10^{-6}\ m^4$

$I_{yy} = \dfrac{\pi}{64}(D^4 - d^4)$  $\therefore 1{,}216 \times 10^{-6}\ m^4 = \dfrac{\pi}{64}\left(0{,}08^4 - \dfrac{\sigma}{4}\right) = 200\ GPa$

$\therefore d^4 = 0{,}08^6 - 2{,}477 \times 10^{-5}$

$= 1{,}619 \times 10^{-}5$

$d = 63{,}43\ mm$

$t = \dfrac{(D - d)}{2} = 8{,}285\ mm$

6.2 $P_R = \dfrac{\sigma A}{1 + a\left(\frac{l_e}{k}\right)^2}$

$$150k = \frac{\sigma\frac{\pi}{4}(0{,}08^2 - 0{,}06343^2)}{\left(1 + \frac{1}{7\,500}\left(\frac{4}{k}\right)^2\right)}$$

$= \dfrac{\sigma 1{,}8667 \times 10^{-3}}{1 + \frac{1}{7\,500}\left(\frac{4}{0{,}0255}\right)^2}$  $k = \sqrt{\dfrac{I_{yg}}{A}}$

$\dfrac{\sigma 1{,}8667 \times 10^{-3}}{4{,}275} = \dfrac{\sqrt{(1{,}216 \times 10^{-4})}}{1{,}877 \times 10^{-3}}$

$\therefore \sigma = 343{,}5\ MPa$  $= 0{,}0255$

$\therefore FoS = \dfrac{350}{343{,}5} = 1{,}018$

7. 7.1 $l_e = \text{L} = 3 \text{ m}$

$$\text{I}_{yy} = l_e \frac{0{,}05 \times 0{,}02^3}{12} = 3{,}333 \times 10^{-8} \text{ m}^4$$

$$\text{P}_\text{E} = \frac{\pi^2 \text{EI}}{l_e^2} = \frac{\pi^2 \times 200\text{G} \times 3{,}333 \times 10^{-8}}{3^2} = 7{,}31 \text{ kN}$$

7.2 $l_e = 0{,}5\text{L} = 1{,}5 \text{ m}$

$$\text{I}_{xx} = \frac{0{,}02 \times 0{,}05^3}{12} = 2{,}083 \times 10^{-7} \text{ m}^4$$

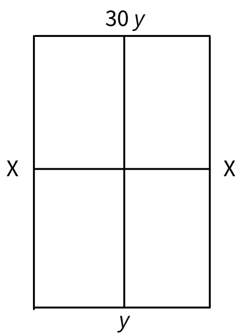
$$\text{P}_\text{E} = \frac{\pi^2 \text{EI}}{l_e^2} = \frac{\pi^2 \times 200\text{G} \times 2{,}083 \times 10^{-7}}{1{,}5^2} = 182{,}741 \text{ kN}$$

7.3 $\text{SR} = \frac{l_e}{k}$

$$k = \sqrt{\frac{\text{I}_{yy}}{\text{A}}} = \sqrt{\frac{3{,}333 \times 10^{-8}}{0{,}02 \times 0{,}05}} = 5{,}773 \times 10^{-3} \text{ m}$$

$$\text{SR} = \frac{l_e}{k} = \frac{3}{5{,}773 \times 10^{-3}} = 519{,}66$$

8.




$$\text{P}_\text{R} = \frac{\sigma \text{A}}{1 + a\left(\frac{le}{k}\right)^2}$$

$$k = \sqrt{\frac{\text{I}_{yy}}{\text{A}}} = \frac{\frac{\cancel{0{,}09} \times 0{,}03^3}{12}}{\cancel{0{,}09} \times \cancel{0{,}03}}$$

$$= \sqrt{\frac{0{,}03^2}{12}}$$

$$= 8{,}66 \times 10^{-3} \text{ m}$$

$$\therefore 100 \times 10^3 = \frac{130 \times 10^6 \times 0{,}09 \times 0{,}03}{1 + 58\,800a} = 351\,000$$

$$1 + 58\,800a = 3{,}51$$

$$\therefore 58\,800a = 2{,}51$$

$$a = \frac{1}{23\,426{,}295}$$

$$= 4{,}269 \times 10^{-5}$$

9. 9.1 Euler crippling load

Inside diagram = $d = 60 - (2 \times 6) = 48$ mm

$l_e = 2L = 2 \times 3 = 6$ m

$I_{yy} = \frac{\pi}{64}(0{,}06^4 - 0{,}048^4) = 37{,}562 \times 10^{-6}\ m^4$

$P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 200G \times 37{,}562 \times 10^{-6}}{6^2} = 20{,}595$ kN

9.2 $k = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{37{,}562 \times 10^{-6}}{\frac{\pi}{4}(0{,}06^2 - 0{,}048^2)}} = \sqrt{\frac{37{,}562 \times 10^{-6}}{1{,}0178 \times 10^{-3}}} = 0{,}192$ m

$P_R = \frac{\sigma_C A}{1 + a\left(\frac{l_e}{k}\right)^2} = \frac{350M \times 1{,}0178 \times 10^{-3}}{1 + \frac{1}{7\,500}\left(\frac{6}{0{,}192}\right)^2} = 315{,}164$ kN

10. 10.1 $I_{yy} = \frac{\pi 0{,}2^4}{64} = 7{,}854 \times 10^{-5}\ m^4$

$A = \frac{\pi 0{,}2^2}{4} = 31{,}416 \times 10^{-3}\ m^2$

$l_e = 0{,}707 \times 3 = 2{,}121$ m

$k = \sqrt{\frac{7{,}854 \times 10^{-5}}{31{,}416 \times 10^{-3}}} = 0{,}05$ m

$P_R = \frac{\sigma_C A}{1 + a\left(\frac{l_e}{k}\right)^2} = \frac{560M \times 31{,}416 \times 10^{-3}}{1 + \frac{1}{1\,600}\left(\frac{2{,}121}{0{,}05}\right)^2} = 8{,}28$ MN

10.2 $P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 90G \times 7{,}854 \times 10^{-5}}{2{,}121^2} = 15{,}508$ MN

10.3 $SR = \frac{l_e}{k} = \frac{2{,}121}{0{,}05} = 42{,}42$

10.4 $P_R = P_E$

$$\frac{\sigma_C A}{1 + a\left(\frac{l_e}{k}\right)^2} = \frac{\pi^2 EI}{l_e^2}$$

$$\frac{\sigma_C A}{1 + \frac{ale^2}{k^2}} = \frac{\pi^2 EI}{l_e^2}$$

$$\pi^2 EI\left(1 + \frac{ale^2}{k^2}\right) = le^2 \sigma_C A$$

$\div\ \pi^2 EI$ $\qquad \therefore 1 + \frac{ale^2}{k^2} = le^2 \frac{17\,592\,960}{\pi^2 \times 90G \times 31{,}416 \times 10^{-3}}$

$$1 + \frac{ale^2}{k^2} = 0{,}252le^2$$

$\times\ k^2$ $\qquad \therefore k^2 + \frac{le^2}{1\,600} = k^2 \times 0{,}252le^2$

$\times\ 1\,600;$ $\qquad \therefore 1\,600 \times 0{,}05^2 + le^2 = 0{,}05^2 \times 0{,}252le^2$

$2{,}008le^2 = 4$

$le = 1{,}411$

and $le = 0{,}707\text{L}$

Length of column $= \text{L} = \frac{1{,}411}{0{,}707} = 1{,}996$ m

11. 11.1 $\text{I}_{yy} = \frac{1}{12}(0{,}08^4 - 0{,}07^4) = 1{,}4125 \times 10^{-6}\ \text{m}^4$

$le = 0{,}5\text{L} = 0{,}5 \times 2 = 1$ m

$$\text{P}_\text{E} = \frac{\pi^2 \text{EI}}{l_e^2} = \frac{\pi^2 \times 210\text{G} \times 1{,}4125 \times 10^{-6}}{1^2} = 2{,}928\ \text{MN}$$

$$\text{Safe load} = \frac{\text{P}_\text{E}}{\text{FoS}} = \frac{1{,}928\text{M}}{6} = 487{,}929\ \text{kN}$$

11.2 $k = \sqrt{\frac{\text{I}_{yy}}{\text{A}}} = \sqrt{\frac{1{,}4125 \times 10^{-6}}{0{,}08^2 - 0{,}07^2}} = 0{,}0307$ m

$$\text{P}_\text{R} = \frac{\sigma_\text{C}\text{A}}{1 + a\left(\frac{l_e}{k}\right)^2} = \frac{200\text{M}(0{,}08^2 - 0{,}07^2)}{1 + \frac{1}{7\,500}\left(\frac{1}{0{,}0307}\right)^2} = 262{,}812\ \text{kN}$$

$$\text{Safe load} = \frac{\text{P}_\text{R}}{\text{FoS}} = \frac{262{,}812k}{6} = 43{,}802\ \text{kN}$$

The crippling stress can also be divided by the factor of safety to calculate the safe load.

MODULE

# 7 Shafts

**By the end of this module, students should be able to**:

- explain the two critical limits that must be considered when designing shafts, namely the shear stress limit and the twisting angle limit;
- apply torque and power equations;
- calculate maximum torque when starting torque is not considered;
- calculate maximum torque when starting torque is considered as a percentage of the average torque;
- calculate the dimensions of solid and hollow shafts when a stress limit or twisting angle limit is given;
- calculate the percentage saving in weight if a solid shaft is replaced by a hollow shaft for the same shear stress limit; and
- do calculations on compound shafts connected in series and parallel.

When shafts are subjected to a twisting moment or torque about the longitudinal axis, the shaft is subjected to pure or simple torsion.

## Exercise 7.1 SB page 233

1. L = 2 m    D = ?    T = 20 kNm    G = 80 GPa

   $\tau$ = 60 MPa    $\theta = 2°$

   1.1 $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$

   Shear stress:

   $$\frac{T}{J} = \frac{\tau^2}{D}$$

   $$J = \frac{TD}{\tau^2}$$

   $$= \frac{20 \times 10^3 \times D}{60 \times 10^6 \times 2}$$

$\frac{\pi}{32}D^4 = 1{,}667 \times 10^{-4} D$

$\therefore D^3 = 1{,}677 \times 10^{-3}$

D = 119,3 mm

Angle of twist:

$\frac{T}{J} = \frac{G\theta}{L}$

$J = \frac{TL}{G\theta} = \frac{20 \times 10^3 \times 2 \times 188}{80 \times 10^9 \times 2 \times \pi}$

$\therefore \frac{\pi}{32}D^4 = 1{,}43^2 \times 10^{-5}$

$D^4 = 1{,}459 \times 10^{-4}$

D = 109,9 mm

Use D = 109,9 $\theta = 2°$ and $\tau < 60$ MPa

1.2 $P = 2\pi NT$

$= \frac{2\pi \times 900 \times 20}{60}$

= 1 884,96 kW

= 1,885 MW

2. P = 2 MW @ 400 rpm 13% D = 100 $d = 90$

L = 2 m G = 80 GPa

2.1 $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$

$T_{max} = \frac{P}{2\pi N} = \frac{2{,}010^3 \times 60}{2\pi 400}$

= 477,465 Nm

$T_{max} = 477{,}465 \times 1{,}13$

= 539,54 Nm

$\therefore \tau = \frac{T \times R}{J} = \frac{TD}{\frac{\pi}{32}(D^4 - d^4) \times 2}$

$= \frac{539{,}54 \times 0{,}1}{\frac{\pi}{32}(0{,}1^4 - 0{,}09^4) \times 2}$

= 7,99 or 8 MPa

2.2 Angle of twist

$\therefore \frac{T}{J} = \frac{G\theta}{L}$

$\therefore = \frac{TL}{JG}$

$$\theta = \frac{539{,}54 \times 2}{\frac{\pi}{32}(0{,}1^4 - 0{,}09^4)80 \times 10^9}$$

$$= 0{,}004 \text{ rad}$$

$$= 0{,}229°$$

3. $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$

3.1 $\frac{T}{J} = \frac{\tau}{R}$

Stress:

$$J = \frac{TR}{\tau}$$

$$= 20 \times 10^3 \times \frac{D}{2} \times \frac{1}{75 \times 10^6}$$

$$\frac{\pi}{32} D^4 = 1{,}33^1 \times 10^{-4} D$$

$$\therefore D^3 = 1{,}358 \times 10^{-3}$$

$$\frac{D^4 - d^4}{D} = 1{,}358 \times 10^{-3}$$

$$\frac{(2{,}5d)^4 - d^4}{2{,}5d} = 1{,}358 \times 10^{-3}$$

$$15{,}225d^3 = 1{,}358 \times 10^{-3}$$

$$d^3 = 8{,}92 \times 10^{-5}$$

$$d = 44{,}68 \text{ mm}$$

$$\therefore D = 111{,}7 \text{ mm}$$

$\theta$ of twist $\frac{T}{J} = \frac{G\theta}{L}$

$$J = \frac{TL}{G\theta}$$

$$= \frac{20 \times 10^3 \times 1{,}5 \times 180}{80 \times 10^9 \times 2{,}1 \times \pi}$$

$$\frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right) = 1{,}023 \times 10^{-5}$$

$$\therefore \frac{(2{,}5d)^4 - d^4}{2{,}5d} = 1{,}04216 \times 10^{-4}$$

$$15{,}225d^3 = 1{,}04216 \times 10^{-4}$$

$$d^3 = 6{,}845$$

$$d = 19 \text{ mm}$$

and $D = 47{,}5$ mm

Use $d = 44{,}68$ mm

$D = 111{,}7$ mm

$\therefore \tau = 75$ MPa $\quad\quad \theta < 1{,}5°$

**Important**

For Question 3.1, you must check both limits to see which is the strongest.

3.2 $T_{mean} = \frac{T_{max}}{1,12}$

$= \frac{20 \times 10^3}{1,12}$

$= 17,86$ kNm

$P = 2\pi NT_{mean}$

$= 2\pi\frac{700}{60} \times 17,86 \times 10^3$

$= 1,31$ MW

4. D = 160 G = 85 GPa L = 3 m $\theta = 1,1°$ N = 400 rpm

4.1 $\frac{T}{R} = \frac{T}{I}$

$\therefore \tau = \frac{RT}{J}$

$\frac{T}{J} = \frac{G\theta}{L}$

$T = \frac{G\theta J}{L}$

$= \frac{85G}{3} \times \frac{1,1 \times \pi}{180} \times \frac{\pi 0,16^4}{3^2}$

$= 34,998$ kNm

$\therefore \tau = \frac{RT}{J}$

$= \frac{0,8 \times 34,998}{\frac{\pi}{32}0,16^4}$

$= 43,52$ MPa

4.2 Power $= 2\pi NT$

$= 2\pi \times \frac{400}{60} \times 34,998$

$= 1,466$ MW

4.3 $P_H = 1,15\ P_s$

$= 1,15 \times 1,466$

$= 1,6859$ MW

$D = 2d$ … ①

$$T_H = \frac{P_H}{2\pi N}$$

$$= \frac{1{,}6859 \times 60}{2\pi \times 400}$$

$$= 40{,}248 \text{ kNm}$$

$$T = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)\tau \ \ldots \text{②}$$

Substitute ① into ②: $\therefore 40\ 248 = \frac{\pi}{16}\left(\frac{2d^4 - d^4}{2d}\right)43{,}52 \times 10^6$

$$7{,}5d^3 = 4{,}71 \times 10^{-3}$$

$$d^3 = 6{,}28 \times 10^{-4}$$

$$d = 85{,}63 \text{ mm}$$

$$D = 171{,}27 \text{ mm}$$

5. $T = \frac{\pi}{16} d^3 \tau$

5.1 $\frac{\pi}{16} \times 0{,}13^3 \times 9 \times 10^6$

$= 29{,}765$ kNm

$P = 2\pi NT = 2\pi \times \frac{140}{60} \times 29{,}765$

$= 436{,}381$ kW

5.2 $T_s = T_H$

$\therefore 29\ 765 = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)84 \times 10^6$

$1{,}805 \times 10^{-3} = \frac{0{,}13^4 - d^4}{0{,}13}$

$2{,}346 \times 10^{-4} = 0{,}134 - d^4$

$d^4 = 5{,}1 \times 10^{-5}$

$d = 84{,}51$ mm

6. $d = 30$  D = 60 mm  $\frac{T}{S} = \frac{\tau}{R} = \frac{G\theta}{L}$

6.1 $T = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)\tau$

$\therefore 5 \times 10^3 = \frac{\pi}{16}\left(\frac{0{,}06^4 - 0{,}03^4}{0{,}06}\right)\tau$

$\therefore \tau = 125{,}75$ MPa

6.2 Shear strain $\delta$

$\delta = \frac{\theta R}{L}$

$= \frac{1 \times \pi \times 0{,}03}{180 \times 1}$

$= 5{,}236 \times 10^{-4}$

6.3 $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$

Any one:

$\therefore G = \frac{\tau L}{R\theta}$

$= \frac{125{,}75 \times 10^6 \times 1 \times 180}{0{,}03 \times 1 \times \pi}$

$= 240{,}16$ GPa

6.4 $T_{max} = \frac{5}{1{,}2} = 4{,}167$ kNm

$\therefore P = 2\pi NT$

$= \frac{2\pi 200 \times 4{,}167}{60}$

$= 87{,}27$ kW

7. P = 7,5 MW @ 100 rpm  D = 2*d*  *d* = 200  θ = 3°  80 GPa

7.1 $P = 2\pi NT$

$T = \frac{7{,}5 \times 10^6 \times 60}{2\pi \times 100}$

$= 509{,}86$ kW

$\frac{T}{J} = \frac{\tau}{R} = 716{,}197$ MNm

$\therefore \frac{\pi}{16}\left(\frac{(D^4 - d^4)}{D}\right)\tau$

$716{,}197 = \frac{\pi}{16}\left(\frac{0{,}4^4 - 0{,}2^4}{0{,}4}\right)\tau$

$= 60{,}79$ MPa

7.2 $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$

Any one in this case:

$\therefore \frac{G\theta}{L} = \frac{\tau}{R}$

$\therefore L = \frac{G\theta R}{\tau}$

$= \frac{80 \times 10^9 \times 3 \times \pi \times 0{,}2}{60{,}79 \times 10^6 \times 180}$

$= 13{,}78$ m

**Note**

All values are available, so any combination of the bending equation can be used to find L.

OR

$$\frac{G\theta}{L} = \frac{T}{J}$$

$$L = \frac{G\theta J}{T}$$

$$= \frac{\left(80 \times 10^9 \times \pi \times 3\pi(D^4 - d^4)\right)}{(716\ 197 \times 180 \times 32)}$$

$$= 13{,}78 \text{ m}$$

8. $T = \frac{P}{2\pi N} = \frac{500k \times 60}{2\pi 120}$

$= 38{,}789$ kNm

8.1 $T_{max} = 39{,}789 \times 1{,}14$

$= 45{,}359$ kNm

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

$D = 2d$

Stress: $T = \frac{J\tau}{R}$

$$45\ 359 = \frac{J \times 55 \times 10^6 \times 2}{D}$$

$$D = 2\ 425{,}0891\ J$$

$$= 2\ 425{,}0891 \times \frac{\pi}{32}(D^4 - d^4)$$

$$\therefore I = 238{,}083\left(\frac{D^4 - d^4}{D}\right)$$

$\div\ 238{,}083 \quad \therefore 4{,}2002 \times 10^{-3} = \left(\frac{(2d^4 - d^4)}{2d}\right)$

$= 7{,}5\ d^3$

$d^3 = 5{,}6 \times 10^{-4}$

$d = 82{,}43$ mm

$D = 164{,}86$ mm

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$J = \frac{TL}{G\theta}$$

$$= \frac{45{,}359 \times 4 \times 180}{80 \times 10^9 \times 1{,}1 \times \pi}$$

**Important**

You must check both, as each is a limit in the shaft:

① $\tau$ shear stress

② $\theta$ angle of twist.

$\frac{\pi}{32}(D^4 - d^4) = 1{,}181 \times 10^{-4}$

$(2d^4 - d^4) = 1{,}203 \times 10^{-3}$

$d^4 = 8{,}022 \times 10^{-5}$

$d = 94{,}64$ mm

D = 189,28 mm (use this shaft)

8.2 $\delta = \frac{\theta R}{L}$

$= \frac{1{,}1 \times \pi \times 0{,}18928}{180 \times 4 \times 2}$

$= 4{,}542 \times 10^{-4}$

9. T = 120 kNm @ 150 rpm  $\tau = 75$ MPa

9.1 $T_{max} = 120 \times 1{,}1$

$= 132$ kNm

$\frac{T}{J} = \frac{\tau}{R} \quad \therefore T = \frac{\pi}{16}(D^3)\tau$

$\therefore 132k = \frac{\pi}{16} D^3 \times 75 \times 10^6$

$D^3 = 8{,}964 \times 10^{-3}$

D = 207,73 mm

9.2 $D^3 = \frac{D^4 - d^4}{D}$

$\therefore 8{,}964 \times 10^{-3} = \frac{D^4 - 0{,}65^4 D^4}{D}$

$= 0{,}8215\ D^3$

$D^3 = 0{,}0109$

D = 221,8 mm

$d = 144{,}17$ mm

9.3 $\frac{A_s - A_H}{A_s} = \frac{207{,}73^2 - (221{,}8^2 - 144{,}17^2)}{207{,}73^2}$

$= 34{,}16\%$

10. $P_H = 1{,}18\ P_s$

$2\pi(1{,}5N_s)T_H = (2\pi N_s T_s)1{,}18$

$\therefore T_H = 0{,}787\ T_s \ldots$ ①

$V_H = 0{,}75\ V_s$

$\therefore A_H = 0{,}75\ A_s$

$D_2 - d_2 = 0{,}75 \times 0{,}14^2$

$D_2 - d^2 = 0{,}0147 \ldots$ ②

$T_H = \tau_s$

$\therefore \frac{16 T_H D}{\pi(D^4 - d^4)} = \frac{16 T_s}{\pi D_s^3} \ldots$ ③

Substitute into: $\frac{0,787D}{D^4 - d^4} = \frac{1}{0,14^3}$

$\therefore 2,1586 \times 10^{-3}\ D = D^4 - d^4 \ldots$ ④

Square ②:

$\therefore d^4 = (D^2 - 0,0147)^2$

$= D^4 - 0,0294\ D^2 + 2,161 \times 10^{-4} \ldots$ ⑤

Substitute ① into ②:

$\therefore 2,1586 \times 10^{-3}\ D = D^4 - (D^4 - 0,0294\ D^2 + 2,161 \times 10^{-4})$

$= D^4 - D^4 + 0,0294\ D^2 - 2,161 \times 10^{-4}$

$\therefore 0,0294\ D^2 - 2,1586 \times 10^{-3}\ D - 2,161 \times 10^{-4} = 0$

$\div\ 0,0294 \therefore D^2 - 0,0734D - 7,35 \times 10^{-3} = 0$

$$D = \frac{0,0734 \pm \sqrt{(0,0734^2 - 4(1)(-7,35 \times 10^{-3})}}{2}$$

$$= \frac{0,0734 \pm 0,1865}{2}$$

$D = 129,96$ mm

From ②:

$D^2 = d^2 + 0,0147$

$0,12996^2 = d^2 + 0,0147$

$\therefore d = 46,8$ mm

11. 11.1 $d = 0,378D \ldots$ ①

$T = \frac{400k \times 60}{2\pi \times 110} = 34,725$ kNm

$T_{max} = 1,22 \times 34,725k = 42,364$ kNm

11.2 Stress diameters

$J = \frac{TR}{\tau} = \frac{42\ 364 \times D}{70M \times 2} = 3,026 \times 10^{-4} D$

$J = \frac{\pi}{32}(D^4 - d^4) = 3,026 \times 10^{-4} D \ldots$ ②

Substitute ① in ②: $\therefore \frac{D^4 - (0,378D)^4}{D} = \frac{32 \times 3,026 \times 10^{-4}}{\pi}$

$\therefore 0,9796D^3 = 3,082 \times 10^{-3}$

$D = 146,53$ mm

$d = 0,378 \times 146,53 = 55,39$ mm

11.3 Diameters

$$\theta \text{ rad} = \frac{2\pi}{180} = 0{,}0349 \text{ rad}$$

$$J = \frac{TL}{G\theta} = \frac{42\ 364 \times 3{,}8}{80 \times 10^9 \times 0{,}0349}$$

$$\therefore \frac{\pi}{32}\left(D^4 - (0{,}378D)^2\right) = 5{,}766 \times 10^{-5}$$

$$0{,}9796D^4 = 5{,}873 \times 10^{-4}$$

$$D = \left(\frac{5{,}873 \times 10^{-4}}{0{,}9796}\right)^{0{,}25}$$

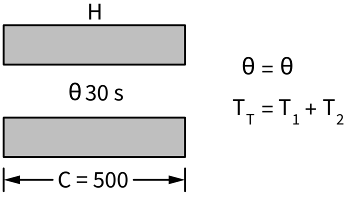
$D = 153{,}8$ mm and $d = 58{,}14$ mm

11.4 153,8 mm and 58,14 mm. Deflection will be the same, but the stress will be less.

12. $G_B = 42$ GPa  $G_{al} = 31$ GPa  $D = 40$  $d = 32$

12.1 $$C_B = \frac{GJ}{L} = \frac{\left(42G \times \frac{\pi}{32}0{,}03^4\right)}{0{,}5}$$

$$= 6{,}68 \text{ kNm/rad}$$




$$C_{AL} = \frac{GJ}{L}$$

$$= \frac{31G \times \frac{\pi}{32}(0{,}04^2 - 0{,}032^4)}{0{,}5}$$

$$= 9{,}2 \text{ kNm/rad}$$

$\therefore$ Total C = 15,88 kNm/rad

12.2 $T_T = T_B + T_{al}$

$$600 = \frac{J_B G_B \theta}{L} + \frac{J_{al} G_{al} \theta}{L}$$

$$= \theta[C_B + C_{al}] \qquad \left(\frac{JG}{L} = C\right)$$

$$= \theta \times 15{,}88k$$

$$\theta = 3{,}778 \times 10^{-2} \text{ rad}$$

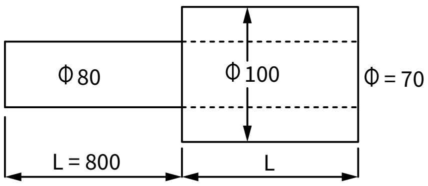
$$= 2{,}16°$$

12.3 $$\tau_B = \frac{RG\theta}{L} = \frac{0{,}015 \times 42G \times 0{,}03778}{0{,}5} = 47{,}66 \text{ MPa}$$

$$\tau_{al} = \frac{RG\theta}{L} = \frac{(0{,}02 \times 31G \times 0{,}03778)}{0{,}5} = 46{,}85 \text{ MPa}$$

12.4 $P = 2\pi NT = \omega T = 82 \times 600 = 49{,}2$ kW

13.




$\theta_T = \theta_S + \theta_H$
$T_S = T_H$
$\theta = 2°$
G = 82 GPa
T = 20 kNm

13.1 $\theta_T = T_s + T_H$

$$= \frac{TL_s}{GJ_s} + \frac{TL_H}{GJ_H}$$

$$\therefore \frac{3 \times \pi}{180} = \frac{20k \times 0{,}8}{82G \times \frac{\pi}{32} 0{,}08^4} + \frac{20k L_H}{82G \times \frac{\pi}{32}(0{,}1^4 - 0{,}07^4)}$$

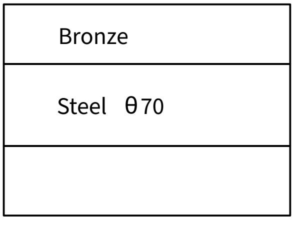
$$\therefore 0{,}05236 = 0{,}04852 + 3{,}8399 \times 10^{-3} = 0{,}03269\ L_H$$

$$\therefore L_H = 117{,}45 \text{ mm}$$

13.2 $T_s = T_H \quad \therefore C = \frac{T}{\theta} = \frac{20k}{3 \times \frac{\pi}{180}}$

$$= 381{,}982 \text{ kNm (rad)}$$

14.




$T_T = T_1 + T_2$

$\theta_1 = \theta_2$

$(L_B = L_S)$

14.1 $T_B = 1{,}45\ T_s$

$$\therefore \theta_B = \theta_S$$

$$\therefore \frac{T_B \not{L}_B}{J_B G_B} = \frac{T_S \not{L}_S}{J_S G_S}$$

$$\therefore \frac{1{,}45}{\left(\frac{\not{\pi}}{32}(D^4 - 0{,}07^4)32\right)} = \frac{1}{\frac{\not{\pi}}{32} 0{,}07^4 \times 80}$$

$$\therefore 1{,}45 \times 0{,}07^4 \times 80 = 32(D^4 - 0{,}07^4)$$

$$8{,}704 \times 10^{-5} = D^4 - 0{,}07^4$$

$$D^4 = 6{,}303 \times 10^{-5}$$

$$D = 89{,}1 \text{ mm}$$

14.2 $T_s = \frac{\pi}{16} D^3 \tau$

$$= \frac{\pi}{16} \times 0{,}07^3 \times 90 \times 10^6$$

$$= 6{,}061 \text{ kNm}$$

$$T_H = \frac{\pi}{16}\left(\frac{D^4 - d^6}{D}\right)\tau$$

$$= \frac{\pi}{16}\left(\frac{0{,}0891 \times -0{,}07^6}{0{,}0891}\right)55 \times 10^6$$

$$= 4{,}729 \text{ kNm}$$

(A) $T_H = 1{,}45\ T_s$ But (B) $\therefore T_s = \frac{T_H}{1{,}45}$

$= 6{,}061 \times 1{,}45$ $= \frac{4{,}729}{1{,}45}$

$= 8{,}788$ kNm $= 3{,}26$ Nm

$$\therefore T_{total} = T_H + T_s$$

$$= 4{,}729 + 3{,}26$$

$$= 7{,}989 \text{ kNm}$$

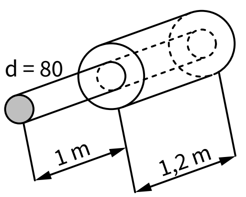
14.3 $T_{mean} = \frac{7{,}989}{1{,}12}$

$$= 7{,}133 \text{ kNm}$$

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 900 \times 7{,}133}{60}$$

$$= 672{,}269 \text{ kW}$$

15.




D = 120 $d$ = ?

T = 20 kNm $\theta > 2°$

G = 80 GPa $\theta_T = \theta_1 + \theta_2$

$T_s = T_H = 20$ kNm

$$\frac{\pi}{16} D^3 \tau = \frac{\pi D^4 - d^4}{D}\tau$$

$$80^3 = \frac{120^4 - d^4}{120}$$

$$61{,}44 \times 10^6 = 120^4 - d^4$$

$$d^4 = 120^4 - 61{,}44 \times 10^6$$

$$= 1{,}4592 \times 10^8$$

$$d = 110 \text{ mm}$$

$\theta_T = \theta_1 + \theta_2$

$\frac{2 \times \pi}{180} = \frac{T_H L_H}{J_H G_H} = \frac{T_s L_s}{J_s G_s}$

$\frac{2 \times \pi}{180} = \frac{20k \times 1{,}2}{J_H 80G} = \frac{20k \times 1}{J_s 80G}$

$\therefore \times 80G$ and $\div 20k$

$\therefore 139\,626{,}34 = \frac{1{,}2}{J_H} + \frac{1}{J_s}$

$139\,626{,}34 = \frac{1{,}2}{\frac{\pi}{32}(0{,}12^4 - d^4)} + \frac{1}{\frac{\pi}{32}0{,}08^4}$

$-109\,053{,}2 = \frac{(1{,}2 \times 32)}{\pi(0{,}12^4 - d^4)}$

$-8\,921{,}89 = \frac{1}{0{,}12^4 - d^4}$

$\therefore 0{,}12^4 - d^4 = \frac{1}{-8\,921{,}89}$

$= 1{,}121 \times 10^{-4}$

$\therefore d^4 = 0{,}12^4 + 1{,}121 \times 10^{-4}$

$d^4 = 3{,}194 \times 10^{-4}$

$d = 133{,}69$ mm

16. $d = 0{,}66\,D \ldots$ ①

$\therefore V_s = V_H \qquad (L_s = L_H)$

$\therefore A_s = A_H$

$\therefore D_1^2 = D^2 - d^2 \ldots$ ②

Substitute into:

$\therefore D_s^2 = D^2 - (0{,}66D)^2$

$= 0{,}5644\,D_H^2$

Take $\sqrt{\ }$: $D_s = 0{,}7513\,D_H \ldots$ ③

$T_s{:}T_H$

$\therefore D_s^3{:}\frac{D_H^4 - d_H^4}{D_H} \ldots$ ④

Substitute ① and ③ into ④:

$(0{,}7513\,D_H)^3{:}\frac{(D_H^4 - (0{,}66D)^4)}{D_H}$

$0{,}424\,D_H^3{:}\ 0{,}8103\,D_H^3$

$\therefore 1{:}1{,}911$

17.1 Solid $\frac{T}{\theta} = \frac{TGJ}{TL}$

$$= \frac{30 \times 10^9 \times \pi 0{,}008^4}{0{,}22 \times 32}$$

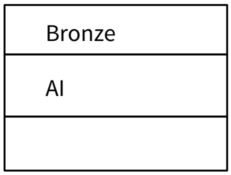
$$= 54{,}84 \text{ Nm/rad}$$

Hollow $\frac{T}{\theta} = \frac{TGJ}{TL}$

$$= \frac{40 \times 10^9 \times \pi(0{,}012^4 - 0{,}01^4)}{0{,}22 \times 32}$$

$$= 191{,}64 \text{ Nm/rad}$$

17.2 17.2.1




$T_T = T_1 + T_2$

$\theta_1 = \theta_2$

$$\theta_s = \theta_H \dots ①$$

$$\text{But } \theta_s = \frac{T_s}{\theta_s} = 54{,}84$$

$$\therefore \theta_s = \frac{T_s}{54{,}84} \dots ②$$

$$\frac{T_H}{\theta_H} = 191{,}64$$

$$\therefore \theta_H = \frac{T_H}{191{,}64} \dots ③$$

Substitute ② and ③ into ①:

$$\therefore \frac{T_s}{54{,}84} = \frac{T_H}{191{,}64}$$

$$\therefore T_H = 3{,}495\ T_s \dots ④$$

$$\therefore T_T = T_H + T_s \dots ⑤$$

$$\therefore 15 = 3{,}495\ T_s + T_s$$

$$T_s = 3{,}34 \text{ Nm}$$

$$\therefore T_H = 11{,}66 \text{ Nm}$$

17.2.2 $T_H = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)\tau$

$11{,}66 = \frac{\frac{\pi}{16}(0{,}012^4 - 0{,}01^4)}{0{,}012}\tau$

$\tau_{Hd} = 66{,}38$ MPa

$T_s = \frac{\pi}{16}D^3\tau$

$3{,}34 = \frac{\pi}{16}0{,}008^3\tau$

$\therefore \tau_{sd} = 33{,}22$ MPa

17.2.3 $\theta_{sd} = \frac{T_s}{54{,}84}$

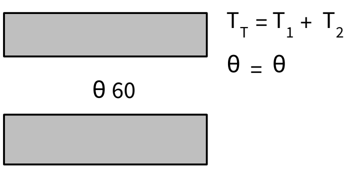
$= \frac{3{,}34}{54{,}84} = 0{,}061$ rad

$\theta_{Hd} = \frac{11{,}66}{191{,}64} = 0{,}061$ rad

$0{,}061 \times \frac{180}{\pi} = 3{,}5°$

18. $G_s = 2{,}5\ G_H$

$T_H = 3T_s \ldots$ ①




18.1 $\therefore \theta_s = \theta_H$

$\therefore \frac{T_s L_s}{J_s G_s} = \frac{T_H L_H}{J_H G_H}$

$\div T_s \times G_H: \frac{T_s}{J_s 2{,}5 G_H} = \frac{3T_s}{J_H G_H}$

$\frac{1}{\frac{\pi}{32}0{,}06 \times 2{,}5} = \frac{1}{\frac{\pi}{32}(D^4 - 0{,}06^4)}$

$3 \times 0{,}06^4 \times 2{,}5 = D^4 - 0{,}06^4$

$D^4 = 1{,}1016 \times 10^{-4}$

$D = 102{,}45$ mm

18.2 $T_H = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)\tau$

$= \frac{\pi}{16}\left(\frac{0{,}10245^4 - 0{,}06^4}{0{,}10245}\right)49M$

$= 9{,}129$ kNm

**Note**

The value of the ratio $T_H$ is bigger than the allowable value of 9,129 kNm.

Therefore, 10,689 kNm cannot be used and will damage the shaft.

$T_s = \frac{\pi}{16}D^3\tau$

$= \frac{\pi}{16} \times 0{,}06^3 \times 84M$

$= 3{,}563$ kNm

$T_H = 3T_s$

$= 3 \times 3{,}563 = 10{,}689$

$10{,}689 > 9{,}129$

$\therefore T_s = \frac{T_H}{3} = \frac{9{,}129}{3}$

$= 3{,}043$ kNm

$\therefore T_T = T_H + T_s$

$= 9{,}129 + 3{,}043$

$= 12{,}172$ kNm

18.3 $P = \frac{2\pi NT}{60}$

$= 2\pi \times \frac{400}{60} \times 12{,}172$

$= 509{,}86$ kW

19. 19.1 $\theta_s = \theta_B$

Given: $T_s = \frac{T_B}{3} \therefore 3T_s = T_B \ldots$ ①

$G_s = 2{,}2G_B \ldots$ ②

$\therefore \theta_s = \theta_B$

$\left(\frac{TL}{JG}\right)_s = \left(\frac{TL}{JG}\right)_B$

$\left(\frac{T}{\frac{\pi}{32}0{,}048^4 \times 2{,}2G_B}\right)_s = \left(\frac{3T_s}{\frac{\pi}{32}(D^4 - 0{,}048^4)G}\right)_B$

$\frac{1}{0{,}048^4 \times 2{,}2} = \frac{3}{D^4 - 0{,}048^4}$

$3 \times 0{,}048^4 \times 2.2 = D^4 - 0{,}048^4$

$D = \sqrt[4]{3{,}5036 \times 10^{-5} + 0{,}048^4} = 79{,}7$ mm

19.2 Torque steel = $T_s = \frac{\pi}{16}0{,}048^3 \times 84M = 1{,}824$ kNm

Torque bronze = $T_B = \frac{\pi}{16}\left(\frac{(0{,}0797^4 - 0{,}048^4)}{0{,}0797}\right)46M = 3{,}971$ kNm

According to ratio $3T_s = T_B$ $\therefore 3 \times 1{,}824k = 5{,}472$ kNm

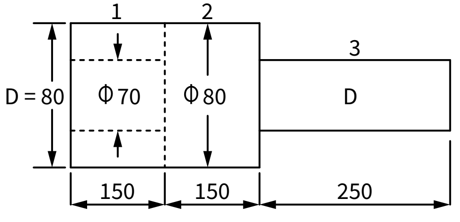
This is more than what the bronze can take.

$\therefore T_s = \frac{T_B}{3} = \frac{3{,}971k}{3} = 1{,}324$ kNm, which is less than what the steel can take.

$\therefore T_{total} = 3{,}971k_B + 1{,}324k_S = 6{,}792$ kNm

19.3 Power transmitted = $P = \frac{2\pi \times 288 \times 6{,}792k}{60} = 204{,}841$ kW

20. G = 84 GPa   T = T = T   $\theta_T = \theta_1 + \theta_2 + \theta_3$




20.1 $C_H = C_{s_3}$

$$\frac{\cancel{G}J}{L} = \frac{\cancel{G}J}{L}$$

$$\frac{\frac{\pi}{32}(0{,}08^3 - 0{,}07^4)}{0{,}15} = \frac{\frac{\pi}{32}D^4}{0{,}25}$$

$$\therefore D^4 = \frac{0{,}25(0{,}08^4 - 0{,}07^4)}{0{,}15}$$

$$D = (2{,}825 \times 10^{-5})^{0{,}25}$$

$$= 72{,}9 \text{ mm}$$

20.2 $\theta_T = \theta_1 + \theta_2 + \theta_3$

$$= \frac{\tau_L}{GR} + \frac{\tau_L}{GR} + \frac{\tau_L}{GR}$$

$$= \frac{\tau}{G}\left(\frac{L_1}{R_1} + \frac{L_2}{R_2} + \frac{L_3}{R_3}\right)$$

$$= \frac{60M}{84G}\left(\frac{0{,}25}{0{,}0729} + \frac{0{,}15}{0{,}04} + \frac{0{,}15}{0{,}04}\right)$$

$$= 0{,}0103 \text{ rad} = 0{,}588°$$

$$T_3 = \frac{\tau J_3}{R_3} = \frac{60M \times \frac{\pi}{32}0{,}0729^4 \times 2}{0{,}0729} = 4{,}564 \text{ kNm}$$

$$T_1 = \frac{\tau J_1}{R_1} = \frac{\left(60M \times \frac{\pi}{32}(0{,}08^4 \times 0{,}07^4) \times 2\right)}{0{,}08} = 2{,}496 \text{ kNm}$$

$\therefore$ Maximum allowable torque = 2,496 kNm $\because T_1 = T_2 = T_3$

20.3 $P = 2\pi NT = \omega T = 20\pi \times 2\,496$

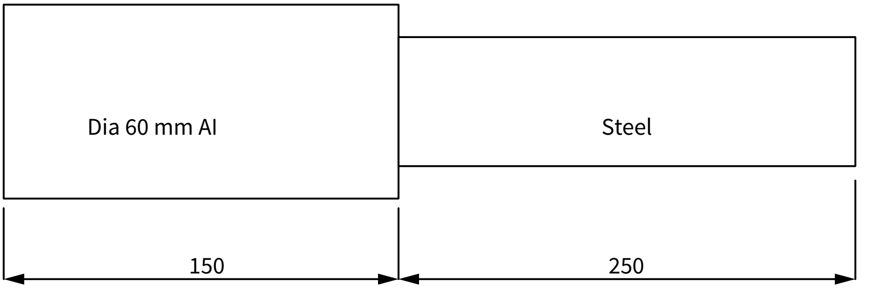
$= 156{,}828$ kW

20.4 $\omega = 2\pi N \quad \therefore 20\pi = 2\pi N$

$\therefore N = 10$ revolutions per second

$= 10 \times 60 = 600$ rpm

21.




21.1 Diameter shaft steel

$$T_s = T_{al}$$

$$\frac{\tau_s J_s}{R_s} = \frac{\tau_{al} J_{al}}{R_{al}} = \frac{60M \times \pi \times D^4 \times 2}{D \times 32} = \frac{40M \times \pi \times 0{,}06^4 \times 2}{32 \times 0{,}06}$$

$$D^3 = \frac{40M \times 0{,}06^3}{60M}$$

$$D = \sqrt[3]{1{,}44 \times 10^{-4}} = 52{,}41 \text{ mm}$$

21.2 Angle of twist per shaft

$$\theta_{al} = \frac{2\tau L}{DG} = \frac{2 \times 40M \times 0{,}15}{0{,}06 \times 30 \times 10^9} = 6{,}667 \times 10^{-3} \text{ rad}$$

$$\theta_{steel} = \frac{2\tau L}{DG} = \frac{2 \times 60M \times 0{,}25}{0{,}05241 \times 80 \times 10^9} = 7{,}155 \times 10^{-3} \text{ rad}$$

21.3 Torque in shaft

$$\text{Stress: } T_s = \frac{\tau_s J_s}{R_s} = \frac{60M \times \pi \times 0{,}05241^4 \times 2}{0{,}5241} = 1{,}696 \text{ kNm} = T_{al} \text{ (from 21.1)}$$

$$\text{Angle of twist: } = \frac{JG\theta}{L} = \frac{\pi \times 0{,}05241^4 \times 80 \times 10^9 \times 7{,}155 \times 10^{-3}}{32 \times 0{,}25} = 1{,}696 \text{ kNm}$$

Maximum torque = 1,696 kNm

21.4 Replace shaft with hollow shaft

$$T_s = T_H$$

$$1\,696 = \frac{\tau J}{R}$$

$$1\,696 = \frac{60M \times \pi(D^4 - d^4) \times 2}{D \times 32}$$

$$1\,696 = \frac{60M \times \pi((2d)^4 - d^4) \times 2}{2d \times 32}$$

$$2{,}879 \times 10^{-4} = 15d^3$$

$d^3 = 1{,}9195 \times 10^{-5}$

$d = 26{,}78$ mm and D = 53,56 mm

21.5 Saving in weight

Volume solid shaft:

$$V_{al} = \frac{\pi}{4} 0{,}06^2 \times 0{,}15 = 4{,}421 \times 10^{-4}\ \text{m}^3$$

$$V_S = \frac{\pi}{4} 0{,}05241^2 \times 0{,}25 = 5{,}393 \times 10^{-4}\ \text{m}^3$$

$$V_{total} = 9{,}634 \times 10^{-4}\ \text{m}^3$$

$$V_{hollow} = \frac{\pi}{4}\left(0{,}05356^2 - 0{,}02678^2\right) \times 0{,}4 = 6{,}759 \times 10^{-4}\ \text{m}^3$$

$$\%\ \text{saving} = \frac{V_{solid} - V_{hollow}}{V_{solid}} \times \frac{100}{1}$$

$$\%\ \text{saving} = \frac{(9{,}634 - 6{,}759)10^{-4}}{9{,}634 \times 10^{-4}} \times \frac{100}{1} = 29{,}84\%$$

MODULE

# 8 *Structural frameworks*

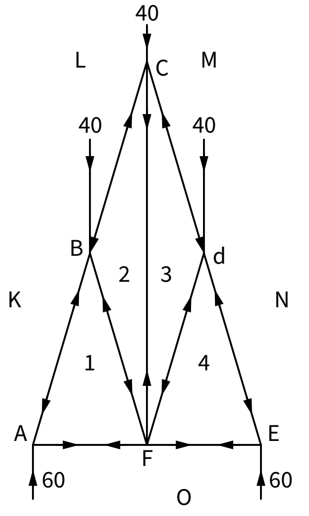
**By the end of this module, students should be able to**:

- explain the rules for statically determinate plane frameworks;
- differentiate between a *tie* and a *strut*;
- apply mathematical triangles;
- apply the equilibrium law;
- calculate reactions at the supports of structural frameworks with vertical loads;
- calculate reactions at the supports of structural frameworks with vertical as well as horizontal loads or angled loads with one of the supports on rollers;
- draw a space diagram of the framework to scale;
- draw a vector diagram to scale and graphically determine forces in simply supported structural frameworks or cantilevers; and
- tell whether a member is a strut or a tie.

A structural framework is a system of three or more straight members capable of transmitting structural loads. These frameworks can be simple or complex systems. The frameworks we are going to discuss all lie in the same plane and will not be three-dimensional.

## Exercise 8.1 SB page 265

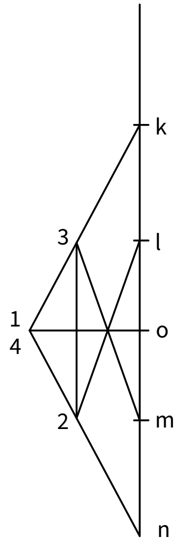
1. Note: Drawings are not to scale. Scale 1 cm = 1 m, 1 cm = 10 N.




**Note**

It is a symmetrical loaded structure, therefore each support will be: 60 N = $\frac{3 \times 40}{2}$.

| Member | Vector | Force (N) | Type |
|---|---|---|---|
| AB | *k*1 | 65 | S |
| BC | *l*2 | 43 | S |
| CD | *m*3 | 43 | S |
| DE | *n*4 | 65 | S |
| EF | *o*4 | 24 | T |
| FA | *ol* | 24 | T |
| CF | 23 | 40 | T |
| FB | 12 | 22 | S |
| FD | 34 | 22 | S |




*Vector diagram*

2. 2.1 Moment on A

$\therefore 13D = 100 \times 5 + 50 \times 9 + 20 \times 13$

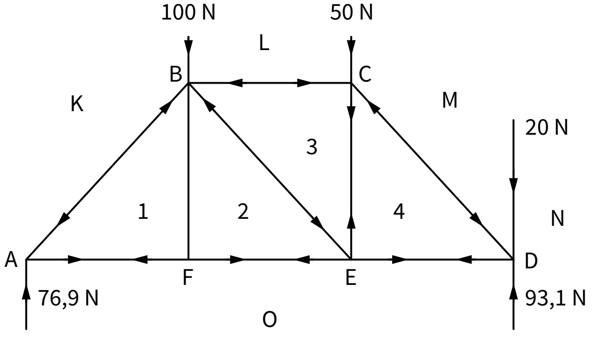
$D = 93{,}1$ N

Moment on D

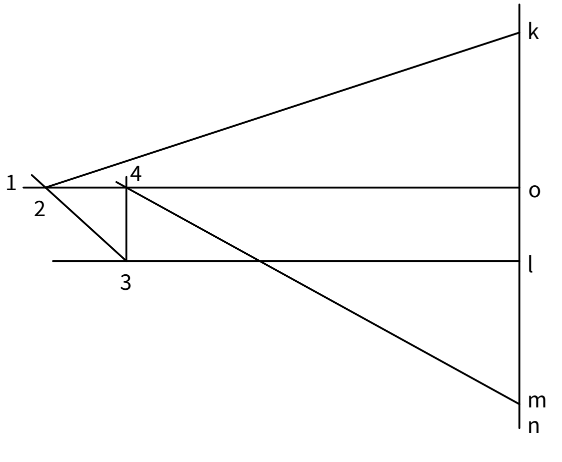
$\therefore 13A = 50 \times 4 + 100 \times 8$

$A = 76{,}9$

2.2 Scale 1 cm = 1 m; 1 cm = 10 N

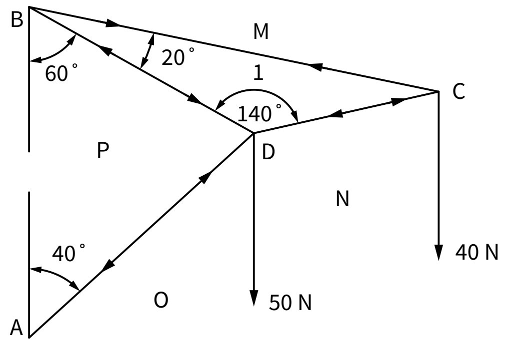



*Space diagram*



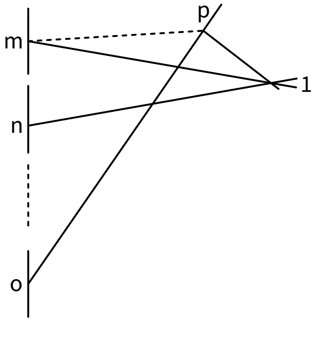

*Vector diagram*

| Member | Vector | Force (N) | Type |
|---|---|---|---|
| AB | *k*1 | 151 | S |
| BC | *l*3 | 98 | S |
| CD | *m*4 | 122 | S |
| DE | *o*4 | 98 | T |
| EF | *o*2 | 130 | T |
| FA | *o*1 | 10 | T |
| FB | 12 | 0 | xx |
| EB | 23 | 39 | S |
| EC | 34 | 23 | T |




*Space diagram*

3. Scale: 1 cm = 10 N




*Vector diagram*

| Member | Vector | Force (N) | Type |
|---|---|---|---|
| AD | *op* | 112 | S |
| DB | *p*1 | 41 | T |
| BC | *m*1 | 114 | T |
| CD | *n*1 | 113 | S |

Reaction B = *pm* = 76 N

Reaction A = *op* = 118 N

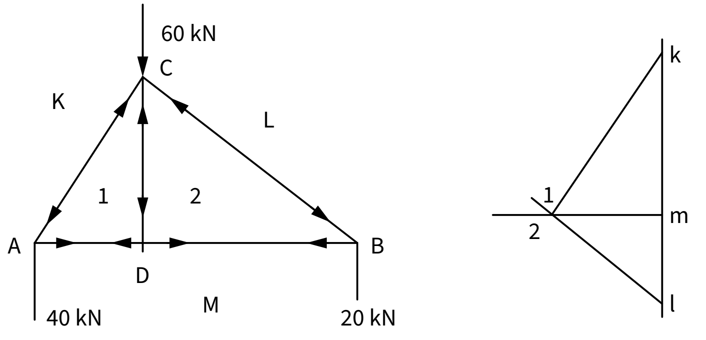
In this case use only length for space diagram with correct angles.
No calculations.

4. 4.1 Moments about A; 6B = 60 × 2

B = 20 kN

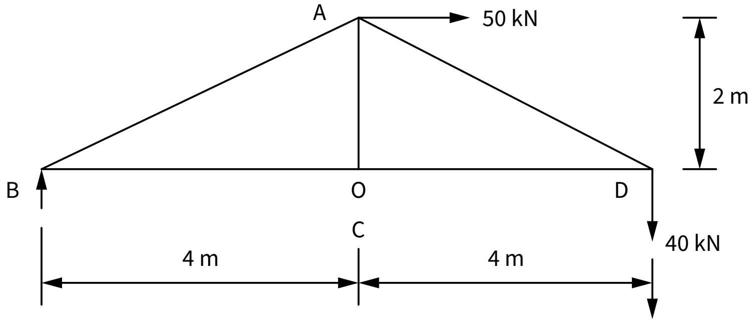
Reaction A = 60 – 20 = 40 kN (F down = F up)

4.2 Scale 1 cm = 1 m and 1 cm = 10 kN




| Member | Vector | Magnitude | Nature |
|---|---|---|---|
| AC | *k*1 | 48 kN | S |
| CB | *l*2 | 33 kN | S |
| BD | 2*m* | 26 kN | T |
| DA | 1*m* | 26 kN | T |
| CD | 12 | 0 | 0 |

5.



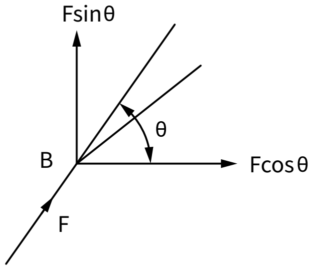

5.1 Fixed at B rollers at C

Moments about B; $4C = 50 \times 2 + 40 \times 8$

$C = 105$ kN

5.2 Draw space and vector diagrams. Scale 1 cm = 1 m and 1 cm = 10 kN.

To calculate reaction at B:



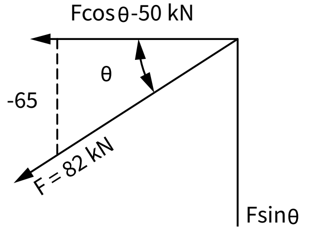

5.3 Reaction at B with its components

Equilibrium; $\Sigma HC = 0$ $\quad \therefore F \cos \theta + 50k = 0$

$\therefore F \cos \theta = -50$ kN

$\Sigma VC = 0$ $\quad \therefore F \sin \theta - 40k + 105 = 0$

$\therefore F \sin \theta = -65$ kN

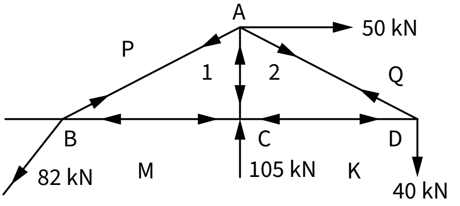



The negative values show that the reaction is downwards to keep the structure in equilibrium.

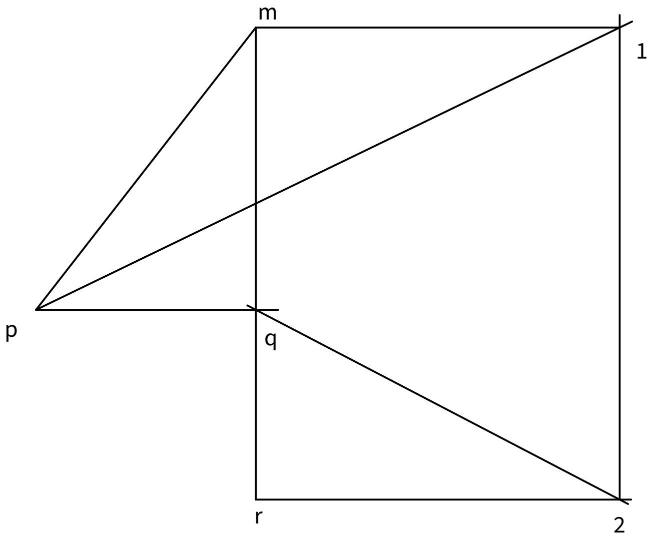
Reaction $F = \sqrt{50k^2 + 65k^2} = 82$ kN downwards

5.4 Angle with horizontal $\tan\theta = \frac{65}{50}$ = W 52,43° S

Draw diagrams:



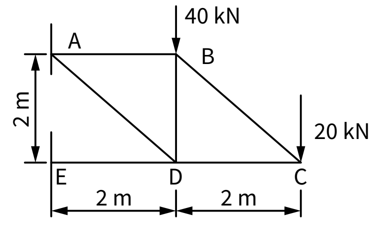

*Space diagram*

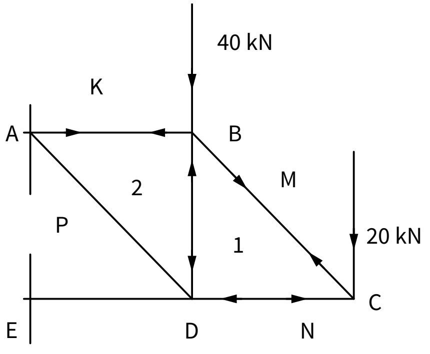


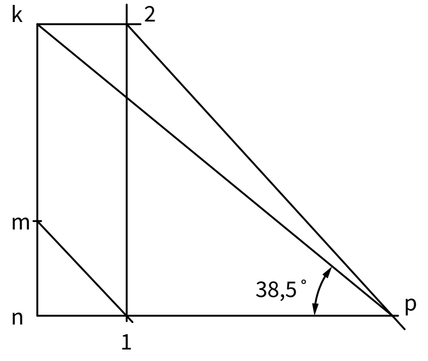

*Vector diagram*

| Member | Vector | Magnitude | Nature |
|---|---|---|---|
| AB | P1 | 147 kN | T |
| BC | 1*m* | 82 kN | S |
| CD | 2*k* | 82 kN | S |
| DA | 2*q* | 92 kN | T |
| AC | 12 | 106 kN | S |

6. 6.1 Scale: 1 cm =10 N and 1 cm = 1 m




6.2 Reaction at A = PK =95 kN at 38,5° with horizontal
Reaction at E =NP = 74 kN horizontal

| Member | Vector | Magnitude | Nature |
|---|---|---|---|
| AB | *k*2 | 18 kN | T |
| BC | *m*1 | 26 kN | T |
| CD | 1*n* | 18 kN | S |
| DB | 12 | 60 kN | S |

6.3 38,5°

7. 7.1 Moments about B

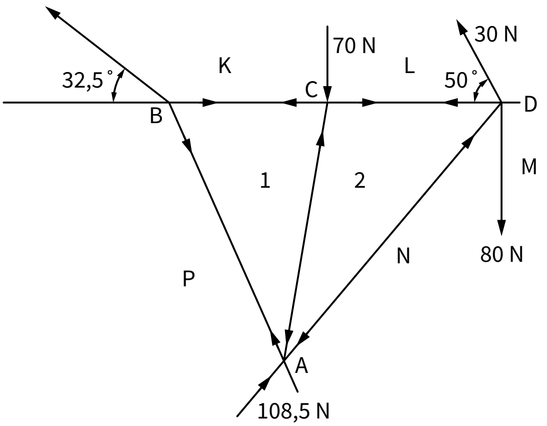
$\therefore$ 6A + 30 sin 50° × 8 = 70 × 4,5 + 80 × 8

A = 315 + 640 − 183,85

= 128,53 N

7.2 Reaction B = $pk$ = 110 N

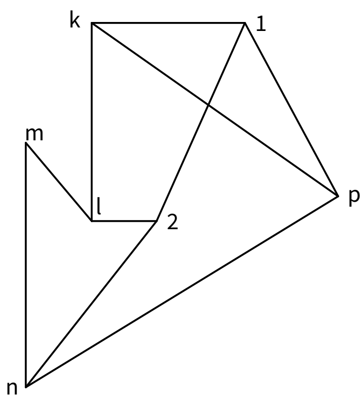
| Member | Vector | Force (N) | Type |
|---|---|---|---|
| AB | $p1$ | 70 | T |
| BC | $k1$ | 56 | T |
| CD | $l2$ | 34 | T |
| AD | $n2$ | 80 | S |
| AC | 12 | 74 | S |




*Space diagram*

7.3 W32,5° N

7.4 (Scale 1 cm = 20 N)




*Vector diagram*

Use length for space diagram.

8. 8.1 Moments about F

CCWM = CWM

$6D + (25 \times 1,5 = (30 \times 3) + (40 \sin 60 \times 3) + (40 \cos 60 \times 1,5) + (20 \times 6))$

D = 51,07 kN

8.2 Reaction at F

$\Sigma VC = 0$

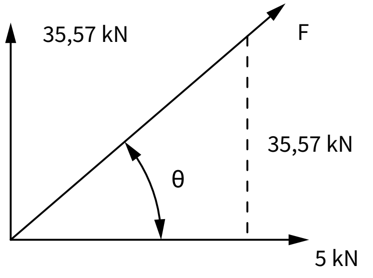
$F \sin \theta - 30 + 51,07 - 20 - 40 \sin 60 = 0$

$F \sin \theta = +35,57$ kN

$\Sigma HC = 0$

$F \cos \theta + 40 \cos 60 - 25 = 0$

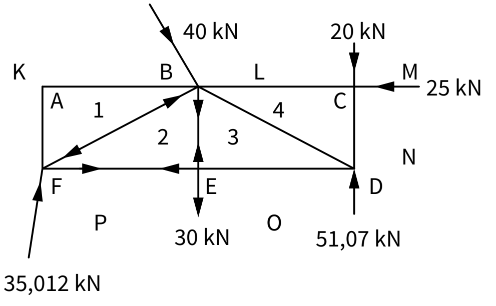
$F \cos \theta = +5$ kN



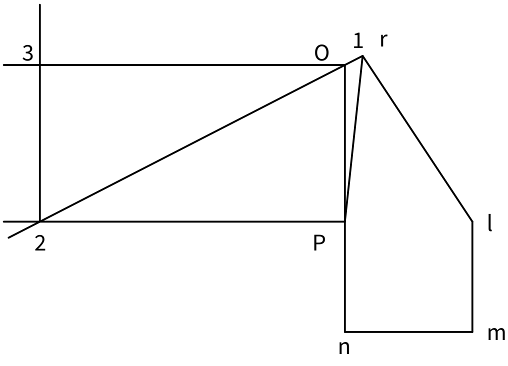

$F = \sqrt{5k^2 + 35,57k^2} = 35,92$ kN

$\tan \theta = \frac{35,57}{5}$ $\therefore \theta = 82°$ with horizontal

Draw diagrams. Scale: 1 cm = 1 m and 1 cm = 10 kN



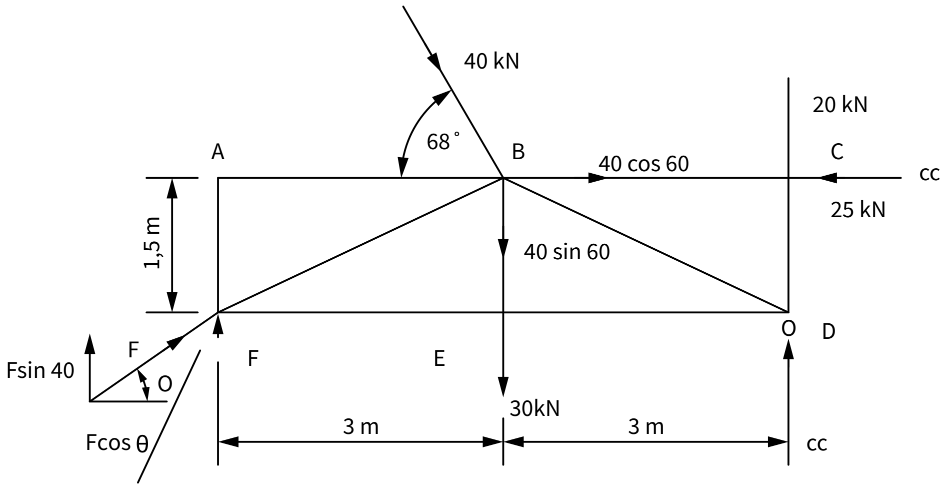

*Space diagram*




*Vector diagram*

| Member | Vector | Magnitude | Nature |
|---|---|---|---|
| AB | $k1$ | 0 | 0 |
| BE | 23 | 30 kN | T |
| EF | $p2$ | 56,5 kN | T |
| FA | $k1$ | 0 | 0 |
| FB | 12 | 68 kN | S |

8.3




9. 9.1 Moment on A

$$\therefore 12B + (50 \times 6) = (40 \sin 60 \times 3) + (4 \cos 60 \times 6) + (80 \times 6) + (30 \cos 45 \times 6) + (30 \sin 45 \times 6)$$

$$= 103{,}92 + 120 + 480 + 127{,}28 + 127{,}28$$

$$12B + 300 = 958{,}48$$

$$B = 54{,}87 \text{ N}$$

$\Sigma VC = 0$

$\therefore E \sin\theta - 40 \sin 60 - 60 - 30 \sin 45 + 54{,}87 - 20 = 0$

$\therefore E \sin\theta = +34{,}64 + 60 + 21{,}21 - 54{,}87 + 20$

$= +80{,}98 \text{ N}$

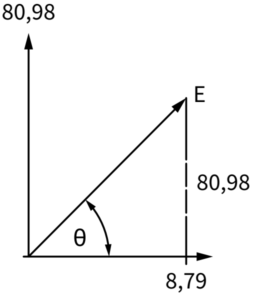
$\Sigma HC = 0$

$\therefore E \cos\theta + 40 \cos 60 + 30 \cos 45 - 50 = 0$

$\therefore E \cos\theta = -20 - 21{,}21 + 50 = 0$

$= +8{,}79 \text{ N}$

$\therefore E = \sqrt{80{,}98^2 + 8{,}79^2} = 81{,}5 \text{ N}$

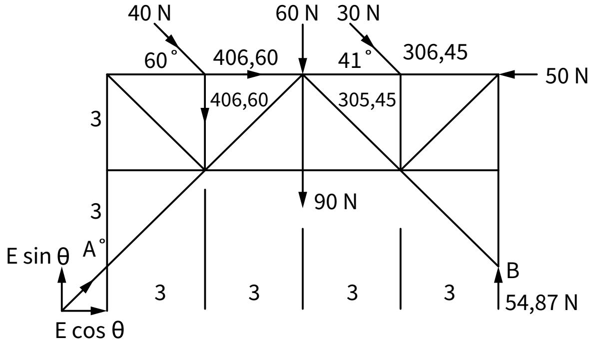



$\tan\theta = \frac{80{,}98}{8{,}79}$ $\therefore \theta = 83{,}8°$

E = 81,5 N E 83,8° N

| Member | Vector | Force (N) | Type |
|---|---|---|---|
| BC | 01 | 55 | S |
| CD | 02 | 55 | S |
| DE | *n*3 | 105 | S |
| EF | *m*4 | 85 | S |
| FG | 45 | 45 | S |
| GB | *p*1 | 0 | xx |
| GE | 43 | 22 | S |
| GD | 32 | 78 | T |
| GC | 12 | 0 | xx |

9.2




10. 10.1 First calculate the reaction at 'E' roller support. Take each length on 2 units (1 unit = 1 cm).

Moment about E $\therefore \overline{M} = \overline{M}$

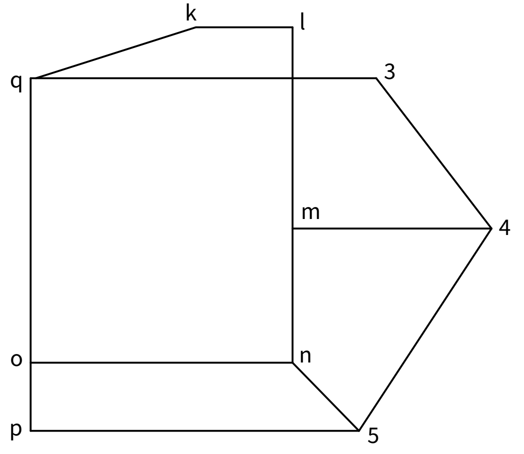
$\therefore 4C = (30 \times \sqrt{3}) + (60N) + (40 \times 5) + (20 \times 6)$

$\therefore 4C = 51{,}96 + 60 + 200 + 120$

$C = 107{,}99$ N

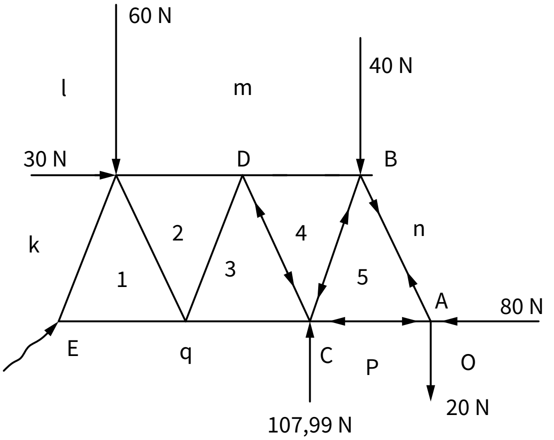
10.2 Draw the vector diagram and measure length *qk* for the reaction at E = 51,42 N (E 13,5° N).

| Member | Vector | Force (N) | Type |
|---|---|---|---|
| AB | *n*5 | 22 | T |
| BC | 45 | 69 | S |
| CD | 43 | 55 | S |
| DB | *m*4 | 46 | T |
| CA | *p*5 | 91 | S |




*Vector diagram*

10.3

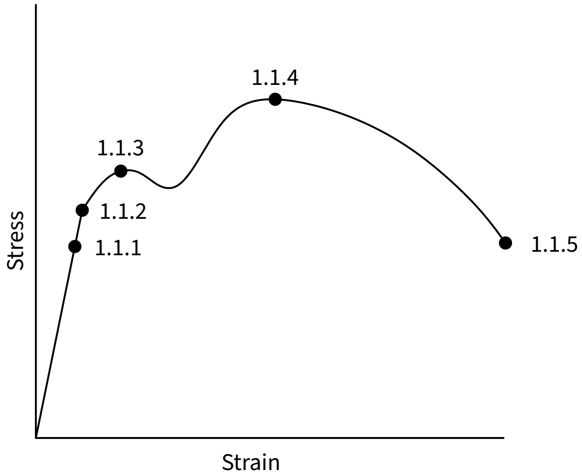



*Space diagram*

# Exemplar examination paper memorandum

## QUESTION 1

1.1




1.1.1 Proportional limit

1.1.2 Elastic limit

1.1.3 Yield point

1.1.4 Maximum load point

1.1.5 Point of fracture

1.2 1.2.1 The stress at the proportional limit (lop)

$$\sigma = \frac{F}{A} = \frac{32k \times 4}{\pi \times 0{,}013^2} = 241{,}087 \text{ MPa}$$

1.2.2 Young's modulus of elasticity

$$E = \frac{\sigma_{lop} \times l}{x_{lop}} = \frac{241{,}087M \times 0{,}07}{0{,}22 \times 10^{-3}} = 76{,}71 \text{ GPa}$$

1.2.3 The ultimate tensile strength

$$\sigma_{ult} = \frac{F_{ult}}{A} = \frac{70k \times 4}{\pi 0{,}013^2} = 527r, 377 \text{ MPa}$$

1.2.4 The fracture stress

$$\sigma_{frac} = \frac{F_{frac}}{A} = \frac{52k \times 4}{\pi 0{,}013^2} = 391{,}766 \text{ MPa}$$

1.2.5 The percentage elongation

$$\%X = \frac{x}{l} = \frac{8{,}33}{70} \times \frac{100}{1} = 11{,}9 \ \%$$

## QUESTION 2

2.1 The length of 70 mm diameter section:

$$U_T = U_1 + U_2 = 0{,}5Fx_1 + 0{,}5Fx_2 = 0{,}5F\left(\frac{FL_1}{A_1E} + \frac{FL_2}{A_2E}\right)$$

$$\therefore 12 = \frac{0{,}5F^2}{215G}\left(\frac{0{,}9 \times 4}{\pi \times 0{,}05^2} + \frac{L_2 \times 4}{\pi \times 0{,}07^2}\right)$$

$$\therefore \frac{12 \times 215G}{0{,}5(2\,038 \times 9{,}81)^2} = 45{,}837 + 259{,}845L_2$$

$$\therefore L_2 = \frac{12\,909{,}319 - 45{,}837}{259{,}845} = 49{,}504 \text{ m}$$

2.2 The total change in length

$$U_T = 0{,}5Fx_T \therefore X_T = \frac{12}{0{,}5(2\,038 \times 9{,}81)} = 1{,}2 \text{ mm}$$

2.3 The total strain

$$\varepsilon_T = \varepsilon_1 + \varepsilon_2 = \left(\frac{x}{L}\right)_1 + \left(\frac{x}{L}\right)_2 = \frac{FL_1}{L_1A_1E} + \frac{FL_2}{L_2A_2E}$$

$$\therefore \varepsilon_T = \frac{F}{E}\left(\frac{1}{A_1} + \frac{1}{A_2}\right) = \frac{2\,038 \times 9{,}81}{215G}\left(\frac{4}{\pi \times 0{,}05^2} + \frac{4}{\pi \times 0{,}07^2}\right) = 7{,}152 \times 10^{-5}$$

2.4 The maximum stress in the smallest area

$$\therefore \sigma_{max} = \frac{F}{A} = \frac{2\,038 \times 9{,}81 \times 4}{\pi \times 0{,}05^2} = 10{,}182 \text{ MPa}$$

## QUESTION 3

3.1 3.1.1 The stress in each material under load 65 kN

$$F_T = 65k = F_c + F_s \ldots (1)$$

$$\text{And: } x_c = x_s \therefore \frac{F_CL_c}{A_cE_c} = \frac{F_sL_s}{A_sE_s} \; (L_c = L_s)$$

$$\div\text{by L: } \therefore F_c = \frac{F_sA_cE_c}{A_sE_s} = \frac{F_s \times 0{,}02^2 \times 145}{0{,}015^2 \times 215}$$

$$\therefore F_c = 1{,}199F_s \ldots (2)$$

Substitute: (2) into (1) $\therefore 65k = 1{,}199F_s + F_s$

$\therefore F_s = 29{,}559$ kN

$F_c = 29{,}559k \times 1{,}199 = 35{,}441$ kN

$$\sigma_s = \frac{F_s}{A_s} = \frac{29\,559 \times 4}{\pi \times 0{,}015^2} = 167{,}27 \text{ MPa (C)}$$

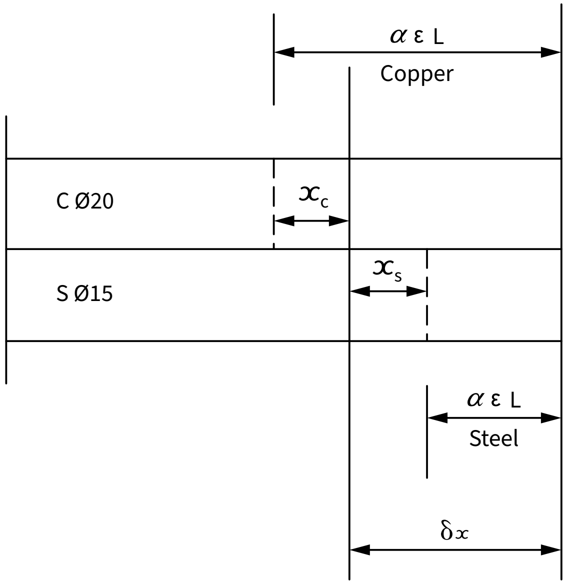
$$\sigma_c = \frac{F_c}{A_c} = \frac{35\,441 \times 4}{\pi \times 0{,}02^2} = 112{,}812 \text{ MPa (C)}$$

3.1.2 Final length: $L_F$

$$x_c = x_s \therefore X_C = \frac{\sigma_c L_c}{E_c} = \frac{112{,}812M \times 0{,}127}{145G} = 0{,}098 \text{ mm}$$

$$\therefore L_F = L_{original} + x_c = 127 + 0{,}098 = 127{,}098 \text{ mm}$$

3.2.1




$$\delta_{XC} = \delta_{XS}$$

$$\therefore (\alpha tL)_c - x_c = (\alpha tL)_s + x_s$$

$$\therefore X_s + X_c = (\alpha tL)_c - (\alpha tL)_s$$

$$\therefore \frac{F_s L}{A_s E_s} + \frac{F_c L}{A_c E_c} = tL(\alpha_c - \alpha_S)$$

$$\div L: \frac{F_s \times 4}{\pi \times 0{,}015^2 \times 215G} + \frac{F_c \times 4}{\mu \times 0{,}02^2 \times 145G} = 37(18 - 12)\, 10^{-6}$$

$$\therefore 2{,}632 \times 10^{-8} F_s + 2{,}195 \times 10^{-8} F_c = 2{,}22 \times 10^{-4}$$

$$\therefore F_s = F_c = 4{,}599 \text{ kN}$$

$$\therefore \sigma_c = \frac{4\,599 \times 4}{\pi \times 0{,}02^2} = 14{,}639 \text{ MPa (T)}$$

$$\therefore \sigma_s = \frac{4\,599 \times 4}{\pi \times 0{,}015^2} = 26{,}025 \text{ MPa (C)}$$

The resultant stresses: load and temperature

$$\sigma_{RC} = \sigma_L - \sigma_T = 112{,}812 - 14{,}639 = 98{,}173 \text{ MPa (C)}$$

$$\sigma_{RS} = \sigma_L + \sigma_T = 167{,}27 + 26{,}025 = 193{,}295 \text{ MPa (C)}$$

3.2.2 The final length after load and temperature

$$\therefore x_T = \delta_{xc} = (\alpha tL)_c - x_c = \left(18 \times 10^{-6} \times 37 \times 0{,}127\right) - \left(\frac{F_c L_c}{A_c E_c}\right)$$

$$\therefore X_T = 8{,}458 \times 10^{-5} - \left(\frac{4\,599 \times 0{,}127 \times 4}{\pi \times 0{,}02^2 \times 145G}\right) = 0{,}072 \text{ m}$$

The final length:

$$L_{final} = L_{original} + x_{load} - X_{temp} = 127 + 0{,}098 - 0{,}072 = 127{,}026 \text{ mm}$$

## QUESTION 4

4.1 The allowable stress means it must be taken for longitudinal stress as well as tensile stress or hoop stress.

Considered as tensile stress:

$$\therefore \sigma_t = \frac{p_i D}{2t\cap_L} = 150M = \frac{3Md}{2 \times 0{,}018 \times 0{,}85}$$

$$\therefore d = \frac{150M \times 2 \times 0{,}018 \times 0{,}85}{3\ M} = 1{,}53 \text{ m}$$

Considered as longitudinal stress:

$$\therefore \sigma_L = \frac{p_i D}{4t\cap_c} = 150M = \frac{3Md}{4 \times 0{,}018 \times 0{,}52}$$

$$\therefore d = \frac{150M \times 4 \times 0{,}018 \times 0{,}52}{3M} = 1{,}872 \text{ m}$$

Use a diameter of 1,53 m. Because of a diameter of 1,872 m, the tensile stress will be more than 150 MPa and the cylinder will fail.

4.2.1 The longitudinal strain

Longitudinal stress for the diameter of 1,53 m:

$$\therefore \sigma_L = \frac{pd}{4t\cap_c} = \frac{3M \times 1{,}53}{4 \times 0{,}018 \times 0{,}52} = 122{,}6 \text{ MPa}$$

$$\text{Longitudinal strain} = \varepsilon_L = \frac{(\sigma_L - \gamma\sigma_H)}{E}$$

$$\varepsilon_L = \frac{(122{,}6M - 0{,}3 \times 150M)}{200G} = 3{,}88 \times 10^{-4}$$

4.2.2 The circumferential strain

$$\text{Circumferential or hoop strain} = \varepsilon_H = \frac{(\sigma_H - \gamma\sigma_L)}{E}$$

$$\varepsilon_H = \frac{(150M - 0{,}3 \times 122{,}6M)}{200G} = 5{,}661 \times 10^{-4}$$

4.2.3 The change in volume

Original volume = $V = \frac{\pi d^2 L}{4}$

$V = \frac{\pi \times 1{,}53^2 \times 3}{4} = 5{,}516 \text{ m}^3$

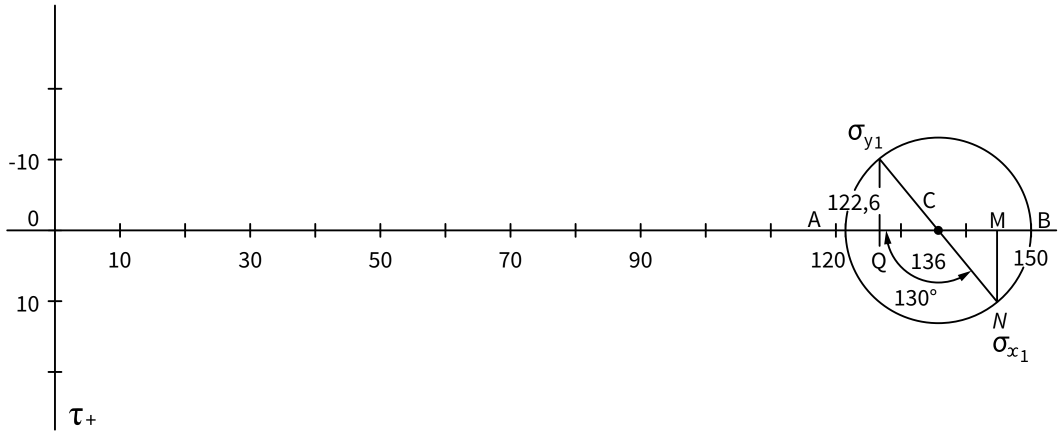
Change in volume = $\delta_V = \frac{pdV}{4tE}(5 - \gamma 4)$

$\delta_V = \frac{3M \times 1{,}53 \times 5{,}516}{4 \times 0{,}018 \times 200G}(5 - 0{,}3 \times 4) = 6{,}681 \times 10^{-3} \text{ m}^3$

4.3 Mohr's circle

Scale: 1 cm = 10 MPa

$C = \frac{150 + 122}{2} = 136{,}3 \text{ MPa}$




Normal stress on X face = $\sigma_{X1}$ = 141 MPa (T)

Normal stress on Y face = $\sigma_{Y1}$ = 127 MPa (T)

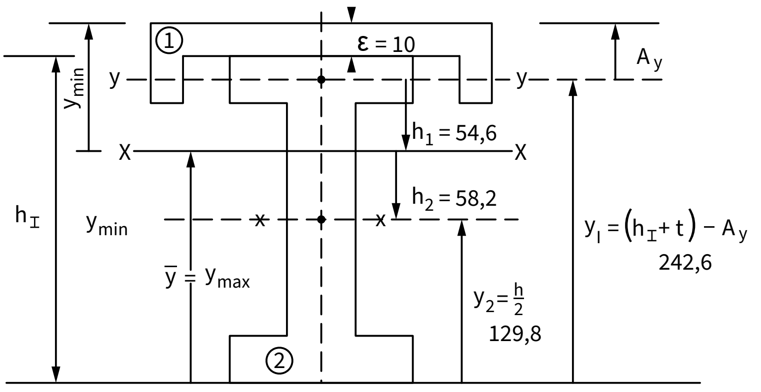
## QUESTION 5

5.1 The bending moment for beam

$\therefore M_{max} = M_{pl} + M_{udl} = \frac{WL}{4} + \frac{wL^2}{8}$

$M_{max} = \frac{17k \times 1{,}2}{4} + \frac{25k \times 1{,}2^2}{8} = 8{,}1 \text{ kNm}$

5.2 The position of the *xx*-axis




$\bar{y}A_T = \Sigma$area moments

$y_2 = \frac{h_{\text{I-section}}}{2} = \frac{259,6}{2} = 129,8$ mm

$y_1 = \left(h_{\text{I-section}} + t - A_y\right) = 259,6 + 10 - 27 = 242,6$ mm

| **Number** | **Area** | **Y** | **A × *y*** |
|---|---|---|---|
| 1 | $5,876 \times 10^{-3}$ | 0,2426 | $1,426 \times 10^{-3}$ |
| 2 | $5,501 \times 10^{-3}$ | 0,1298 | $7,14 \times 10^{-4}$ |
| Total area | $11,377 \times 10^{-3}$ | ΣArea moments | $2,14 \times 10^{-3}$ |

$\bar{y} = \frac{\Sigma\text{area moments}}{\text{total area}} = \frac{2,14 \times 10^{-3}}{11,377 \times 10^{-3}} = 188$ mm

5.3 The bending moment resistance (moment of inertia)

From drawing at Question 5.2:

$h_1 = y - \bar{y} = 242,6 - 188 = 54,6$ mm

$h_2 = \bar{y} - y_2 = 188 - 129,8 = 58,2$ mm

$I_{xx\,\text{total}} = \left(I_{1\,yy} + A_1 h_1^2\right)_{\text{channel}} + \left(I_{2\,xx} + A_2 h_2^2\right)_{\text{I-section}}$

For channel: $I_{\text{channel}} = 4,931 \times 10^{-6} + \left(5,876 \times 10^{-3} \times 0,0546^2\right)$

$= 2,245 \times 10^{-5}\ \text{m}^4$

For I-section: $I_{\text{I-section}} = 65,54 \times 10^{-6} + \left(5,501 \times 10^{-3} \times 0,0582^2\right)$

$= 8,417 \times 10^{-5}\ \text{m}^4$

$\therefore I_{xx} = I_{\text{channel}} + I_{\text{I-section}} = 2,245 \times 10^{-5} + 8,417 \times 10^{-5} = 106,62 \times 10^{-6}\ \text{m}^4$

5.4 The maximum and minimum stress

$y_{max} = 188$ mm and $y_{min} = h_1 + a_y = 54,6 + 27 = 81,6$ mm

$$\therefore \sigma_{min} = \frac{My_{min}}{I_{xx}} = \frac{8,1k \times 0,0816}{106,62 \times 10^{-6}} = 6,199 \text{ MPa}$$

$$\therefore \sigma_{max} = \frac{My_{max}}{I_{xx}} = \frac{8,1k \times 0,188}{106 \times 10^{-6}} = 14,282 \text{ MPa}$$

## QUESTION 6

6.1 The maximum shear stress

$$\frac{\tau}{R} = \frac{G\theta}{L} \therefore \tau = \frac{dG\theta}{2L}$$

$$\therefore \tau = \frac{0,6 \times 85 \times 10^9 \times 1,8 \times \pi}{2 \times 1,55 \times 180} = 51,684 \text{ MPa}$$

6.2 The power transmitted

$$T = \frac{\tau J}{R} = \frac{51,684 \times 10^6 \times \pi \times 0,06^4 \times 2}{32 \times 0,06} = 2,192 \text{ kNm}$$

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 900 \times 2\,192}{60} = 206,591 \text{ kW}$$

6.3 The diameters of the hollow shaft

Maximum power = 206 591 × 1,2 = 247,909 kW

$$\text{Torque transmitted} = T = \frac{P60}{2\pi N} = \frac{247\,909 \times 60}{2\pi \times 900} = 2,63 \text{ kNm}$$

$$J = \frac{TR}{\tau} = \frac{2\,630 \times D}{51,684 \times 10^6 \times 2} = 2,544 \times 10^{-5} D \ldots (1)$$

$$\text{But } J = \frac{\pi}{32}[D^4 - d^4] = 2,544 \times 10^{-5} D \ldots (2)$$

$$D = 2d \ldots (3)$$

$$\text{Substitute (3)into (2)} \therefore \frac{\pi}{32}\left[(2d)^4 - d^4\right] = 3,544 \times 10^{-5} \times 2d$$

$$\therefore 16d^4 - d^4 = 5,183 \times 10^{-4} d$$

$$\therefore 15d^3 = 5,183 \times 10^{-4}$$

$$\therefore d = \sqrt[3]{\frac{5,183 \times 10^{-4}}{15}} = 32,571 \text{ mm}$$

$$\therefore D = 65,142 \text{ mm}$$

## QUESTION 7

7.1 True

7.2 True

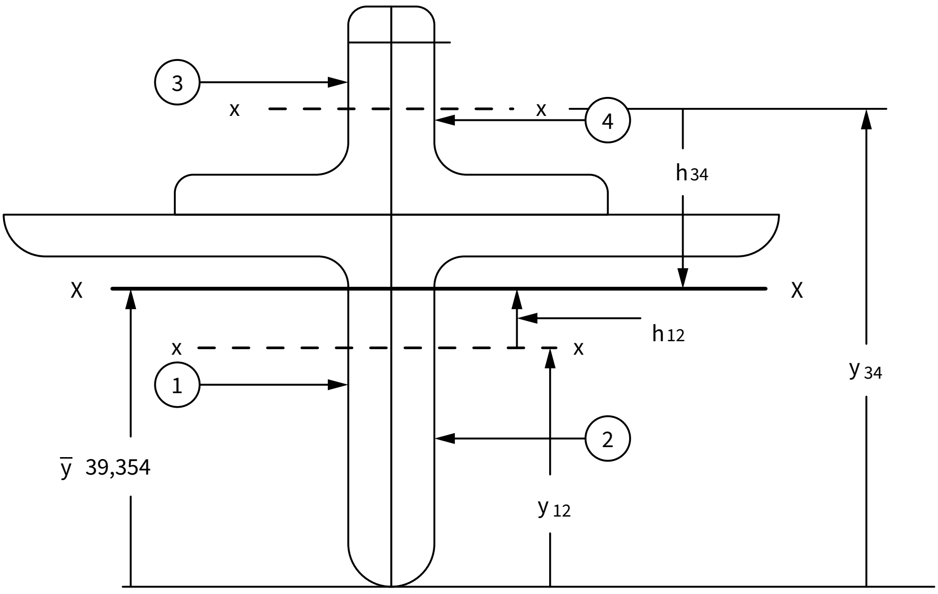
7.3 True

7.4 True

7.5 False

## QUESTION 8

The moment of inertia about the *xx*-axis




$h_{12} = \bar{y} - \left(h - A_x\right) = 39{,}354 - (45 - 12{,}8) = 7{,}154$ mm

$h_{34} = \left(h + A_x\right) - \bar{y} = (45 + 7{,}98) - 39{,}354 = 13{,}626$ mm

$y_{12} = h - A = 45 - 12{,}8 = 32{,}2$ mm

$y_{34} = 45 + 7{,}98 = 52{,}98$ mm

$\therefore I_{xx} = 2I_{xx12} + 2I_{xx34}$

$\therefore 2\,I_{xx12} = 2\left[I_{xx} + \left(A\,h_{12}^2\right)\right] = 2\left[0{,}0784 \times 10^{-6} + \left(0{,}4303 \times 10^{-3} \times 0{,}007154^2\right)\right]$

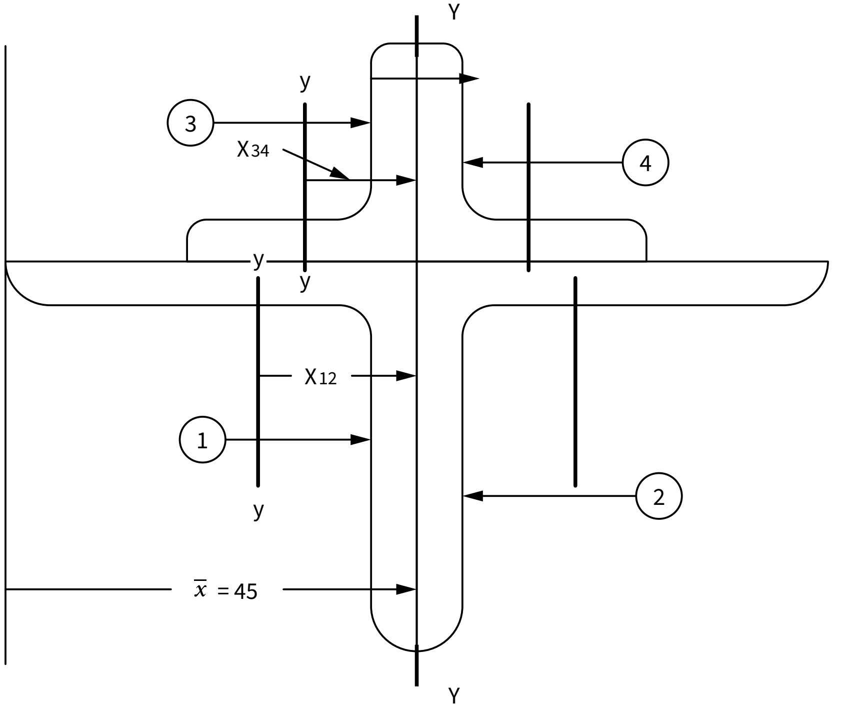
$\therefore 2I_{xx12} = 2{,}008 \times 10^{-7}$

$\therefore 2I_{xx34} = 2\left[I_{xx} + \left(A\,h_{34}^2\right)\right] = 2\left[0{,}012 \times 10^{-6} + \left(0{,}2259 \times 10^{-3} \times 0{,}013626^2\right)\right]$

$2I_{xx34} = 1{,}079 \times 10^{-7}$

$\therefore I_{xx} = 2{,}008 \times 10^{-7} + 1{,}079 \times 10^{-7} = 0{,}3087 \times 10^{-6}\ m^4$

The moment of inertia about the *yy*-axis




$x_{12} = Ay = 12{,}8$ mm

$x_{34} = Ay = 7{,}98$ mm

$I_{yy} = 2I_{yy12} + 2I_{yy34}$

$\therefore 2I_{yy12} = 2\left[I_{yy} + \left(A + x_{12}^2\right)\right] = 2\left[0{,}0784 \times 10^{-6} + \left(0{,}4303 \times 10^{-3} \times 0{,}0128^2\right)\right]$

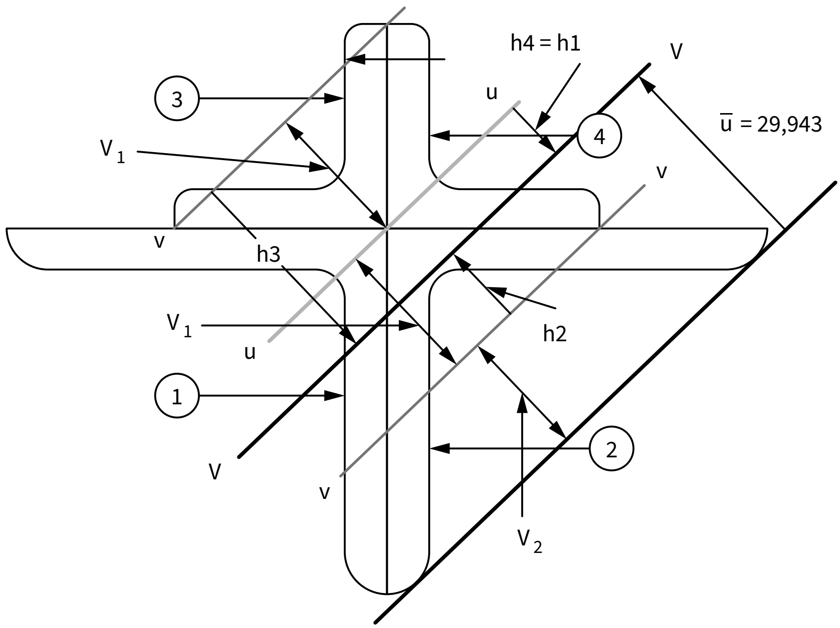
$2I_{yy12} = 2{,}978 \times 10^{-7}$

$\therefore 2I_{yy34} = 2\left[I_{yy} + \left(A + x_{34}^2\right)\right] = 2\left[0{,}012 \times 10^{-6} + \left(0{,}2259 \times 10^{-3} \times 0{,}00798^2\right)\right]$

$\therefore 2I_{yy34} = 5{,}277 \times 10^{-8}$

$\therefore I_{yy} = 2{,}978 \times 10^{-7} + 5{,}277 \times 10^{-8} = 0{,}35057 \times 10^{-6}$ m$^4$

The moment of inertia about the UU-axis and VV-axis




$h_2 = 29{,}943 - v_2 = 29{,}943 - 15{,}8 = 14{,}143$ mm

$h_3 = \left[\left(v_2 + v_1\right) + v_1\right] - 29{,}943 = [15{,}8 + 18{,}1 + 11{,}3] - 29{,}943 = 15{,}257$ mm

$h_{14} = \left(v_2 + v_1\right) - 29{,}943 = (15{,}8 + 18{,}1) - 29{,}943 = 3{,}957$ mm

$I_{VV} = I_{vv2} + I_{uu1} + I_{uu4} + I_{vv3}$

$\therefore I_{vv2} = I_{vv} + \left(A_2 \times h_2^2\right) = 0{,}0326 \times 10^{-6} + \left(0{,}4303 \times 10^{-3} \times 0{,}014143^2\right) = 1{,}187 \times 10^{-7}$

$\therefore I_{uu1} = I_{uu} + \left(A + h_1^2\right) = 0{,}1243 \times 10^{-6} + \left(0{,}4303 \times 10^{-3} \times 0{,}003957^2\right) = 1{,}31 \times 10^{-7}$

$\therefore I_{uu4} = I_{uu} + \left(A \times h_4^2\right) = 0{,}0189 \times 10^{-6} + \left(0{,}2259 \times 10^{-3} \times 0{,}003957^2\right) = 2{,}244 \times 10^{-8}$

$\therefore I_{vv3} = I_{vv} + \left(A + h_3^2\right) = 0{,}0052 \times 10^{-6} + \left(0{,}2259 \times 10^{-3} \times 0{,}015257^2\right) = 5{,}778 \times 10^{-8}$

$\therefore I_{VV} = 0{,}32992 \times 10^{-6}$

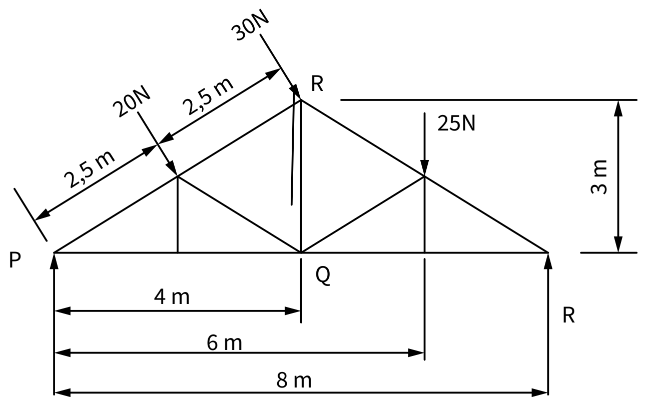
The smallest moment of inertia = $0{,}3087 \times 10^{-6}$ m$^4$

Effective length = $l_e = 2L = 2 \times 3 = 6$ m

Maximum buckling load = $P_E = \frac{\pi^2 EI}{l_e} = \frac{\pi^2 \times 200 \times 10^9 \times 0{,}3087 \times 10^{-6}}{6} = 101{,}59$ MN

## QUESTION 9

The reaction at the roller



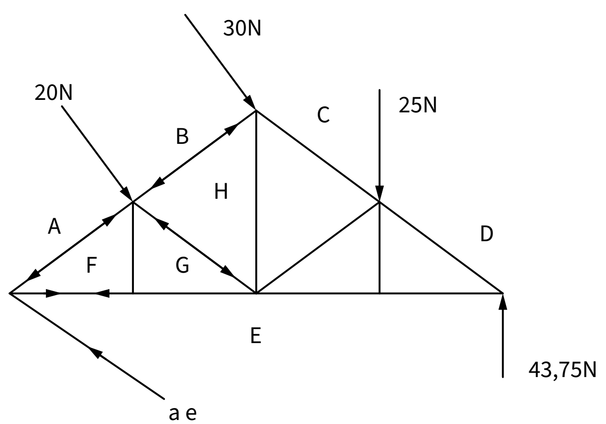

From triangle PQR:

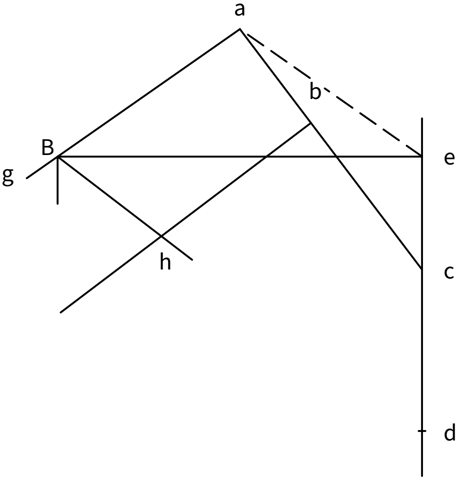
$\therefore$ PR = $\sqrt{3^2 + 4^2} = 5$ m

Moments about the fixed end: $\therefore 8R = (20 \times 2{,}5) + (30 \times 5) + (25 \times 6)$

$\therefore R = \frac{50 + 150 + 150}{8} = 43{,}75$ N

Scale: 1 cm = 1 m and 1 cm = 10 N






The reaction at the fixed support = 32 N

| Member | Magnitude | Nature |
|---|---|---|
| af | 35 N | Strut |
| ef | 60 N | Tie |
| gh | 22 N | Strut |

# Glossary

## A

**Abscissa** – the horizontal or *x*-coordinate within the Cartesian coordinate system

**Angle of distortion** – represented by the angle delta (δ)

**Angle of twist** – the angular deformation in an object due to twisting torques applied

## B

**Bending stress** – when two or more forces work against one or more forces opposing them. The forces are far enough apart that bending rather than shearing occurs

**Bow's notation** – a method of lettering the cells and outside spaces formed by the directions of the loads on a framed structure so that these loads can be traced by similar letters in the reciprocal diagram

**Bush** – a type of bearing fixed between two parts (which may be moving) or a strengthened point for mechanical fixing

## C

**Concentric** – when two objects (usually circular) share the same axis or centre

## D

**Diametral** – of, relating to or measured across a diameter

## E

**Effective length** – the length between two points of inflection on a buckled member

## F

**Factor of safety (FoS)** – that which allows a safe margin for possible defects in the manufacturing of components and also gives a safe margin in the case of unexpected overloading or the deterioration of the material due to rust

## H

**Homogeneous** – the nature of a material that has properties that do not vary depending on the location

**Hooke's law** – strain is directly proportional to the stress that causes it

**Hoop stress** – the stress in the circumferential (tangential) direction in cylindrical and non-cylindrical vessels when loaded by internal or external pressure in a closed vessel

**Hot-rolled press** – a metalworking process wherein metals are processed (heated at high temperatures and cooled down) to reproduce specific structures with specific properties

## I

**Inextensible** – unable to stretch or be drawn out (in length)

**Isotropic** – a material with properties that do not vary when tested in different directions

## L

**Limit of elasticity** – the maximum stress a material can withstand without becoming deformed

## M

**Modular ratio** – the ratio of Young's modulus of two different materials

**Modulus of rigidity** – a constant defined by the ratio of shear stress to shear strain

**Moment** – a force times the perpendicular distance from the pivot to the line of action

**Moment of inertia** – a measure of a resistance to rotational change, or in this case, bending stress in a material

## N

**Non-ferrous** – does not have iron as its prime component

**Notation** – a system of written symbols used to represent numbers or amounts in mathematics and other fields

## P

**Parabolic** – like a parabola

**Poisson's ratio** – a constant that relates longitudinal strain in the direction of the load to lateral strain perpendicular to the load; defines the ratio of how a material will increase lengthwise and contract width wise when stretched

**Polar moment of inertia** – a shaft's resistance to being distorted by torsion as a function of its shape

**Principal stress** – the normal stress perpendicular to the plane on which the shear stress is zero

**Proportional limit** – the point on a stress-strain curve where the linear, elastic deformation region transitions into a non-linear, plastic deformation region; the proportional limit determines the greatest stress that is directly proportional to strain; also known as limit of proportionality

## R

**Radian** – a unit of plane angular measurement

**Radius of gyration** – the measure of the elastic stability of a cross section against buckling, which is calculated using the mass, moment of inertia and perpendicular distances from the axis of rotation

## S

**Section modulus** – a geometric property used when designing beams or flexural members

**Shear strain** – the ratio of relative displacement of any layer to its perpendicular distance from the fixed layer

**Shear stress ($\tau$)** – stress on an object caused by one or more forces that act to tear or separate the object into two or more pieces in a movement that is parallel to each other

**Sign convention** – a way of representing whether something is in compression or tensile, or displacing upwards or downwards, by assigning a positive or negative symbol

**Slenderness** – the ratio of the length of a column (L) to the minimum radius of gyration of the cross section; slenderness is the quality of being thin

**Stiffness** – the material property of a component based on its ability to bend under load and still return to its original shape after the load is removed

**Strain (ε)** – the deformation of an object due to an internal state of stress

**Strain energy (U)** – energy stored in a body due to a force apply; causes extension or shorting of the body. Due to this deformation, work is done and energy is stored in the body, which is called strain energy, expressed in joules

**Stress (σ)** – the ability of an object to resist the effects of an external force; stress is given by the amount of load per unit area

## T

**Tensile strength** – the maximum stress, caused by the maximum load before the material fractures

**Tensile stress (σ T)** – stress on an object caused by an extensive force that acts to increase the original length of the object

**Torque** – the amount of force needed to change the angular acceleration of an object moving in a circle

**Torsion** – action of twisting an object by applying equal and opposite torque at either end

**Transverse** – placed across something

## Y

**Young's modulus** – the property of a material that indicates how easily it can stretch and deform. It is defined as a constant given the ratio of tensile stress (σ) to tensile strain (ε)