N4
Mathematics
Lecturer Guide

Nigel Solomon and Jolandi Daniels

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1. Subject aims

1.1 General subject aims

Mathematics N4 aims to provide students with the skills to identify and calculate mathematical problems in N4 and the content forms part of engineering calculation problems from industry.

Furthermore, Mathematics N4 will equip students with relevant knowledge to enable them to integrate meaningfully into their trade subjects and also form the foundation for the N5–N6 syllabuses to finally achieve a National diploma.

Upon completion of this subject the student should be able to apply:

- the necessary knowledge of Mathematics to various engineering fields in their respective working environments;
- higher cognitive skills pertaining to application, analysis, synthesis and evaluation, logical and critical thought processes;
- their understanding in the interpretation of real world problems;
- promote Mathematics as a tool to be used to troubleshoot in different fields of study; and
- certain theorems that are not examinable to be calculated.

1.2 Specific subject aims

The specific aims of Mathematics N4 is to conclude precalculus and introduce differential and integral calculus thereby serving as a prerequisite for Mathematics N5 and Mathematics N6.

Mathematics N4 strives to assist students to obtain trade-specific calculation knowledge.

Other specific aims of Mathematics N4 also include:

- promote correct mathematical terminology;
- promote and focus on word problems and the problem-solving thereof, in order to prepare the students for their relevant careers; and
- use technology in Mathematic and apply Mathematics to further technology.

2. Admission requirements

For admission to Mathematics N4, students must have passed:

- Grade 12 pure Mathematics
- NC(V) Level 4 Mathematics
- N3 Mathematics.

3. Duration of course

The duration of the subject is one trimester on full time, part time or distance learning mode.

4. Evaluation

Students must be evaluated continually as follows:

4.1 ICASS trimester mark
- assessment marks are valid for a period of one year and are referred to as ICASS trimester marks
- a minimum of 40% is required for a student to qualify for entry to the final examination
- two formal class tests for full time and part time students (or two assignments for distance learning students only).

4.2 Calculation of trimester mark will be as follows:
- weight of test or assignment 1 = 30% of the syllabus
- weight of test or assignment 2 = 70% of the syllabus.

5. Examination

A final examination will be conducted in April, August and November of each year. The pass requirement is 40%.

The final examination will consist of 100% of the syllabus.

The duration of the final examination will be 3 hours.

The final examination will be a closed book examination.

Minimum pass percentage is 40%.

Assessments will be based on the cognitive domain of Bloom's Taxonomy, that is remember, understand, apply, analyse, evaluate and create.

The division of these aspects are as follows:

Remember	Understand	Apply	Analyse	Evaluate	Create
20%	20%	20%	10%	20%	10%

6. General information

Problems should be based on real-world scenarios allowing students to relate theory to practice.

Emphasis of correct mathematical terminology should be encouraged and promoted at all times.

A systematical approach to problem-solving should be adhered to.

Students should be encouraged to understand rather than memorise the basic formulae applicable to Mathematics N4.

Calculators may be used to do mathematical calculations.

Answers to all calculations must be approximated correctly to three decimal places, unless otherwise stated. Unless otherwise stated, approximations may not be done during calculations. The final answer must be approximated to the stipulated degree of accuracy.

The weight value of a module gives an indication of the time to be spent on teaching the module as well as the relative percentage of the total marks allocated to the module in the final examination (1 mark = 1,8 minutes).

7. Subject matter

Mathematics N4 strives to assist students to obtain trade-specific calculation knowledge. Students should be able to acquire in-depth knowledge of the following content:

Module	Weighted value
1. Determinants	8
2. Complex numbers	12
3. Sketch graphs	10
4. Trigonometry	20
5. Differential calculus	25
6. Integral calculus	25
Total	**100**

8. Workschedule

Week	Module	Topic	Activities	Hours
1	**Module 1** Determinants	1.1 Determinants 1.2 Determining the value of second order determinants 1.3 Determining the value of third order determinants	Activity 1.1 Activity 1.2 Activity 1.3 Activity 1.4 Activity 1.5 Activity 1.6 Summative assessment: Module 1	8 hours
2–3	**Module 2** Complex numbers	2.1 Defining complex numbers (ℂ) 2.2 Working with complex numbers 2.3 Solving complex equations with two variables	Activity 2.1 Activity 2.2 Activity 2.3 Activity 2.4 Activity 2.5 Activity 2.6 Activity 2.7 Activity 2.8 Activity 2.9 Activity 2.10 Activity 2.11 Summative assessment: Module 2	12 hours

Week	Module	Topic	Activities	Hours
3–4	**Module 3** Sketch graphs	3.1 Relations and functions 3.2 Independent and dependent variable 3.3 Domain and range 3.4 Functions 3.5 Symmetry 3.6 Continuous and discontinuous functions or non-functions 3.7 Inverse functions and relations 3.8 Sketch graphs of functions and relations	Activity 3.1 Activity 3.2 Activity 3.3 Activity 3.4 Activity 3.5 Activity 3.6 Activity 3.7 Activity 3.8 Activity 3.9 Activity 3.10 Activity 3.11 Activity 3.12 Activity 3.13 Activity 3.14 Summative assessment: Module 3	10 hours
4–6	**Module 4** Trigonometry	4.1 Reduction formulae 4.2 Negative and positive angles 4.3 Compound angles 4.4 Co-ratios 4.5 Double angles 4.6 Half angles 4.7 Trigonometric identities 4.8 Trigonometric graphs 4.9 Draw reciprocal trigonometric sketch graphs of $y = \text{cosec } x$, $y = \sec x$ and $y = \cot x$ for $-2\pi \leq x \leq 2\pi$	Activity 4.1 Activity 4.2 Activity 4.3 Activity 4.4 Activity 4.5 Activity 4.6 Activity 4.7 Activity 4.8 Activity 4.9 Summative assessment: Module 4	20 hours

Week	Module	Topic	Activities	Hours
6–8	**Module 5** Differential calculus	5.1 Limits 5.2 The binomial theorem 5.3 Differentiation as a rate of change 5.4 Standard derivatives 5.5 Chain rule 5.6 Product and quotient rules 5.7 Second order derivatives	Activity 5.1 Activity 5.2 Activity 5.3 Activity 5.4 Activity 5.5 Activity 5.6 Activity 5.7 Activity 5.8 Activity 5.9 Activity 5.10 Activity 5.11 Summative assessment: Module 5	25 hours
9–10	**Module 6** Integral calculus	6.1 Understanding the concept of integration 6.2 Applying standard forms of integrals 6.3 Applying the rules for integration 6.4 Integrate composite functions 6.5 Integrating polynomials 6.6 Applying integration to determine the magnitude of an area 6.7 Areas under a curve	Activity 6.1 Activity 6.2 Activity 6.3 Activity 6.4 Activity 6.5 Activity 6.6 Activity 6.7 Activity 6.8 Summative assessment: Module 6	25 hours
TOTAL				**100 hours**

MODULE

1 Determinants

After they have completed this module, students should be able to:

- convert equations with either two or three variables into a determinant;
- calculate second and third order determinants using row elimination, followed by the application of Cramer's rule;
- state and calculate the minor of a third order determinant; and
- determine the co-factor of the minor.

Introduction

In previous years students learnt how to solve simultaneous equations with two or three variables by using the elimination or substitution methods. Many problems in engineering can be described with systems of linear equations.

The standard form of a linear equation with two variables is $ax + by = c$ and a linear equation with three variable is $ax + by + cz = 0$.

Students need the following pre-knowledge to successfully complete this module.

Pre-knowledge

Solve for x and y in the following linear simultaneous equations:

$3y - 2x = 11$ … Equation (1)

$y + 2x = 9$ … Equation (2)

Solution

From (2): $y = 9 - 2x$ • Subtract $2x$ from both sides

Substitute this y-value into (1): • Substitution method

$$\therefore 3(9 - 2x) - 2x = 11$$
$$27 - 6x - 2x = 11$$
$$27 - 8x = 11$$
$$-8x = 11 - 27$$
$$-8x = -16$$
$$\therefore x = 2$$

Calculate the value of y:

Substitute $x = 2$ into (2):

$y = 9 - 2(2)$

$= 9 - 4$

$\therefore y = 5$

Solutions: $x = 2$; $y = 5$ or $(2; 5)$

Activity 1.1 SB page 7

1. $\begin{vmatrix} 3 & 5 \\ 2 & 3 \end{vmatrix} = (3)(3) - (2)(5) = -1$

2. $\begin{vmatrix} -5 & 12 \\ -2 & 5 \end{vmatrix} = (-5)(5) - (-2)(12) = -1$

3. $\begin{vmatrix} -13 & -5 \\ 5 & 2 \end{vmatrix} = (-13)(2) - (5)(-5) = -1$

4. $\begin{vmatrix} 8 & 3 \\ 7 & 2 \end{vmatrix} = (8)(2) - (7)(3) = -5$

5. $\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} = (-1)(1) - (2)(2) = -5$

6. $\begin{vmatrix} 3 & 1 \\ 2 & 5 \end{vmatrix} = (3)(5) - (2)(1) = 13$

7. $\begin{vmatrix} -6 & 1 \\ 2 & 3 \end{vmatrix} = (-6)(3) - (2)(1) = -20$

8. $\begin{vmatrix} 3 & 1 \\ 4 & 3 \end{vmatrix} = (3)(3) - (4)(1) = 5$

9. $\begin{vmatrix} -1 & 3 \\ 1 & 2 \end{vmatrix} = (-1)(2) - (1)(3) = -5$

10. $\begin{vmatrix} 6 & 2 \\ 8 & 1 \end{vmatrix} = (6)(1) - (8)(2) = -10$

Activity 1.2 SB page 10

1. $D = \begin{vmatrix} 3 & -5 \\ 4 & -3 \end{vmatrix} = (3)(-3) - (4)(-5) = 11$

$D_x = \begin{vmatrix} 12 & -5 \\ 15 & -3 \end{vmatrix} = (12)(-3) - (15)(-5) = 39$ $\qquad \therefore x = \frac{D_x}{D} = \frac{39}{11} = 3\frac{6}{11}$

$D_y = \begin{vmatrix} 3 & 12 \\ 4 & 15 \end{vmatrix} = (3)(15) - (4)(12) = -3$ $\qquad \therefore y = \frac{D_y}{D} = \frac{-3}{11} = -\frac{3}{11}$

2. $D = \begin{vmatrix} 5 & 2 \\ 3 & 4 \end{vmatrix} = (5)(4) - (3)(2) = 14$

$D_a = \begin{vmatrix} -19 & 2 \\ -17 & 4 \end{vmatrix} = (-19)(4) - (-17)(2) = -42$ $\qquad \therefore a = \frac{D_a}{D} = \frac{-42}{14} = -3$

$D_b = \begin{vmatrix} 5 & -19 \\ 3 & -17 \end{vmatrix} = (5)(-17) - (3)(-19) = -28$ $\qquad \therefore b = \frac{D_b}{D} = \frac{-28}{14} = -2$

3. $D = \begin{vmatrix} 3 & 4 \\ 2 & -6 \end{vmatrix} = (3)(-6) - (2)(4) = -26$

$D_k = \begin{vmatrix} 5 & 4 \\ -1 & -6 \end{vmatrix} = (5)(-6) - (-1)(4) = -26$ $\qquad \therefore k = \frac{D_k}{D} = \frac{-26}{-26} = 1$

$D_l = \begin{vmatrix} 3 & 5 \\ 2 & -1 \end{vmatrix} = (3)(-1) - (2)(5) = -13$ $\qquad \therefore l = \frac{D_\ell}{D} = \frac{-13}{-26} = \frac{1}{2}$

4. $D = \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} = (2)(-4) - (1)(3) = -11$

$D_r = \begin{vmatrix} 4 & 3 \\ 5 & -4 \end{vmatrix} = (4)(-4) - (5)(3) = -31$ $\quad \therefore r = \frac{D_r}{D} = \frac{-31}{-11} = 2\frac{9}{11}$

$D_s = \begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix} = (2)(5) - (1)(4) = 6$ $\quad \therefore s = \frac{D_s}{D} = \frac{6}{-11} = -\frac{6}{11}$

5. $D = \begin{vmatrix} 9 & -15 \\ 3 & -15 \end{vmatrix} = (9)(-15) - (3)(-15) = -90$

$D_{x_1} = \begin{vmatrix} 8 & -15 \\ -2 & -15 \end{vmatrix} = (8)(-15) - (-2)(-15) = -150$ $\quad \therefore x_1 = \frac{D_{x_1}}{D} = \frac{-150}{-90} = 1\frac{2}{3}$

$D_{x_2} = \begin{vmatrix} 9 & 8 \\ 3 & -2 \end{vmatrix} = (9)(-2) - (3)(8) = -42$ $\quad \therefore x_2 = \frac{D_{x_2}}{D} = \frac{-42}{-90} = \frac{7}{15}$

6. $D = \begin{vmatrix} 2 & -1 \\ 6 & -5 \end{vmatrix} = (2)(-5) - (6)(-1) = -4$

$D_x = \begin{vmatrix} 8 & -1 \\ 32 & -5 \end{vmatrix} = (8)(-5) - (32)(-1) = -8$ $\quad \therefore x = \frac{D_x}{D} = \frac{-8}{-4} = 2$

$D_y = \begin{vmatrix} 2 & 8 \\ 6 & 32 \end{vmatrix} = (2)(32) - (6)(8) = 16$ $\quad \therefore y = \frac{D_y}{D} = \frac{16}{-4} = -4$

7. $D = \begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = (2)(2) - (5)(-1) = 9$

$D_a = \begin{vmatrix} 7 & -1 \\ 4 & 2 \end{vmatrix} = (7)(2) - (4)(-1) = 18$ $\quad \therefore a = \frac{D_a}{D} = \frac{18}{9} = 2$

$D_b = \begin{vmatrix} 2 & 7 \\ 5 & 4 \end{vmatrix} = (2)(4) - (5)(7) = -27$ $\quad \therefore b = \frac{D_b}{D} = \frac{-27}{9} = -3$

8. $D = \begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = (3)(1) - (2)(-4) = 11$

$D_k = \begin{vmatrix} -5 & -4 \\ 4 & 1 \end{vmatrix} = (-5)(1) - (4)(-4) = 11$ $\quad \therefore k = \frac{D_k}{D} = \frac{11}{11} = 1$

$D_l = \begin{vmatrix} 3 & -5 \\ 2 & 4 \end{vmatrix} = (3)(4) - (2)(-5) = 22$ $\quad \therefore l = \frac{D_\ell}{D} = \frac{22}{11} = 2$

9. $D = \begin{vmatrix} -1 & 1 \\ -\frac{3}{2} & 1 \end{vmatrix} = (-1)(1) - (-\frac{3}{2})(1) = \frac{1}{2}$

$D_r = \begin{vmatrix} -3 & 1 \\ -1 & 1 \end{vmatrix} = (-3)(1) - (-1)(1) = -2$ $\quad \therefore r = \frac{D_r}{D} = \frac{-2}{\frac{1}{2}} = -4$

$D_s = \begin{vmatrix} -1 & -3 \\ -\frac{3}{2} & -1 \end{vmatrix} = (-1)(-1) - (-\frac{3}{2})(-3) = -\frac{7}{2}$ $\quad \therefore s = \frac{D_s}{D} = \frac{-\frac{7}{2}}{\frac{1}{2}} = -7$

10. $D = \begin{vmatrix} -1 & 3 \\ -1 & 2 \end{vmatrix} = (-1)(2) - (-1)(3) = 1$

$D_{x_1} = \begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} = (2)(2) - (0)(3) = 4$ $\quad \therefore x_1 = \frac{D_{x_1}}{D} = \frac{4}{1} = 4$

$D_{x_2} = \begin{vmatrix} -1 & 2 \\ -1 & 0 \end{vmatrix} = (-1)(0) - (-1)(2) = 2$ $\quad \therefore x_2 = \frac{D_{x_2}}{D} = \frac{2}{1} = 2$

11. $\frac{1}{x} = 3 + \frac{1}{y}$

$\frac{1}{x} - 6 = -\frac{1}{y}$

$\frac{1}{x} - \frac{1}{y} = 3$

$\frac{1}{x} + \frac{1}{y} = 6$

Let $a = \frac{1}{x}$ and $b = \frac{1}{y}$

$\therefore a - b = 3$

$a + b = 6$

$|D| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$

$= 1(1) - (1)(-1)$

$= 1 + 1$

$= 2$

$|D_a| = \begin{vmatrix} 3 & -1 \\ 6 & 1 \end{vmatrix}$

$= 3(1) - 6(-1)$

$= 3 + 6$

$= 9$

$|D_b| = \begin{vmatrix} 1 & 3 \\ 1 & 6 \end{vmatrix}$

$= 1(6) - (1)(3)$

$= 6 - 3$

$= 3$

$\therefore \frac{a}{|D_a|} = \frac{b}{|D_b|} = \frac{1}{|D|}$

$\frac{a}{9} = \frac{1}{2}$ $\qquad$ $\frac{b}{3} = \frac{1}{2}$

$\therefore a = \frac{9}{2}$ $\qquad$ $\therefore b = \frac{3}{2}$

$\therefore x = \frac{2}{9}$ $\qquad$ $\therefore y = \frac{2}{3}$

12. $\frac{5}{3}a + b = 3$

$a = 6 + \frac{3}{2}b$

$\frac{5}{3}a + b = 3$

$a - \frac{3}{2}b = 6$

$|D| = \begin{vmatrix} \frac{5}{3} & 1 \\ 1 & -\frac{3}{2} \end{vmatrix}$

$= \frac{5}{3}\left(-\frac{3}{2}\right) - 1(1)$

$= -\frac{5}{2} - 1$

$= -1\frac{5}{2}$

$= -\frac{7}{2}$

$$|D_a| = \begin{vmatrix} 3 & 1 \\ 6 & -\frac{3}{2} \end{vmatrix}$$

$$= 3\left(-\tfrac{3}{2}\right) - 6(1)$$

$$= -\tfrac{9}{2} - 6$$

$$= \frac{-9 - 12}{2}$$

$$= -\tfrac{21}{2}$$

$$|D_b| = \begin{vmatrix} \frac{5}{3} & 3 \\ 1 & 6 \end{vmatrix}$$

$$= \tfrac{5}{3}(6) - 1(3)$$

$$= 10 - 3$$

$$= 7$$

$$\therefore \frac{1}{|D|} = \frac{a}{|D_a|} \qquad \therefore \frac{1}{|D|} = \frac{b}{|D_b|}$$

$$\frac{1}{-\frac{7}{2}} = \frac{a}{-\frac{21}{2}} \qquad \frac{1}{-\frac{7}{2}} = \frac{b}{7}$$

$$\therefore a = \frac{-\frac{21}{2}}{-\frac{7}{2}} \qquad \therefore b = 7 \times \frac{1}{-\frac{7}{2}}$$

$$= -\tfrac{21}{2} \times -\tfrac{2}{7} \qquad = \frac{7}{-\frac{7}{2}} = 7 \times -\tfrac{2}{7}$$

$$= 3 \qquad = -2$$

Activity 1.3 — SB page 14

1. 1.1 $\begin{vmatrix} 2 & -4 & 1 \\ 1 & -2 & 3 \\ 5 & 1 & -1 \end{vmatrix}$

a) $M_{11} = \begin{vmatrix} -2 & 3 \\ 1 & -1 \end{vmatrix}$

$= (-2)(-1) - (1)(3)$

$M_{11} = -1$

b) $M_{32} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}$

$= (2)(3) - (1)(1)$

$M_{32} = 5$

c) $M_{22} = \begin{vmatrix} 2 & 1 \\ 5 & -1 \end{vmatrix}$

$= (2)(-1) - (5)(1)$

$M_{22} = -7$

1.2 $\begin{vmatrix} 3 & -2 & 1 \\ 4 & -1 & 2 \\ 5 & 6 & 7 \end{vmatrix}$

a) $M_{13} = \begin{vmatrix} 4 & -1 \\ 5 & 6 \end{vmatrix}$

$= (4)(6) - (5)(-1)$

$M_{13} = 29$

b) $M_{31} = \begin{vmatrix} -2 & 1 \\ -1 & 2 \end{vmatrix}$

$= (-2)(2) - (-1)(1)$

$M_{31} = -3$

c) $M_{23} = \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix}$

$= (3)(6) - (5)(-2)$

$M_{23} = 28$

1.3 $\begin{vmatrix} 1 & 2 & 3 \\ 4 & -1 & -2 \\ -3 & -4 & -5 \end{vmatrix}$

a) $M_{12} = \begin{vmatrix} 4 & -2 \\ -3 & -5 \end{vmatrix}$

$= (4)(-5) - (-3)(-2)$

$M_{12} = -26$

b) $M_{33} = \begin{vmatrix} 1 & 2 \\ 4 & -1 \end{vmatrix}$

$= (1)(-1) - (4)(2)$

$M_{33} = -9$

c) $M_{21} = \begin{vmatrix} 2 & 3 \\ -4 & -5 \end{vmatrix}$

$= (2)(-5) - (-4)(3)$

$M_{21} = 2$

2. 2.1 $\begin{vmatrix} 6 & 2 & -3 \\ 2 & 3 & -5 \\ 1 & -1 & 1 \end{vmatrix}$

a) $M_{22} = \begin{vmatrix} 6 & -3 \\ 1 & 1 \end{vmatrix}$

$= (6)(1) - (1)(-3)$

$M_{22} = 9$

b) $M_{23} = \begin{vmatrix} 6 & 2 \\ 1 & -1 \end{vmatrix}$

$= (6)(-1) - (1)(2)$

$M_{23} = -8$

c) $M_{11} = \begin{vmatrix} 3 & -5 \\ -1 & 1 \end{vmatrix}$

$= (3)(1) - (-1)(-5)$

$M_{11} = -2$

2.2 $\begin{vmatrix} 2 & 3 & -1 \\ 3 & 5 & 2 \\ 1 & -2 & -3 \end{vmatrix}$

a) $M_{32} = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$

$= (2)(2) - (3)(-1)$

$M_{32} = 7$

b) $M_{31} = \begin{vmatrix} 3 & -1 \\ 5 & 2 \end{vmatrix}$

$= (3)(2) - (5)(-1)$

$M_{31} = 11$

c) $M_{13} = \begin{vmatrix} 3 & 5 \\ 1 & -2 \end{vmatrix}$

$= (3)(-2) - (1)(5)$

$M_{13} = -11$

2.3 $\begin{vmatrix} -1 & -2 & 1 \\ 3 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$

a) $M_{21} = \begin{vmatrix} -2 & 1 \\ -1 & 2 \end{vmatrix}$

$= (-2)(2) - (-1)(1)$

$M_{21} = -3$

b) $M_{12} = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$

$= (3)(2) - (1)(1)$

$M_{12} = 5$

c) $M_{33} = \begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix}$

$= (-1)(1) - (3)(-2)$

$M_{33} = 5$

Activity 1.4 SB page 17

1. $\begin{vmatrix} 2 & -1 & 3 \\ 1 & -6 & -1 \\ 3 & 6 & 2 \end{vmatrix}$

1.1 $C_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 3 \\ 6 & 2 \end{vmatrix}$

$= (-1)^3[(-1)(2) - (6)(3)]$

$= -20$

1.2 $C_{23} = (-1)^{2+3}\begin{vmatrix} 2 & -1 \\ 3 & 6 \end{vmatrix}$

$= (-1)^5[(2)(6) - (3)(-1)]$

$C_{23} = -15$

1.3 $C_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$

$= (-1)^5[(2)(-1) - (1)(3)]$

$C_{32} = 5$

2. $\begin{vmatrix} -2 & 0 & 1 \\ -3 & 2 & -5 \\ 4 & -2 & 6 \end{vmatrix}$

2.1 $C_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 1 \\ -2 & 6 \end{vmatrix}$

$= (-1)^3[(0)(6) - (-2)(1)]$

$C_{21} = -2$

2.2 $C_{11} = (-1)^{1+1}\begin{vmatrix} 2 & -5 \\ -2 & 6 \end{vmatrix}$

$= (-1)^2[(2)(6) - (-2)(-5)]$

$C_{11} = 2$

2.3 $C_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 1 \\ 2 & -5 \end{vmatrix}$

$= (-1)^4[(0)(-5) - (2)(1)]$

$C_{31} = -2$

3. 3.1 $\begin{vmatrix} 3 & 2 & -1 \\ 3 & -2 & 1 \\ 4 & -5 & -1 \end{vmatrix}$

a) $C_{32} = (-1)^{3+2}\begin{vmatrix} 3 & -1 \\ 3 & 1 \end{vmatrix}$

$= (-1)^5[(3)(1) - (3)(-1)]$

$C_{32} = -6$

b) $C_{23} = (-1)^{2+3}\begin{vmatrix} 3 & 2 \\ 4 & -5 \end{vmatrix}$

$= (-1)^5[(3)(-5) - (4)(2)]$

$C_{23} = 23$

c) $C_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 1 \\ 4 & -1 \end{vmatrix}$

$= (-1)^3[(3)(-1) - (4)(1)]$

$C_{12} = 7$

3.2 $\begin{vmatrix} 1 & 1 & 4 \\ 2 & -3 & -1 \\ -4 & 2 & 2 \end{vmatrix}$

a) $C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 4 \\ -4 & 2 \end{vmatrix}$
$= (-1)^4[(1)(2) - (-4)(4)]$
$C_{22} = 18$

b) $C_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 4 \\ -3 & -1 \end{vmatrix}$
$= (-1)^4[(1)(-1) - (-3)(4)]$
$C_{31} = 11$

c) $C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -3 \\ -4 & 2 \end{vmatrix}$
$= (-1)^4[(2)(2) - (-4)(-3)]$
$C_{13} = -8$

3.3 $\begin{vmatrix} 3 & 1 & -2 \\ 4 & -1 & 1 \\ 1 & -3 & -4 \end{vmatrix}$

a) $C_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 1 \\ -3 & -4 \end{vmatrix}$
$= (-1)^2[(-1)(-4) - (-3)(1)]$
$C_{11} = 7$

b) $C_{21} = (-1)^{2+1}\begin{vmatrix} 1 & -2 \\ -3 & -4 \end{vmatrix}$
$= (-1)^3[(1)(-4) - (-3)(-2)]$
$C_{21} = 10$

c) $C_{33} = (-1)^{3+3}\begin{vmatrix} 3 & 1 \\ 4 & -1 \end{vmatrix}$
$= (-1)^6[(3)(-1) - (4)(1)]$
$C_{33} = -7$

Activity 1.5 SB page 21

1.

1.1 $\begin{vmatrix} 3 & -5 \\ -1 & 3 \end{vmatrix} = (3)(3) - (-1)(-5) = 4$

1.2 $\begin{vmatrix} 2 & 7 \\ -1 & 0 \end{vmatrix} = (2)(0) - (-1)(7) = 7$

1.3 $\begin{vmatrix} 2 & 3 \\ 6 & -12 \end{vmatrix} = (2)(-12) - (6)(3) = -42$

1.4 $\begin{vmatrix} -2 & 4 \\ 3 & -7 \end{vmatrix} = (-2)(-7) - (3)(4) = 2$

1.5 $\begin{vmatrix} -5 & -3 \\ 2 & -2 \end{vmatrix} = (-5)(-2) - (2)(-3) = 16$

1.6 $\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} = (1)(4) - (3)(-2) = 10$

2.

2.1 $\begin{vmatrix} 3 & -1 & 2 \\ 1 & -1 & 2 \\ -2 & 3 & 1 \end{vmatrix}$

$= 3\begin{vmatrix} -1 & 2 \\ 3 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix}$
$= 3(-1 - 6) + 1(1 + 4) + 2(3 - 2)$
$= 3(-7) + 1(5) + 2(1)$
$= -21 + 5 + 2$
$= -14$

2.2 $\begin{vmatrix} 0 & 1 & -3 \\ 2 & 4 & -1 \\ 4 & -2 & 5 \end{vmatrix}$

$= 0 - 1\begin{vmatrix} 2 & -1 \\ 4 & 5 \end{vmatrix} + (-3)\begin{vmatrix} 2 & 4 \\ 4 & -2 \end{vmatrix}$
$= 0 - 1(10 + 4) - 3(-4 - 16)$
$= -14 - 3(-20)$
$= -14 + 60$
$= 46$

2.3 $\begin{vmatrix} 4 & -4 & 5 \\ 3 & 1 & 4 \\ -3 & 4 & 1 \end{vmatrix}$

$= -(-4)\begin{vmatrix} 3 & 4 \\ -3 & 1 \end{vmatrix} + (1)\begin{vmatrix} 4 & 5 \\ -3 & 1 \end{vmatrix} - 4\begin{vmatrix} 4 & 5 \\ 3 & 4 \end{vmatrix}$

$= 4(3 + 12) + (4 + 15) - 4(16 - 15)$

$= 4(15) + 19 - 4$

$= 60 + 19 - 4$

$= 75$

2.4 $\begin{vmatrix} 2 & 3 & -1 \\ 2 & -1 & 2 \\ 3 & -1 & 1 \end{vmatrix}$

$= 3\begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 2 & -1 \\ 2 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix}$

$= 3(6 - 1) + 1(4 + 2) + 1(-2 - 6)$

$= 3(5) + 1(6) + 1(-8)$

$= 15 + 6 - 8$

$= 13$

3. 3.1 $\begin{vmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{vmatrix} = (3)\begin{vmatrix} 1 & 5 \\ 1 & 3 \end{vmatrix} + (4)\begin{vmatrix} -1 & 3 \\ 1 & 3 \end{vmatrix} + (2)\begin{vmatrix} -1 & 3 \\ 1 & 5 \end{vmatrix}$

$= 3[(1)(3) - (1)(5)] - 4[(-1)(3) - (1)(3)] + 2[(-1)(5) - (1)(3)]$

$= -6 + 24 - 16$

$\begin{vmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{vmatrix} = 2$

3.2 $\begin{vmatrix} -2 & 1 & 0 \\ 0 & 3 & -1 \\ 5 & 0 & 6 \end{vmatrix} = (5)\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix} + (0)\begin{vmatrix} -2 & 0 \\ 0 & -1 \end{vmatrix} + (6)\begin{vmatrix} -2 & 1 \\ 0 & 3 \end{vmatrix}$

$= 5[(1)(-1) - (3)(0)] + 0 + 6\,[(-2)(3) - (0)(1)]$

$= -5 + 0 - 36$

$\begin{vmatrix} -2 & 1 & 0 \\ 0 & 3 & -1 \\ 5 & 0 & 6 \end{vmatrix} = -41$

3.3 $\begin{vmatrix} 3 & -2 & 1 \\ 2 & 1 & -5 \\ 1 & 1 & -6 \end{vmatrix} = (-2)\begin{vmatrix} 2 & -5 \\ 1 & -6 \end{vmatrix} + (1)\begin{vmatrix} 3 & 1 \\ 1 & -6 \end{vmatrix} + (1)\begin{vmatrix} 3 & 1 \\ 2 & -5 \end{vmatrix}$

$= 2[(2)(-6) - (1)(-5)] + 1[(3)(-6) - (1)(1)] - 1[(3)(-5) - (2)(1)]$

$= -14 - 19 + 17$

$\begin{vmatrix} 3 & -2 & 1 \\ 2 & 1 & -5 \\ 1 & 1 & -6 \end{vmatrix} = -16$

3.4 $\begin{vmatrix} 1 & 4 & 2 \\ -1 & 2 & -1 \\ 3 & 2 & 1 \end{vmatrix} = (-1)\begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix} + (2)\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix}$

$= 1[(4)(1) - (2)(2)] + 2[(1)(1) - (3)(2)] + 1[(1)(2) - (3)(4)]$

$= 0 - 10 - 10$

$\begin{vmatrix} 1 & 4 & 2 \\ -1 & 2 & -1 \\ 3 & 2 & 1 \end{vmatrix} = -20$

Activity 1.6 SB page 32

1. 1.1 $D = \begin{vmatrix} 1 & 2 & 2 \\ 3 & -1 & 4 \\ 3 & 2 & -1 \end{vmatrix}$

$= 1\begin{vmatrix} -1 & 4 \\ 2 & -1 \end{vmatrix} - 2\begin{vmatrix} 3 & 4 \\ 3 & -1 \end{vmatrix} + 2\begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix}$

$= 1(1 - 8) - 2(-3 - 12) + 2(6 + 3)$

$= -7 - 2(-15) + 2(9)$

$= -7 + 30 + 18$

$= 41$

$D_x = \begin{vmatrix} 4 & 2 & 2 \\ 25 & -1 & 4 \\ -4 & 2 & -1 \end{vmatrix}$

$= 4\begin{vmatrix} -1 & 4 \\ 2 & -1 \end{vmatrix} - 2\begin{vmatrix} 25 & 4 \\ -4 & -1 \end{vmatrix} + 2\begin{vmatrix} 25 & -1 \\ -4 & 2 \end{vmatrix}$

$= 4(1 - 8) - 2(-25 + 16) + 2(50 - 4)$

$= 4(-7) - 2(-9) + 2(46)$

$= -28 + 18 + 92$

$= 82$

$D_y = \begin{vmatrix} 1 & 4 & 2 \\ 3 & 25 & 4 \\ 3 & -4 & -1 \end{vmatrix}$

$= 1\begin{vmatrix} 25 & 4 \\ -4 & -1 \end{vmatrix} - 4\begin{vmatrix} 3 & 4 \\ 3 & -1 \end{vmatrix} + 2\begin{vmatrix} 3 & 25 \\ 3 & -4 \end{vmatrix}$

$= 1(-25 + 16) - 4(-3 - 12) + 2(-12 - 75)$

$= -9 - 4(-15) + 2(-87)$

$= -9 + 60 - 174$

$= -123$

$D_z = \begin{vmatrix} 1 & 2 & 4 \\ 3 & -1 & 25 \\ 3 & 2 & -4 \end{vmatrix}$

$= 1\begin{vmatrix} -1 & 25 \\ 2 & -4 \end{vmatrix} - 2\begin{vmatrix} 3 & 25 \\ 3 & -4 \end{vmatrix} + 4\begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix}$

$= 1(4 - 50) - 2(-12 - 75) + 4(6 + 3)$

$= -46 - 2(-87) + 4(9)$

$= -46 + 174 + 36$

$= 164$

$\therefore x = \frac{D_x}{D} = \frac{82}{41} = 2$

$\therefore y = \frac{D_y}{D} = \frac{-123}{41} = -3$

$\therefore z = \frac{D_z}{D} = \frac{164}{41} = 4$

1.2 $D = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & 2 \\ 4 & 1 & -4 \end{vmatrix}$

$= 2\begin{vmatrix} 2 & 2 \\ 1 & -4 \end{vmatrix} - (-3)\begin{vmatrix} 3 & 2 \\ 4 & -4 \end{vmatrix} + 5\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

$= 2(-8 - 2) + 3(-12 - 8) + 5(3 - 8)$

$= 2(-10) + 3(-20) + 5(-5)$

$= -20 - 60 - 25$

$= -105$

$D_a = \begin{vmatrix} 4 & -3 & 5 \\ 3 & 2 & 2 \\ -6 & 1 & -4 \end{vmatrix}$

$= 4\begin{vmatrix} 2 & 2 \\ 1 & -4 \end{vmatrix} - (-3)\begin{vmatrix} 3 & 2 \\ -6 & -4 \end{vmatrix} + 5\begin{vmatrix} 3 & 2 \\ -6 & 1 \end{vmatrix}$

$= 4(-8 - 2) + 3(-12 + 12) + 5(3 + 12)$

$= 4(-10) + 3(0) + 5(15)$

$= -40 + 75$

$= 35$

$D_b = \begin{vmatrix} 2 & 4 & 5 \\ 3 & 3 & 2 \\ 4 & -6 & -4 \end{vmatrix}$

$= 2\begin{vmatrix} 3 & 2 \\ -6 & -4 \end{vmatrix} - (4)\begin{vmatrix} 3 & 2 \\ 4 & -4 \end{vmatrix} + 5\begin{vmatrix} 3 & 3 \\ 4 & -6 \end{vmatrix}$

$= 2(-12 + 12) - 4(-12 - 8) + 5(-18 - 12)$

$= -4(-20) + 5(-30)$

$= 80 - 150$

$= -70$

$D_c = \begin{vmatrix} 2 & -3 & 4 \\ 3 & 2 & 3 \\ 4 & 1 & -6 \end{vmatrix}$

$= 2\begin{vmatrix} 2 & 3 \\ 1 & -6 \end{vmatrix} - (-3)\begin{vmatrix} 3 & 3 \\ 4 & -6 \end{vmatrix} + 4\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

$= 2(-12 - 3) + 3(-18 - 12) + 4(3 - 8)$

$= 2(-15) + 3(-30) + 4(-5)$

$= -30 - 90 - 20$

$= -140$

$\therefore a = \frac{D_a}{D} = \frac{35}{-105} = -\frac{1}{3}$

$\therefore b = \frac{D_b}{D} = \frac{-70}{-105} = \frac{2}{3}$

$\therefore c = \frac{D_c}{D} = \frac{-140}{-105} = 1\frac{1}{3}$

2. 2.1 $|D| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ -1 & 2 & 2 \end{vmatrix} = +(1)\begin{vmatrix} -1 & 3 \\ 2 & 2 \end{vmatrix} - (1)\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} + (1)\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$

$= 1[(-1)(2) - (2)(3)] - 1[(2)(2) - (-1)(3)] + 1[(2)(2) - (-1)(-1)]$

$= -8 - 7 + 3$

$\therefore |D| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ -1 & 2 & 2 \end{vmatrix} = -12$

$|D_x| = \begin{vmatrix} 6 & 1 & 1 \\ 9 & -1 & 3 \\ 9 & 2 & 2 \end{vmatrix} = +(6)\begin{vmatrix} -1 & 3 \\ 2 & 2 \end{vmatrix} - (1)\begin{vmatrix} 9 & 3 \\ 9 & 2 \end{vmatrix} + (1)\begin{vmatrix} 9 & -1 \\ 9 & 2 \end{vmatrix}$

$= 6[(-1)(2) - (2)(3)] - 1[(9)(2) - (9)(3)] + 1[(9)(2) - (9)(-1)]$

$= -48 + 9 + 27$

$\therefore |D_x| = \begin{vmatrix} 6 & 1 & 1 \\ 9 & -1 & 3 \\ 9 & 2 & 2 \end{vmatrix} = -12$

$|D_y| = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 9 & 3 \\ -1 & 9 & 2 \end{vmatrix} = +(1)\begin{vmatrix} 9 & 3 \\ 9 & 2 \end{vmatrix} - (6)\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} + (1)\begin{vmatrix} 2 & 9 \\ -1 & 9 \end{vmatrix}$

$= 1[(9)(2) - (9)(3)] - 6[(2)(2) - (-1)(3)] + 1[(2)(9) - (-1)(9)]$

$= -9 - 42 + 27$

$\therefore |D_y| = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 9 & 3 \\ -1 & 9 & 2 \end{vmatrix} = -24$

$|D_z| = \begin{vmatrix} 1 & 1 & 6 \\ 2 & -1 & 9 \\ -1 & 2 & 9 \end{vmatrix} = +(1)\begin{vmatrix} -1 & 9 \\ 2 & 9 \end{vmatrix} - (1)\begin{vmatrix} 2 & 9 \\ -1 & 9 \end{vmatrix} + (6)\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$

$= 1[(-1)(9) - (2)(9)] - 1[(2)(9) - (-1)(9)] + 6[(2)(2) - (-1)(-1)]$

$= -27 - 27 + 18$

$\therefore |D_z| = \begin{vmatrix} 1 & 1 & 6 \\ 2 & -1 & 9 \\ -1 & 2 & 9 \end{vmatrix} = -36$

$\therefore x = \frac{D_x}{D} = \frac{-12}{-12} = 1$

$\therefore y = \frac{D_y}{D} = \frac{-24}{-12} = 2$

$\therefore z = \frac{D_z}{D} = \frac{-36}{-12} = 3$

2.2 $|D| = \begin{vmatrix} 4 & -3 & 1 \\ 1 & 2 & -5 \\ 3 & 1 & 2 \end{vmatrix} = +(3)\begin{vmatrix} -3 & 1 \\ 2 & -5 \end{vmatrix} - (1)\begin{vmatrix} 4 & 1 \\ 1 & -5 \end{vmatrix} + (2)\begin{vmatrix} 4 & -3 \\ 1 & 2 \end{vmatrix}$

$= 3[(-3)(-5) - (2)(1)] - 1[(4)(-5) - (1)(1)] + 2[(4)(2) - (1)(-3)]$

$= 39 + 21 + 22$

$\therefore |D| = \begin{vmatrix} 4 & -3 & 1 \\ 1 & 2 & -5 \\ 3 & 1 & 2 \end{vmatrix} = 82$

$|D_k| = \begin{vmatrix} -1 & -3 & 1 \\ 11 & 2 & -5 \\ 20 & 1 & 2 \end{vmatrix} = +(20)\begin{vmatrix} -3 & 1 \\ 2 & -5 \end{vmatrix} - (1)\begin{vmatrix} -1 & 1 \\ 11 & -5 \end{vmatrix} + (2)\begin{vmatrix} -1 & -3 \\ 11 & 2 \end{vmatrix}$

$= 20[(-3)(-5) - (2)(1)] - 1[(-1)(-5) - (11)(1)] + 2[(-1)(2) - (11)(-3)]$

$= 260 + 6 + 62$

$$\therefore |D_k| = \begin{vmatrix} -1 & -3 & 1 \\ 11 & 2 & -5 \\ 20 & 1 & 2 \end{vmatrix} = 328$$

$$|D_l| = \begin{vmatrix} 4 & -1 & 1 \\ 1 & 11 & -5 \\ 3 & 20 & 2 \end{vmatrix} = +(3)\begin{vmatrix} -1 & 1 \\ 11 & -5 \end{vmatrix} - (20)\begin{vmatrix} 4 & 1 \\ 1 & -5 \end{vmatrix} + (2)\begin{vmatrix} 4 & -1 \\ 1 & 11 \end{vmatrix}$$

$$= 3[(-1)(-5) - (11)(1)] - 20[(4)(-5) - (1)(1)] + 2[(4)(11) - (1)(-1)]$$

$$= -18 + 420 + 90$$

$$\therefore |D_l| = \begin{vmatrix} 4 & -1 & 1 \\ 1 & 11 & -5 \\ 3 & 20 & 2 \end{vmatrix} = 492$$

$$|D_m| = \begin{vmatrix} 4 & -3 & -1 \\ 1 & 2 & 11 \\ 3 & 1 & 20 \end{vmatrix} = +(3)\begin{vmatrix} -3 & -1 \\ 2 & 11 \end{vmatrix} - (1)\begin{vmatrix} 4 & -1 \\ 1 & 11 \end{vmatrix} + (20)\begin{vmatrix} 4 & -3 \\ 1 & 2 \end{vmatrix}$$

$$= 3[(-3)(11) - (2)(-1)] - 1[(4)(11) - (1)(-1)] + 20[(4)(2) - (1)(-3)]$$

$$= -93 - 45 + 220$$

$$\therefore |D_m| = \begin{vmatrix} 4 & -3 & -1 \\ 1 & 2 & 11 \\ 3 & 1 & 20 \end{vmatrix} = 82$$

$$\therefore k = \frac{D_k}{D} = \frac{328}{82} = 4$$

$$\therefore l = \frac{D_\ell}{D} = \frac{492}{82} = 6$$

$$\therefore m = \frac{D_m}{D} = \frac{82}{82} = 1$$

2.3 $$|D| = \begin{vmatrix} 2 & -1 & 4 \\ 3 & -2 & 1 \\ 1 & -5 & 3 \end{vmatrix} = +(2)\begin{vmatrix} -2 & 1 \\ -5 & 3 \end{vmatrix} - (3)\begin{vmatrix} -1 & 4 \\ -5 & 3 \end{vmatrix} + (1)\begin{vmatrix} -1 & 4 \\ -2 & 1 \end{vmatrix}$$

$$= 2[(-2)(3) - (-5)(1)] - 3[(-1)(3) - (-5)(4)] + 1[(-1)(1) - (-2)(4)]$$

$$= -2 - 51 + 7$$

$$\therefore |D| = \begin{vmatrix} 2 & -1 & 4 \\ 3 & -2 & 1 \\ 1 & -5 & 3 \end{vmatrix} = -46$$

$$|D_r| = \begin{vmatrix} 6 & -1 & 4 \\ 2 & -2 & 1 \\ 9 & -5 & 3 \end{vmatrix} = +(6)\begin{vmatrix} -2 & 1 \\ -5 & 3 \end{vmatrix} - (2)\begin{vmatrix} -1 & 4 \\ -5 & 3 \end{vmatrix} + (9)\begin{vmatrix} -1 & 4 \\ -2 & 1 \end{vmatrix}$$

$$= 6[(-2)(3) - (-5)(1)] - 2[(-1)(3) - (-5)(4)] + 9[(-1)(1) - (-2)(4)]$$

$$= -6 - 34 + 63$$

$$\therefore |D_r| = \begin{vmatrix} 6 & -1 & 4 \\ 2 & -2 & 1 \\ 9 & -5 & 3 \end{vmatrix} = 23$$

$$|D_s| = \begin{vmatrix} 2 & 6 & 4 \\ 3 & 2 & 1 \\ 1 & 9 & 3 \end{vmatrix} = +(2)\begin{vmatrix} 2 & 1 \\ 9 & 3 \end{vmatrix} - (3)\begin{vmatrix} 6 & 4 \\ 9 & 3 \end{vmatrix} + (1)\begin{vmatrix} 6 & 4 \\ 2 & 1 \end{vmatrix}$$

$$= 2[(2)(3) - (9)(1)] - 3[(6)(3) - (9)(4)] + 1[(6)(1) - (2)(4)]$$

$$= -6 + 54 - 2$$

$$\therefore |D_s| = \begin{vmatrix} 2 & 6 & 4 \\ 3 & 2 & 1 \\ 1 & 9 & 3 \end{vmatrix} = 46$$

$$|D_t| = \begin{vmatrix} 2 & -1 & 6 \\ 3 & -2 & 2 \\ 1 & -5 & 9 \end{vmatrix} = +(2)\begin{vmatrix} -2 & 2 \\ -5 & 9 \end{vmatrix} - (3)\begin{vmatrix} -1 & 6 \\ -5 & 9 \end{vmatrix} + (1)\begin{vmatrix} -1 & 6 \\ -2 & 2 \end{vmatrix}$$

$$= 2[(-2)(9) - (-5)(2)] - 3[(-1)(9) - (-5)(6)] + 1[(-1)(2) - (-2)(6)]$$

$$= -16 - 63 + 10$$

$$\therefore |D_t| = \begin{vmatrix} 2 & -1 & 6 \\ 3 & -2 & 2 \\ 1 & -5 & 9 \end{vmatrix} = -69$$

$$\therefore r = \frac{D_r}{D} = \frac{23}{-46} = -\frac{1}{2}$$

$$\therefore s = \frac{D_s}{D} = \frac{46}{-46} = -1$$

$$\therefore t = \frac{D_t}{D} = \frac{-69}{-46} = 1\tfrac{1}{2}$$

3. 3.1 $|D| = \begin{vmatrix} 2 & 4 & -1 \\ 1 & 2 & -3 \\ 3 & -1 & 1 \end{vmatrix} = (2)\begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix} + (4)\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$

$$= 2[(2)(1) - (-1)(-3)] - 4[(1)(1) - (3)(-3)] - 1[(1)(-1) - (3)(2)]$$

$$= -2 - 40 + 7$$

$$\therefore |D| = \begin{vmatrix} 2 & 4 & -1 \\ 1 & 2 & -3 \\ 3 & -1 & 1 \end{vmatrix} = -35$$

$$|D_x| = \begin{vmatrix} 2 & 4 & -1 \\ -4 & 2 & -3 \\ 1 & -1 & 1 \end{vmatrix} = (2)\begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix} + (4)\begin{vmatrix} -4 & -3 \\ 1 & 1 \end{vmatrix} + (-1)\begin{vmatrix} -4 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= 2[(2)(1) - (-1)(-3)] - 4[(-4)(1) - (1)(-3)] - 1[(-4)(-1) - (1)(2)]$$

$$= -2 + 4 - 2$$

$$\therefore |D_x| = \begin{vmatrix} 2 & 4 & -1 \\ -4 & 2 & -3 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$$|D_y| = \begin{vmatrix} 2 & 2 & -1 \\ 1 & -4 & -3 \\ 3 & 1 & 1 \end{vmatrix} = (2)\begin{vmatrix} -4 & -3 \\ 1 & 1 \end{vmatrix} + (2)\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & -4 \\ 3 & 1 \end{vmatrix}$$

$$= 2[(-4)(1) - (1)(-3)] - 2[(1)(1) - (3)(-3)] - 1[(1)(1) - (3)(-4)]$$

$$= -2 - 20 - 13$$

$$\therefore |D_y| = \begin{vmatrix} 2 & 2 & -1 \\ 1 & -4 & -3 \\ 3 & 1 & 1 \end{vmatrix} = -35$$

$$|D_z| = \begin{vmatrix} 2 & 4 & 2 \\ 1 & 2 & -4 \\ 3 & -1 & 1 \end{vmatrix} = (2)\begin{vmatrix} 2 & -4 \\ -1 & 1 \end{vmatrix} + (4)\begin{vmatrix} 1 & -4 \\ 3 & 1 \end{vmatrix} + (2)\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= 2[(2)(1) - (-1)(-4)] - 4[(1)(1) - (3)(-4)] + 2[(1)(-1) - (3)(2)]$$

$$= -4 - 52 - 14$$

$$\therefore |D_z| = \begin{vmatrix} 2 & 4 & 2 \\ 1 & 2 & -4 \\ 3 & -1 & 1 \end{vmatrix} = -70$$

$$\therefore x = \frac{D_x}{D} = \frac{0}{-35} = 0$$

$$\therefore y = \frac{D_y}{D} = \frac{-35}{-35} = 1$$

$$\therefore z = \frac{D_z}{D} = \frac{-70}{-35} = 2$$

3.2 $|D| = \begin{vmatrix} 5 & -6 & 3 \\ 2 & -3 & 2 \\ 3 & -7 & 5 \end{vmatrix} = (-6)\begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix} + (-3)\begin{vmatrix} 5 & 3 \\ 3 & 5 \end{vmatrix} + (-7)\begin{vmatrix} 5 & 3 \\ 2 & 2 \end{vmatrix}$

$= 6[(2)(5) - (3)(2)] - 3[(5)(5) - (3)(3)] + 7[(5)(2) - (2)(3)]$

$= 24 - 48 + 28$

$\therefore |D| = \begin{vmatrix} 5 & -6 & 3 \\ 2 & -3 & 2 \\ 3 & -7 & 5 \end{vmatrix} = 4$

$|D_{x_1}| = \begin{vmatrix} -9 & -6 & 3 \\ -5 & -3 & 2 \\ -16 & -7 & 5 \end{vmatrix} = (-6)\begin{vmatrix} -5 & 2 \\ -16 & 5 \end{vmatrix} + (-3)\begin{vmatrix} -9 & 3 \\ -16 & 5 \end{vmatrix} + (-7)\begin{vmatrix} -9 & 3 \\ -5 & 2 \end{vmatrix}$

$= 6[(-5)(5) - (-16)(2)] - 3[(-9)(5) - (-16)(3)] + 7[(-9)(2) - (-5)(3)]$

$= 42 - 9 - 21$

$\therefore |D_{x_1}| = \begin{vmatrix} -9 & -6 & 3 \\ -5 & -3 & 2 \\ -16 & -7 & 5 \end{vmatrix} = 12$

$|D_{x_2}| = \begin{vmatrix} 5 & -9 & 3 \\ 2 & -5 & 2 \\ 3 & -16 & 5 \end{vmatrix} = (-9)\begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix} + (-5)\begin{vmatrix} 5 & 3 \\ 3 & 5 \end{vmatrix} + (-16)\begin{vmatrix} 5 & 3 \\ 2 & 2 \end{vmatrix}$

$= 9[(2)(5) - (3)(2)] - 5[(5)(5) - (3)(3)] + 16[(5)(2) - (2)(3)]$

$= 36 - 80 + 64$

$\therefore |D_{x_2}| = \begin{vmatrix} 5 & -9 & 3 \\ 2 & -5 & 2 \\ 3 & -16 & 5 \end{vmatrix} = 20$

$|D_{x_3}| = \begin{vmatrix} 5 & -6 & -9 \\ 2 & -3 & -5 \\ 3 & -7 & -16 \end{vmatrix} = (-6)\begin{vmatrix} 2 & -5 \\ 3 & -16 \end{vmatrix} + (-3)\begin{vmatrix} 5 & -9 \\ 3 & -16 \end{vmatrix} + (-7)\begin{vmatrix} 5 & -9 \\ 2 & -5 \end{vmatrix}$

$= 6[(2)(-16) - (3)(-5)] - 3[(5)(-16) - (3)(-9)] + 7[(5)(-5) - (2)(-9)]$

$= -102 + 159 - 49$

$\therefore |D_{x_3}| = \begin{vmatrix} 5 & -6 & -9 \\ 2 & -3 & -5 \\ 3 & -7 & -16 \end{vmatrix} = 8$

$\therefore x_1 = \frac{D_{x_1}}{D} = \frac{12}{4} = 3$

$\therefore x_2 = \frac{D_{x_2}}{D} = \frac{20}{4} = 5$

$\therefore x_3 = \frac{D_{x_3}}{D} = \frac{8}{4} = 2$

3.3 $D = \begin{vmatrix} 2 & 1 & -1 \\ 3 & -4 & 5 \\ 4 & 0 & 2 \end{vmatrix}$

$= 2\begin{vmatrix} -4 & 5 \\ 0 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & 5 \\ 4 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 3 & -4 \\ 4 & 0 \end{vmatrix}$

$= 2[-8] - [6 - 20] - [0 + 16]$

$= -16 - [-14] - 16$

$= -16 + 14 - 16$

$= -18$

$$|D_{I_1}| = \begin{vmatrix} 2 & 1 & -1 \\ 7 & -4 & 5 \\ 9 & 0 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} -4 & 5 \\ 0 & 2 \end{vmatrix} - 1\begin{vmatrix} 7 & 5 \\ 9 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 7 & -4 \\ 9 & 0 \end{vmatrix}$$

$= 2[-8] - [14 - 45] + [0 - (-36)]$
$= -16 - [-31] + 36$
$= 51$

$\therefore \frac{1}{D} = \frac{I_1}{|D_{I_1}|}$

$\frac{1}{-18} = \frac{I_1}{51}$

$I_1 = \frac{51}{-18}$
$= -2{,}833$

3.4 $\frac{1}{a} + \frac{2}{b} + \frac{1}{c} = 0$

$\frac{1}{a} = \frac{1}{b} - \frac{2}{c} + 2$

$-\frac{1}{a} + \frac{1}{b} - \frac{1}{c} + 3 = 0$

Let $\frac{1}{a} = x$, $\frac{1}{6} = y$ and $\frac{1}{c} = z$
$\therefore x + 2y + z = 0$
$x - y + 2z = 2$
$-x + y - z = -3$

$$|D| = \begin{vmatrix} 1 & 2 & 1 \\ 1 & -1 & 2 \\ -1 & 1 & -1 \end{vmatrix}$$

$$= -1\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + (-1)\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}$$

$= -[1 - 2] - [-2 - 1] - 1[4 + 1]$
$= -1 - [-3] - 5$
$= -6 + 3$
$= -3$

$$|D_z| = \begin{vmatrix} 1 & 2 & 0 \\ 1 & -1 & 2 \\ -1 & 1 & -3 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 2 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 2 & 0 \\ 1 & -3 \end{vmatrix} + (-1)\begin{vmatrix} 2 & 0 \\ -1 & 2 \end{vmatrix}$$

$= [3 - 2] - [-6] - 1[4]$
$= 1 + 6 - 4$
$= 3$

$\therefore \frac{1}{|D|} = \frac{z}{|D_z|}$

$\frac{1}{-3} = \frac{z}{3}$

$\therefore z = -1$

$\therefore \frac{1}{c} = z$

$\therefore \frac{1}{c} = -1$

$\therefore c = -1$

Summative assessment: Module 1 **SB page 33**

1. 1.1 $\begin{vmatrix} 2 & 6 \\ 1 & -1 \end{vmatrix} = (2)(-1) - (1)(6) = -8$ (2)

1.2 $\begin{vmatrix} -5 & -1 & 1 \\ 9 & 1 & -1 \\ -1 & 1 & -5 \end{vmatrix} = -5\begin{vmatrix} 1 & -1 \\ 1 & -5 \end{vmatrix} - (-1)\begin{vmatrix} 9 & -1 \\ -1 & -5 \end{vmatrix} + 1\begin{vmatrix} 9 & 1 \\ -1 & 1 \end{vmatrix}$

$= -5(-5 + 1) + 1(-45 + 1) + 1(9 + 1)$

$= -5(-4) - 44 + 10$

$= 20 - 44 + 10$

$= -16$ (3)

2. 2.1 $D = 2\begin{vmatrix} -1 & 0 \\ -3 & 2 \end{vmatrix} - 2\begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 3 & -3 \end{vmatrix}$

$= 2[-2] - 2[2] + [-3 + 3]$

$= -4 - 4$

$= -8$ (2)

2.2 $+\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$

$= 2(2) - 3(1)$

$= 4 - 3$

$= 1$ (2)

2.3 $\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$

$= 2(0) - 1(1)$

$= -1$ (2)

3. $|D| = \begin{vmatrix} 6 & -7 & 1 \\ 1 & -3 & -8 \\ -2 & -1 & 3 \end{vmatrix}$

$= -2\begin{vmatrix} -7 & 1 \\ -3 & -8 \end{vmatrix} - (-1)\begin{vmatrix} 6 & 1 \\ 1 & -8 \end{vmatrix} + 3\begin{vmatrix} 6 & -7 \\ 1 & -3 \end{vmatrix}$

$= -2[56 + 3] + [-48 - 1] + 3[-18 + 7]$

$= -2[59] - 49 + 3[-11]$

$= -200$

$|D_{I_3}| = \begin{vmatrix} 6 & -7 & 0,5 \\ 1 & -3 & -9 \\ -2 & -1 & 4 \end{vmatrix}$

$= -2\begin{vmatrix} -7 & 0,5 \\ -3 & -9 \end{vmatrix} - (-1)\begin{vmatrix} 6 & 0,5 \\ 1 & -9 \end{vmatrix} + 4\begin{vmatrix} 6 & -7 \\ 1 & -3 \end{vmatrix}$

$= -2[63 + 3(0,5)] + [-56 - 0,5] + 4[-18 + 7]$

$= -2[64,5] - 56,5 + 4[-11]$

$= -36$

$\therefore \frac{1}{|D|} = \frac{I_3}{|D_{I_3}|}$

$\frac{1}{-200} = \frac{I_3}{-36}$

$\therefore I_3 = 0{,}18$ (8)

4. $D = \begin{vmatrix} -2 & -3 \\ 3 & -5 \end{vmatrix} = (-2)(-5) - (3)(-3) = 19$

$D_r = \begin{vmatrix} -5 & -3 \\ -2 & -5 \end{vmatrix} = (-5)(-5) - (-2)(-3) = 19$ $\quad \therefore r = \frac{D_r}{D} = \frac{19}{19} = 1$

$D_s = \begin{vmatrix} -2 & -5 \\ 3 & -2 \end{vmatrix} = (-2)(-2) - (3)(-5) = 19$ $\quad \therefore s = \frac{D_s}{D} = \frac{19}{19} = 1$ (10)

5. 5.1 $\begin{vmatrix} 3 & x \\ -x & 2 \end{vmatrix} = 5x$

$(3)(2) - (-x)(x) = 5x$

$6 + x^2 = 5x$

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x - 2 = 0$ or $x - 3 = 0$

$\therefore x = 2$ or $x = 3$ (3)

5.2 $\begin{vmatrix} y-3 & -2 \\ y+4 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 7 \\ -3 & y \end{vmatrix}$

$(y - 3)(4) - (y + 4)(-2) = (1)(y) - (-3)(7)$

$4y - 12 + 2y + 8 = y + 21$

$6y - 4 = y + 21$

$5y = 25$

$\therefore y = 5$ (3)

5.3 $\begin{vmatrix} z & 3 & 2 \\ 3 & -2 & 5 \\ 2 & z & 6 \end{vmatrix} = -120$

$+ (z)\begin{vmatrix} -2 & 5 \\ z & 6 \end{vmatrix} - (3)\begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix} + (2)\begin{vmatrix} 3 & -2 \\ 2 & z \end{vmatrix} = -120$

$z[(-2)(6) - (z)(5)] - 3[(3)(6) - (2)(5)] + 2[(3)(z) - (2)(-2)] = -120$

$z[-12 - 5z] - 3[18 - 10] + 2\,[3z + 4] = -120$

$-12z - 5z^2 - 54 + 30 + 6z + 8 = -120$

$-5z^2 - 6z - 16 = -120$

$-5z^2 - 6z + 104 = 0$

$5z^2 + 6z - 104 = 0$

$(5z + 26)(z - 4) = 0$

$5z + 26 = 0$ or $z - 4 = 0$

$5z = -26$

$\therefore z = -5\frac{1}{5}$ or $\therefore z = 4$

(4)

6. 6.1 $|D| = \begin{vmatrix} 5 & -6 & 3 \\ 2 & -3 & 2 \\ 3 & -7 & 5 \end{vmatrix}$

$= 5\begin{vmatrix} -3 & 2 \\ -7 & 5 \end{vmatrix} - (-6)\begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix} + 3\begin{vmatrix} 2 & -3 \\ 3 & -7 \end{vmatrix}$

$= 5[-15 + 14] + 6[10 - 6] + 3[-14 + 9]$

$= -5[-1] + 6[4] + 3[-5]$

$= 5 + 24 - 15$

$= 14$ (3)

6.2 $|D_c| = \begin{vmatrix} 5 & -6 & -9 \\ 2 & -3 & -5 \\ 3 & -7 & -16 \end{vmatrix}$

$= 5\begin{vmatrix} -3 & -5 \\ -7 & -16 \end{vmatrix} - (-6)\begin{vmatrix} 2 & -5 \\ 3 & -16 \end{vmatrix} + (-9)\begin{vmatrix} 2 & -3 \\ 3 & -7 \end{vmatrix}$

$= 5[48 - 35] + 6[-32 + 15] - 9[-14 + 9]$

$= 5[13] + 6[-17] - 9[-5]$

$= 8$

$\therefore \frac{1}{|D|} = \frac{c}{|D_c|}$

$\frac{1}{14} = \frac{c}{8}$

$\therefore c = 0{,}571$ (3)

7. 7.1 $|D| = \begin{vmatrix} -2 & -3 & -3 \\ 5 & -1 & -6 \\ 3 & 2 & 2 \end{vmatrix} = +(-2)\begin{vmatrix} -1 & -6 \\ 2 & 2 \end{vmatrix} - (-3)\begin{vmatrix} 5 & -6 \\ 3 & 2 \end{vmatrix} + (-3)\begin{vmatrix} 5 & -1 \\ 3 & 2 \end{vmatrix}$

$= -2[(-1)(2) - (2)(-6)] + 3[(5)(2) - (3)(-6)] - 3[(5)(2) - (3)(-1)]$

$= -20 + 84 - 39$

$\therefore |D| = \begin{vmatrix} -2 & -3 & -3 \\ 5 & -1 & -6 \\ 3 & 2 & 2 \end{vmatrix} = 25$ (3)

7.2 $|D_{k_i}| = \begin{vmatrix} -4 & -3 & -3 \\ 3 & -1 & -6 \\ 1 & 2 & 2 \end{vmatrix} = -4\begin{vmatrix} -1 & -6 \\ 2 & 2 \end{vmatrix} - (3)\begin{vmatrix} -3 & -3 \\ 2 & 2 \end{vmatrix} + 1\begin{vmatrix} -3 & -3 \\ -1 & -6 \end{vmatrix}$

$= -4(-2 + 12) - 3(-6 + 6) + (18 - 3)$

$= -4(10) + 15$

$= -25$ (3)

7.3 $|D_\ell| = \begin{vmatrix} -2 & -4 & -3 \\ 5 & 3 & -6 \\ 3 & 1 & 2 \end{vmatrix} = +(-3)\begin{vmatrix} 5 & 3 \\ 3 & 1 \end{vmatrix} - (-6)\begin{vmatrix} -2 & -4 \\ 3 & 1 \end{vmatrix} + (2)\begin{vmatrix} -2 & -4 \\ 5 & 3 \end{vmatrix}$

$= -3[(5)(1) - (3)(3)] + 6[(-2)(1) - (3)(-4)] + 2[(-2)(3) - (5)(-4)]$

$= 12 + 60 + 28$

$\therefore |D_\ell| = \begin{vmatrix} -2 & -4 & -3 \\ 5 & 3 & -6 \\ 3 & 1 & 2 \end{vmatrix} = 100$ (3)

7.4 $|D_m| = \begin{vmatrix} -2 & -3 & -4 \\ 5 & -1 & 3 \\ 3 & 2 & 1 \end{vmatrix}$

$= (5)\left[(-1)^{2+1}\begin{vmatrix} -3 & -4 \\ 2 & 1 \end{vmatrix}\right] + (-1)\left[(-1)^{2+2}\begin{vmatrix} -2 & -4 \\ 3 & 1 \end{vmatrix}\right] + (3)\left[(-1)^{2+3}\begin{vmatrix} -2 & -3 \\ 3 & 2 \end{vmatrix}\right]$

$= -5[(-3)(1) - (2)(-4)] - 1[(-2)(1) - (3)(-4)] - 3[(-2)(2) - (3)(-3)]$

$= -25 - 10 - 15$

$\therefore |D_m| = \begin{vmatrix} -2 & -3 & -4 \\ 5 & -1 & 3 \\ 3 & 2 & 1 \end{vmatrix} = -50$ (3)

7.5 $k = \frac{|D_k|}{|D|} = \frac{-25}{25} = -1$

$l = \frac{|D_\ell|}{|D|} = \frac{100}{25} = 4$

$m = \frac{|D_m|}{|D|} = \frac{-50}{25} = -2$ (3)

TOTAL: [60]

MODULE

2 Complex numbers

After they have completed this module, students should be able to:

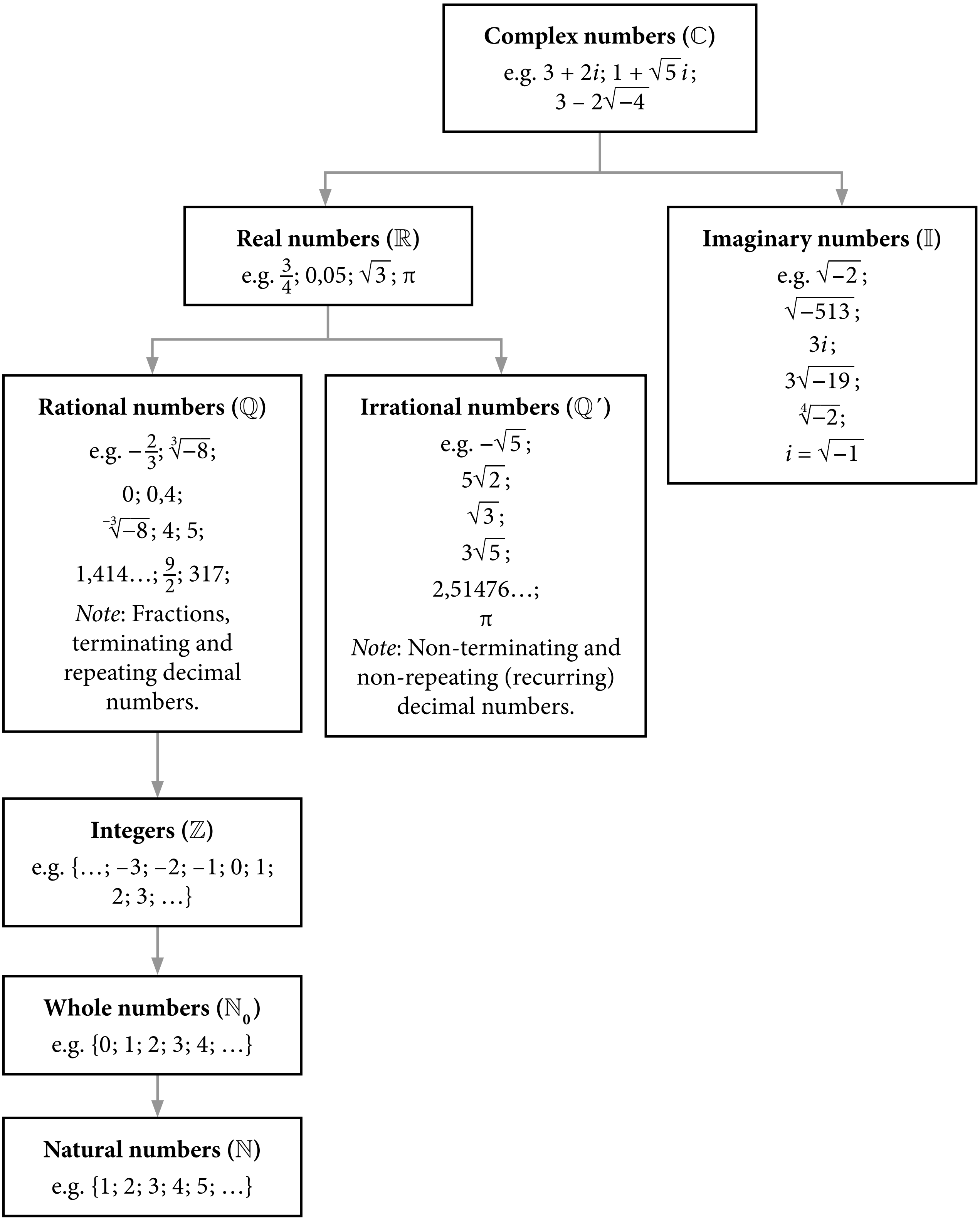
- define an imaginary number;
- identify real and imaginary parts of a complex number in rectangular form;
- simplify complex powers;
- add, subtract and multiply complex numbers in rectangular form;
- define and determine the conjugate of a complex number;
- divide complex numbers in rectangular for using the conjugate;
- define the modulus and argument of the complex number and plot them on an Argand diagram;
- convert a complex number from rectangular form to polar form and vice versa, using a pocket calculator or any analytical method;
- multiply and divide complex numbers in polar form;
- state and apply De Moivre's theorem to products, quotients and powers of complex numbers; and
- solve complex equations in rectangular or polar form.

Introduction

Complex numbers were encountered previously when solving x-intercepts for $ax^2 + bx + c = 0$ using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. When it yields a negative number inside the square root it implies that no solution exists in the real number system, that is, the parabola does not intersect the x-axis.

To accommodate the square root of negative numbers the real number system needs to be extended to include pure imaginary numbers. The expanded number system is called the complex number system.

Before working with complex numbers, we need to revise the relationships between several types of numbers.


Complex numbers ($\mathbb{C}$)
e.g. $3 + 2i$; $1 + \sqrt{5}\,i$; $3 - 2\sqrt{-4}$
Real numbers ($\mathbb{R}$)
e.g. $\frac{3}{4}$; 0,05; $\sqrt{3}$; π
Imaginary numbers ($\mathbb{I}$)
e.g. $\sqrt{-2}$; $\sqrt{-513}$; $3i$; $3\sqrt{-19}$; $\sqrt[4]{-2}$; $i = \sqrt{-1}$
Rational numbers ($\mathbb{Q}$)
e.g. $-\frac{2}{3}$; $\sqrt[3]{-8}$; 0; 0,4; $-\sqrt[3]{-8}$; 4; 5; 1,414…; $\frac{9}{2}$; 317;
Note: Fractions, terminating and repeating decimal numbers.
Irrational numbers ($\mathbb{Q}'$)
e.g. $-\sqrt{5}$; $5\sqrt{2}$; $\sqrt{3}$; $3\sqrt{5}$; 2,51476…; π
Note: Non-terminating and non-repeating (recurring) decimal numbers.
Integers ($\mathbb{Z}$)
e.g. {…; −3; −2; −1; 0; 1; 2; 3; …}
Whole numbers ($\mathbb{N}_0$)
e.g. {0; 1; 2; 3; 4; …}
Natural numbers ($\mathbb{N}$)
e.g. {1; 2; 3; 4; 5; …}

Complex numbers can be represented in three forms, rectangular form, polar form and exponential form.

Complex numbers		
Rectangular form $z = a + bi$ or $z = a + bj$	**Polar form** $z = r(\cos\theta + i\sin\theta)$ or $z = r(\cos\theta + j\sin\theta)$	**Exponential form** $z = re^{i\theta}$ or $z = re^{j\theta}$

In this module we only represent complex numbers in rectangular (standard) form and polar (trigonometric) form with the imaginary unit represented as *i* instead of *j* as in some texts.

Students need the following pre-knowledge to successfully complete this module.

Pre-knowledge

Laws of exponents:

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $(a.b)^m = a^m.b^m$
5. $\left(\frac{a^m}{a^n}\right)^b = \frac{a^{mb}}{a^{nb}}$
6. $a^{-n} = \frac{1}{a^n}$
7. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ or $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$
8. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ or $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
9. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$
10. $(\sqrt[m]{a})^n = \sqrt[m]{a^n}$

Addition and subtraction of surds

$\sqrt{a} + \sqrt{a} = 2\sqrt{a}$

$\sqrt{a} - \sqrt{a} = 0$

$\sqrt{a} + \sqrt{b} = \sqrt{a} + \sqrt{b}$

$\sqrt{a} - \sqrt{b} = \sqrt{a} - \sqrt{b}$

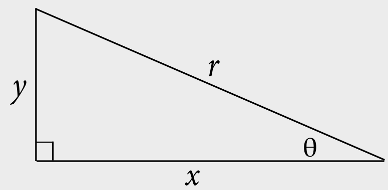
Multiplication and division of surds

$a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$

$a\sqrt{b} \div c\sqrt{d} = \frac{a\sqrt{b}}{c\sqrt{d}} = \frac{a}{c}\sqrt{\frac{b}{d}}$

- Perfect squares: $1 = 1^2; 4 = 2^2; 9 = 3^2; 16 = 4^2; 25 = 5^2; \ldots$
- $\sqrt{4} = \sqrt{2 \times 2} = 2; \sqrt{9} = \sqrt{3 \times 3} = 3; \sqrt{a^2} = \sqrt{a \times a} = a; \sqrt[3]{27} = \sqrt[3]{3.3.3} = 3$
- $(\sqrt{6})^2 = \sqrt{6}.\sqrt{6} = \sqrt{36} = 6$

- $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$
- $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$
- $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$
- Theorem of Pythagoras: $x^2 + y^2 = r^2$



To divide $\frac{r_1\angle\theta_1}{r_2\angle\theta_2}$ the moduli (r) will be divided and the arguments (θ) subtracted.

For example:

Given $z_1 = r_1\angle\theta_1$ and $z_2 = r_2\angle\theta_2$. Calculate $z_1 \div z_2$.

$\therefore \frac{z_1}{z_2} = \frac{r_1\angle\theta_1}{r_2\angle\theta_2}$

$= \frac{r_1}{r_2}\angle\theta_1 - \theta_2$ • Theorem

Activity 2.1 SB page 44

1. $i^2 = -1$

2. $i^7 = (i^2)^3 i$
   $= (-1)^3 i$
   $= -i$

3. $i^8 = (i^2)^4$
   $= (-1)^4$
   $= 1$

4. $3i^9 = 3(i^2)^4 i$
   $= 3(-1)^4 i$
   $= 3i$

5. $i^{26} = (i^2)^{13}$
   $= (-1)^{13}$
   $= -1$

6. $-i^{11} = -(i^2)^5 i$
   $= -(-1)^5 i$
   $= i$

7. $\frac{5}{i} = \frac{5}{i} \times \frac{-i}{-i}$
   $= \frac{-5i}{-i^2}$
   $= \frac{-5i}{-(-1)}$
   $= -5i$

8. $\frac{1}{-i} = \frac{1}{-i} \times \frac{i}{i}$
   $= \frac{i}{-i^2}$
   $= \frac{i}{-(-1)}$
   $= i$

9. $\frac{1}{2i} = \frac{1}{2i} \times \frac{-i}{-i}$
   $= \frac{-i}{-2i^2}$
   $= \frac{-i}{-2(-1)}$
   $= \frac{-i}{2}$ or $-\frac{1}{2}i$

10. $-\frac{3}{4i} = -\frac{3}{4i} \times \frac{-i}{-i}$
   $= \frac{3i}{-4i^2}$
   $= \frac{3i}{-4(-1)}$
   $= \frac{3i}{4}$ or $\frac{3}{4}i$

Activity 2.2 SB page 47

1\. 1.1 $\sqrt{-49}$
$= \sqrt{49}.\sqrt{-1}$
$= 7i$

1.2 $\sqrt{-45}$
$= \sqrt{9.5}.\sqrt{-1}$
$= 3\sqrt{5}i$

1.3 $-\sqrt{-100}$
$= -\sqrt{100}.\sqrt{-1}$
$= -10i$

1.4 $\sqrt{-32}$
$= \sqrt{32}.\sqrt{-1}$
$= \sqrt{16.2}i$
$= 4\sqrt{2}i$

1.5 $\sqrt{-9} \times \sqrt{-6}$
$= \sqrt{9}.\sqrt{-1} \times \sqrt{6}\sqrt{-1}$
$= 3i \times \sqrt{6}i$
$= 3\sqrt{6}i^2$
$= 3\sqrt{6}(-1)$
$= -3\sqrt{6}$

1.6 $-\sqrt{-64}.\sqrt{-3}$
$= -\sqrt{64}.\sqrt{-1}.\sqrt{3}.\sqrt{-1}$
$= -8i\sqrt{3}i$
$= -8\sqrt{3}i^2$
$= -8\sqrt{3}(-1)$
$= 8\sqrt{3}$

1.7 $\frac{\sqrt{-12}.\sqrt{-6}}{\sqrt{-2}}$
$= \frac{\sqrt{12}\sqrt{-1}.\sqrt{6}\sqrt{-1}}{\sqrt{2}.\sqrt{-1}}$
$= \frac{\sqrt{12}i\sqrt{6}i}{\sqrt{2}i}$
$= \sqrt{\frac{12.6}{2}}i$
$= \sqrt{36}i$
$= 6i$

1.8 $(\sqrt{-125})(\sqrt{5})$
$= (\sqrt{25.5}.\sqrt{-1})(\sqrt{5})$
$= 5\sqrt{5}i\sqrt{5}$
$= 5\sqrt{25}i$
$= 25i$

1.9 $(\sqrt{-4})^5$
$= (\sqrt{4}.\sqrt{-1})^5$
$= (2i)^5$
$= 32i^5$
$= 32(i^2)^2i$
$= 32(-1)^2i$
$= 32i$

1.10 $\sqrt{-5}(\sqrt{-3}.\sqrt{-4})$
$= \sqrt{5}.\sqrt{-1}(\sqrt{3}\sqrt{-1})(\sqrt{4}.\sqrt{-1})$
$= \sqrt{5}i(\sqrt{3}i)(2i)$
$= 2\sqrt{15}i^3$
$= 2\sqrt{15}(i^2)i$
$= 2\sqrt{15}(-1)i$
$= -2\sqrt{15}i$

1.11 $\frac{\sqrt{-12}.\sqrt{-4}}{\sqrt{-6}}$

$= \frac{\sqrt{12}\sqrt{-1}.\sqrt{4}.\sqrt{-1}}{\sqrt{6}.\sqrt{-1}}$

$= \frac{\sqrt{12}.i.2i}{\sqrt{6}i}$

$= \frac{2\sqrt{12}i^2}{\sqrt{6}\,i}$

$= 2\sqrt{\frac{12}{6}}i$

$= 2\sqrt{2}i$

1.12 $7i^2\sqrt{3}(4i\sqrt{-27})$

$= 7(-1)\sqrt{3}(4i\sqrt{27}\sqrt{-1})$

$= -7\sqrt{3}(4i\sqrt{9.3}i)$

$= -7\sqrt{3}(4i.3\sqrt{3}i)$

$= -7.4.3.3.i^2$

$= -252(-1)$

$= 252$

1.13 $\frac{-6 + \sqrt{-169}}{3}$

$= \frac{-6 + \sqrt{169}\sqrt{-1}}{3}$

$= \frac{-6 + 13i}{3}$

$= -2 + \frac{13}{3}i$

or $-2 + 4{,}33i$

1.14 $(\sqrt{-144})(\sqrt{-32})(-\sqrt{-2})$

$= (\sqrt{144}.\sqrt{-1})(\sqrt{32}.\sqrt{-1})(-\sqrt{2}.\sqrt{-1})$

$= (12i)(\sqrt{16.2}i)(-\sqrt{2}i)$

$= (12i)(4\sqrt{2}i)(-\sqrt{2}i)$

$= 12.4.2.i^3$

$= 96(i^2)i$

$= 96(-1)i$

$= -96i$

2. 2.1 $\sqrt{-4} + \sqrt{-16} - \sqrt{81}$

$= \sqrt{4}.\sqrt{-1} + \sqrt{16}.\sqrt{-1} - 9$

$= 2i + 4i - 9$

$= 6i - 9$ or $-9 + 6i$

2.2 $-\sqrt{-9} - \sqrt{-25} + \sqrt{-64} - \sqrt{-1}$

$= -\sqrt{9}\sqrt{-1} - \sqrt{25}\sqrt{-1} + \sqrt{64}.\sqrt{-1} - i$

$= -3i - 5i + 8i - i$

$= -i$

2.3 $\sqrt{-3} + \sqrt{-48}$

$= \sqrt{3}\sqrt{-1} + \sqrt{16.3}\sqrt{-1}$

$= \sqrt{3}i + 4\sqrt{3}i$

$= 5\sqrt{3}i$

2.4 $\sqrt{-12} + \sqrt{-36} + \sqrt{-28}$

$= \sqrt{12}\sqrt{-1} + \sqrt{36}\sqrt{-1} + \sqrt{28}\sqrt{-1}$

$= \sqrt{4.3}i + 6i + \sqrt{7.4}i$

$= 2\sqrt{3}i + 6i + 2\sqrt{7}i$

$= (2\sqrt{3} + 6 + 2\sqrt{7})i$

2.5 $\frac{\sqrt{-18}}{\sqrt{-2}} + \sqrt{-144}$

$= \frac{\sqrt{9.2}\sqrt{-1}}{\sqrt{2}.\sqrt{-1}} + \sqrt{144}.\sqrt{-1}$

$= \frac{3\sqrt{2}i}{\sqrt{2}i} + 12i$

$= 3 + 12i$

Activity 2.3 SB page 53

1.

	Number	Rectangular form	Real part	Imaginary (non-real) part
1.1	$-5 + 3i$	$-5 + 3i$	-5	$3i$
1.2	π	$\pi + 0i$	π	$0i$
1.3	$-2i$	$0 - 2i$	0	$-2i$
1.4	$3 - i9$	$3 - 9i$	3	$-9i$
1.5	$4 + \sqrt[3]{-27}$	$1 + 0i$	1	$0i$
1.6	$5 - \sqrt{-100}$	$5 - 10i$	5	$-10i$
1.7	$-5\sqrt{64}$	$-40 + 0i$	-40	$0i$
1.8	$-8i - 1$	$-1 - 8i$	-1	$-8i$
1.9	51	$51 + 0i$	51	$0i$
1.10	$\sqrt{-81} - \sqrt{-25}$	$0 + 4i$	0	$4i$

2. 2.1 $i + 2 - 3i - 1 - 5i$

$= 1 - 7i$

2.2 $(4 + i) - (4 - i)$

$= 4 + i - 4 + i$

$= 0 + 2i$

2.3 $(10 + 20i) - (-11 - 12i)$

$= 10 + 20i + 11 + 12i$

$= 21 + 32i$

2.4 $(3 - 2i) - (4 - 3i) + (-2 + 3i)$

$= 3 - 2i - 4 + 3i - 2 + 3i$

$= -3 + 4i$

2.5 $(-27 - 17i) - (-19i)$

$= -27 - 17i + 19i$

$= 2i - 27$

$= -27 + 2i$

2.6 $(5 - 2i) - (3 - 4i) - (4 - i)i$

$= 5 - 2i - 3 + 4i - 4i + i^2$

$= 5 - 2i - 3 + 4i - 4i + (-1)$

$= 1 - 2i$

2.7 $(4 - 5i) - (2i^4 - i^2)$
$= (4 - 5i) - (2(i^2)^2 - (-1))$
$= (4 - 5i) - (2(-1)^2 + 1)$
$= (4 - 5i) - (2 + 1)$
$= 4 - 5i - 3$
$= 1 - 5i$

2.8 $30i^9 + 2i - 2^5 i + 25i^2 + 50i^{100}$
$= 30(i^2)^4 i + 2i - 32i + 25(-1) + 50(i^2)^{50}$
$= 30(-1)^4 i + 2i - 32i - 25 + 50(-1)^{50}$
$= 30i + 2i - 32i - 25 + 50$
$= 25 + 0i$

3. 3.1 $15 + 40i - (-41 + 16i)$
$= 15 + 40i + 41 - 16i$
$= 56 + 24i$

3.2 $20i - 12 - (-12 - 15i)$
$= 20i - 12 + 12 + 15i$
$= 0 + 35i$

4. 4.1 $-5i(4 + 3i)$
$= -20i - 15i^2$
$= -20i - 15(-1)$
$= -20i + 15$
$= 15 - 20i$

4.2 $(1 + 3i)(5 + 5i)$
$= 5 + 5i + 15i + 15i^2$
$= 5 + 20i + 15(-1)$
$= 5 + 20i - 15$
$= -10 + 20i$

4.3 $(i - 1)(3 - 2i)$
$= 3i - 2i^2 - 3 + 2i$
$= 3i - 2(-1) - 3 + 2i$
$= 3i + 2 - 3 + 2i$
$= -1 + 5i$

4.4 $(3 + 2i)(2 - 4i)$
$= 6 - 12i + 4i - 8i^2$
$= 6 - 8i - 8(-1)$
$= 6 - 8i + 8$
$= 14 - 8i$

4.5 $(3 + i)^2 - (3 - i)^2$
$= (3 + i)(3 + i) - (3 - i)(3 - i)$
$= 9 + 3i + 3i + i^2 - [9 - 3i - 3i + i^2]$
$= 9 + 6i + (-1) - [9 - 6i + (-1)]$
$= 9 + 6i - 1 - [9 - 6i - 1]$
$= 9 + 6i - 1 - [8 - 6i]$
$= 9 + 6i - 1 - 8 + 6i$
$= 0 + 12i$

4.6 $(2 + \sqrt{5}i)(1 - \sqrt{5}i)$
$= 2 - 2\sqrt{5}i + \sqrt{5}i - \sqrt{25}i^2$
$= 2 - \sqrt{5}i - 5(-1)$
$= 2 - \sqrt{5}i + 5$
$= 7 - \sqrt{5}i$

4.7 $(\frac{1}{2} - \frac{1}{2}i)(1 - i)$
$= \frac{1}{2} - \frac{1}{2}i - \frac{1}{2}i + \frac{1}{2}i^2$
$= \frac{1}{2} - i + \frac{1}{2}(-1)$
$= 0 - i$

4.8 $2(3 - 4i)(i - 5)$
$= 2(3i - 15 - 4i^2 + 20i)$
$= 2[23i - 15 - 4(-1)]$
$= 2(23i - 15 + 4)$
$= 2(23i - 11)$
$= 46i - 22$
$= -22 + 46i$

4.9 $(2 + i)^2(3 - i)$
$= (2 + i)(2 + i)(3 - i)$
$= (4 + 2i + 2i + i^2)(3 - i)$
$= [4 + 4i + (-1)](3 - i)$
$= (4 + 4i - 1)(3 - i)$
$= (3 + 4i)(3 - i)$
$= 9 - 3i + 12i - 4i^2$
$= 9 + 9i - 4(-1)$
$= 9 + 9i + 4$
$= 13 + 9i$

4.10 $(2 - i)(-3 + 3i)(4 - 5i)$
$= (-6 + 6i + 3i - 3i^2)(4 - 5i)$
$= (-6 + 9i - 3(-1))(4 - 5i)$
$= (-3 + 9i)(4 - 5i)$
$= -12 + 15i + 36i - 45i^2$
$= -12 + 51i - 45(-1)$
$= -12 + 51i + 45$
$= 33 + 51i$

5. $I_1 + I_2 = I_3$
$I_2 = I_3 - I_1$
$= 25 + 11i - (18 - 13i)$
$= 25 + 11i - 18 + 13i$
$= 7 + 24i$

6. 6.1 $z_1 + z_2 - z_3$
$= 2 - 3i + (-7i) - (-4 + 2i)$
$= 2 - 3i - 7i + 4 - 2i$
$= 6 - 12i$

6.2 $(z_1)(z_3)$
$= (2 - 3i)(-4 + 2i)$
$= -8 + 4i + 12i - 6i^2$
$= -8 + 16i - 6(-1)$
$= -8 + 16i + 6$
$= -2 + 16i$

6.3 $2z_2 - 2z_1$
$= 2(-7i) - 2(2 - 3i)$
$= -14i - 4 + 6i$
$= -4 - 8i$

6.4 $z_1.z_2.z_3$
$= (2 - 3i)(-7i)(-4 + 2i)$
$= (-14i + 21i^2)(-4 + 2i)$
$= [-14i + 21(-1)][-4 + 2i]$
$= [-14i - 21][-4 + 2i]$
$= 56i - 28i^2 + 84 - 42i$
$= 14i - 28(-1) + 84$
$= 14i + 28 + 84$
$= 14i + 112$
$= 112 + 14i$

Activity 2.4 — SB page 57

1.

	Complex number (z)	Conjugate ($\bar{z}$)
1.1	$-7i$	$7i$
1.2	$-3 + 3i$	$-3 - 3i$
1.3	$-i + 2$	$i + 2$
1.4	$4i - 5$	$-4i - 5$
1.5	-5	$-5 - 0i$
1.6	i	$-i$
1.7	$-6 - \frac{2i}{3}$	$-6 + \frac{2}{3}i$
1.8	$\frac{5-3i}{8}$	$\frac{5}{8} + \frac{3}{8}i$
1.9	$-\sqrt{-64}$	$8i$
1.10	$3i - \sqrt{-25}$	$2i$

2. 2.1 $\bar{z} = -2{,}3 - 2{,}5i$

2.2 $-z = -(-2{,}3 + 2{,}5i)$
$= 2{,}3 - 2{,}5i$

2.3 $-\bar{z} = -(-2{,}3 - 2{,}5i)$
$= 2{,}3 + 2{,}5i$

2.4 $\bar{\bar{z}} = -2{,}3 + 2{,}5i$

3. 3.1 $z_1 + z_2 - z_3 + \bar{z}_2 - \bar{z}_3$
$= 2 + 5i + 3 - i - (-2i - 4) + (3 + i) - (2i - 4)$
$= 2 + 5i + 3 - i + 2i + 4 + 3 + i - 2i + 4$
$= 16 + 5i$

3.2 $\bar{z}_1.z_3$
$= (2 - 5i)(-2i - 4)$
$= -4i - 8 + 10i^2 + 20i$
$= -4i - 8 + 10(-1) + 20i$
$= 16i - 8 - 10$
$= -18 + 16i$

3.3 $\bar{z}_2.\bar{z}_3$
$= (3 + i)(2i - 4)$
$= 6i - 12 + 2i^2 - 4i$
$= 2i - 12 + 2(-1)$
$= -14 + 2i$

3.4 $\bar{z}_1.\bar{z}_3.\bar{z}_4$
$= (2 - 5i)(2i - 4)(-5i)$
$= (4i - 8 - 10i^2 + 20i)(-5i)$
$= (24i - 8 - 10(-1))(-5i)$
$= (2 + 24i)(-5i)$
$= -10i - 120i^2$
$= -10i - 120(-1)$
$= 120 - 10i$

4. 4.1 $\overline{(12 + 4i) + (14 + 3i)} + (-2 + 11i)$
$= \overline{12 + 4i + 14 + 3i} + (-2 + 11i)$
$= \overline{26 + 7i} + (-2 + 11i)$
$= 26 - 7i - 2 + 11i$
$= 24 + 4i$

4.2 $\overline{(-1 + 2\sqrt{2}\,i)}(1 + \sqrt{2}\,i)^2$
$= (-1 - 2\sqrt{2}\,i)(1 + \sqrt{2}\,i)(1 + \sqrt{2}\,i)$
$= (-1 - 2\sqrt{2}\,i)(1 + \sqrt{2}\,i + \sqrt{2}\,i + 2\,i^2)$
$= (-1 - 2\sqrt{2}\,i)(1 + 2\sqrt{2}\,i - 2)$
$= (-1 - 2\sqrt{2}\,i)(-1 + 2\sqrt{2}\,i)$
$= 1 - 2\sqrt{2}\,i + 2\sqrt{2}\,i - 8\,i^2$
$= 1 + 8$
$= 9 + 0i$

4.3 $(-4 + 5i)\overline{(-3 - i)(2 - 2i)}$
$= (-4 + 5i)\overline{(-6 + 6i - 2i + 2\,i^2)}$
$= (-4 + 5i)\overline{(-6 + 4i - 2)}$
$= (-4 + 5i)\overline{(-8 + 4i)}$
$= (-4 + 5i)(-8 - 4i)$
$= 32 + 16i - 40i - 20\,i^2$
$= 32 - 24i + 20$
$= 52 - 24i$

Activity 2.5 SB page 63

1. 1.1 $\frac{-3 - 4i}{-12}$
$= \frac{-3}{-12} - \frac{4i}{-12}$
$= \frac{1}{4} + \frac{1}{3}i$ or $0{,}25 + 0{,}333i$

1.2 $\frac{5}{i} - \frac{i}{5}$
$= \frac{5}{i} \times \frac{i}{i} - \frac{i}{5}$
$= \frac{5i}{i^2} - \frac{i}{5}$
$= \frac{5i}{(-1)} - \frac{i}{5}$
$= -5i - \frac{1}{5}i$
$= -5\frac{1}{5}i$
$= 0 - \frac{26}{5}i$ or $0 - 5{,}2i$

1.3 $\frac{2-3i}{i}$

$= \frac{2-3i}{i} \times \frac{-i}{-i}$

$= \frac{-2i + 3i^2}{-i^2}$

$= \frac{-2i + 3(-1)}{-(-1)}$

$= \frac{-2i - 3}{1}$

$= -3 - 2i$

1.4 $\frac{-3-i}{-3i}$

$= \frac{-3-i}{-3i} \times \frac{3i}{3i}$

$= \frac{-3i - i^2}{-3i^2}$

$= \frac{-3i - (-1)}{-3(-1)}$

$= \frac{-3i + 1}{3}$

$= \frac{-3i}{3} + \frac{1}{3}$

$= -i + \frac{1}{3}$

$= \frac{1}{3} - i$

1.5 $2i^8 - 3i^7 + \frac{4i^6}{5i^9}$

$= \frac{2(i^2)^4 - 3(i^2)^3 i + 4(i^2)^3}{5(i^2)^4\, i}$

$= \frac{2(-1)^4 - 3(-1)^3 i + 4(-1)^3}{5(-1)^4 i}$

$= \frac{2(1) - 3(-1)i + 4(-1)}{5(1)i}$

$= \frac{2 + 3i - 4}{5i}$

$= \frac{-2 + 3i}{5i} \times \frac{-5i}{-5i}$

$= \frac{10i - 15i^2}{-25i^2}$

$= \frac{10i - 15(-1)}{-25(-1)}$

$= \frac{10i + 15}{25}$

$= \frac{3}{5} + \frac{2}{5}i$ or $0{,}6 + 0{,}4i$

2. 2.1 $z_1 + z_2 \div z_3$

$= 5 - 3i + \frac{(-3 + 4i)}{i}$

$= 5 - 3i + \left(\frac{-3 + 4i}{i} \times \frac{-i}{-i}\right)$

$= 5 - 3i + \left(\frac{3i - 4i^2}{-i^2}\right)$

$= 5 - 3i + \left(\frac{3i - 4(-1)}{-(-1)}\right)$

$= 5 - 3i + \left(\frac{3i + 4}{1}\right)$

$= 5 - 3i + 3i + 4$

$= 5 - 3i + 3i + 4$

$= 9 + 0i$

2.2 $z_1.\bar{z}_2.z_3$

$= (2 + 3i)(i - 3)(-4i)$

$= (2i - 6 + 3i^2 - 9i)(-4i)$

$= (-7i - 6 + 3(-1))(-4i)$

$= (-7i - 9)(-4i)$

$= 28i^2 + 36i$

$= -28 + 36i$

3.

3.1 $\frac{-i}{3-5i}$

$= \frac{-i}{3-5i} \times \frac{3+5i}{3+5i}$

$= \frac{-3i-5i^2}{9+15i-15i-25i^2}$

$= \frac{-3i-5(-1)}{9-25(-1)}$

$= \frac{-3i+5}{9+25}$

$= \frac{5-3i}{34}$

$= \frac{5}{34} - \frac{3}{34}i$ or

$= 0{,}147 - 0{,}088i$

3.2 $\frac{3-5i}{2+i}$

$= \frac{3-5i}{2+i} \times \frac{2-i}{2-i}$

$= \frac{(3-5i)(2-i)}{(2+i)(2-i)}$

$= \frac{6-3i-10i+5i^2}{4-i^2}$

$= \frac{6-13i+5(-1)}{4-(-1)}$

$= \frac{1-13i}{5}$

$= \frac{1}{5} - \frac{13}{5}i$ or

$= 0{,}2 - 2{,}6i$

3.3 $\frac{1-2i}{i-5}$

$= \frac{1-2i}{-5+i} \times \frac{-5-i}{-5-i}$

$= \frac{(1-2i)(-5-i)}{(-5+i)(-5-i)}$

$= \frac{-5-i+10i+2i^2}{25-i^2}$

$= \frac{-5+9i+2(-1)}{25-(-1)}$

$= \frac{-5+9i-2}{26}$

$= \frac{-7+9i}{26}$

$= -\frac{7}{26} + \frac{9}{26}i$

or $-0{,}269 + 0{,}346i$

3.4 $\frac{1-7i}{-2-4i}$

$= \frac{1-7i}{-2-4i} \times \frac{-2+4i}{-2+4i}$

$= \frac{-2+4i+14i-28i^2}{4-16i^2}$

$= \frac{-2+18i+28}{4+16}$

$= \frac{26+18i}{20}$

$= \frac{26}{20} + \frac{18}{20i}$

$= \frac{13}{10} + \frac{9}{10}i$

$= 1{,}3 + 0{,}9i$

3.5 $\frac{\sqrt{2}+5i}{-3i+\sqrt{2}}$

$= \frac{\sqrt{2}+5i}{\sqrt{2}-3i}$

$= \frac{\sqrt{2}+5i}{\sqrt{2}-3i} \times \frac{\sqrt{2}+3i}{\sqrt{2}+3i}$

$= \frac{2+3\sqrt{2}i+5\sqrt{2}i+15i^2}{2-9i^2}$

$= \frac{2+8\sqrt{2}i-15}{2+9}$

$= -\frac{13}{11} + \frac{8\sqrt{2}}{11}i$ or

$= -1{,}182 + 1{,}029i$

4. 4.1 $(3 - i)^{-2}$

$= \frac{1}{(3 - i)^2}$

$= \frac{1}{(3 - i)(3 - i)}$

$= \frac{1}{9 - 6i + i^2}$

$= \frac{1}{9 - 6i - 1}$

$= \frac{1}{8 - 6i} \times \frac{8 + 6i}{8 + 6i}$

$= \frac{8 + 6i}{64 - 36i^2}$

$= \frac{8 + 6i}{64 + 36}$

$= \frac{8}{100} + \frac{6}{100i}$

$= 0{,}08 + 0{,}06i$

4.2 $\frac{2 - i}{(2 + i)^2}$

$= \frac{(2 - i)}{(2 + i.(2 + i)}$

$= \frac{2 - i}{4 + 4i + i^2}$

$= \frac{2 - i}{4 + 4i - 1}$

$= \frac{2 - i}{3 + 4i} \times \frac{3 - 4i}{3 - 4i}$

$= \frac{6 - 8i - 3i + 4i^2}{9 - 16i^2}$

$= \frac{6 - 11i - 4}{9 + 16}$

$= \frac{2 - 11i}{25}$

$= \frac{2}{25} - \frac{11}{25}i$ or

$= 0{,}08 - 0{,}44i$

4.3 $\frac{(3 - 2i)(-2 + i)}{3 - i}$

$= \frac{-6 + 3i + 4i - 2i^2}{3 - i}$

$= \frac{-6 + 7i - 2(-1)}{3 - i}$

$= \frac{-4 + 7i}{3 - i} \times \frac{3 + i}{3 + i}$

$= \frac{-12 - 4i + 21i + 7i^2}{9 - i^2}$

$= \frac{-12 + 17i - 7}{9 + 1}$

$= -\frac{19}{10} + \frac{17}{10}i$

$= -1{,}9 + 1{,}7i$

4.4 $\frac{(5 - 4i)(3 + i)}{i^3}$

$= \frac{15 + 5i - 12i - 4i^2}{(i^2)i}$

$= \frac{15 - 7i + 4}{(-1)i}$

$= \frac{19 - 7i}{-i} \times \frac{i}{i}$

$= \frac{19i - 7\,i^2}{-i^2}$

$= \frac{19i - 7(-1)}{-(-1)}$

$= \frac{19i + 7}{1}$

$= 7 + 19i$

4.5 $\frac{2-3i}{4-i} - \frac{1-i}{2+i}$

$= \left(\frac{2-3i}{4-i} \times \frac{4+i}{4+i}\right) - \left(\frac{1-i}{2+i} \times \frac{2-i}{2-i}\right)$

$= \left(\frac{8-10i-3i^2}{16-i^2}\right) - \left(\frac{2-3i+i^2}{4-i^2}\right)$

$= \left(\frac{8-10i+3}{17}\right) - \left(\frac{2-3i-1}{5}\right)$

$= \left(\frac{11-10i}{17}\right) - \left(\frac{1-3i}{5}\right)$

$= \frac{11}{17} - \frac{10}{17}i - \frac{1}{5} + \frac{3}{5}i$

$= 0{,}447 + 0{,}012i$

4.6 $\frac{(2+i)^3}{(3-4i)(2+i)}$

$= \frac{(2+i)^2}{3-4i}$

$= \frac{(2+i)(2+i)}{3-4i}$

$= \frac{4+4i+i^2}{3-4i}$

$= \frac{4+4i-1}{3-4i}$

$= \frac{3+4i}{3-4i} \times \frac{3+4i}{3+4i}$

$= \frac{9+24i+16i^2}{9-16i^2}$

$= \frac{9+24i+16(-1)}{9-16(-1)}$

$= \frac{-7+24i}{25}$

$= \frac{-7}{25} + \frac{24}{25}i$

$= -0{,}28 + 0{,}96i$

4.7 $\frac{2i^{20} - i^{19}}{3i-1}$

$= \frac{2(i^2)^{10} - (i^2)^9 i}{3i-1}$

$= \frac{2(-1)^{10} - (-1)^9 i}{3i-1}$

$= \frac{2+i}{3i-1}$

$= \frac{2+i}{-1+3i} \times \frac{-1-3i}{-1-3i}$

$= \frac{-2-6i-i-3i^2}{1-9i^2}$

$= \frac{-2-7i+3}{1+9}$

$= \frac{1-7i}{10}$

$= \frac{1}{10} - \frac{7}{10i}$ or

$= 0{,}1 - 0{,}7i$

4.8 $\frac{3i}{(1-i)^2}$

$= \frac{3i}{(1-i)(1-i)}$

$= \frac{3i}{1-2i+i^2}$

$= \frac{3i}{1-2i+(-1)}$

$= \frac{3i}{-2i} \times \frac{2i}{2i}$

$= \frac{6i^2}{-4i^2}$

$= \frac{6(-1)}{-4(-1)}$

$= \frac{-6}{4}$

$= -\frac{3}{2} + 0i$

4.9 $\dfrac{\sqrt{2} + 3i}{-3i + \sqrt{2}}$

$= \dfrac{\sqrt{2} + 3i}{\sqrt{2} - 3i} \times \dfrac{\sqrt{2} + 3i}{\sqrt{2} + 3i}$

$= \dfrac{2 + 3\sqrt{2}i + 3\sqrt{2}i + 9i^2}{2 - 9i^2}$

$= \dfrac{2 + 6\sqrt{2}i + 9(-1)}{2 - 9(-1)}$

$= \dfrac{2 + 6\sqrt{2}i - 9}{11}$

$= \dfrac{-7 + 6\sqrt{2}\,i}{11}$

$= -\dfrac{7}{11} + \dfrac{6\sqrt{2}}{11}i$

4.10 $\dfrac{(2 - 2i)(3 + i)}{-i + 2} - \dfrac{2 + 3i}{1 + i}$

$= \dfrac{6 + 2i - 6i - 2i^2}{2 - i} - \dfrac{2 + 3i}{1 + i}$

$= \dfrac{6 - 4i - 2(-1)}{2 - i} - \dfrac{2 + 3i}{1 + i}$

$= \left(\dfrac{2 + i}{2 + i} \times \dfrac{8 - 4i}{2 - i}\right) - \left(\dfrac{2 + 3i}{1 + i} \times \dfrac{1 - i}{1 - i}\right)$

$= \left(\dfrac{16 - 8i + 8i - 4i^2}{4 - i^2}\right) - \left(\dfrac{2 - 2i + 3i - 3i^2}{1 - i^2}\right)$

$= \left(\dfrac{16 + 4}{5}\right) - \left(\dfrac{2 + i + 3}{2}\right)$

$= \dfrac{20}{5} - \dfrac{5}{2} - \dfrac{1}{2}i$

$= \dfrac{15}{10} - \dfrac{1}{2}i$

$= 1\tfrac{1}{2} - \tfrac{1}{2}i$

5. $z_p = \dfrac{z_1 \times z_2}{z_1 + z_2} + z_3$

$z_p = \dfrac{(-3 + 2i)(-2 + 5i)}{(-3 + 2i) + (-2 + 5i)} + (-2 - 3i)$

$= \dfrac{6 - 15i - 4i + 10i^2}{-3 + 2i - 2 + 5i} + (-2 - 3i)$

$= \dfrac{6 - 19i + 10(-1)}{-5 + 7i} + (-2 - 3i)$

$= \dfrac{-4 - 19i}{-5 + 7i} \times \dfrac{-5 - 7i}{-5 - 7i} + (-2 - 3i)$

$= \dfrac{20 + 123i + 133i^2}{25 - 49i^2} + (-2 - 3i)$

$= \dfrac{20 + 123i + 133(-1)}{25 - 49(-1)} + (-2 - 3i)$

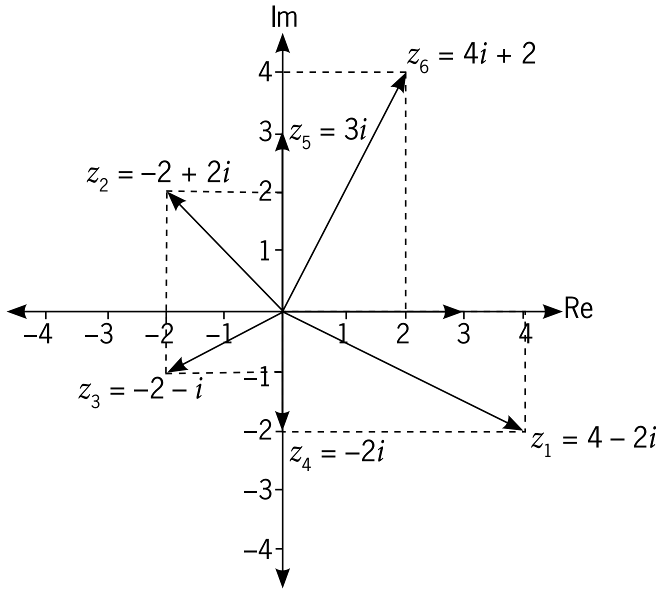
$= \dfrac{-113 + 123i}{74} + (-2 - 3i)$

$= \dfrac{-113}{74} + \dfrac{123i}{74} - 2 - 3i$

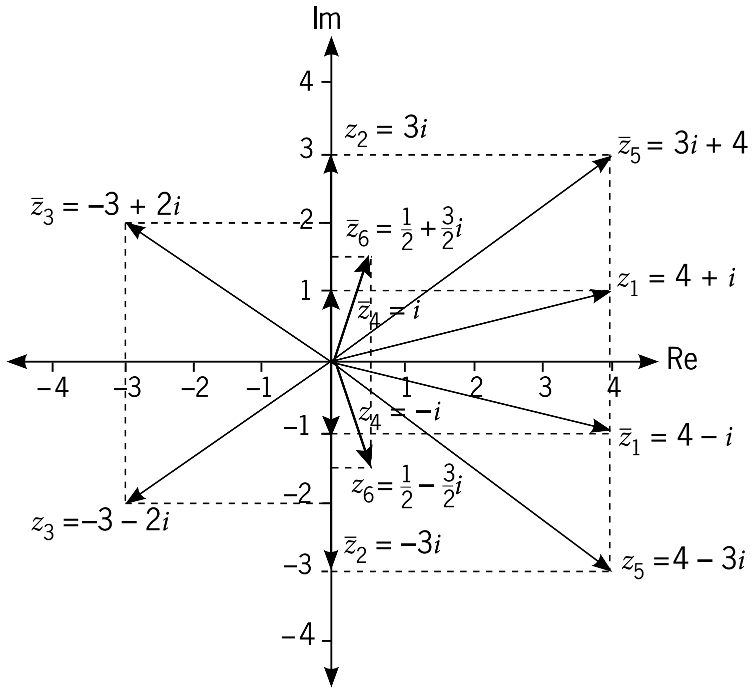
$= -3{,}53 - 1{,}34i$

Activity 2.6 SB page 68

1.



2.



Activity 2.7 SB page 80

1. 1.1 $z = -2 + 3i$ 1.2 $z = -2$

   1.3 $z = -i - 3$ 1.4 $z = 2 - 4i$

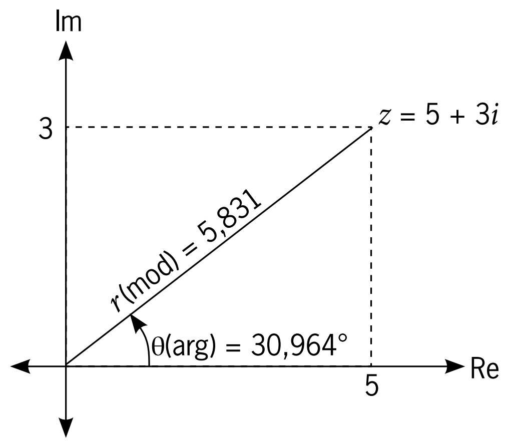
   1.5 $z = i + 6$ 1.6 $z = -5i$

2. 2.1 $z = 5 + 3i$

$r(\text{mod}) = \sqrt{(5)^2 + (3)^2}$
$= \sqrt{25 + 9}$
$= \sqrt{34}$
$= 5{,}831$

$\theta(\text{arg}) = \tan^{-1}\left(\frac{b}{a}\right)$
$= \tan^{-1}\left(\frac{3}{5}\right)$
$= 30{,}964°$

$r\underline{|\theta} = 5{,}831\underline{|30{,}964°}$



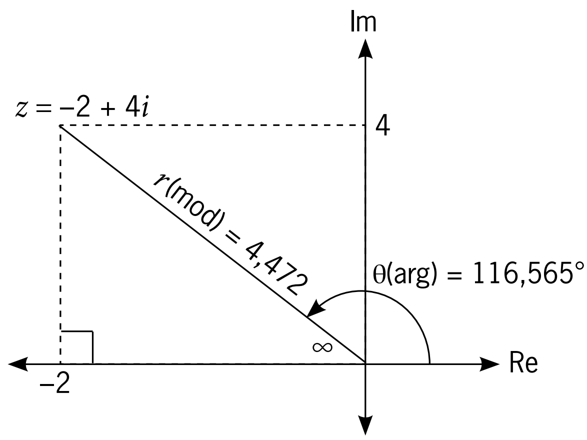
2.2 $z = 4i - 2$

$\therefore z = -2 + 4i$

$r(\text{mod}) = \sqrt{(-2)^2 + (4)^2}$
$= \sqrt{4 + 16}$
$= \sqrt{20}$
$= 4{,}472$

$\theta(\text{arg}) = 180° - \tan^{-1}\left(\frac{4}{2}\right)$
$= 180° - 63{,}435°$
$= 116{,}565°$

$r\underline{|\theta} = 4{,}472\underline{|116{,}565°}$



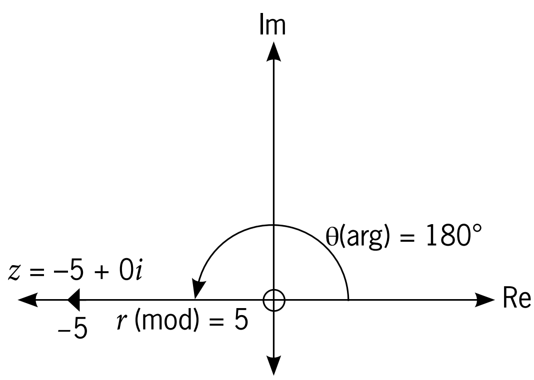
2.3 $z = -5$

$\therefore z = -5 + 0i$

$r(\text{mod}) = \sqrt{(-5)^2 + (0)^2}$
$= \sqrt{25}$
$= 5$

$\theta(\text{arg}) = 180° - \tan^{-1}\left(\frac{0}{5}\right)$
$= 180° - 0°$
$= 180°$

$r\underline{|\theta} = 5\underline{|180°}$



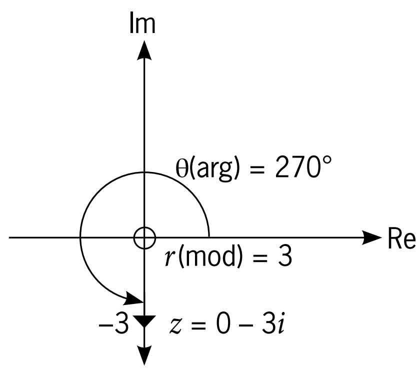
2.4 $z = -3i$

$\therefore z = 0 - 3i$

$r(\text{mod}) = \sqrt{(0)^2 + (-3)^2}$
$= \sqrt{9}$
$= 3$

$\theta(\text{arg}) = 270°$

$r\underline{|\theta} = 3\underline{|270°}$



2.5 $z = 3\sqrt[3]{8} - 5i$

$\therefore z = 6 - 5i$

$r(\text{mod}) = \sqrt{(6)^2 + (-5)^2}$

$= \sqrt{36 + 25}$

$= \sqrt{61}$

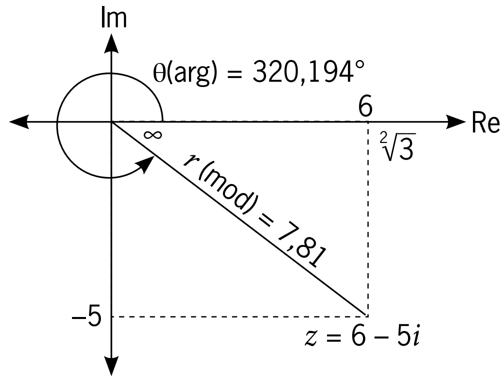
$= 7{,}81$

4[th] quadrant: $\theta(\text{arg}) = 360° - \tan^{-1}\left(\frac{5}{6}\right)$

$= 360° - 39{,}806°$

$= 320{,}194°$

$\therefore r\underline{|\theta} = 7{,}81\underline{|320{,}194°}$



3. 3.1 $r(\text{mod}) = \sqrt{(-1)^2 + (-\sqrt{3})^2}$

$= \sqrt{1 + 3}$

$= 2$

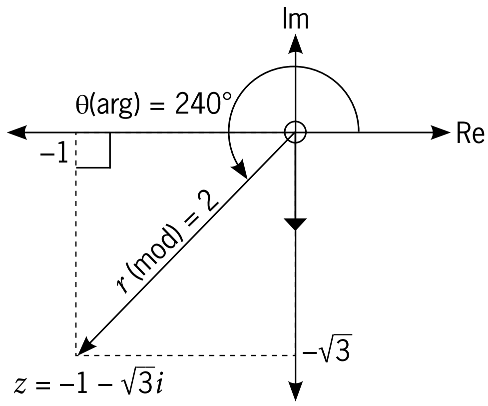
3.2 $\theta(\text{arg}) = 180° + \tan^{-1}\frac{\sqrt{3}}{1}$

$= 180° + 60°$

$= 240°$

3.3 $2\underline{|240°}$

3.4



4. 4.1 $z = -2 - 4i$

4.2 $r(\text{mod}) = \sqrt{(-2)^2 + (-4)^2}$

$= \sqrt{20}$

$= 4{,}472$

4.3 $\theta(\text{arg}) = 180° + \tan^{-1}(\frac{4}{2})$

$= 243{,}435°$

4.4 Negative argument = $-116{,}565°$

4.5 $4{,}472\underline{|243{,}435°}$

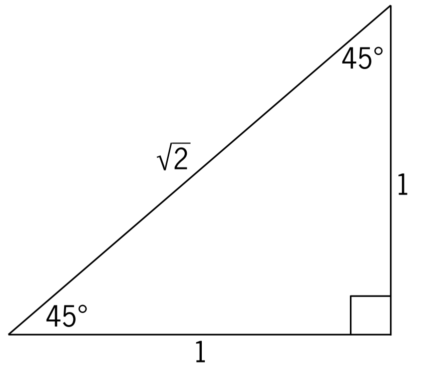
Activity 2.8 SB page 83

1. 1.1 $\sqrt{2}\angle 45^\circ$

$= \sqrt{2}(\cos 45^\circ + i \sin 45^\circ)$

$= \sqrt{2}(\frac{1}{\sqrt{2}} + i.\frac{1}{\sqrt{2}})$

$= 1 + i$

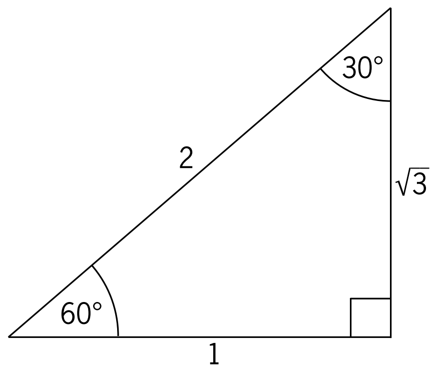


1.2 2 cis 30°

$= 2(\cos 30^\circ + i \sin 30^\circ)$

$= 2\ (\frac{\sqrt{3}}{2} + i.\frac{1}{2})$

$= \sqrt{3} + i$



1.3 $\sqrt{3}(\cos 60^\circ + i \sin 60^\circ)$

$= \sqrt{3}\ (\frac{1}{2} + i.\frac{\sqrt{3}}{2})$

$= \frac{\sqrt{3}}{2} + \frac{3}{2}i$

1.4 $\angle 45^\circ$

$= 1(\cos 45^\circ + i \sin 45^\circ)$

$= 1(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ or

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

1.5 $\frac{\sqrt{3}}{2}$ cis 60°

$= \frac{\sqrt{3}}{2}(\cos 60^\circ + i \sin 60^\circ)$

$= \frac{\sqrt{3}}{2}(\frac{1}{2} + i.\frac{\sqrt{3}}{2})$

$= \frac{\sqrt{3}}{4} + \frac{3}{4}i$

2. 2.1 $4\angle 135^\circ$

$= 4(\cos 135^\circ + i \sin 135^\circ)$

$= 4(-0{,}707 + i.0{,}707)$

$= -2{,}828 + 2{,}828i$

2.2 $\angle 90^\circ$

$= 1(\cos 90^\circ + i \sin 90^\circ)$

$= 0 + i$

2.3 $2\angle -300^\circ$

$= 2(\cos(-300^\circ) + i \sin(-300^\circ))$

$= 2(0{,}5 + i.0{,}866)$

$= 1 + 1{,}732i$

2.4 $\sqrt{2}(\cos 140^\circ + i \sin 140^\circ)$

$= \sqrt{2}(-0{,}766 + i.0{,}643)$

$= -1{,}083 + 0{,}909i$

2.5 $3{,}4 \text{ cis } 330°$
$= 3{,}4(\cos 330° + i \sin 330°)$
$= 3{,}4(0{,}866 + i(-0{,}5))$
$= 2{,}944 - 1{,}7i$

2.6 $5 \text{ cis } (-220°)$
$= 5(\cos(-220°) + i \sin(-220°))$
$= 5(0{,}766 + i(0{,}643))$
$= 3{,}83 + 3{,}215i$

Activity 2.9 SB page 91

1. $z_1 = 6 \text{ cis } 90°;\ z_2 = 2 \text{ cis } 30°$
$z_1 = 6(\cos 90° + i \sin 90°)$
$= 0 + 6i$
$z_2 = 2(\cos 30° + i \sin 30°)$
$= 2\left(\frac{\sqrt{3}}{2} + i.\frac{1}{2}\right)$
$= \sqrt{3} + i$
$\therefore z_1 + z_2 = 0 + 6i + \sqrt{3} + i$
$= \sqrt{3} + 7i$

2. 2.1 $8\underline{|30°} \times 2\ \underline{|60°}$
$= 8 \times 2\underline{|30° + 60°}$
$= 16\underline{|90°}$

2.2 $(4\underline{|50°})(6\underline{|30°})(\frac{1}{2}\underline{|-25°})$
$= 4 \times 6 \times \frac{1}{2}\underline{|50° + 30° - 25°}$
$= 12\underline{|55°}$

2.3 $\frac{\sqrt{3}\underline{|86{,}34°}}{\sqrt{2}\underline{|40{,}44°}}$
$= \frac{\sqrt{3}}{\sqrt{2}}\underline{|86{,}34° - 40{,}44°}$
$= 1{,}225\underline{|45{,}9°}$

2.4 $\frac{6{,}8 \text{ cis } 40°}{4{,}2 \text{ cis } -130°}$
$= \frac{6{,}8\underline{|40°}}{4{,}2\underline{|-130°}}$
$= \frac{6{,}8}{4{,}2}\underline{|40° - (-130°)}$
$= 1{,}619\underline{|170°}$

2.5 $\frac{2 \text{ cis } 55° \times 6{,}2 \text{ cis } 67{,}3°}{0{,}2 \text{ cis } 22° \times 4 \text{ cis } (-220°)}$
$= \frac{2\underline{|55°} \times 6{,}2\underline{|67{,}3°}}{0{,}2\underline{|22°} \times 4\underline{|-220°}}$
$= \frac{2 \times 6{,}2\underline{|55° + 67{,}3°}}{0{,}2 \times 4\underline{|22° - 220°}}$
$= \frac{12{,}4}{0{,}8}\underline{|122{,}3° - (-198°)}$
$= 15{,}5\underline{|320{,}3°}$

3. 3.1 $\sqrt{3}\left(\cos 30° + i \sin 30°\right)$
$= \sqrt{3}\left(\frac{\sqrt{3}}{2} + i.\frac{1}{2}\right)$
$= \frac{3}{2} + \frac{\sqrt{3}}{2}i$

3.2 $\underline{|45°}$
$= 1(\cos 45° + i \sin 45°)$
$= 1\left(\frac{1}{\sqrt{2}} + i.\frac{1}{\sqrt{2}}\right)$
$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ or $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

3.3 $\sqrt{3}\angle 60° + \sqrt{2}\angle 45°$

$= \sqrt{3}(\cos 60° + i \sin 60°) + \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$

$= \sqrt{3}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$

$= \frac{\sqrt{3}}{2} + \frac{\sqrt{9}}{2}i + \frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}}i$

$= \frac{\sqrt{3}}{2} + \frac{3}{2}i + 1 + i$

$= \left(\frac{\sqrt{3}}{2} + 1\right) + \left(\frac{3}{2} + 1\right)i$

$= \frac{\sqrt{3}+2}{2} + \frac{5}{2}i$

3.4 $3\angle 45°.2 \text{ cis } -15°$

$= 3 \times 2\angle 45° - 15°$

$= 6\angle 30°$

$= 6(\cos 30° + i \sin 30°)$

$= 6\left(\frac{\sqrt{3}}{2} + i.\frac{1}{2}\right)$

$= 3\sqrt{3} + 3i$

3.5 $\frac{9 \text{ cis } 90°}{\frac{1}{3} \text{ cis } 30°}$

$= \frac{9}{\frac{1}{3}}\angle 90° - 30°$

$= 27\angle 60°$

$= 27(\cos 60° + i \sin 60°)$

$= 27\left(\frac{1}{2} + i.\frac{\sqrt{3}}{2}\right)$

$= \frac{27}{2} + \frac{27\sqrt{3}}{2}i$

4. 4.1 $2 \text{ cis } 30° - 3 \text{ cis } 70°$

$= 2(\cos 30° + i \sin 30°) - 3(\cos 70° + i \sin 70°)$

$= 1{,}732 + i - 1{,}026 - 2{,}819i$

$= 0{,}706 - 1{,}819i$

4.2 $0{,}59 \text{ cis } 314{,}27° \times 0{,}74 \text{ cis } 16{,}3°$

$= 0{,}59 \times 0{,}74\angle 314{,}27° + 16{,}3°$

$= 0{,}437\angle 330{,}57°$

$= 0{,}437(\cos 330{,}57° + i \sin 330{,}57°)$

$= 0{,}381 - 0{,}215i$

4.3 $\frac{8\angle 45°.2\angle 60°}{3\angle -75°}$

$= \frac{8 \times 2\angle 45° + 60°}{3\angle -75°}$

$= \frac{16\angle 105°}{3\angle -75°}$

$= \frac{16}{3}\angle 105° - (-75°)$

$= 5{,}333\angle 180°$

$= 5{,}333(\cos 180° + i \sin 180°)$

$= -5{,}333 + 0i$

4.4 $\frac{2 \text{ cis } 40°.3 \text{ cis } 60°}{3\angle 20°.2\angle -35°}$

$= \frac{2\angle 40°.3\angle 60°}{3\angle 20°.2\angle -35°}$

$= \frac{2 \times 3\angle{40° + 60°}}{3 \times 2\angle{20° + (-35°)}}$

$= \frac{6\angle{100°}}{6\angle{-15°}}$

$= \angle{100° - (-15°)}$

$= \angle{115°}$

$= (\cos 115° + i \sin 115°)$

$= -0{,}423 + 0{,}906i$

4.5 $\frac{3\angle{130°}.4\angle{40°}}{6\angle{20°}.8\angle{80°}}$

$= \frac{3 \times 4\angle{130° + 40°}}{6 \times 8\angle{20° + 80°}}$

$= \frac{12\angle{170°}}{48\angle{100°}}$

$= \frac{12}{48}\angle{170° - 100°}$

$= 0{,}25\angle{70°}$

$= 0{,}25(\cos 70° + i \sin 70°)$

$= 0{,}086 + 0{,}235i$

5. $\frac{2 + i}{10 \text{ cis } 20°}$

$= \frac{\sqrt{5}\angle{26{,}565°}}{10\angle{20°}}$

$= \frac{\sqrt{5}}{10}26{,}565° - 20°$

$= 0{,}224\angle{6{,}565°}$

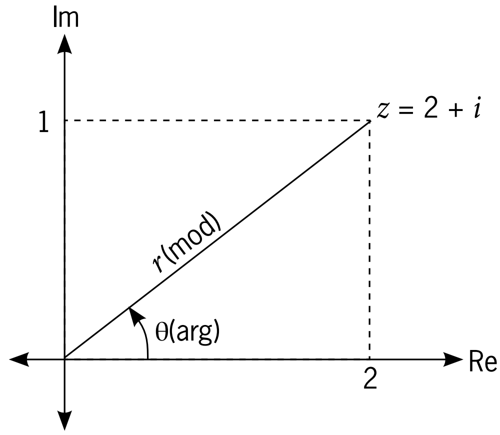
$r(\text{mod}) = \sqrt{(2)^2 + (-1)^2}$

$= \sqrt{5}$

$\theta(\text{arg}) = \tan^{-1}\left(\frac{1}{2}\right)$

$= 26{,}565°$

$\therefore r\angle{\theta} = \sqrt{5}\angle{26{,}565°}$



6. $(-3 - i)(-2 + i)$

$(-3 - i)$

$r(\text{mod}) = \sqrt{(-3)^2 + (-1)^2}$

$= \sqrt{9 + 1}$

$= \sqrt{10}$

$= 3{,}162$

$\theta(\text{arg}) = 180° + \tan^{-1}\left(\frac{1}{3}\right)$

$= 198{,}435°$

$(-2 + i)$

$r(\text{mod}) = \sqrt{(-2)^2 + (1)^2}$

$= \sqrt{4 + 1}$

$= \sqrt{5}$

$= 2{,}236$

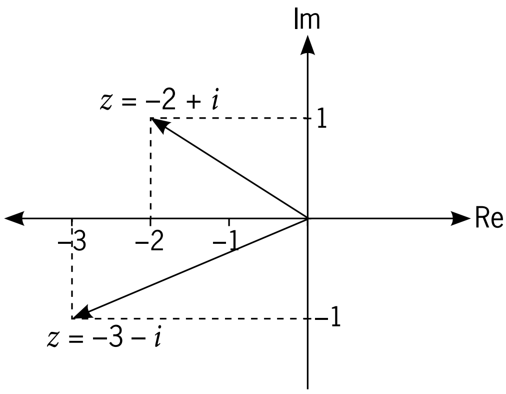
$\theta(\text{arg}) = 180° - \tan^{-1}\left(\frac{1}{2}\right)$

$= 153{,}435°$

$\therefore r\angle\theta . r\angle\theta$

$= 3{,}162 . 2{,}236 \angle 198{,}435° + 153{,}435°$

$= 7{,}071 \angle 351{,}78°$



Activity 2.10 | SB page 97

1\.

1.1 $(3\angle 50°)^4$

$= 3^4 \angle 50° \times 4$

$= 81 \angle 200°$

1.2 $(2{,}5 \text{ cis } 60{,}3°)^5$

$= (2{,}5 \angle 60{,}3°)^5$

$= 2{,}5^5 \angle 60{,}3° \times 5$

$= 97{,}656 \angle 301{,}5°$

1.3 $(\sqrt{4{,}2} \angle -80°)^3$

$= (\sqrt{4{,}2})^3 \angle -80° \times 3$

$= 8{,}607 \angle -240°$

1.4 $\frac{(5\angle 85°)^3}{(2\angle 20°)^2}$

$= \frac{5^3 \angle 85° \times 3}{2^2 \angle 20° \times 2}$

$= \frac{125 \angle 255°}{4 \angle 40°}$

$= \frac{125}{4} \angle 255° - 40°$

$= 31{,}25 \angle 215°$

1.5 $\frac{4 \text{ cis } 45° \times (3 \text{ cis } 60°)^3}{(2 \text{ cis } -50°)^2}$

$= \frac{4 \text{ cis } 45° \times 27 \text{ cis } 180°}{4 \text{ cis } -100°}$

$= \frac{4 \times 27 \angle 45° + 180°}{4 \angle -100°}$

$= \frac{108}{4} \angle 225° - (-100°)$

$= 27 \angle 325°$

2. $z_1 = 21 \text{ cis } 120°$

$z_2 = 3 \text{ cis } 80°$

$z_3 = \underline{|-108°}$

$$\frac{(z_1)(z_2)^3}{(z_3)^2} = \frac{(21\underline{|120°})(3\underline{|80°})^3}{(\underline{|-108°})^2}$$

$$= \frac{(21\underline{|120°})(27\underline{|240°})}{\underline{|-216°}}$$

$$= \frac{21 \times 27\underline{|120° + 240°}}{\underline{|-216°}}$$

$$= \frac{567\underline{|360°}}{\underline{|-216°}}$$

$$= 567\underline{|360° + 216°}$$

$$= 567\underline{|576°}$$

$$= 567\underline{|216°}$$

3. 3.1 $\left(\frac{6\underline{|50°}}{3\underline{|25°}}\right)^4 \times \left(\frac{2\underline{|60°}}{4\underline{|190°}}\right)^{-3}$

$$= \left(\frac{6\underline{|50°}}{3\underline{|25°}}\right)^4 \times \left(\frac{4\underline{|190°}}{2\underline{|60°}}\right)^3$$

$$= (2\underline{|25°})^4 \times (2\underline{|130°})^3$$

$$= (2^4\underline{|25° \times 4})(2^3\underline{|130° \times 3})$$

$$= (16\underline{|100°})(8\underline{|390°})$$

$$= 16 \times 8\underline{|100° + 390°}$$

$$= 128\underline{|490°}$$

$$= 128\underline{|130°}$$

3.2 $[2(\cos 30° + i \sin 30°)]^3$

$$= (2\underline{|30°})^3$$

$$= 2^3\underline{|30° \times 3}$$

$$= 8\underline{|90°}$$

3.3 $\left(\frac{3 \text{ cis } 78°}{2 \text{ cis } 35°}\right)^3 \times \left(\frac{3 \text{ cis } 25°}{4 \text{ cis } 120°}\right)^{-2}$

$$= \left(\frac{3 \text{ cis } 78°}{2 \text{ cis } 35°}\right)^3 \times \left(\frac{4 \text{ cis } 120°}{3 \text{ cis } 25°}\right)^2$$

$$= \left(\frac{3^3\underline{|78° \times 3}}{2^3\underline{|35° \times 3}}\right) \times \left(\frac{4^2\underline{|120° \times 2}}{3^2\underline{|25° \times 2}}\right)$$

$$= \frac{27\underline{|234°}}{8\underline{|105°}} \times \frac{16\underline{|240°}}{9\underline{|50°}}$$

$$= \left(\frac{27}{8}\underline{|234° - 105°}\right) \times \left(\frac{16}{9}\underline{|240° - 50°}\right)$$

$$= \left(\frac{27}{8}\underline{|129°}\right)\left(\frac{16}{9}\underline{|190°}\right)$$

$$= \left(\frac{27}{8}\right)\left(\frac{16}{9}\right)\underline{|129° + 190°}$$

$$= 6\underline{|319°}$$

4. 4.1 $\dfrac{(\overline{z}_1 . \overline{z}_2)^2}{\overline{\overline{z}}_4} \times z_3$

$= \dfrac{\left(\frac{1}{3}\angle 35^\circ \, . \, 5 \angle -105^\circ\right)^2}{\overline{4\angle 210^\circ}} \times \sqrt{2}\angle 45^\circ$

$= \dfrac{\left(\frac{1}{3} \times 5\angle 35^\circ - 105^\circ\right)^2}{\overline{4\angle 210^\circ}} \times \sqrt{2}\angle 45^\circ$

$= \dfrac{\left(\frac{5}{3}\angle -70^\circ\right)^2}{\overline{4\angle 210^\circ}} \times \sqrt{2}\angle 45^\circ$

$= \dfrac{\left(\frac{5}{3}\angle 70^\circ\right)^2}{4\angle -210^\circ} \times \sqrt{2}\angle 45^\circ$

$= \dfrac{\left(\frac{5}{3}\right)^2 \angle 2 \times 70^\circ}{4\angle -210^\circ} \times \sqrt{2}\angle 45^\circ$

$= \dfrac{\frac{25}{9}\angle 140^\circ}{4\angle -210^\circ} \times \sqrt{2}\angle 45^\circ$

$= \dfrac{\frac{25}{9}}{4}\angle 140^\circ + 210^\circ \times \sqrt{2}\angle 45^\circ$

$= \frac{25}{36}\angle 350^\circ \times \sqrt{2}\angle 45^\circ$

$= \frac{25}{36} \times \sqrt{2}\angle 350^\circ + 45^\circ$

$= \frac{25\sqrt{2}}{36}\angle 395^\circ$

$= \frac{25\sqrt{2}}{36}\angle 35^\circ$

4.2 $\dfrac{(z_1 + z_3)^4}{\overline{z}_2} \div \dfrac{1}{z_4}$

$= \dfrac{\left(\frac{1}{3}\angle 35^\circ + \sqrt{2}\angle 45^\circ\right)^4}{\overline{5\angle -105^\circ}} \div \dfrac{1}{4\angle 210^\circ}$

$= \dfrac{\left[\frac{1}{3}(\cos 35^\circ + i \sin 35^\circ) + \sqrt{2}(\cos 45^\circ + i \sin 45^\circ)\right]^4}{\overline{5\angle -105^\circ}} \div \dfrac{1}{4\angle 210^\circ}$

$= \dfrac{\left[\left(\frac{1}{3}\cos 35^\circ + \sqrt{2}\cos 45^\circ\right) + i\left(\frac{1}{3}\sin 35^\circ + \sqrt{2}\sin 45^\circ\right)\right]^4}{\overline{5\angle -105^\circ}} \div \dfrac{1}{4\angle 210^\circ}$

$= \dfrac{[1{,}273 + 1{,}191i]^4}{5\angle 105^\circ} \times 4\angle 210^\circ$

$= \dfrac{(1{,}743\angle 43{,}097^\circ)^4}{5\angle 105^\circ} \times 4\angle 210^\circ$

$= \dfrac{(1{,}743)^4\angle 4 \times 43{,}097^\circ}{5\angle 105^\circ} \times 4\angle 210^\circ$

$= \dfrac{9{,}239\angle 172{,}390^\circ}{5\angle 105^\circ} \times 4\angle 210^\circ$

$= \frac{9{,}239}{5}\angle 172{,}390^\circ - 105^\circ \times 4\angle 210^\circ$

$= 1{,}848\angle 67{,}390^\circ \times 4\angle 210^\circ$

$= 1{,}848 \times 4\angle 67{,}390^\circ + 210^\circ$

$= 7{,}391\angle 277{,}390^\circ$

Activity 2.11 SB page 106

1\. 1.1 $x - yi = -3 + i$
$\therefore x = -3;\ -y = 1$
$y = -1$
or $(-3; -1)$

1.2 $-3 + 2i = x + 4yi$
$\therefore -3 = x; \quad 2 = 4y$
$\therefore x = -3$ and $y = \frac{1}{2}$
or $\left(-3; \frac{1}{2}\right)$

1.3 $3x - 2i = -yi$
$\therefore 3x = 0; \quad -2 = -y$
$\therefore x = 0$ and $y = 2$
$\therefore (0; 2)$

1.4 $-2x - 8yi = -10 + 16i$
$\therefore -2x = -10; \quad -8y = 16$
$\therefore x = 5$ and $y = -2$
$\therefore (5; -2)$

1.5 $3x + 2yi - 5 = 4 + 6i$
$(3x - 5) + 2yi = 4 + 6i$
$\therefore 3x - 5 = 4; \quad 2y = 6$
$3x = 9; \quad y = 3$
$x = 3$
$\therefore (3; 3)$

1.6 $2x + 3 + i(y + 5) = x - y + ix + iy$
$(2x + 3) + i(y + 5) = (x - y) + i(x + y)$
$\therefore 2x + 3 = x - y; \quad y + 5 = x + y$
$x + y = -3; \quad x = 5$
$\therefore x + y = -3$ … substitute $x = 5$
$5 + y = -3$
$y = -8$ and $x = 5$
$\therefore (5; -8)$

1.7 $4 + 5i = x + yi - (1 + i)$
$= x + yi - 1 - i$
$4 + 5i = (x - 1) + i(y - 1)$
$\therefore 4 = x - 1; \quad 5 = y - 1$
$\therefore x = 5$ and $y = 6$
$\therefore (5; 6)$

1.8 $(3 - 2i)^2 = x - yi$
$(3 - 2i)(3 - 2i) = x - yi$
$9 - 12i + 4i^2 = x - yi$
$9 - 12i + 4(-1) = x - yi$
$5 - 12i = x - yi$
$\therefore 5 = x; \quad -12 = -y$
$\therefore x = 5$ and $y = 12$
$\therefore (5; 12)$

1.9 $(i + 1)^2 + (3 + i)i = x + y + 4yi$

$(i + 1)(i + 1) + 3i + i^2 = (x + y) + 4yi$

$i^2 + 2i + 1 + 3i + i^2 = (x + y) + 4yi$

$(-1) + 5i + 1 + (-1) = (x + y) + 4yi$

$5i - 1 = (x + y) + 4yi$

$\therefore 5 = 4y; \qquad -1 = x + y$

$y = \frac{5}{4}; \qquad x = -1 - y$

$\therefore x = -1 - \frac{5}{4}$ … substitute $y = \frac{5}{4}$

$\therefore x = -\frac{9}{4}$

$\therefore y = \frac{5}{4}$ or $1\frac{1}{4}$ and $x = -\frac{9}{4}$ or $-2\frac{1}{4}$

$\therefore \left(-\frac{9}{4}; \frac{5}{4}\right)$

2. 2.1 $i(x - iy) = i(y - i9) - 3x - i$

$xi - yi^2 = yi - 9i^2 - 3x - i$

$xi - y(-1) = yi - 9(-1) - 3x - i$

$xi + y = yi + 9 - 3x - i$

$xi + y = (9 - 3x) + i(y - 1)$

$\therefore x = y - 1$

$y = 9 - 3x$ … (1)

$y = x + 1$ … (2)

(2) = (1): $x + 1 = 9 - 3x$

$4x = 8$

$x = 2$

$\therefore y = x + 1$ … (2)

$y = 2 + 1$

$y = 3$

$\therefore (2; 3)$

2.2 $(1 + i)(x - iy) = (2 + 3i)^2$

$x - iy + ix - i^2y = (2 + 3i)(2 + 3i)$

$x - iy + ix - (-1)y = 4 + 12i + 9i^2$

$x - iy + ix + y = 4 + 12i + 9(-1)$

$(x + y) + i(-y + x) = -5 + 12i$

$\therefore x + y = -5$ … (1); $-y + x = 12$ … (2)

$\therefore$ (1) + (2): $2x = 7$

$x = \frac{7}{2}$ or $3\frac{1}{2}$

$\therefore x + y = -5$

$\frac{7}{2} + y = -5$

$y = -5 - \frac{7}{2}$

$\therefore y = -\frac{17}{2}$ or $-8\frac{1}{2}$

$\therefore \left(3\frac{1}{2}; -8\frac{1}{2}\right)$

2.3 $(5-2i)(x+yi)=\frac{1+i}{1-i}$

$$5x+5yi-2xi-2yi^2=\frac{1+i}{1-i}\times\frac{1+i}{1+i}$$

$$5x+5yi-2xi-2y(-1)=\frac{1+2i+i^2}{1-i^2}$$

$$(5x+2y)+i(5y-2x)=\frac{1+2i+(-1)}{1-i^2}$$

$$=\frac{2i}{2}$$

$$\therefore (5x+2y)+i(5y-2x)=0+i$$

$$\therefore 5x+2y=0;\ 5y-2x=1$$

$$5x=-2y$$

$$x=-\tfrac{2}{5}y$$

$$\therefore 5y-2(-\tfrac{2}{5})\ y=1$$

$$5y+\tfrac{4}{5}y=1$$

$$\tfrac{29}{5}y=1$$

$$y=\tfrac{5}{29}\ \text{ or }\ 0{,}172$$

$$\therefore x=-\tfrac{2}{5}\left(\tfrac{5}{29}\right)$$

$$x=-\tfrac{2}{29}$$

$$=-0{,}69$$

$$\therefore (-0{,}69;\ 0{,}172)\ \text{ or }\ \left(-\tfrac{2}{29};\ \tfrac{5}{29}\right)$$

2.4 $\frac{2+i}{3-i}=\frac{2y-i}{x}$

$$\frac{3+i}{3+i}\times\frac{2+i}{3-i}=\frac{2y}{x}-\frac{1}{x}i$$

$$\frac{6+5i+i^2}{9-i^2}=\frac{2y}{x}-\frac{1}{x}i$$

$$\frac{6+5i+(-1)}{9-(-1)}=\frac{2y}{x}-\frac{1}{x}i$$

$$\frac{5+5i}{10}=\frac{2y}{x}-\frac{1}{x}i$$

$$\tfrac{1}{2}+\tfrac{1}{2}i=\frac{2y}{x}-\frac{1}{x}i$$

$$\therefore \tfrac{1}{2}=\frac{2y}{x};\quad \tfrac{1}{2}=\frac{1}{x}$$

$$\therefore x=2$$

$$\therefore 2y=\tfrac{1}{2}.2$$

$$y=\tfrac{1}{2}$$

$$\therefore \left(2;\ \tfrac{1}{2}\right)$$

3. 3.1 $(3 - 4i)^2 = \frac{a + bi}{i^2}$

$(3 - 4i)(3 - 4i) = \frac{a + bi}{(-1)}$

$9 - 24i + 16i^2 = -a - bi$

$9 - 24i + 16(-1) = -a - bi$

$-7 - 24i = -a - bi$

$\therefore -7 = -a; \quad -24 = -b$

$\therefore a = 7$ and $b = 24$

3.2 $(a + bi) = \frac{(3 + 5i)(2 - 5i)}{1 - 3i}$

$a + bi = \frac{6 - 15i + 10i - 25i^2}{1 - 3i}$

$= \frac{6 - 5i - 25(-1)}{1 - 3i}$

$= \frac{6 - 5i + 25}{1 - 3i}$

$= \frac{31 - 5i}{1 - 3i} \times \frac{1 + 3i}{1 + 3i}$

$= \frac{31 + 93i - 5i - 15i^2}{1 - 9i^2}$

$= \frac{31 + 88i - 15(-1)}{1 - 9(-1)}$

$a + bi = \frac{46}{10} + \frac{88}{10}i$

$\therefore a = \frac{46}{10}$ and $b = \frac{88}{10}$

$a = 4{,}6$ and $b = 8{,}8$

3.3 $a - bi = \frac{5 - i^5}{1 + i}$

$= \frac{5 - (i^2)^2 i}{1 + i}$

$= \frac{5 - (-1)^2 i}{1 + i}$

$= \frac{5 - i}{1 + i} \times \frac{1 - i}{1 - i}$

$= \frac{5 - 6i + i^2}{1 - i^2}$

$= \frac{5 - 6i + (-1)}{1 - (-1)}$

$= \frac{4}{2} - \frac{6}{2}i$

$\therefore a - bi = 2 - 3\text{i}$

$a = 2; -b = -3$

$b = 3$

3.4 $a + bi = \frac{2 - 3i}{1 + i} + \frac{1 - 2i}{1 + 2i}$

$a + bi = \left(\frac{1 - i}{1 - i} \times \frac{2 - 3i}{1 + i}\right) + \left(\frac{1 - 2i}{1 + 2i} \times \frac{1 - 2i}{1 - 2i}\right)$

$a + bi = \left(\frac{2 - 5i + 3i^2}{1 - i^2}\right) + \left(\frac{1 - 4i + 4i^2}{1 - 4i^2}\right)$

$a + bi = \left(\frac{2 - 5i + 3(-1)}{1 - (-1)}\right) + \left(\frac{1 - 4i + 4(-1)}{1 - 4(-1)}\right)$

$a + bi = \frac{-1}{2} - \frac{5}{2}i + \left(-\frac{3}{5} - \frac{4}{5}i\right)$

$a + bi = -\frac{1}{2} - \frac{5}{2}i - \frac{3}{5} - \frac{4}{5}i$

$a + bi = -\frac{11}{10} - \frac{33}{10}i$

$\therefore a = -\frac{11}{10}$ and $b = -\frac{33}{10}$

$= -1{,}1 \qquad = -3{,}3$

4. $x + y + xi - yi = 8{,}944\angle 63{,}436°$

RHS: $8{,}944\angle 63{,}436° = r(\cos\theta + i\sin\theta)$

$= 8{,}944(\cos 63{,}436° + i\sin 63{,}436°)$

$= 8{,}944(0{,}447 + i\,0{,}894)$

$= 3{,}998 + 8{,}044i$

$\approx 4 + 8i$

LHS: $x + y + xi - yi = (x + y) + (x - y)i$

$\therefore (x + y) + (x - y)i = 4 + 8i$

$x + y = 4 \quad \ldots (1)$

$x - y = 8 \quad \ldots (2)$

$2x = 12 \quad \ldots (3)$

$\therefore x = 6$

Substitute $x = 6$ into (1):

$6 + y = 4$

$\therefore y = -2$

$\therefore (x; y) = (6\,;-2)$

5. $\begin{vmatrix} a+b & 2a-b \\ i & 2 \end{vmatrix} = -5 - 7i$

$(a + b)(2) - (2a - b)(i) = -5 - 7i$

$(2a + 2b) - (2a - b)i = -5 - 7i$

$2a + 2b = -5$ and $-(2a - b) = -7$

$2a + 2(2a - 7) = -5 \qquad b = 2a - 7$

$2a + 4a - 14 = -5 \qquad = 2\left(\frac{3}{2}\right) - 7$

$6a - 14 = -5 \qquad = 3 - 7$

$6a = 9 \qquad = -4$

$a = \frac{3}{2}$

$a = \frac{3}{2};\ b = -4$

Summative assessment: Module 2 — SB page 107

1. 1.1 $12i^2 + 7 + i^{11} + (2i)^3 + 4i$

$= 12(-1) + 7 + (i^2)^5 i + 8(i^2)i + 4i$

$= -12 + 7 + (-1)^5 i + 8(-1)i + 4i$

$= -12 + 7 - i - 8i + 4i$

$= -5 - 5i$ (3)

1.2 $(5 - \sqrt{-50}) - (3 - \sqrt{-8})$

$= (5 - \sqrt{25.2}.\sqrt{-1}) - (3 - \sqrt{8}.\sqrt{-1})$

$= 5 - 5\sqrt{2}i - 3 + \sqrt{4.2}i$

$= 5 - 5\sqrt{2}i - 3 + 2\sqrt{2}i$

$= 2 - 3\sqrt{2}i$ or $2 - 4{,}243i$ (3)

1.3 $\sqrt{-9} - \sqrt{-169} + \sqrt{49}$

$= \sqrt{9}.\sqrt{-1} - \sqrt{169}.\sqrt{-1} + 7$

$= 3i - 13i + 7$

$= 7 - 10i$ (2)

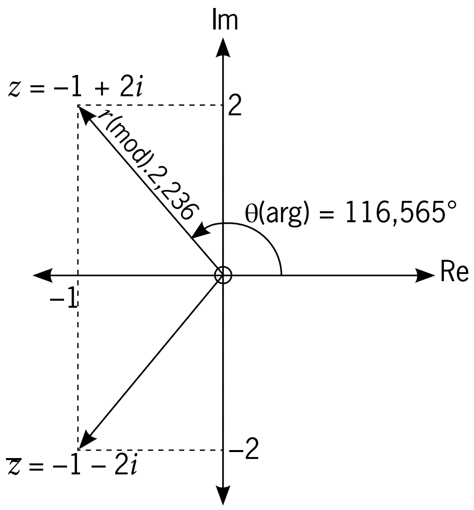
2. $z = \frac{-3 + 6i}{3}$

$\therefore z = -1 + 2i$

2.1 $\bar{z} = -1 - 2i$

or $\bar{z} = \frac{-3 - 6i}{3}$ (2)

2.2 and 2.5



(3)

2.3 $r(\text{mod}) = \sqrt{(-1)^2 + (2)^2}$

$= \sqrt{1 + 4}$

$= \sqrt{5}$

$= 2{,}236$ (2)

2.4 $\theta(\text{arg}) = 180° - \tan^{-1}\left(\frac{2}{1}\right)$

$= 116{,}565°$ (2)

2.5 On Argand diagram (2)

2.6 $z = 2{,}236\underline{|116{,}565°}$ (2)

3. 3.1 $(2 - 3i)(4 - 5i)$

$= 8 - 10i - 12i + 15i^2$

$= 8 - 22i + 15(-1)$

$= 8 - 22i - 15$

$= -7 - 22i$ (3)

3.2 $\frac{-4 + 3i}{-i + 3}$

$= \frac{-4 + 3i}{3 - i} \times \frac{3 + i}{3 + i}$

$= \frac{-12 - 4i + 9i + 3\,i^2}{9 - i^2}$

$= \frac{-12 + 5i + 3(-1)}{9 - (-1)}$

$= \frac{-12 + 5i - 3}{10}$

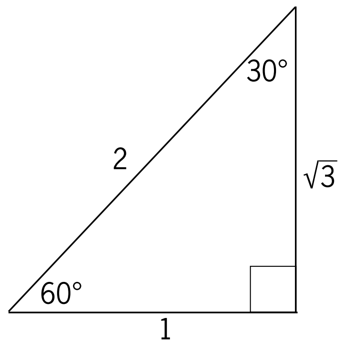
$= \frac{-15 + 5i}{10}$

$= -\frac{3}{2} + \frac{1}{2}i$ or $-1{,}5 + 0{,}5i$ (4)

4. $\sqrt{2}(\cos 60° + i \sin 60°)$

$= \sqrt{2}\left(\frac{1}{2} + i.\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$



(3)

5. 5.1 $2 \text{ cis } 120° + 3 \text{ cis } 65°$

$= 2(\cos 120° + i \sin 120°) + 3(\cos 65° + i \sin 65°)$

$= 2(-0{,}5 + i.0{,}866) + 3(0{,}423 + i.0{,}906)$

$= -1 + 1{,}732i + 1{,}268 + 2{,}72i$

$= 0{,}268 + 4{,}452i$ (3)

5.2 $\frac{5 \text{ cis } 145°}{\sqrt{2} \text{ cis } -45°}$

$= \frac{5}{\sqrt{2}}\underline{|145° - (-45°)}$

$= 3{,}536\underline{|190°}$ or $3{,}536\underline{|-170°}$ (2)

6. $V_T = \frac{V_2}{V_3 + V_1}$

$= \frac{3 + j^2}{2 + j^3 + 3 + j}$

$= \frac{3 + (-1)}{5 - (j^2)j + j}$

$= \frac{2}{5 - (-1)j + j}$

$= \frac{2}{5 + 2j} \times \frac{5 - 2j}{5 - 2j}$

$= \frac{10 - 4j}{25 - 4j^2}$

$= \frac{10 - 4j}{25 - 4(-1)}$

$= \frac{10 - 4j}{29}$

$= \frac{10}{29} - \frac{4}{29}j$

$|r| = \sqrt{\left(\frac{10}{29}\right)^2 + \left(\frac{4}{29}\right)^2}$

$= \sqrt{0{,}119 + 0{,}019}$

$= 0{,}371$

$\theta = \tan^{-1}\left(\frac{\frac{4}{29}}{\frac{10}{29}}\right)$

$= \tan^{-1}0{,}4$

$= 21{,}8°$

$\therefore z = 0{,}371 \text{ cis } 21{,}8°$ (5)

TOTAL: [40]

MODULE

3 *Sketch graphs*

After they have completed this module, students should be able to:

- distinguish between a dependent and an independent variable;
- define a domain and range;
- state the difference and distinguish between functions and relations;
- identify the relevant functions that relate to their graphs;
- identify points of symmetry with reference to an axis or the lines $y = \pm x$;
- calculate inverse functions and relations; and
- draw neat sketch graphs of the following functions/relations:
  - $ax + by + c = 0$
  - $x^2 + y^2 = r^2,\ y = \pm\sqrt{r^2 - x^2},\ x = \pm\sqrt{r^2 - y^2}$
  - $xy = c$
  - $\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$
  - $y = ka^{nx}$, $y = ke^{nx}$, $y = k\log_a(nx)$ and $y = k\log_e(nx)$:
    - $y = ka^{nx}$, with $a > 0$ and a and n positive integers
    - $y = ka^{nx}$, with $0 < a < 1$ rational and n a positive integer
    - $y = k\log_e(nx)$, with n a positive integer
    - $y = k\log_a(nx)$, with $a > 1$ and a and n positive integers
    - $y = k\log_a(nx)$, with $0 < a < 1$ rational and n a positive integer
  - $y = ax^2 + bx + c$.

Introduction

A graph is a visual representation of the relations between certain variable quantities, or the connections that exist between a set of points plotted with reference to a set of axes, namely the x- and y-axes.

In this module we will focus on equations and graphs of the following relations:

1	Straight line	$ax + by + c = 0$
2	Circle	$x^2 + y^2 = r^2$; $y = \pm\sqrt{r^2 - x^2}$; $x = \pm\sqrt{r^2 - y^2}$
3	Ellipse	$\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$
4	Rectangular hyperbola	$xy = c$
5	Exponential	$y = ka^{nx}$; $y = ke^{nx}$; $y = k\log_a(nx)$ and $y = k\log_e(nx)$
6	Logarithmic	$y = ax^2 + bx + c$
7	Parabola	$y = ax^3 + bx^2 + cx + d$

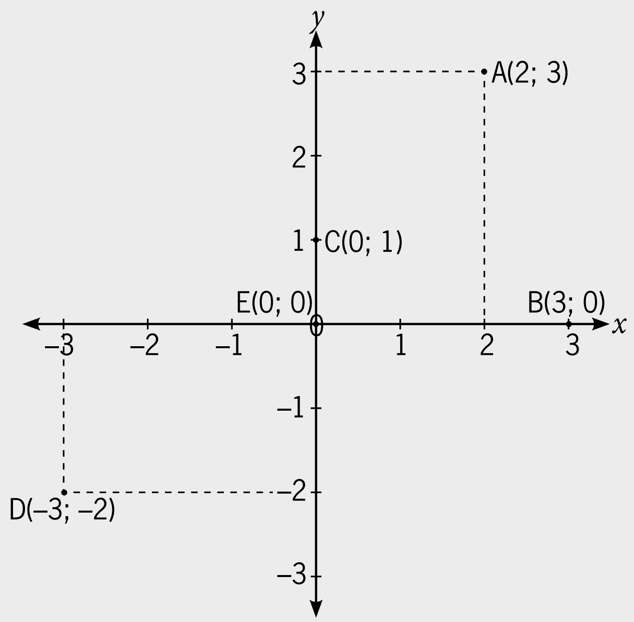
Students need the following pre-knowledge to successfully complete this module.

Pre-knowledge

Cartesian plane

In previous years students learnt about the **Cartesian plane as a** two-dimensional **plane** that is formed by the intersection of two perpendicular lines. The horizontal line is known as the x-axis, and the vertical line is known as the y-axis.

The **coordinates (x; y)** on the **Cartesian plane** is an ordered number pair representing a point. The horizontal distance of the point from the origin is x and the vertical distance is y.



$$y = mx + c$$

m = gradient (slope) c = y-intercept

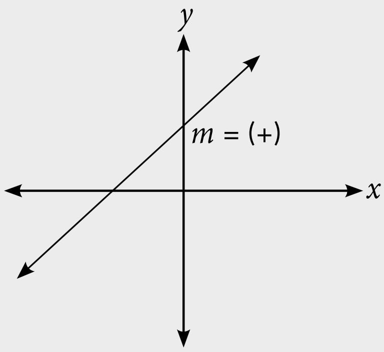
x- and y-intercept method:

Step 1: Find the x-intercept: let $y = 0$.

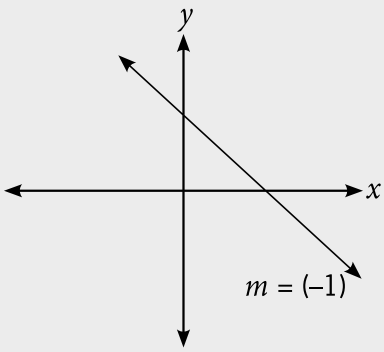
Step 2: Find the y-intercept: let $x = 0$.

Step 3: Plot the two points on a system of axes and draw the straight line graph.

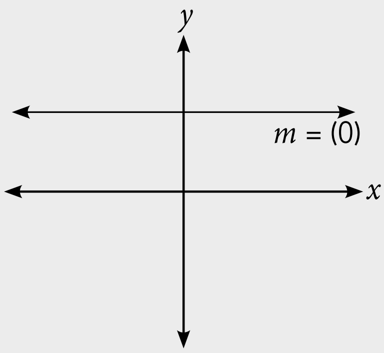
Gradient (m):



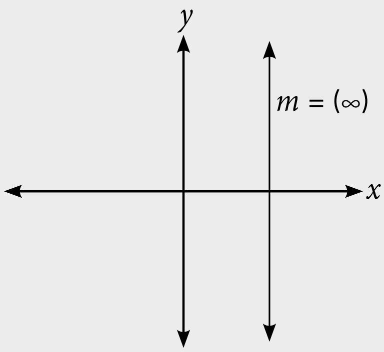
Gradient is positive: $m > 0$



Gradient is negative: $m < 0$



There is no slope or gradient: $m = 0$



If $m = \infty$ the straight line is parallel to the y-axis

Gradient: $m = \frac{y_2 - y_1}{x_2 - x_1} = \tan \theta$

Parallel lines: $m_1 = m_2$

Perpendicular lines: $m_1.m_2 = -1$

All the exponential laws as well as:

- $a^0 = 1$, for example $2^0 = 1$
- $\frac{1}{a^n} = a^{-n}$, for example $\frac{1}{3^2} = 3^{-2}$
- $\frac{1}{a^{-n}} = a^n$, for example $\frac{1}{4^{-2}} = 4^2$, and so on.

Common logarithm: log 100 — • Base 10

Natural logarithm: $\log_e x$ or ln x — • Base $e \approx 2{,}718$

Definition of a logarithm

If $x = a^y$ — • Exponential form

then $y = \log_a x$ — • Logarithmic form; $x > 0$; $a > 0$ and $a \neq 1$

For example: $4^3 = 64$

$\therefore \log_4 64 = 3$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Note

If $a < 0$, then there is a maximum turning point (∩) and if $a > 0$, then there is a minimum turning point (∪).

Activity 3.1 — SB page 116

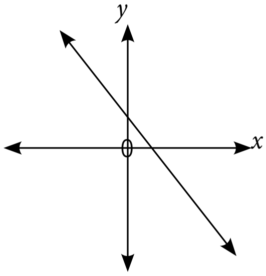
SB page 116

1. Domain: $\{x: x \in \mathbb{R}\}$

Domain: $(-\infty; \infty)$

Range: $\{y, y \in \mathbb{R}\}$

Range: $(-\infty; \infty)$

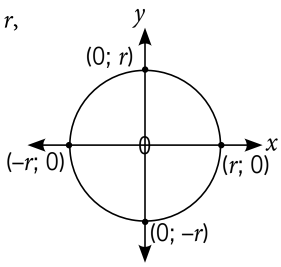


2. Domain: $\{x: -r \leq x \leq r, x \in \mathbb{R}\}$

Domain: $[-r; r]$

Range: $\{y: -r \leq y \leq r, y \in \mathbb{R}\}$

Range: $[-r; r]$

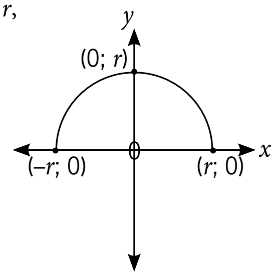


3. Domain: $\{x: -r \leq x \leq r, x \in \mathbb{R}\}$

Domain: $[-r; r]$

Range: $\{y: 0 \leq y \leq r, y \in \mathbb{R}\}$

Range: $[0; r]$

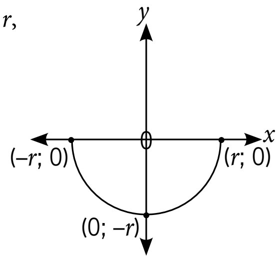


4. Domain: $\{x: -r \leq x \leq r, x \in \mathbb{R}\}$

Domain: $[-r; r]$

Range: $\{y: -r \leq y \leq 0, y \in \mathbb{R}\}$

Range: $[-r; 0]$

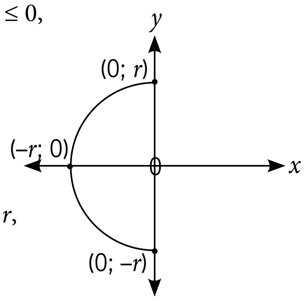


5. Domain: $\{x: -r \leq x \leq 0, x \in \mathbb{R}\}$

Domain: $[-r; 0]$

Range: $\{y: -r \leq y \leq r, y \in \mathbb{R}\}$

Range: $[-r; r]$

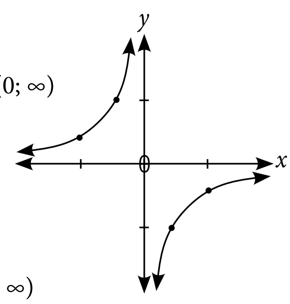


6. Domain: $\{x: x \in \mathbb{R}, x \neq 0\}$

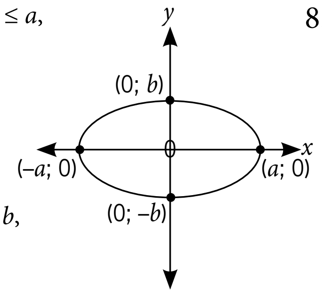
Domain: $(-\infty; 0) \cup (0; \infty)$

Range: $\{y: y \in \mathbb{R}, y \neq 0\}$

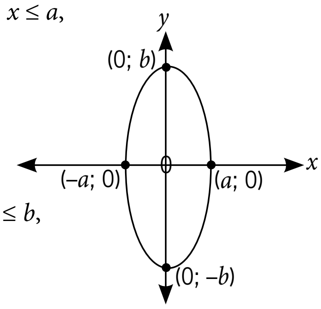
Range: $(-\infty; 0) \cup (0; \infty)$



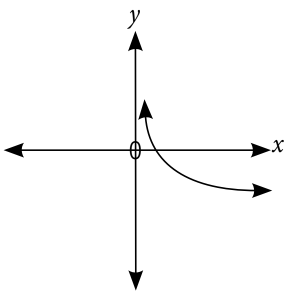
7. Domain: $\{x: -a \leq x \leq a, x \in \mathbb{R}\}$
Domain: $[-a; a]$
Range: $\{y: -b \leq y \leq b, y \in \mathbb{R}\}$
Range: $[-b; b]$



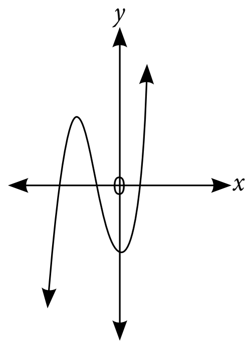
8. Domain: $\{x: -a \leq x \leq a, x \in \mathbb{R}\}$
Domain: $[-a; a]$
Range: $\{y: -b \leq y \leq b, y \in \mathbb{R}\}$
Range: $[-b; b]$



9. Domain: $\{x: x > 0, x \in \mathbb{R}\}$
Domain: $(0; \infty)$
Range: $\{y: y \in \mathbb{R}\}$
Range: $(-\infty; \infty)$



10. Domain: $\{x: x \in \mathbb{R}\}$
Domain: $(-\infty; \infty)$
Range: $\{y: y \in \mathbb{R}\}$
Range: $(-\infty; \infty)$



Activity 3.2 SB page 122

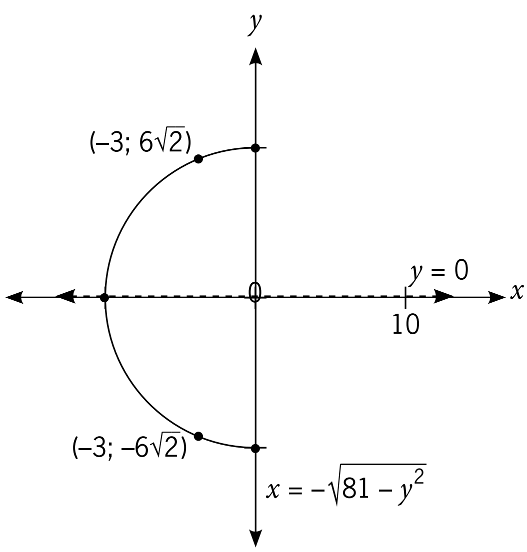
1.	Function	2.	Non-function
3.	Function	4.	Non-function
5.	Function	6.	Non-function
7.	Non-function	8.	Function
9.	Function	10.	Function

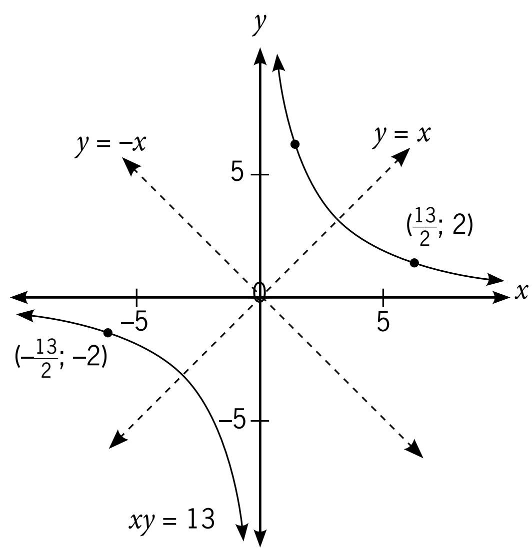
Activity 3.3 SB page 130

1. 1.1 x-axis: $\left(-3; -6\sqrt{2}\right)$

1.2 $y = x$: $\left(\frac{13}{2}; 2\right)$

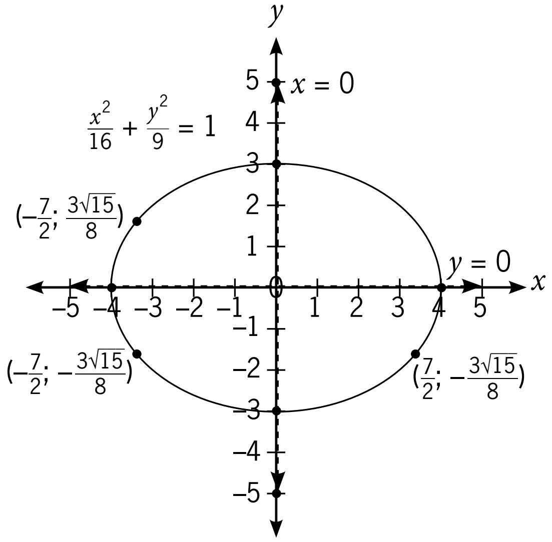
$y = -x$: $\left(-\frac{13}{2}; -2\right)$





1.3 x-axis: $\left(-\frac{7}{2}; \frac{3\sqrt{15}}{8}\right)$

y-axis: $\left(\frac{7}{2}; -\frac{3\sqrt{15}}{8}\right)$

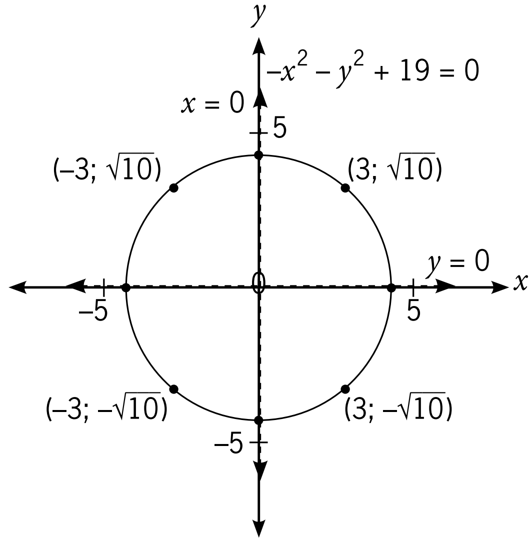


2. 2.1 x-axis: $(-3; -\sqrt{10}) \leftrightarrow (-3; \sqrt{10})$

$(3; -\sqrt{10}) \leftrightarrow (3; \sqrt{10})$

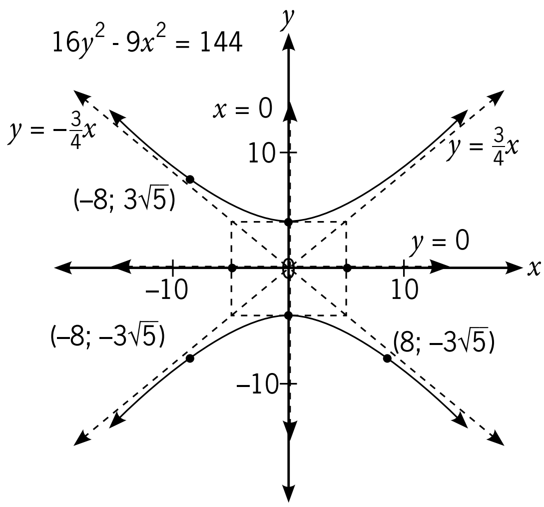
y-axis: $(-3; -\sqrt{10}) \leftrightarrow (3; -\sqrt{10})$

$(-3; \sqrt{10}) \leftrightarrow (3; \sqrt{10})$



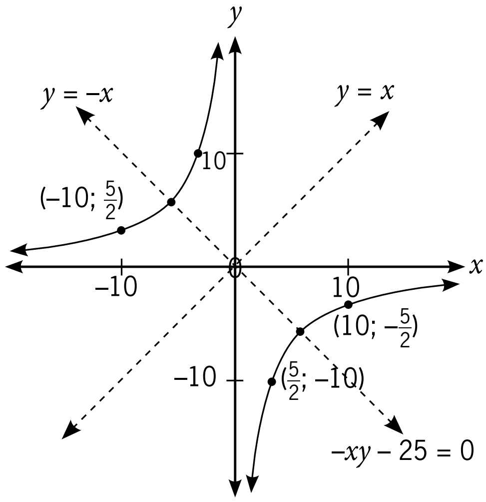
2.2 x-axis: $(-8; -3\sqrt{5}) \leftrightarrow (-8; 3\sqrt{5})$

y-axis: $(-8; -3\sqrt{5}) \leftrightarrow (8; -3\sqrt{5})$



2.3 $y = x$: $\left(\frac{5}{2}; -10\right) \leftrightarrow \left(-10; \frac{5}{2}\right)$

$y = -x$: $\left(\frac{5}{2}; -10\right) \leftrightarrow \left(10; -\frac{5}{2}\right)$



3. 3.1

$$y - \sqrt{75 - x^2} = 0$$
$$y - \sqrt{75 - (-x^2)} = 0$$
$$y - \sqrt{75 - x^2} = 0$$

Since the equation is equivalent, the graph is symmetrical about the y-axis

The tests for symmetry about the x-axis, $y = x$ and $y = -x$ do not yield an equivalent equation, therefore are not lines of symmetry.

3.2 $3x^2 + y^2 = 2$

$3x^2 + (-y^2) = 2$

$3x^2 + y^2 = 2$

Since the equation is equivalent, the graph is symmetrical about the x-axis.

$3x^2 + y^2 = 2$

$3(-x^2) + y^2 = 2$

$3x^2 + y^2 = 2$

Since the equation is equivalent, the graph is symmetrical about the y-axis.

The test for symmetry about the line $y = x$ and the line $y = -x$ do not yield an equivalent equation, therefore are not lines of symmetry.

3.3 $xy - 9 = 0$

$(y)(x) - 9 = 0$

$yx - 9 = 0$

$xy - 9 = 0$

Since the equation is equivalent, the graph is symmetrical about the $y = x$.

$xy - 9 = 0$

$(-y)(-x) - 9 = 0$

$yx - 9 = 0$

$xy - 9 = 0$

Since the equation is equivalent, the graph is symmetrical about the $y = -x$.

The tests for symmetry about the x-axis and y-axis do not yield an equivalent equation, therefore are not lines of symmetry.

3.4 $\frac{x^2}{5} - y^2 = 1$

$\frac{x^2}{5} - (-y)^2 = 1$

$\frac{x^2}{5} - y^2 = 1$

Since the equation is equivalent, the graph is symmetrical about the *x-axis*.

$\frac{x^2}{5} - y^2 = 1$

$\frac{(-x)^2}{5} - y^2 = 1$

$\frac{x^2}{5} - y^2 = 1$

Since the equation is equivalent, the graph is symmetrical about the y-axis.

The tests for symmetry about the line $x = y$ and line $y = -x$ do not yield an equivalent equation, therefore are not lines of symmetry.

Activity 3.4 SB page 134

1. Continuous
2. Continuous
3. Discontinuous
4. Continuous
5. Continuous
6. Continuous
7. Continuous
8. Discontinuous
9. Continuous
10. Continuous

Activity 3.5 SB page 140

1. 1.1 No 1.2 Yes 1.3 No 1.4 Yes

2. 2.1
$$f(x) = 3x - 2$$
$$f^{-1}(x) = 3y - 2$$
$$\therefore x = 3y - 2$$
$$3y = x + 2$$
$$y = \tfrac{1}{3}x + \tfrac{3}{2}$$

2.2
$$xy = 4$$
$$\therefore y = \tfrac{4}{x}$$
$$f^{-1}(x) = \tfrac{4}{y}$$
$$\therefore x = \tfrac{4}{y}$$
$$y = \tfrac{4}{x}$$

2.3
$$x^2 + y^2 = 49$$
$$y = \pm\sqrt{49 - x^2}$$
$$\therefore f^{-1}(x) = \pm\sqrt{49 - y^2}$$
$$x = \pm\sqrt{49 - y^2}$$
$$\therefore x^2 = 49 - y^2$$
$$x^2 + y^2 = 49$$

2.4
$$y = 3^x$$
$$f^{-1}(x) = 3^y$$
$$\therefore x = 3^y$$
$$y = \log_3 x$$

2.5 $\frac{y^2}{4} + \frac{x^2}{25} = 1$

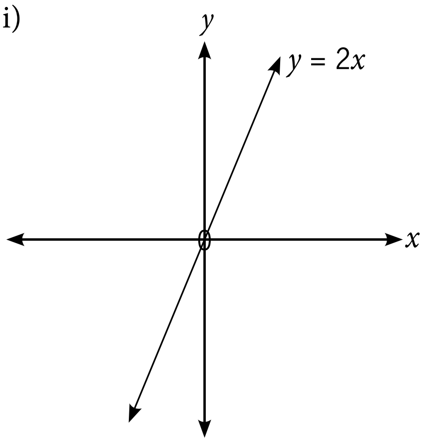
2.6 $y = -3x^2$

2.7 $x = -y$

2.8 $x = \sqrt{16 - y^2}$

Activity 3.6 SB page 147

1. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

b) i) $y = \frac{x}{2}$

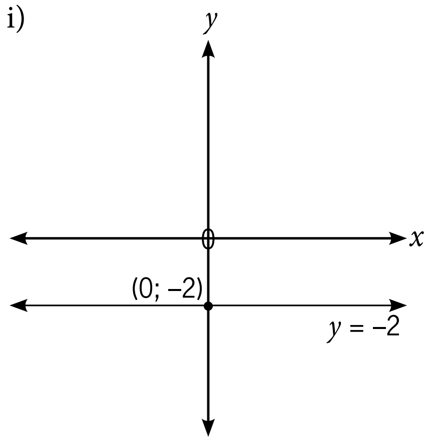
ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

2. a) i)



ii) (–∞; ∞)

iii) (–2)

iv) Function

v) Continuous

b) i) $x = -2$

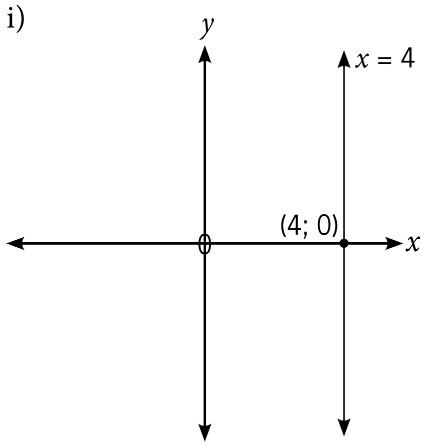
ii) (–2)

iii) (–∞; ∞)

iv) Non-function

v) Continuous

3. a) i)



ii) (4)

iii) (–∞; ∞)

iv) Non-function

v) Continuous

b) i) $y = 4$

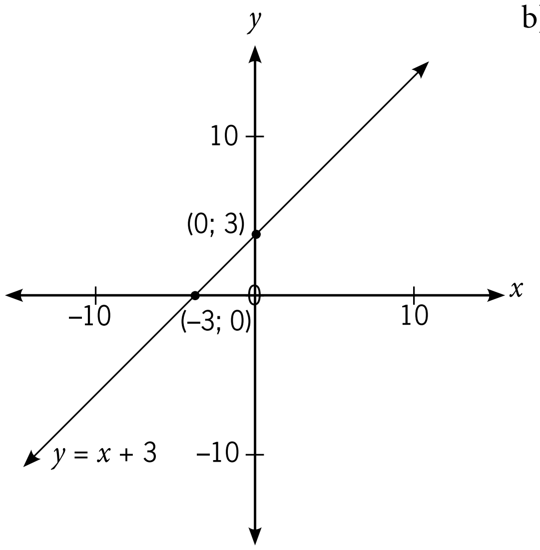
ii) (–∞; ∞)

iii) (4)

iv) Function

v) Continuous

4. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

b) i) $y = x - 3$

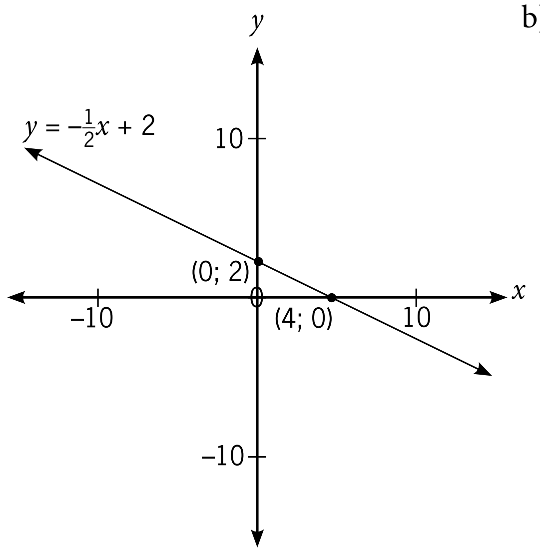
ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

5. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

b) i) $y = -2x + 4$

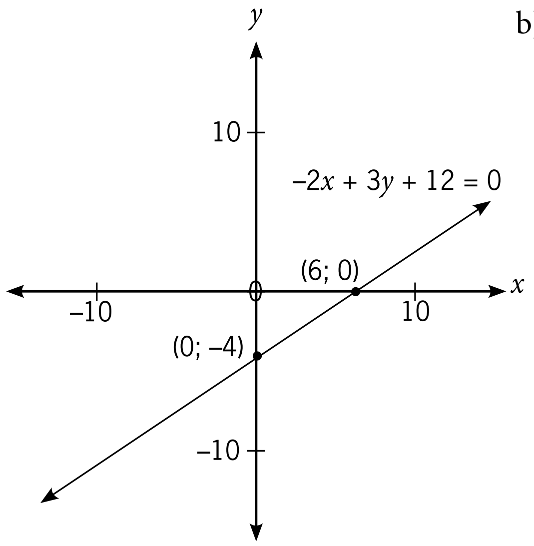
ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

6. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

b) i) $y = \frac{3}{2}x + 6$

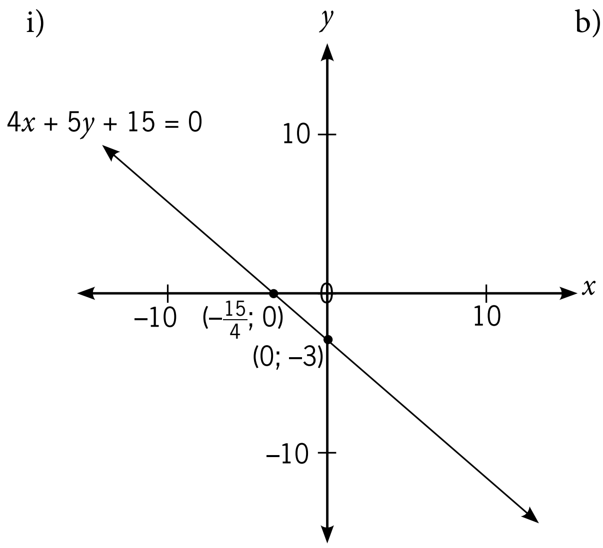
ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

7. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

b) i) $y = -\frac{5}{4}x - \frac{15}{4}$

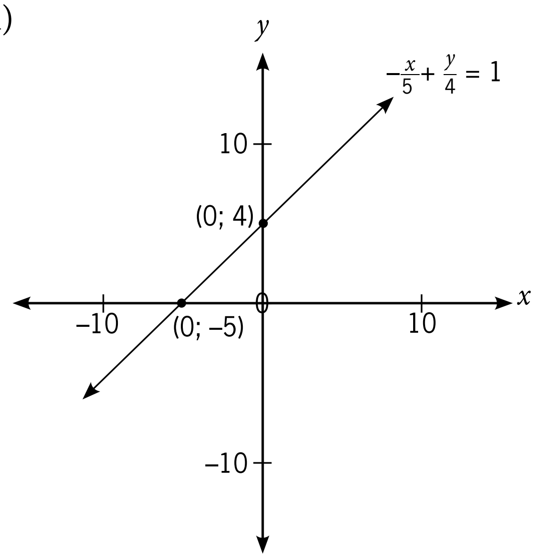
ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

8. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

b) i) $y = \frac{5}{4}x - 5$

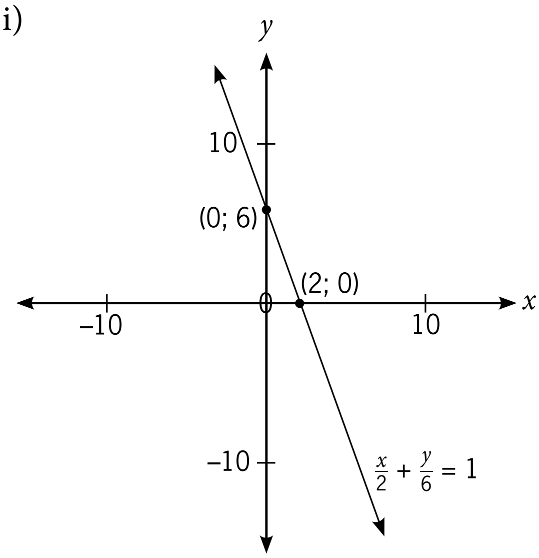
ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

9. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

b) i) $y = -\frac{1}{3}x + 2$

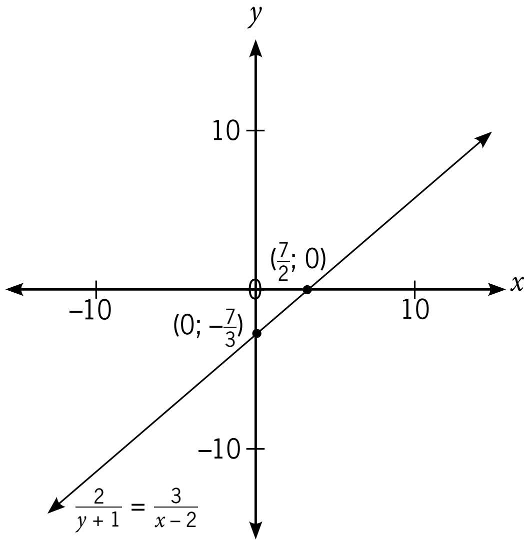
ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

10. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

b) i) $y = \frac{3}{2}x + \frac{7}{2}$

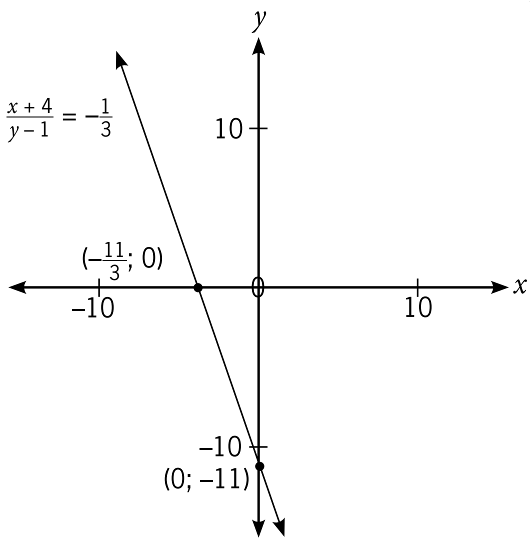
ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

11. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

b) i) $y = -\frac{1}{3}x - \frac{11}{3}$

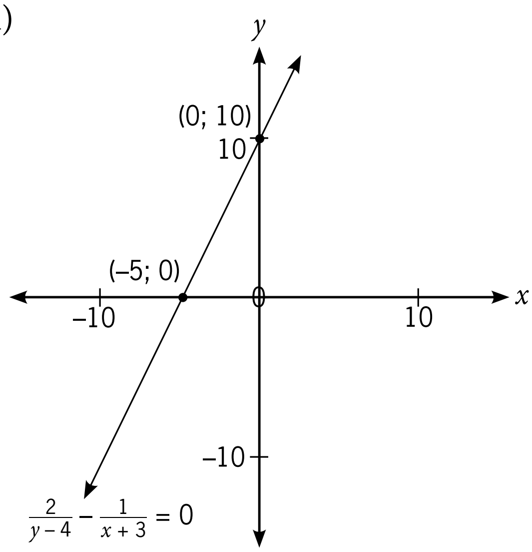
ii) $(-\infty; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

12. a) i)

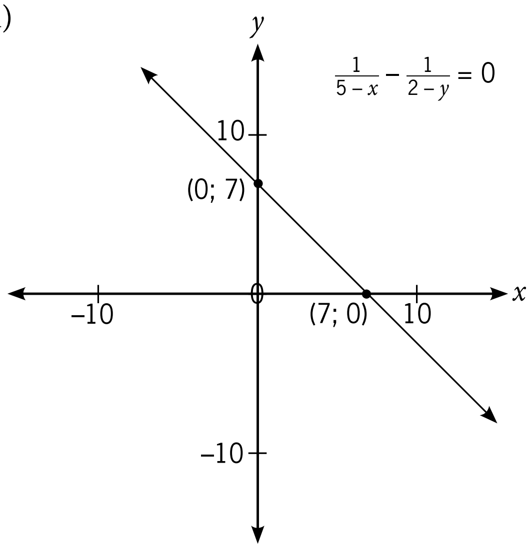


ii) $(-\infty; \infty)$
iii) $(-\infty; \infty)$
iv) Function
v) Continuous

b) i) $y = \frac{1}{2}x - 5$

ii) $(-\infty; \infty)$
iii) $(-\infty; \infty)$
iv) Function
v) Continuous

13. a) i)

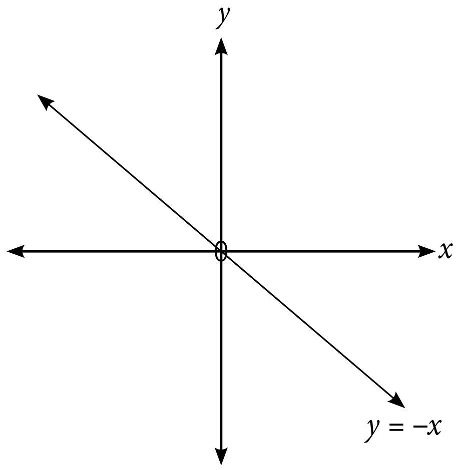


ii) $(-\infty; \infty)$
iii) $(-\infty; \infty)$
iv) Function
v) Continuous

b) i) $y = -x + 7$

ii) $(-\infty; \infty)$
iii) $(-\infty; \infty)$
iv) Function
v) Continuous

14. a) i)



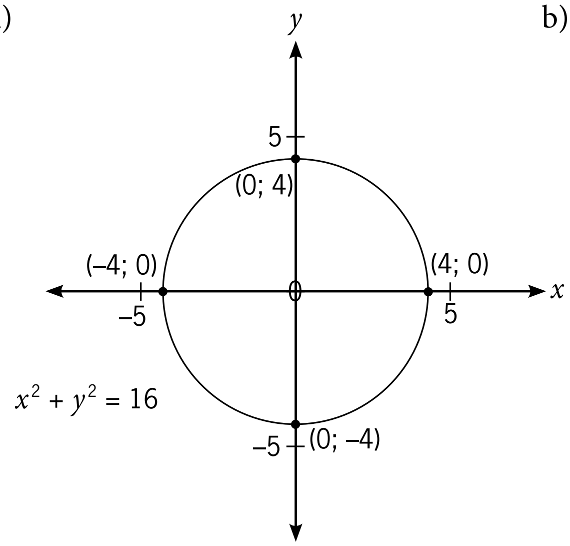
ii) (−∞; ∞)
iii) (−∞; ∞)
iv) Function
v) Continuous

b) i) $y = -x$

ii) (−∞; ∞)
iii) (−∞; ∞)
iv) Function
v) Continuous

Activity 3.7 SB page 151

1. a) i)

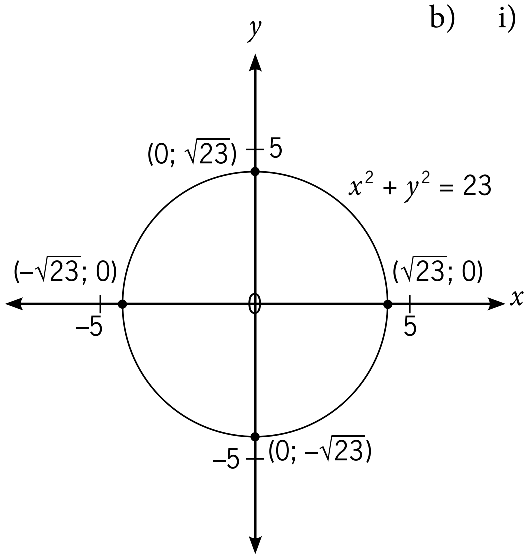


ii) [−4; 4]
iii) [−4; 4]
iv) Non-function
v) Continuous

b) i) $x^2 + y^2 = 16$

ii) [−4; 4]
iii) [−4; 4]
iv) Non-function
v) Continuous

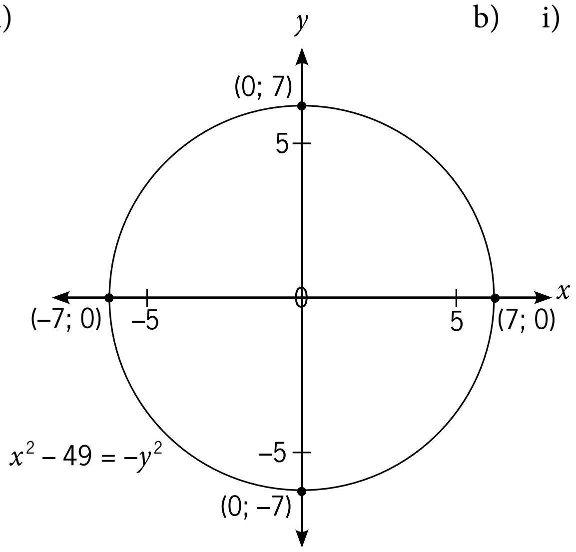
2. a) i)



ii) $[-\sqrt{23};\sqrt{23}]$
iii) $[-\sqrt{23};\sqrt{23}]$
iv) Non-function
v) Continuous

b) i) $x^2 + y^2 = 23$
ii) $[-\sqrt{23};\sqrt{23}]$
iii) $[-\sqrt{23};\sqrt{23}]$
iv) Non-function
v) Continuous

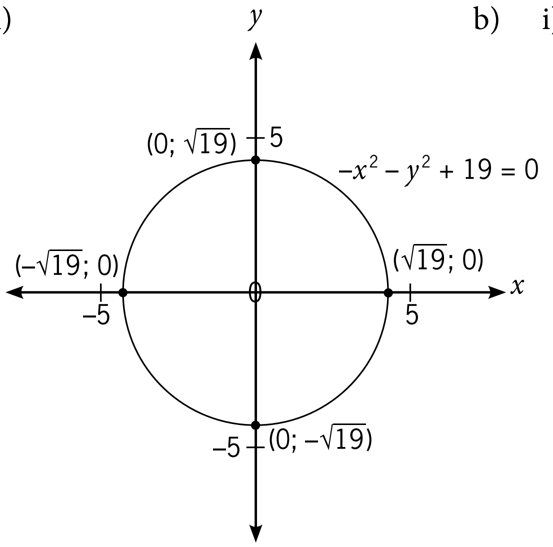
3. a) i)



ii) [–7; 7]
iii) [–7; 7]
iv) Non-function
v) Continuous

b) i) $x^2 + y^2 = 49$
ii) [–7; 7]
iii) [–7; 7]
iv) Non-function
v) Continuous

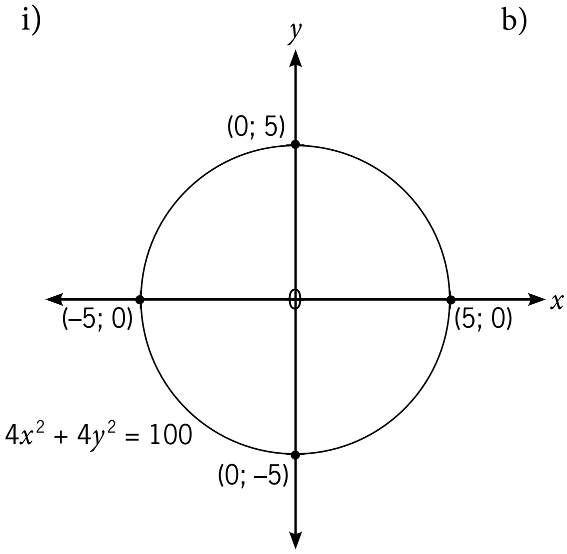
4. a) i)



ii) $[-\sqrt{19};\sqrt{19}]$
iii) $[-\sqrt{19};\sqrt{19}]$
iv) Non-function
v) Continuous

b) i) $x^2 + y^2 = 19$
ii) $[-\sqrt{19};\sqrt{19}]$
iii) $[-\sqrt{19};\sqrt{19}]$
iv) Non-function
v) Continuous

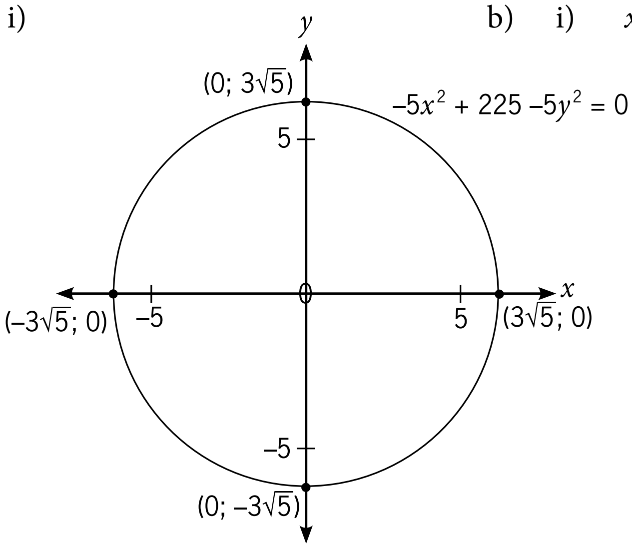
5. a) i)



ii) [−5; 5]
iii) [−5; 5]
iv) Non-function
v) Continuous

b) i) $x^2 + y^2 = 25$
ii) [−5; 5]
iii) [−5; 5]
iv) Non-function
v) Continuous

6. a) i)



ii) $[-3\sqrt{5}; 3\sqrt{5}]$

iii) $[-3\sqrt{5}; 3\sqrt{5}]$

iv) Non-function

v) Continuous

b) i) $x^2 + y^2 = 45$

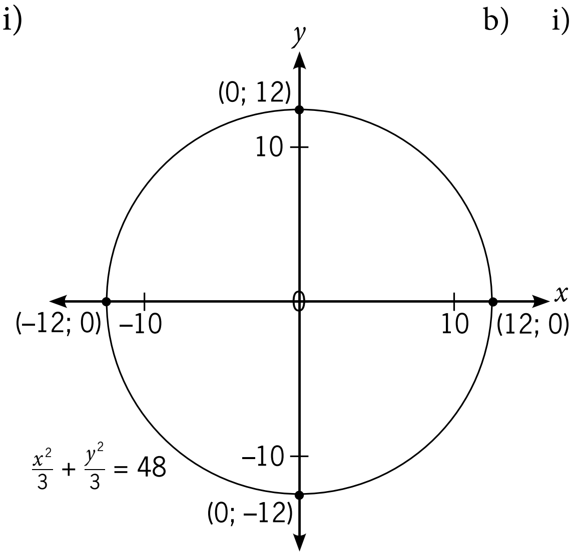
ii) $[-3\sqrt{5};3\sqrt{5}]$

iii) $[-3\sqrt{5}; 3\sqrt{5}]$

iv) Non-function

v) Continuous

7. a) i)



ii) $[-12; 12]$

iii) $[-12; 12]$

iv) Non-function

v) Continuous

b) i) $x^2 + y^2 = 144$

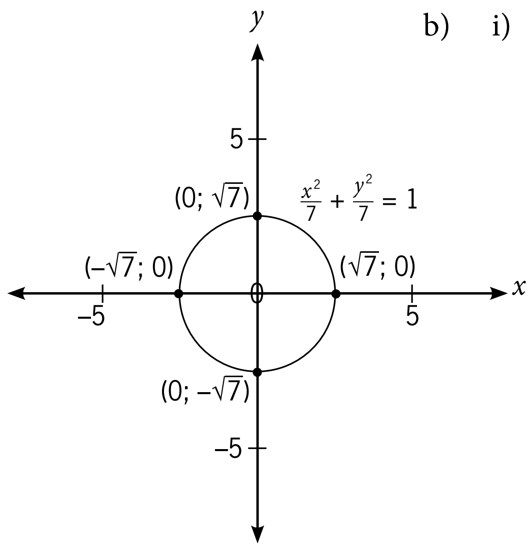
ii) $[-12; 12]$

iii) $[-12; 12]$

iv) Non-function

v) Continuous

8. a) i)

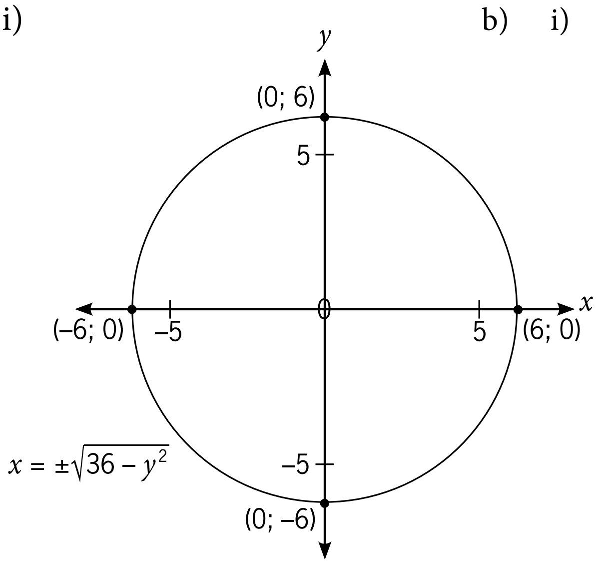


ii) $[-\sqrt{7}; \sqrt{7}]$
iii) $[-\sqrt{7}; \sqrt{7}]$
iv) Non-function
v) Continuous

b) i) $x^2 + y^2 = 7$

ii) $[-\sqrt{7}; \sqrt{7}]$
iii) $[-\sqrt{7}; \sqrt{7}]$
iv) Non-function
v) Continuous

9. a) i)

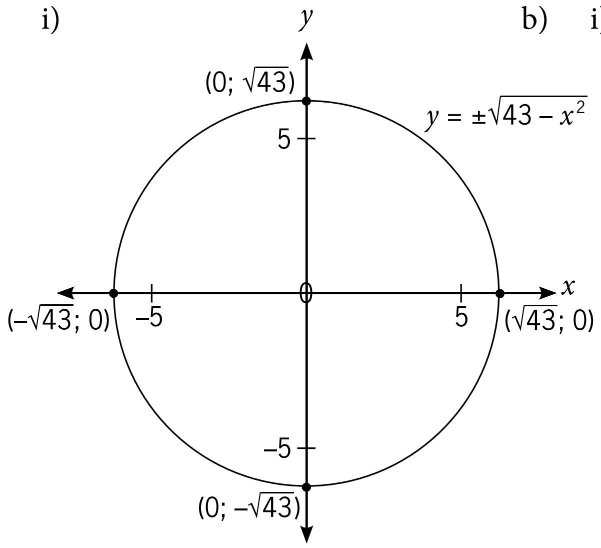


ii) $[-6; 6]$
iii) $[-6; 6]$
iv) Non-function
v) Continuous

b) i) $x^2 + y^2 = 36$

ii) $[-6; 6]$
iii) $[-6; 6]$
iv) Non-function
v) Continuous

10. a) i)



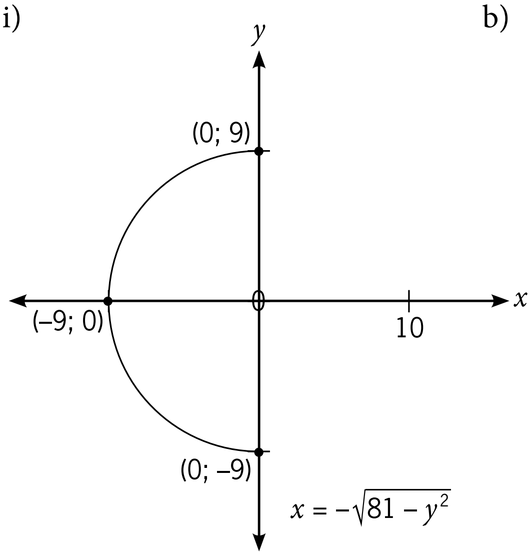
ii) $[-\sqrt{43}; \sqrt{43}]$
iii) $[-\sqrt{43}; \sqrt{43}]$
iv) Non-function
v) Continuous

b) i) $x^2 + y^2 = 43$

ii) $[-\sqrt{43}; \sqrt{43}]$
iii) $[-\sqrt{43}; \sqrt{43}]$
iv) Non-function
v) Continuous

Activity 3.8 SB page 155

1. a) i)

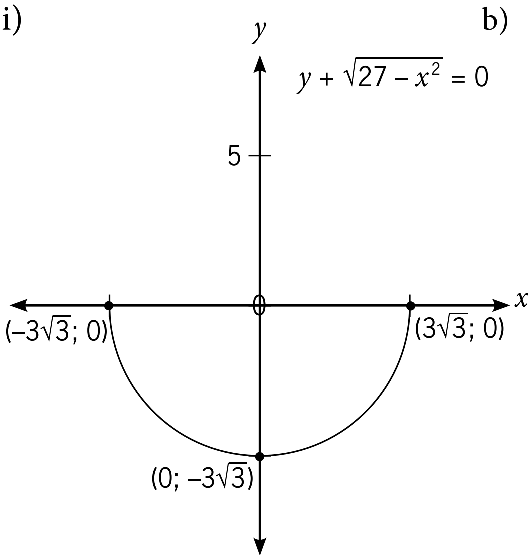


ii) [–9; 0]
iii) [–9; 9]
iv) Non-function
v) Continuous

b) i) $y = -\sqrt{81 - x^2}$

ii) [–9; 9]
iii) [–9; 0]
iv) Function
v) Continuous

2. a) i)



ii) $[-3\sqrt{3}; 3\sqrt{3}]$

iii) $[-3\sqrt{3}; 0]$

iv) Function

v) Continuous

b) i) $x = -\sqrt{27 - y^2}$

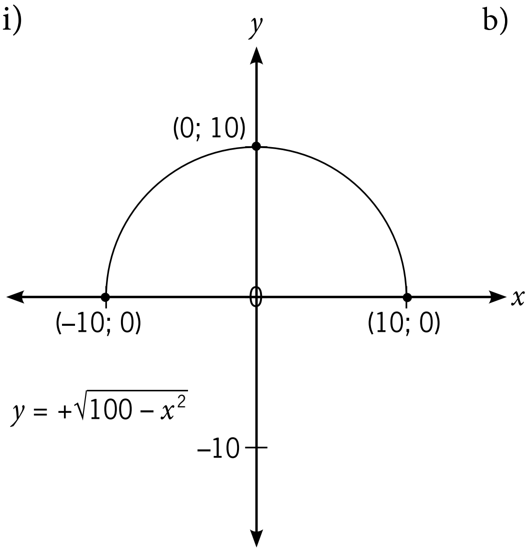
ii) $[-3\sqrt{3}; 0]$

iii) $[-3\sqrt{3}; 3\sqrt{3}]$

iv) Non-function

v) Continuous

3. a) i)



ii) [−10; 10]

iii) [0; 10]

iv) Function

v) Continuous

b) i) $x = +\sqrt{100 - y^2}$

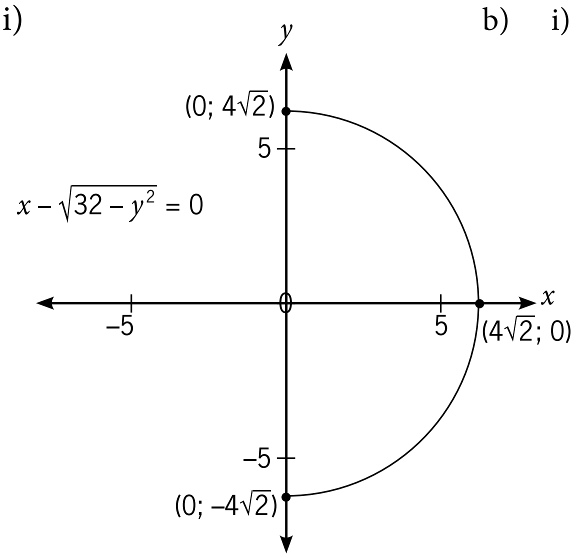
ii) [0; 10]

iii) [−10; 10]

iv) Non-function

v) Continuous

4. a) i)

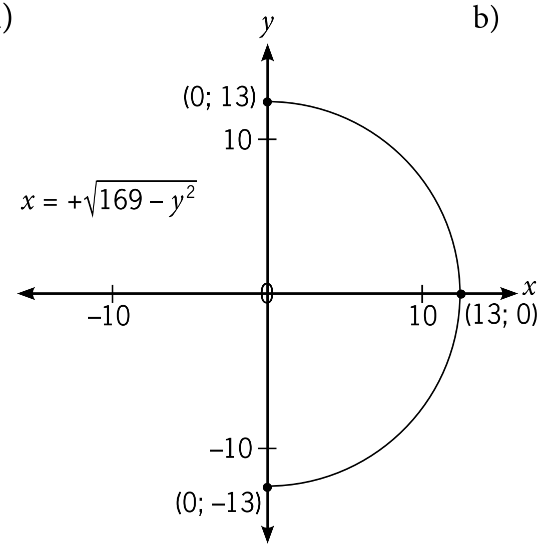


b) i) $y = +\sqrt{32 - x^2}$

a)
ii) $[0; 4\sqrt{2}]$
iii) $[-4\sqrt{2}; 4\sqrt{2}]$
iv) Non-function
v) Continuous

b)
ii) $[-4\sqrt{2}; 4\sqrt{2}]$
iii) $[0; 4\sqrt{2}]$
iv) Function
v) Continuous

5. a) i)

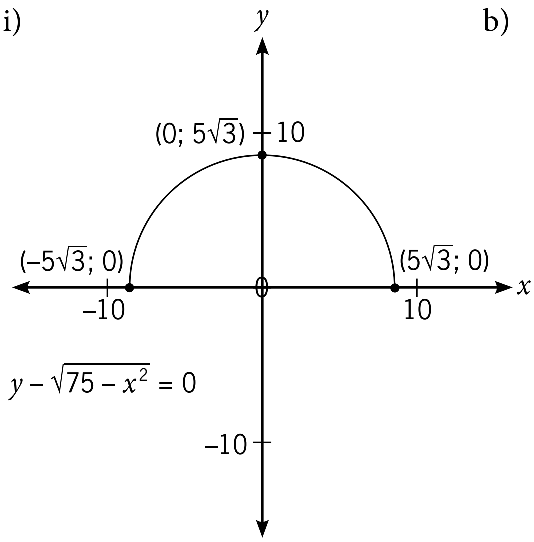


b) i) $y = +\sqrt{169 - x^2}$

a)
ii) [0; 13]
iii) [−13; 13]
iv) Non-function
v) Continuous

b)
ii) [−13; 13]
iii) [0; 13]
iv) Function
v) Continuous

6. a) i)



ii) $[-5\sqrt{3}; 5\sqrt{3}]$

iii) $[0; 5\sqrt{3}]$

iv) Function

v) Continuous

b) i) $x = +\sqrt{75 - y^2}$

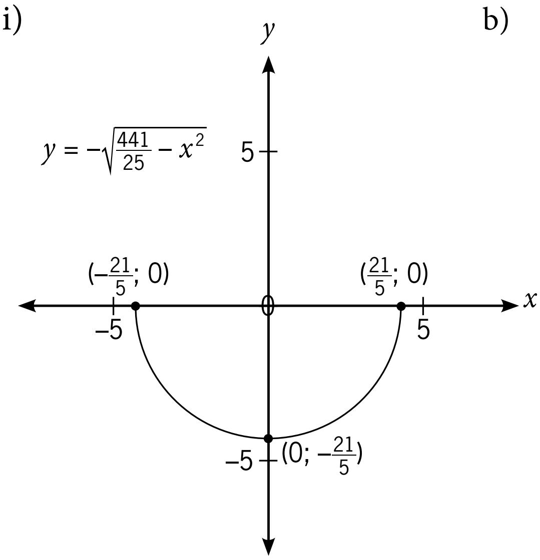
ii) $[0; 5\sqrt{3}]$;

iii) $[-5\sqrt{3}; 5\sqrt{3}]$

iv) Non-function

v) Continuous

7. a) i)



ii) $\left[-\frac{21}{5}; \frac{21}{5}\right]$

iii) $\left[-\frac{21}{5}; 0\right]$

iv) Function

v) Continuous

b) i) $x = -\sqrt{\frac{441}{25} - y^2}$

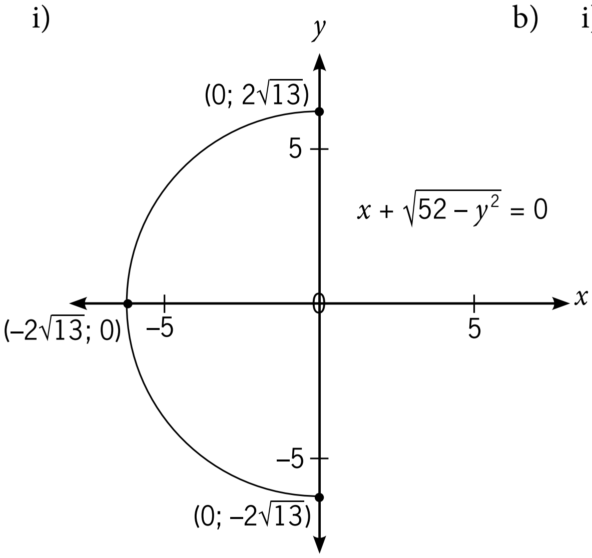
ii) $\left[-\frac{21}{5}; 0\right]$

iii) $\left[-\frac{21}{5}; \frac{21}{5}\right]$

iv) Non-function

v) Continuous

8. a) i)



ii) $[-2\sqrt{13}; 0]$

iii) $[-2\sqrt{13}; 2\sqrt{13}]$

iv) Non-function

v) Continuous

b) i) $y = -\sqrt{52 - x^2}$

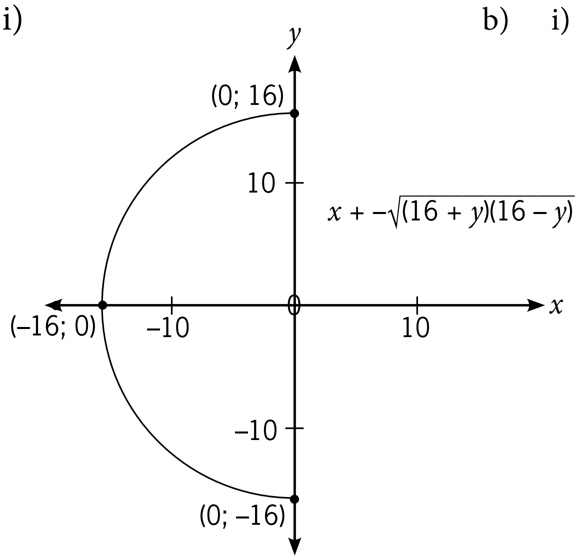
ii) $[-2\sqrt{13}; 2\sqrt{13}]$

iii) $[-2\sqrt{13}; 0]$

iv) Function

v) Continuous

9. a) i)



ii) $[-16; 0]$

iii) $[-16; 16]$

iv) Non-function

v) Continuous

b) i) $y = -\sqrt{256 - x^2}$

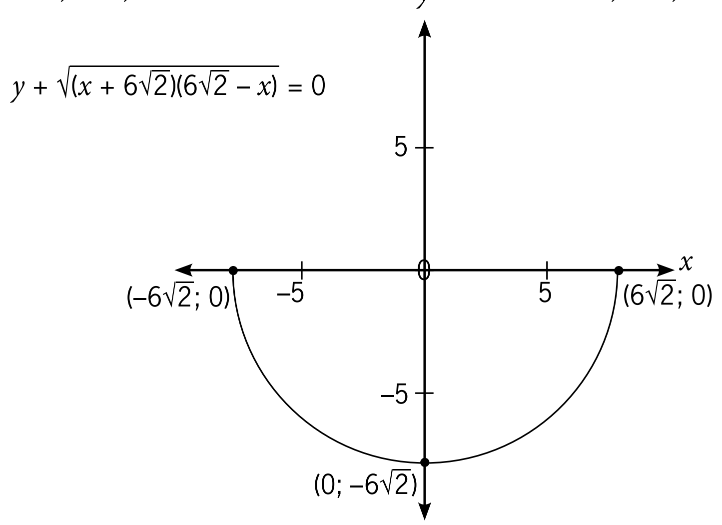
ii) $[-16; 16]$

iii) $[-16; 0]$

iv) Function

v) Continuous

10. a) i)



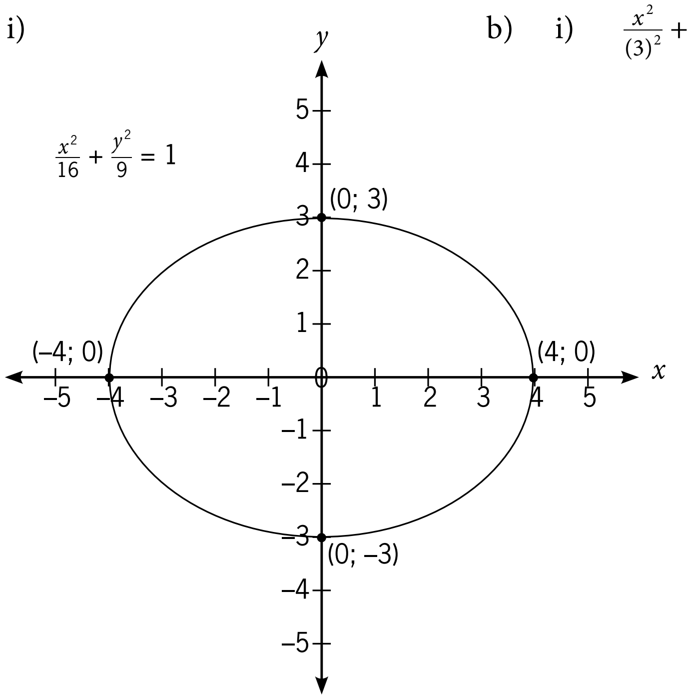
ii) $[-6\sqrt{2}; 6\sqrt{2}]$
iii) $[-6\sqrt{2}; 0]$
iv) Function
v) Continuous

b) i) $x = -\sqrt{72 - y^2}$

ii) $[-6\sqrt{2}; 0]$
iii) $[-6\sqrt{2}; 6\sqrt{2}]$
iv) Non-function
v) Continuous

Activity 3.9 SB page 163

1. a) i)

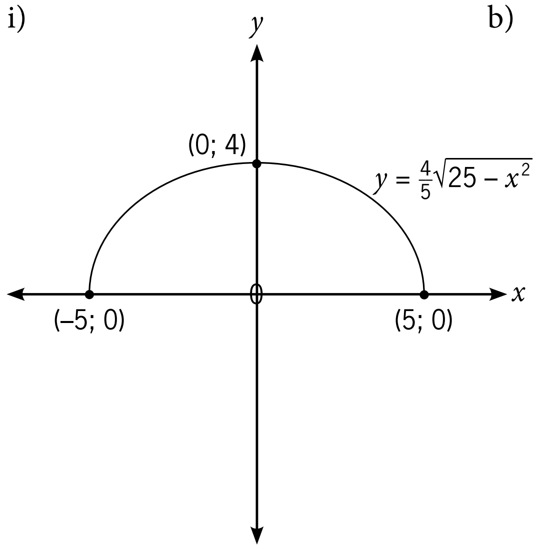


ii) $[-4; 4]$
iii) $[-3; 3]$
iv) Non-function
v) Continuous

b) i) $\frac{x^2}{(3)^2} + \frac{y^2}{(4)^2} = 1$

ii) $[-3; 3]$
iii) $[-4; 4]$
iv) Non-function
v) Continuous

2. a) i)



ii) (–5; 5)

iii) (4; 0)

iv) Function

v) Continuous

b) i) $x = \frac{4}{5}\sqrt{25 - y^2}$

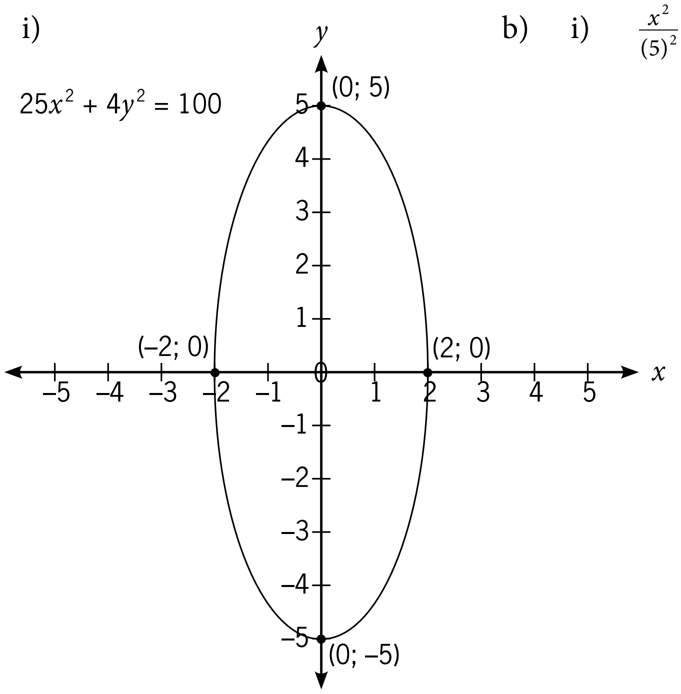
ii) (0; 4)

iii) (–5; 5)

iv) Non-function

v) Continuous

3. a) i)



ii) [–2; 2]

iii) [–5; 5]

iv) Non-function

v) Continuous

b) i) $\frac{x^2}{(5)^2} + \frac{y^2}{(2)^2} = 1$

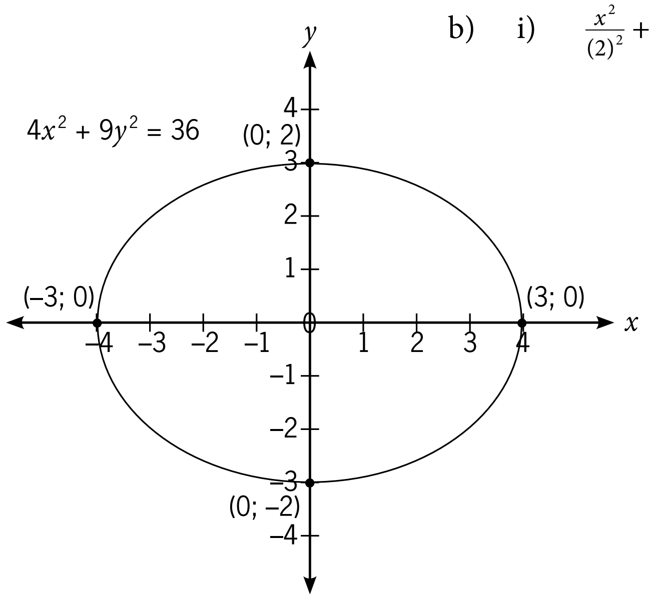
ii) [–5; 5]

iii) [–2; 2]

iv) Non-function

v) Continuous

4. a) i)



ii) [–3; 3]

iii) [–2; 2]

iv) Non-function

v) Continuous

b) i) $\frac{x^2}{(2)^2} + \frac{y^2}{(3)^2} = 1$

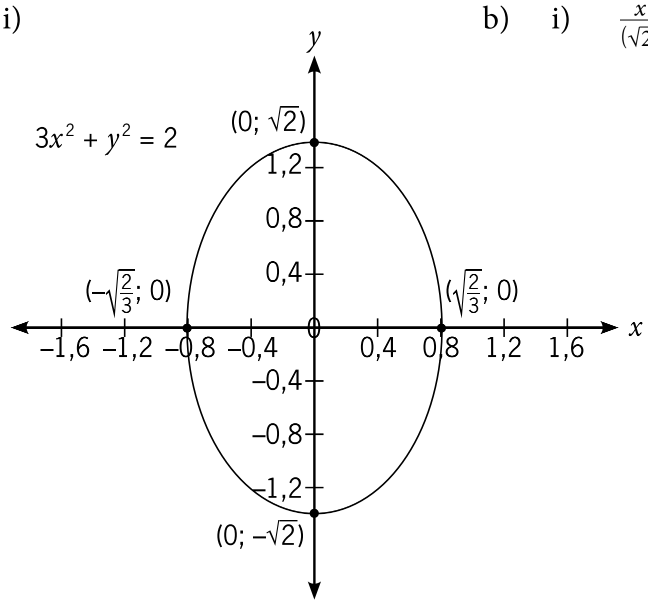
ii) [–2; 2]

iii) [–3; 3]

iv) Non-function

v) Continuous

5. a) i)



ii) $\left[-\sqrt{\frac{2}{3}}; \sqrt{\frac{2}{3}}\right]$

iii) $[-\sqrt{2}; \sqrt{2}]$

iv) Non-function

v) Continuous

b) i) $\frac{x^2}{(\sqrt{2})^2} + \frac{y^2}{\left(\sqrt{\frac{2}{3}}\right)^2} = 1$

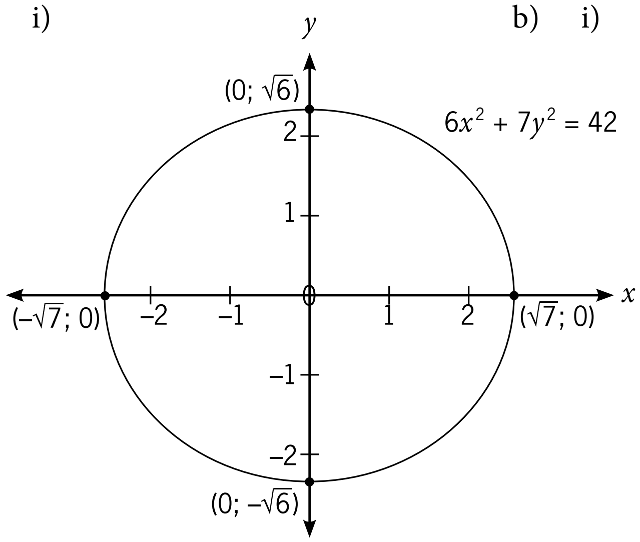
ii) $[-\sqrt{2}; \sqrt{2}]$

iii) $\left[-\sqrt{\frac{2}{3}}; \sqrt{\frac{2}{3}}\right]$

iv) Non-function

v) Continuous

6. a) i)



ii) $[-\sqrt{7}; \sqrt{7}]$

iii) $[-\sqrt{6}; \sqrt{6}]$

iv) Non-function

v) Continuous

b) i) $\frac{x^2}{(\sqrt{6})^2} + \frac{y^2}{(\sqrt{7})^2} = 1$

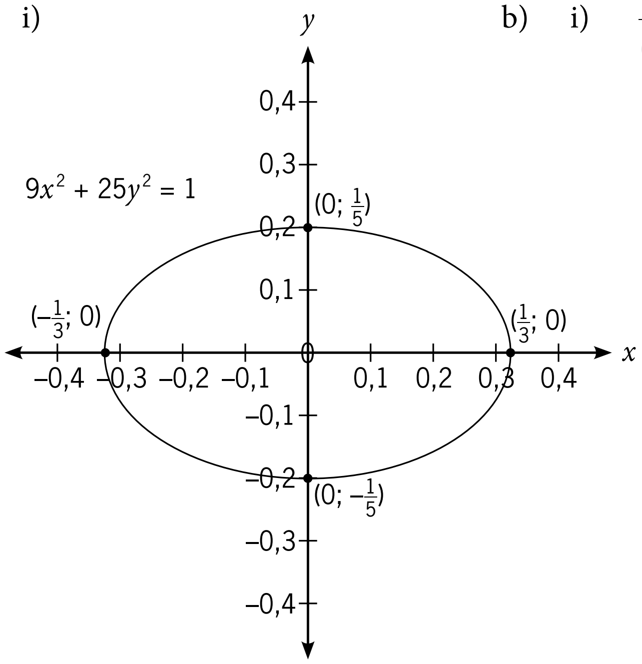
ii) $[-\sqrt{6}; \sqrt{6}]$

iii) $[-\sqrt{7}; \sqrt{7}]$

iv) Non-function

v) Continuous

7. a) i)



ii) $\left[-\frac{1}{3}; \frac{1}{3}\right]$

iii) $\left[-\frac{1}{5}; \frac{1}{5}\right]$

iv) Non-function

v) Continuous

b) i) $\frac{x^2}{\left(\frac{1}{5}\right)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$

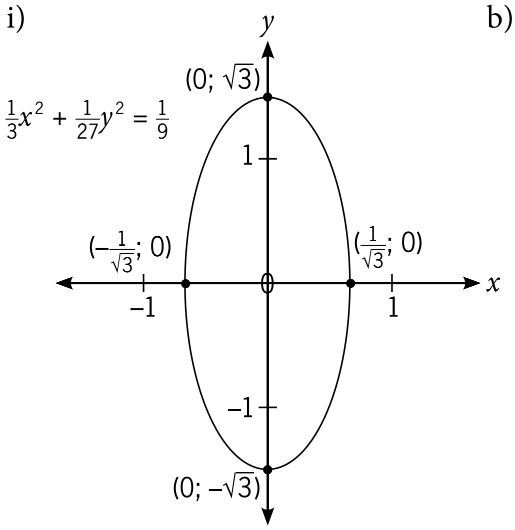
ii) $\left[-\frac{1}{5}; \frac{1}{5}\right]$

iii) $\left[-\frac{1}{3}; \frac{1}{3}\right]$

iv) Non-function

v) Continuous

8. a) i)



ii) $\left[-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right]$;

iii) $[-\sqrt{3}; \sqrt{3}]$

iv) Non-function

v) Continuous

b) i) $\frac{x^2}{(\sqrt{3})^2} + \frac{y^2}{\left(\frac{1}{\sqrt{3}}\right)^2} = 1$

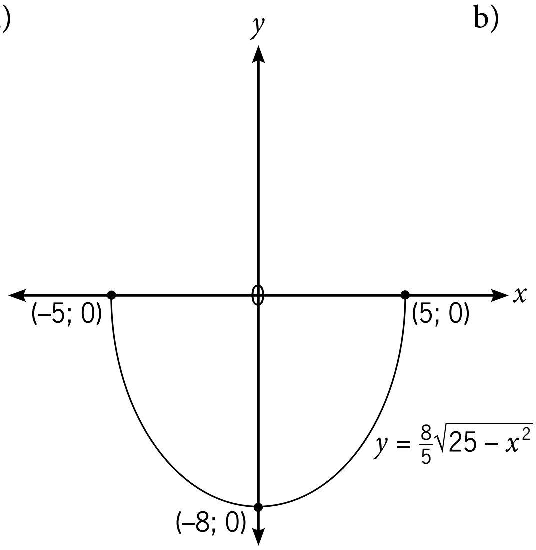
ii) $[-\sqrt{3}; \sqrt{3}]$

iii) $\left[-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right]$

iv) Non-function

v) Continuous

9. a) i)



ii) $(-5; 5)$

iii) $(-8; 0)$

iv) Function

v) Continuous

b) i) $x = \frac{8}{5}\sqrt{25 - y^2}$

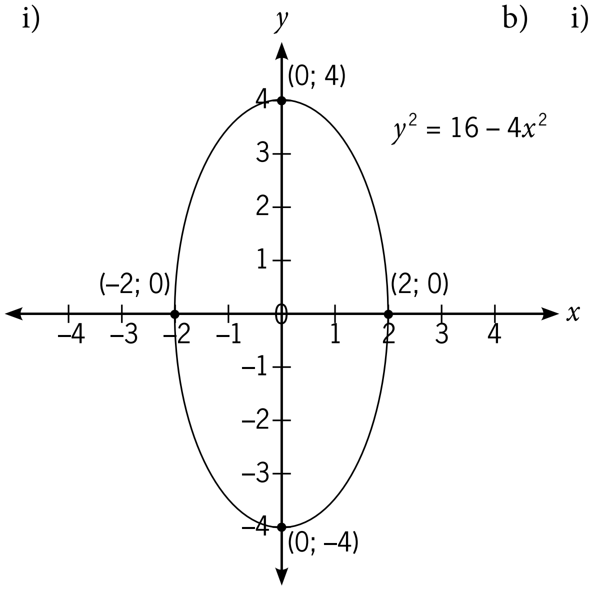
ii) $(-8; 0)$

iii) $(-5; 5)$

iv) Non-function

v) Continuous

10. a) i)



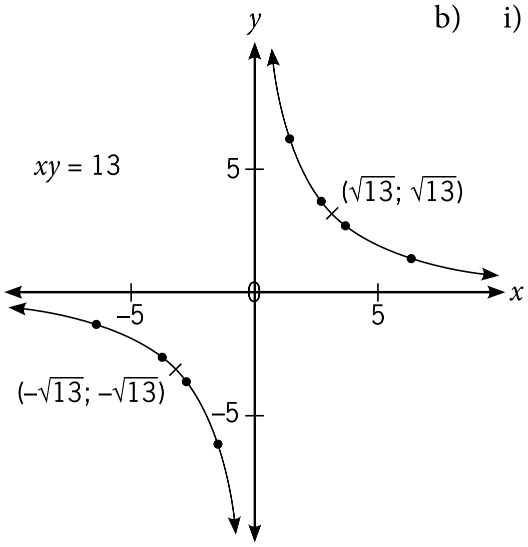
ii) [–2; 2]
iii) [–4; 4]
iv) Non-function
v) Continuous

b) i) $\frac{x^2}{(4)^2} + \frac{y^2}{(2)^2} = 1$

ii) [–4; 4]
iii) [–2; 2]
iv) Non-function
v) Continuous

Activity 3.10 SB page 170

1. a) i)

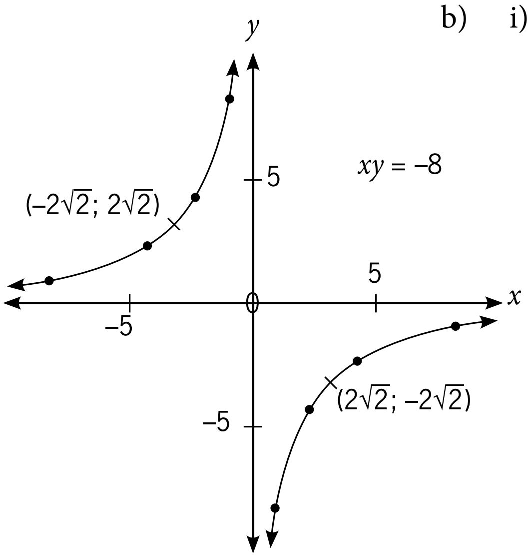


ii) $(-\infty; 0) \cup (0; \infty)$
iii) $(-\infty; 0) \cup (0; \infty)$
iv) Function
v) Discontinuous

b) i) $xy = 13$

ii) $(-\infty; 0) \cup (0; \infty)$
iii) $(-\infty; 0) \cup (0; \infty)$
iv) Function
v) Discontinuous

2. a) i)



ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

b) i) $xy = -8$

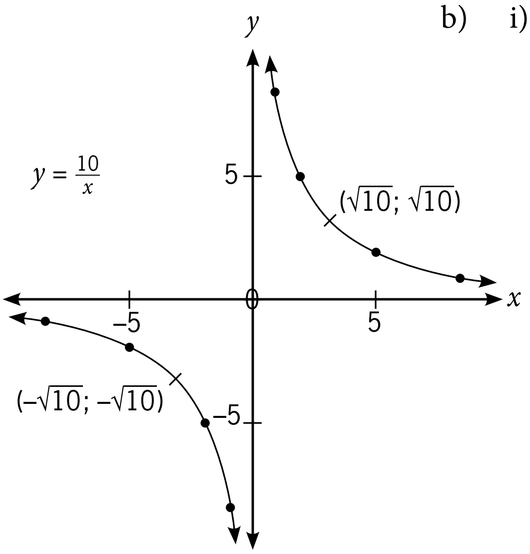
ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

3. a) i)



ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

b) i) $xy = 10$

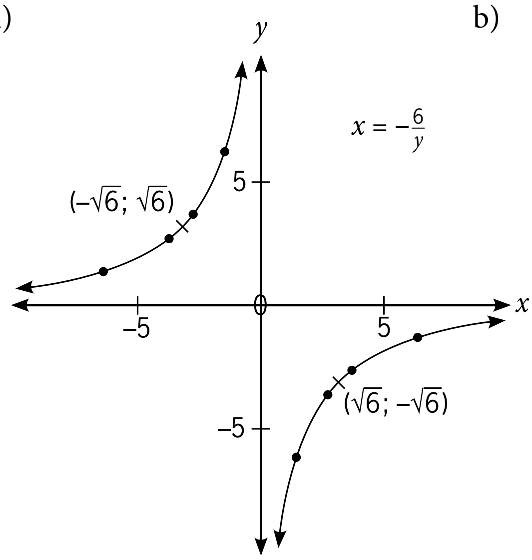
ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

4. a) i)



ii) $(-\infty; 0) \cup (0; \infty)$;

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

b) i) $xy = -6$

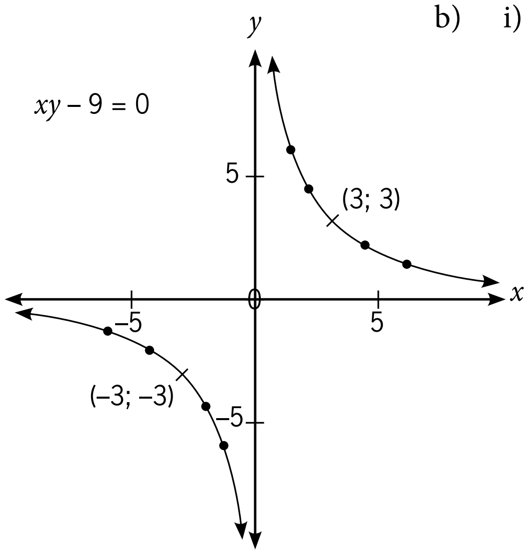
ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

5. a) i)



ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

b) i) $xy = 9$

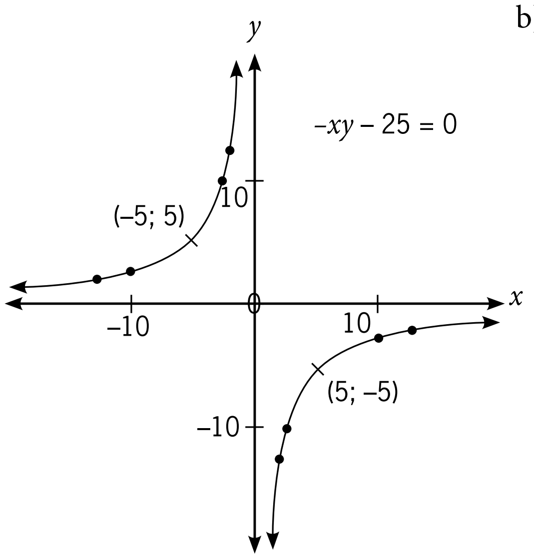
ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

6. a) i)



ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

b) i) $xy = -25$

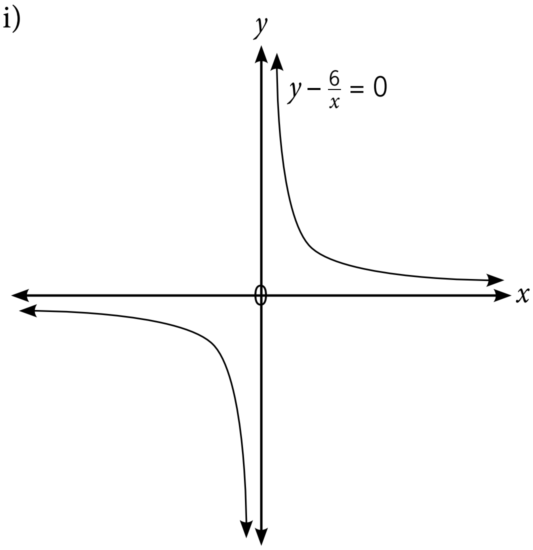
ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

7. a) i)



ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

b) i) $x = \frac{6}{y}$

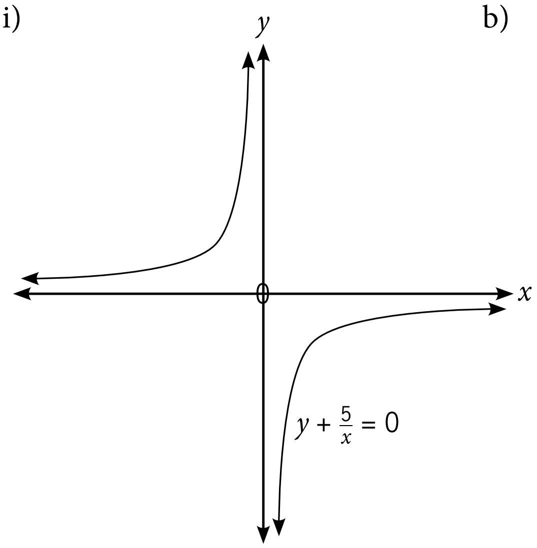
ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

8. a) i)



ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

b) i) $x + \frac{5}{y} = 0$

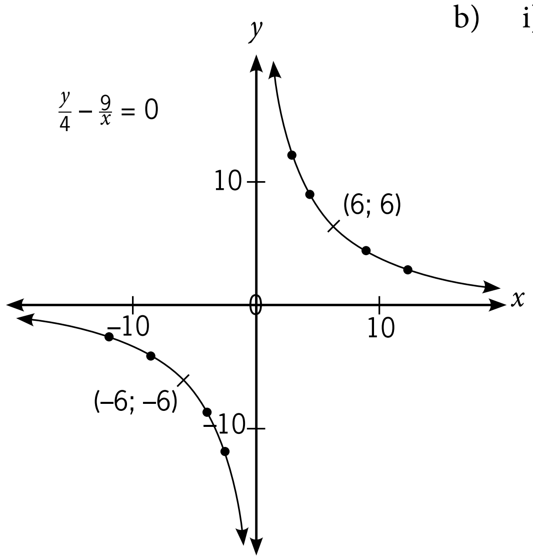
ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

9. a) i)



ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

b) i) $xy = 36$

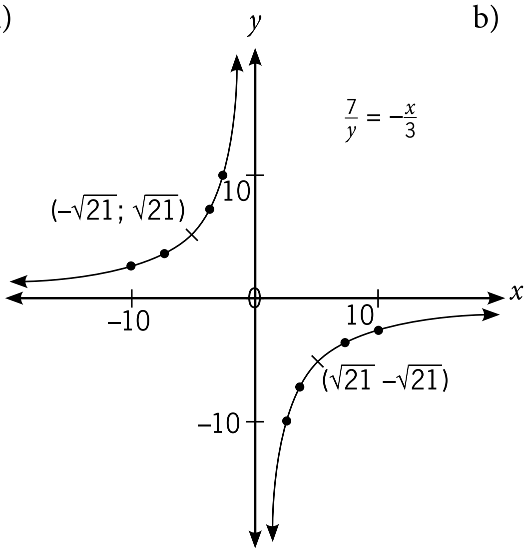
ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

10. a) i)



ii) $(-\infty; 0) \cup (0; \infty)$

iii) $(-\infty; 0) \cup (0; \infty)$

iv) Function

v) Discontinuous

b) i) $xy = -21$

ii) $(-\infty; 0) \cup (0; \infty)$

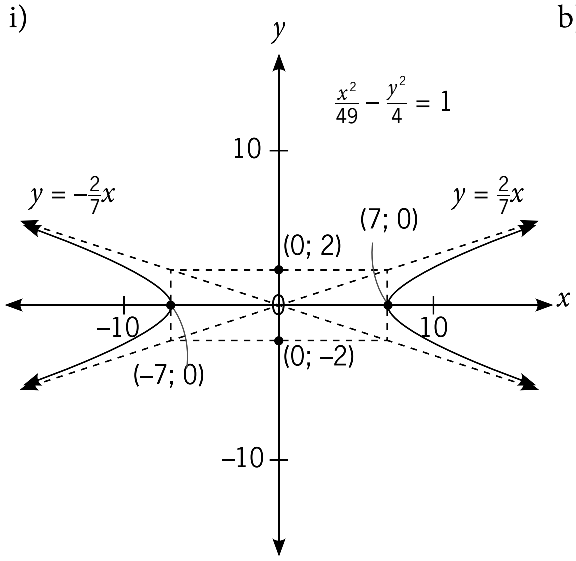
iii) $(-\infty;0) \cup (0; \infty)$

iv) Function

v) Discontinuous

Activity 3.11 SB page 175

1. a) i)



ii) $(-\infty; -7] \cup [7; \infty)$

iii) $(-\infty; \infty)$

iv) Non-function

v) Discontinuous

b) i) $-\frac{x^2}{(2)^2} + \frac{y^2}{(7)^2} = 1$

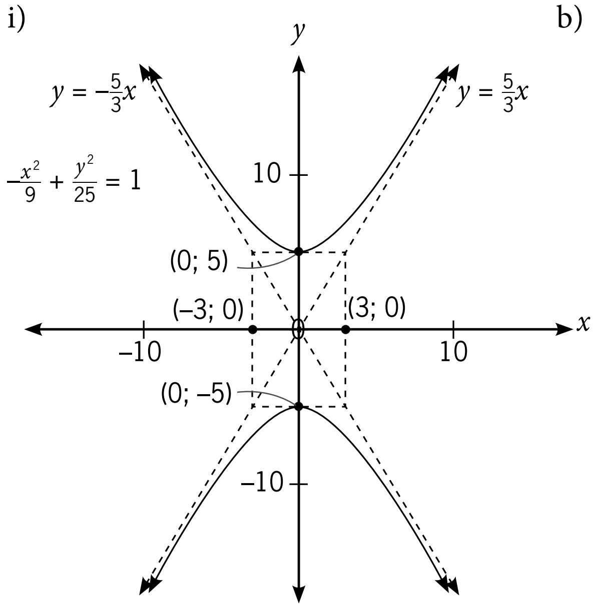
ii) $(-\infty; \infty)$

iii) $(-\infty; -7] \cup [7; \infty)$

iv) Non-function

v) Discontinuous

2. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; -5] \cup [5; \infty)$

iv) Non-function

v) Discontinuous

b) i) $\frac{x^2}{(5)^2} - \frac{y^2}{(3)^2} = 1$

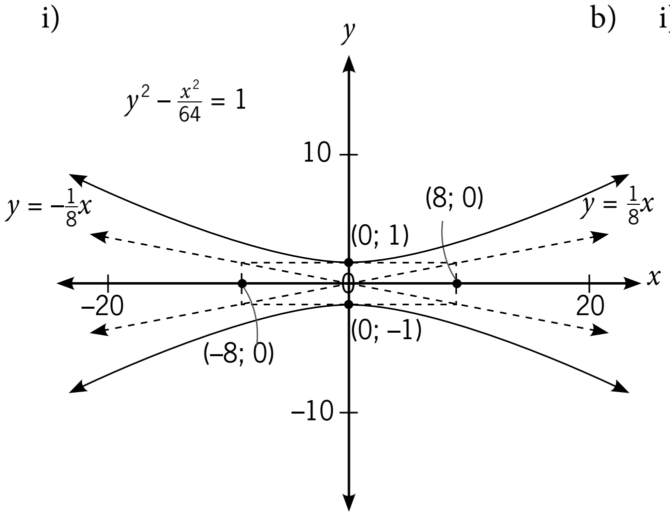
ii) $(-\infty; -5] \cup [5; \infty)$

iii) $(-\infty; \infty)$

iv) Non-function

v) Discontinuous

3. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; -1] \cup [1; \infty)$

iv) Non-function

v) Discontinuous

b) i) $\frac{x^2}{(1)^2} - \frac{y^2}{(8)^2} = 1$

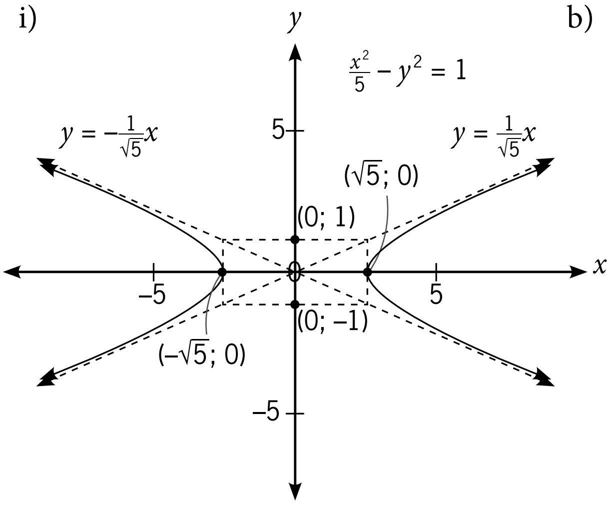
ii) $(-\infty; -1] \cup [1; \infty)$

iii) $(-\infty; \infty)$

iv) Non-function

v) Discontinuous

4. a) i)



ii) $(-\infty; -\sqrt{5}] \cup [\sqrt{5}; \infty)$

iii) $(-\infty; \infty)$

iv) Non-function

v) Discontinuous

b) i) $-\frac{x^2}{(1)^2} + \frac{y^2}{(\sqrt{5})^2} = 1$

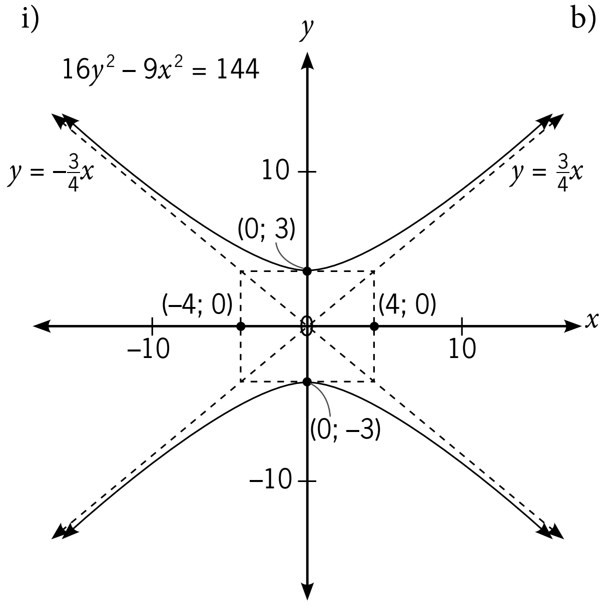
ii) $(-\infty; \infty)$

iii) $[-\infty; -\sqrt{5}] \cup [\sqrt{5}; \infty)$

iv) Non-function

v) Discontinuous

5. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; -3] \cup [3; \infty)$

iv) Non-function

v) Discontinuous

b) i) $\frac{x^2}{(3)^2} - \frac{y^2}{(4)^2} = 1$

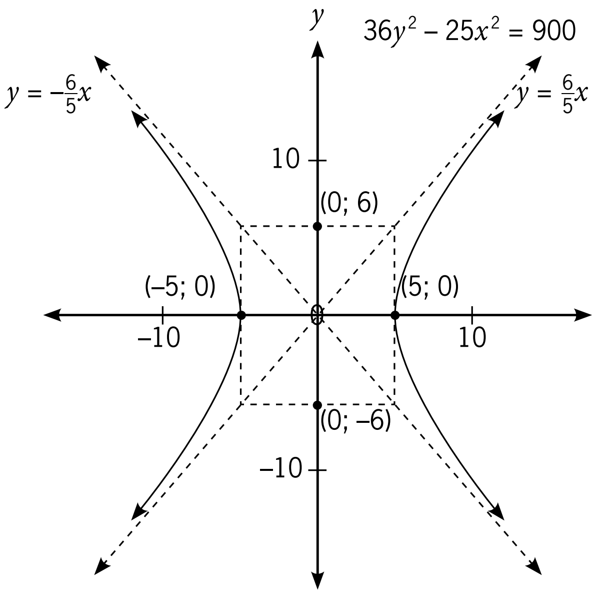
ii) $(-\infty; -3] \cup [3; \infty)$

iii) $(-\infty; \infty)$

iv) Non-function

v) Discontinuous

6. a) i)



ii) $(-\infty; -5] \cup [5; \infty)$

iii) $(-\infty; \infty)$

iv) Non-function

v) Discontinuous

b) i) $-\frac{x^2}{(6)^2} + \frac{y^2}{(5)^2} = 1$

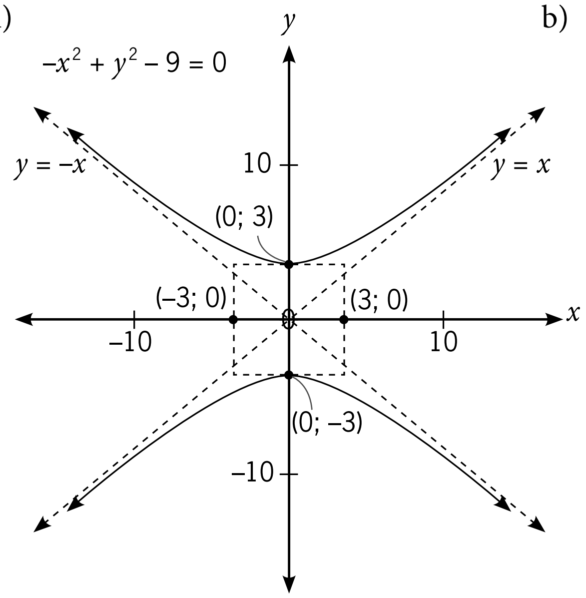
ii) $(-\infty; \infty)$

iii) $(-\infty; -5] \cup [5; \infty)$

iv) Non-function

v) Discontinuous

7. a) i)



ii) $(-\infty;\infty)$

iii) $(-\infty;-3] \cup [3;\infty)$

iv) Non-function

v) Discontinuous

b) i) $\frac{x^2}{(3)^2} - \frac{y^2}{(3)^2} = 1$

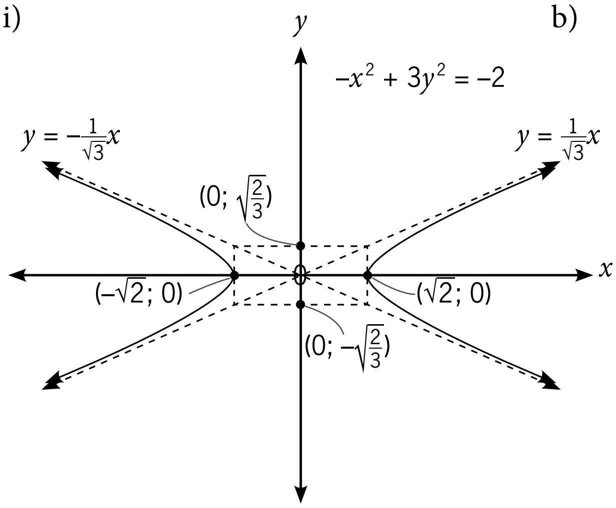
ii) $(-\infty;-3] \cup [3;\infty)$

iii) $(-\infty;\infty)$

iv) Non-function

v) Discontinuous

8. a) i)



ii) $(-\infty; -\sqrt{2}] \cup [\sqrt{2}; \infty)$

iii) $(-\infty; \infty)$

iv) Non-function

v) Discontinuous

b) i) $-\frac{x^2}{\left(\sqrt{\frac{2}{3}}\right)^2} + \frac{y^2}{(\sqrt{2})^2} = 1$

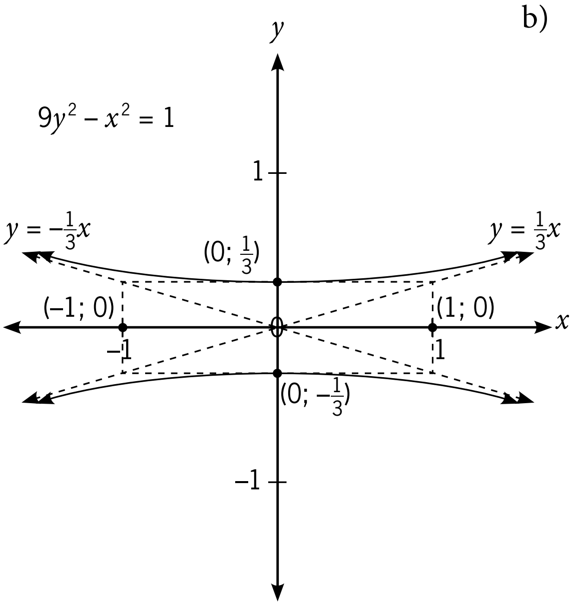
ii) $(-\infty; \infty)$

iii) $(-\infty; -\sqrt{2}] \cup [\sqrt{2}; \infty)$

iv) Non-function

v) Discontinuous

9. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; -\frac{1}{3}] \cup [\frac{1}{3}; \infty)$

iv) Non-function

v) Discontinuous

b) i) $\frac{x^2}{\left(\frac{1}{3}\right)^2} - \frac{y^2}{(1)^2} = 1$

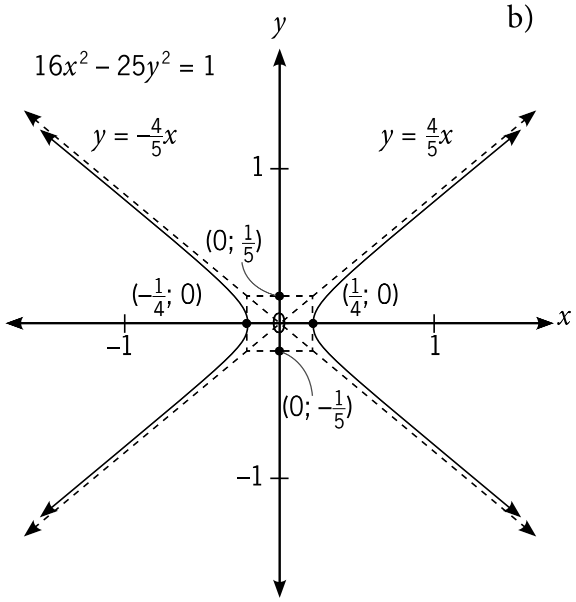
ii) $(-\infty; -\frac{1}{3}] \cup [\frac{1}{3}; \infty)$

iii) $(-\infty;\infty)$

iv) Non-function

v) Discontinuous

10. a) i)



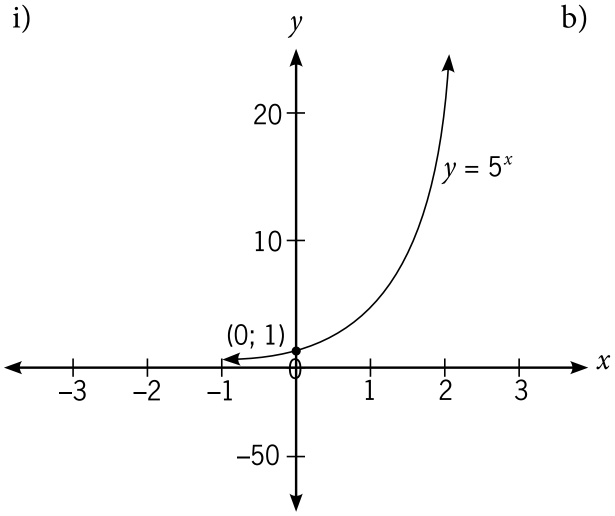
ii) $(-\infty; -\frac{1}{4}] \cup [\frac{1}{4}; \infty)$
iii) $(-\infty; \infty)$
iv) Non-function
v) Discontinuous

b) i) $-\frac{x^2}{\left(\frac{1}{5}\right)^2} + \frac{y^2}{\left(\frac{1}{4}\right)^2} = 1$

ii) $(-\infty; \infty)$
iii) $(-\infty; -\frac{1}{4}] \cup [\frac{1}{4}; \infty)$
iv) Non-function
v) Discontinuous

Activity 3.12 SB page 181

1. a) i)

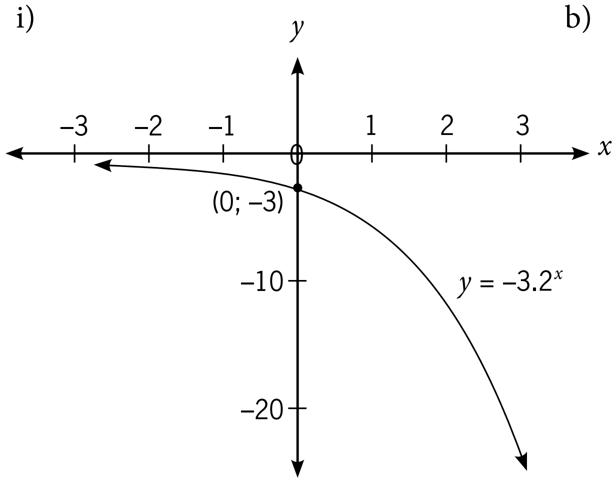


ii) $(-\infty; \infty)$
iii) $(0; \infty)$
iv) Function
v) Continuous

b) i) $y = \log_5 x$

ii) $(0; \infty)$
iii) $(-\infty; \infty)$
iv) Function
v) Continuous

2. a) i)



ii) (−∞; ∞)

iii) (−∞; 0)

iv) Function

v) Continuous

b) i) $y = \log_2(-\frac{x}{3})$

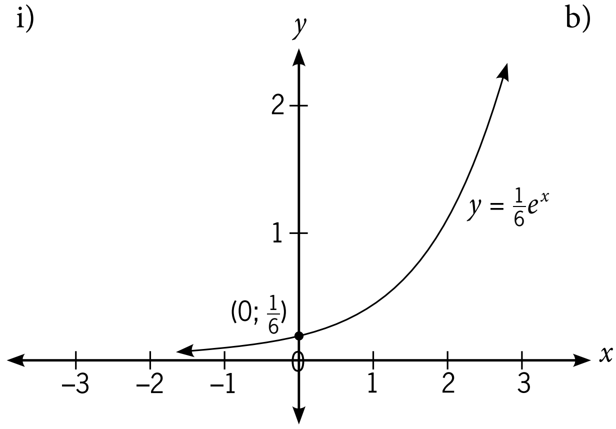
ii) (−∞; 0)

iii) (−∞; ∞)

iv) Function

v) Continuous

3. a) i)



ii) (−∞; ∞);

iii) (0; ∞)

iv) Function

v) Continuous

b) i) $y = \ln(6x)$

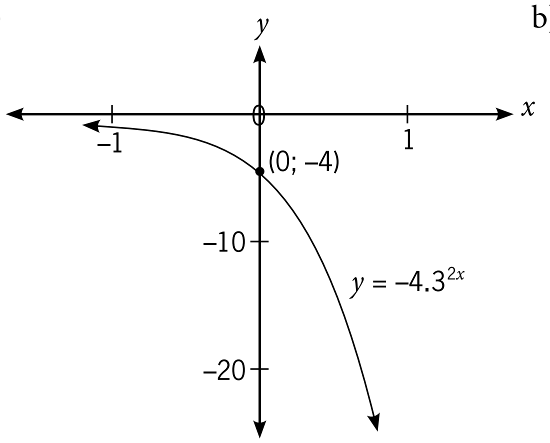
ii) (0; ∞);

iii) (−∞; ∞)

iv) Function

v) Continuous

4. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; 0)$

iv) Function

v) Continuous

b) i) $y = \frac{1}{2}\log_3(-\frac{x}{4})$

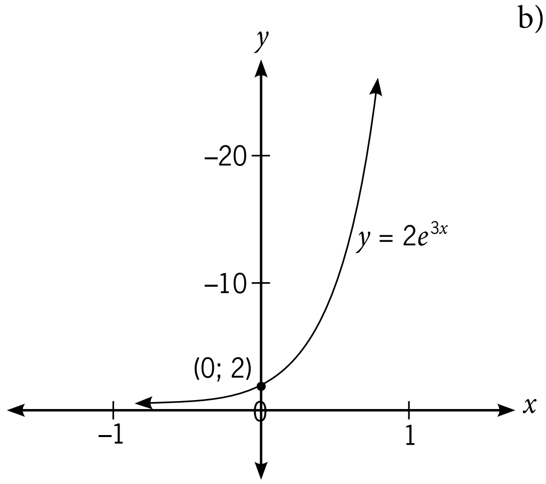
ii) $(-\infty; 0)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

5. a) i)



ii) $(-\infty; \infty)$

iii) $(0; \infty)$

iv) Function

v) Continuous

b) i) $y = \frac{1}{3}\ln\left(\frac{x}{2}\right)$

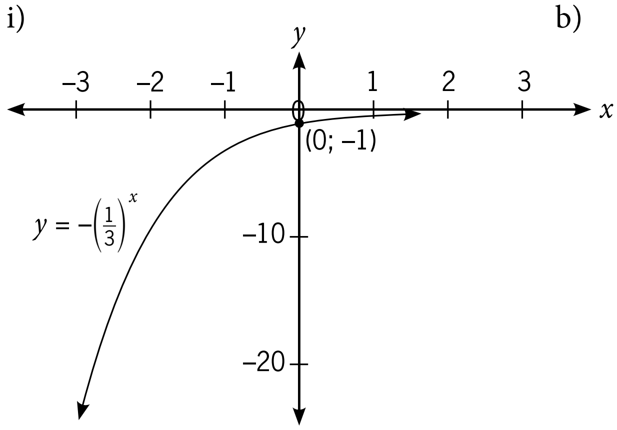
ii) $(0; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

6. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; 0)$

iv) Function

v) Continuous

b) i) $y = \log_{\frac{1}{3}}(-x)$

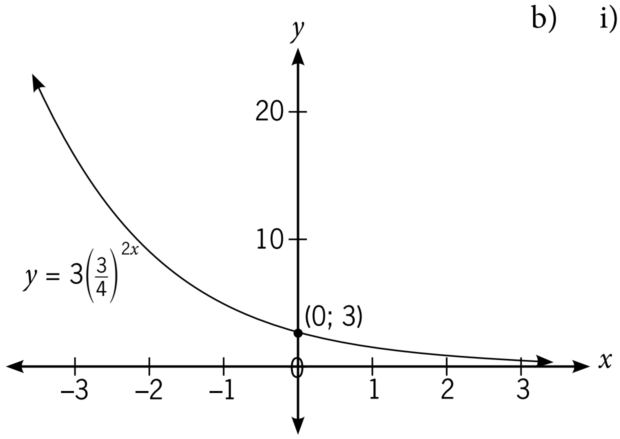
ii) $(-\infty; 0)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

7. a) i)



ii) $(-\infty; \infty)$

iii) $(0; \infty)$

iv) Function

v) Continuous

b) i) $y = \frac{1}{2}\log_{\frac{3}{4}}\left(\frac{x}{3}\right)$

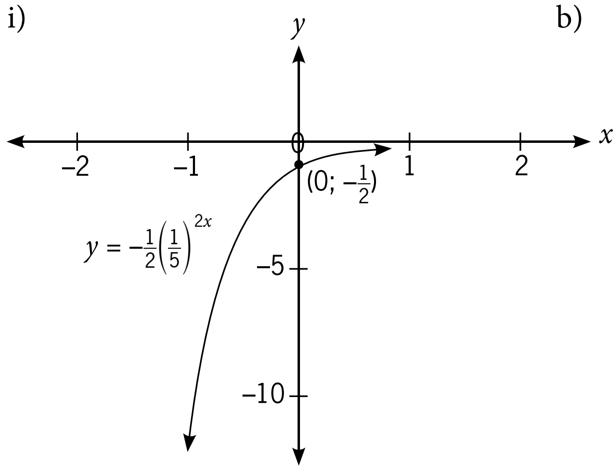
ii) $(0; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

8. a) i)



ii) (–∞; ∞)

iii) (–∞; 0)

iv) Function

v) Continuous

b) i) $y = \frac{1}{2}\log_{\frac{1}{5}}(-2x)$

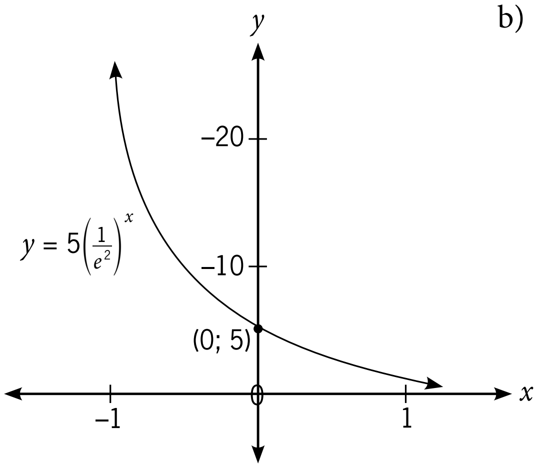
ii) (–∞; 0)

iii) (–∞; ∞)

iv) Function

v) Continuous

9. a) i)



ii) (–∞; ∞)

iii) (0; ∞)

iv) Function

v) Continuous

b) i) $y = -\frac{1}{2}\ln\left(\frac{x}{5}\right)$

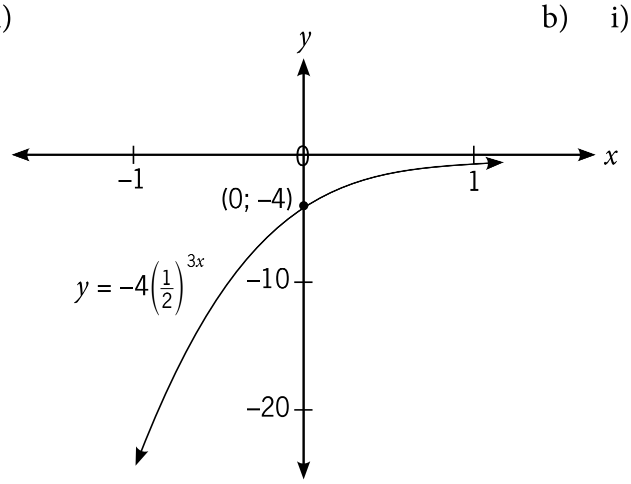
ii) (0; ∞)

iii) (–∞; ∞)

iv) Function

v) Continuous

10. a) i)



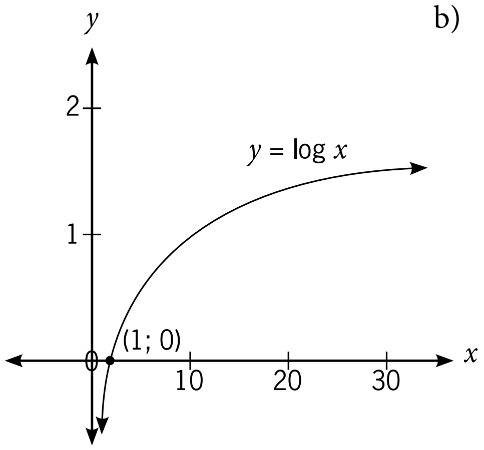
ii) (–∞; ∞)
iii) (–∞; 0)
iv) Function
v) Continuous

b) i) $y = \frac{1}{3}\log_{\frac{1}{2}}\left(-\frac{x}{4}\right)$

ii) (–∞; 0)
iii) (–∞; ∞)
iv) Function
v) Continuous

Activity 3.13 SB page 188

1. a) i)

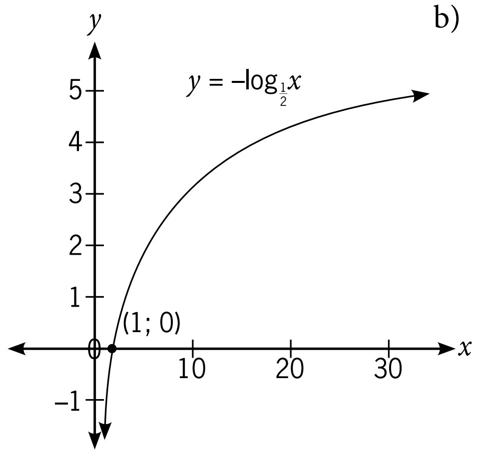


ii) (0; ∞)
iii) (–∞; ∞)
iv) Function
v) Continuous

b) i) $y = 10^x$

ii) (–∞; ∞)
iii) (0; ∞)
iv) Function
v) Continuous

2. a) i)



ii) (0; ∞)

iii) (–∞; ∞)

iv) Function

v) Continuous

b) i) $y = 2^x$

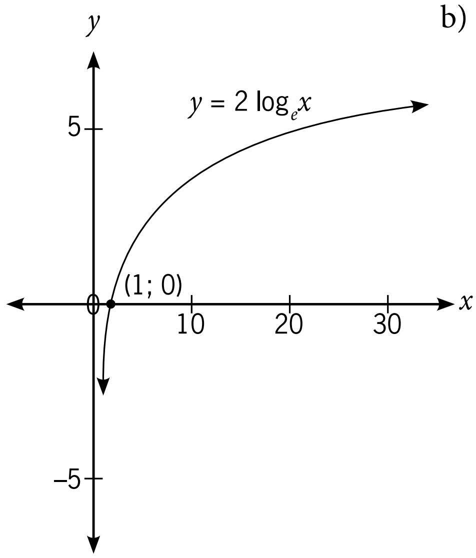
ii) (–∞; ∞)

iii) (0; ∞)

iv) Function

v) Continuous

3. a) i)



ii) (0; ∞)

iii) (–∞; ∞)

iv) Function

v) Continuous

b) i) $y = e^{\frac{x}{2}}$

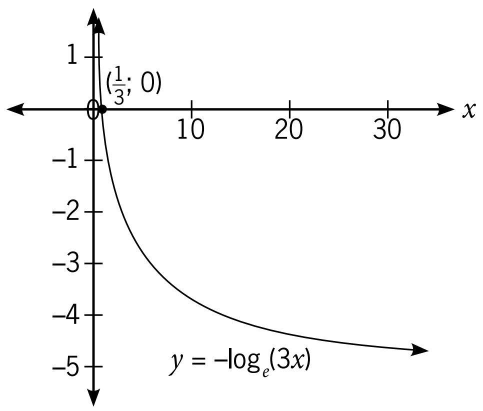
ii) (–∞; ∞)

iii) (0; ∞)

iv) Function

v) Continuous

4. a) i)



ii) (0; ∞)

iii) (−∞; ∞)

iv) Function

v) Continuous

b) i) $y = \frac{1}{3}e^{-x}$

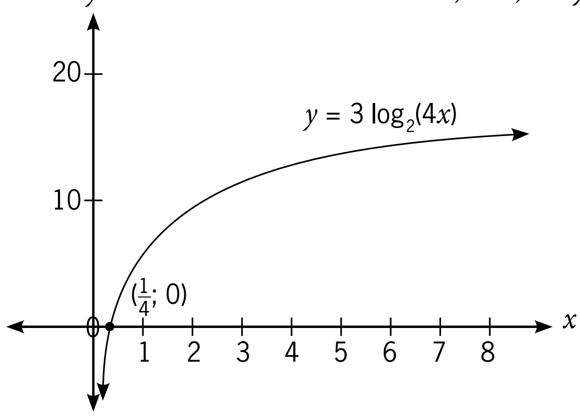
ii) (−∞; ∞)

iii) (0; ∞)

iv) Function

v) Continuous

5. a) i)



ii) (0; ∞)

iii) (−∞; ∞)

iv) Function

v) Continuous

b) i) $y = \frac{1}{4}.2^{\frac{x}{3}}$

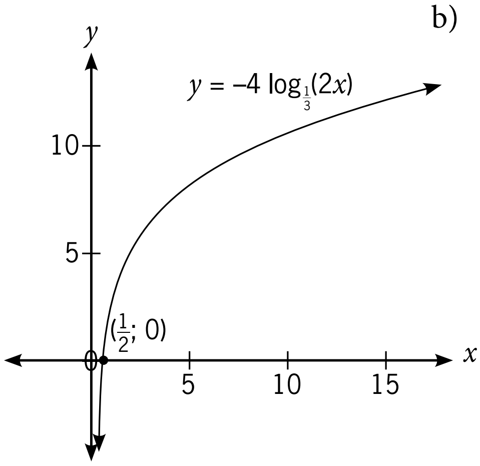
ii) (−∞; ∞)

iii) (0; ∞)

iv) Function

v) Continuous

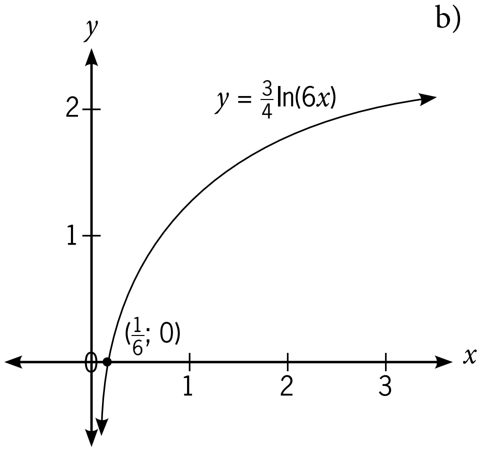
6. a) i)



ii) $(0; \infty)$
iii) $(-\infty; \infty)$
iv) Function
v) Continuous

b) i) $y = \frac{1}{2}.3^{\frac{x}{4}}$
ii) $(-\infty; \infty)$
iii) $(0; \infty)$
iv) Function
v) Continuous

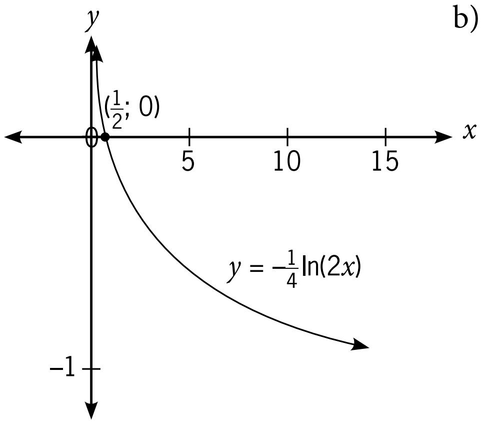
7. a) i)



ii) $(0; \infty)$
iii) $(-\infty; \infty)$
iv) Function
v) Continuous

b) i) $y = \frac{1}{6}e^{\frac{4x}{3}}$
ii) $(-\infty; \infty)$
iii) $(0; \infty)$
iv) Function
v) Continuous

8. a) i)



ii) (0; ∞)

iii) (–∞; ∞)

iv) Function

v) Continuous

b) i) $y = \frac{1}{2}e^{-4x}$

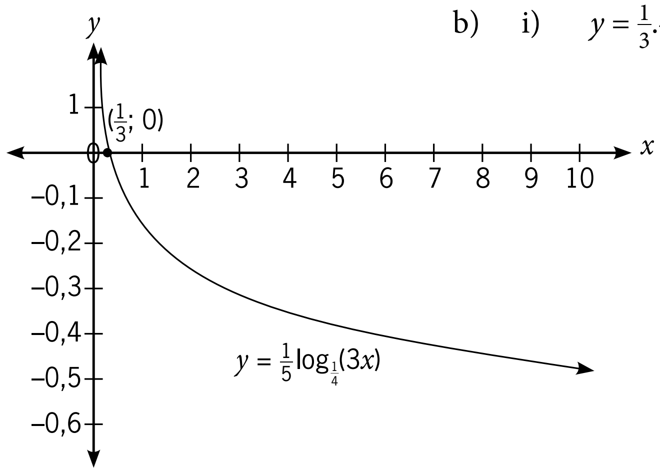
ii) (–∞; ∞)

iii) (0; ∞)

iv) Function

v) Continuous

9. a) i)



ii) (0; ∞)

iii) (–∞; ∞)

iv) Function

v) Continuous

b) i) $y = \frac{1}{3}.4^{-5x}$

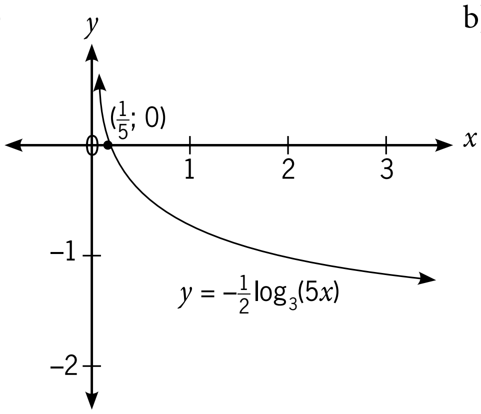
ii) (–∞; ∞)

iii) (0; ∞)

iv) Function

v) Continuous

10. a) i)



ii) $(0; \infty)$

iii) $(-\infty; \infty)$

iv) Function

v) Continuous

b) i) $y = \frac{1}{5}.3^{-2x}$

ii) $(-\infty; \infty)$

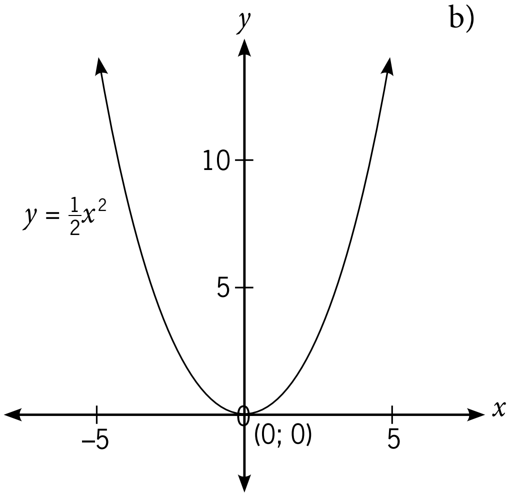
iii) $(0; \infty)$

iv) Function

v) Continuous

Activity 3.14 — SB page 202

1. a) i)



ii) $(-\infty; \infty)$;

iii) $(0; \infty)$

iv) Function

v) Continuous

b) i) $x = \frac{1}{2}y^2$

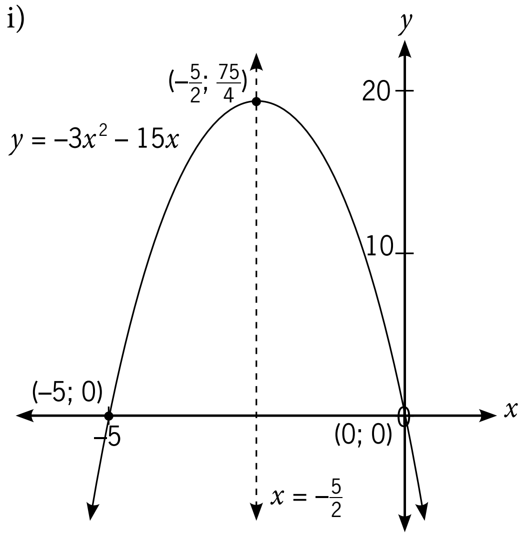
ii) $(0; \infty)$;

iii) $(-\infty; \infty)$

iv) Non-function

v) Continuous

2. a) i)



ii) $(-\infty; \infty)$;

iii) $(-\infty; \frac{75}{4}]$

iv) Function

v) Continuous

b) i) $x = -3y^2 - 15y$

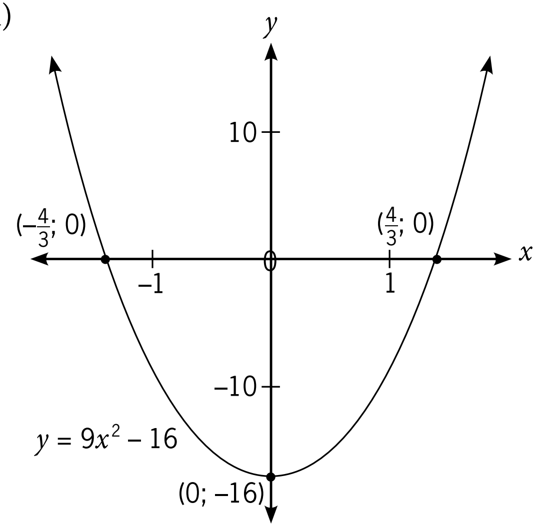
ii) $(-\infty; \frac{75}{4}]$

iii) $(-\infty; \infty)$

iv) Non-function

v) Continuous

3. a) i)



ii) $(-\infty; \infty)$

iii) $[-16; \infty)$

iv) Function

v) Continuous

b) i) $x = 9y^2 - 16$

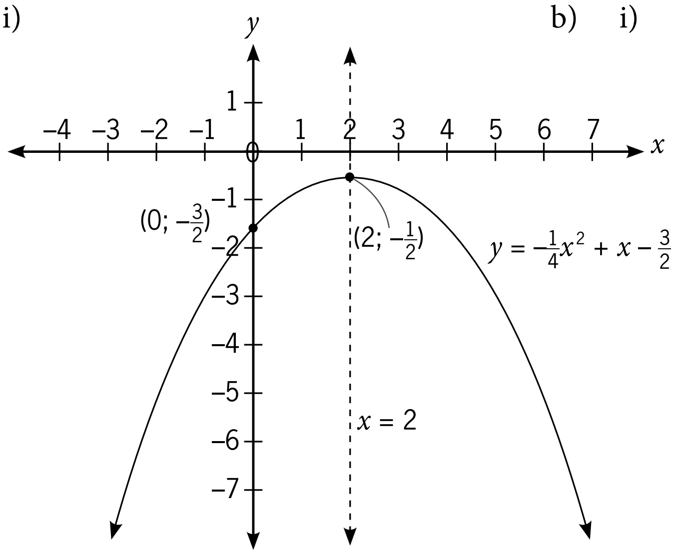
ii) $[-16; \infty)$

iii) $(-\infty; \infty)$

iv) Non-function

v) Continuous

4. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; -\frac{1}{2}]$

iv) Function

v) Continuous

b) i) $x = -\frac{1}{4}y^2 + y - \frac{3}{2}$

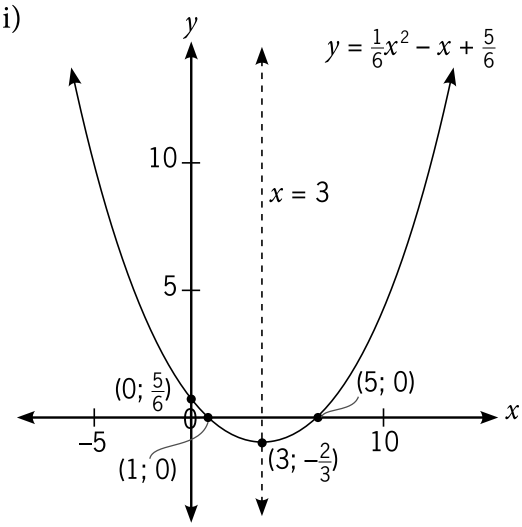
ii) $(-\infty; -\frac{1}{2}]$

iii) $(-\infty; \infty)$

iv) Non-function

v) Continuous

5. a) i)



ii) $(-\infty; \infty)$

iii) $[-\frac{2}{3}; \infty)$

iv) Function

v) Continuous

b) i) $x = \frac{1}{6}y^2 - y + \frac{5}{6}$

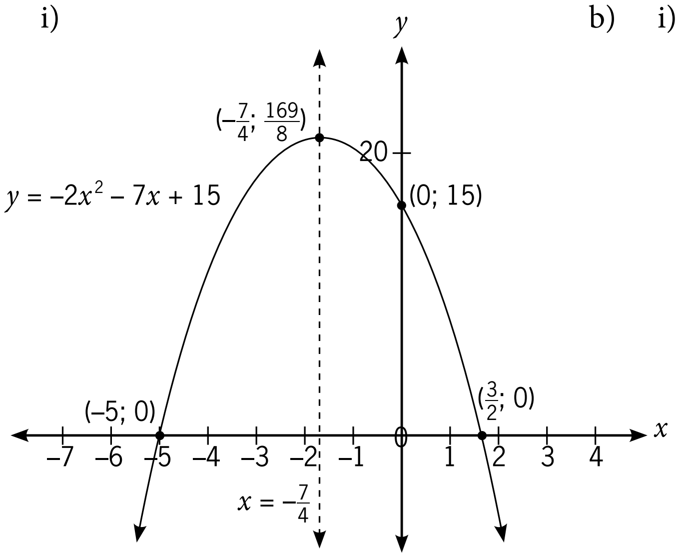
ii) $[-\frac{2}{3}; \infty)$

iii) $(-\infty; \infty)$

iv) Non-function

v) Continuous

6. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; \frac{169}{8}]$

iv) Function

v) Continuous

b) i) $x = -2y^2 - 7y + 15$

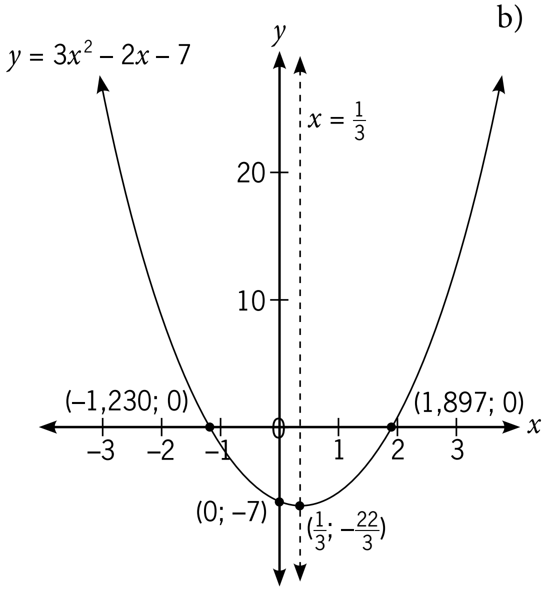
ii) $(-\infty; \frac{169}{8}]$

iii) $(-\infty; \infty)$

iv) Non-function

v) Continuous

7. a) i)



ii) $(-\infty; \infty)$

iii) $\left[-\frac{22}{3}; \infty\right)$

iv) Function

v) Continuous

b) i) $x = 3y^2 - 2y - 7$

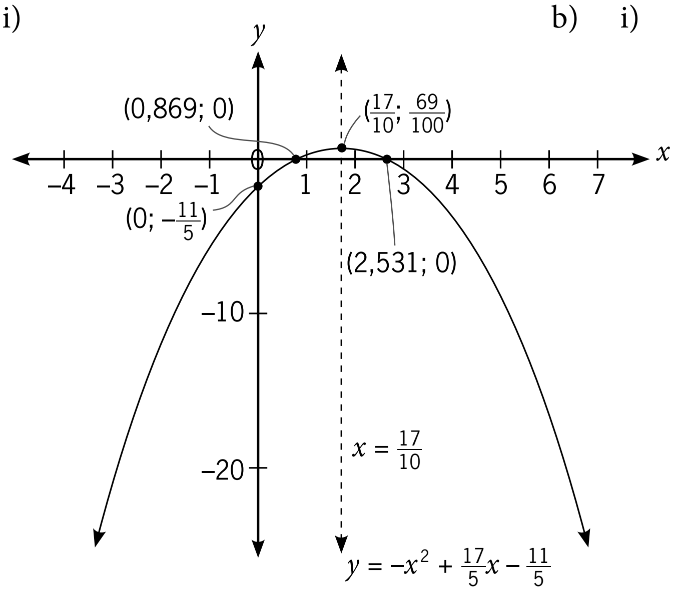
ii) $\left[-\frac{22}{3}; \infty\right)$

iii) $(-\infty; \infty)$

iv) Non-function

v) Continuous

8. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; \frac{69}{100}]$

iv) Function

v) Continuous

b) i) $x = -y^2 + \frac{17}{5}y - \frac{11}{5}$

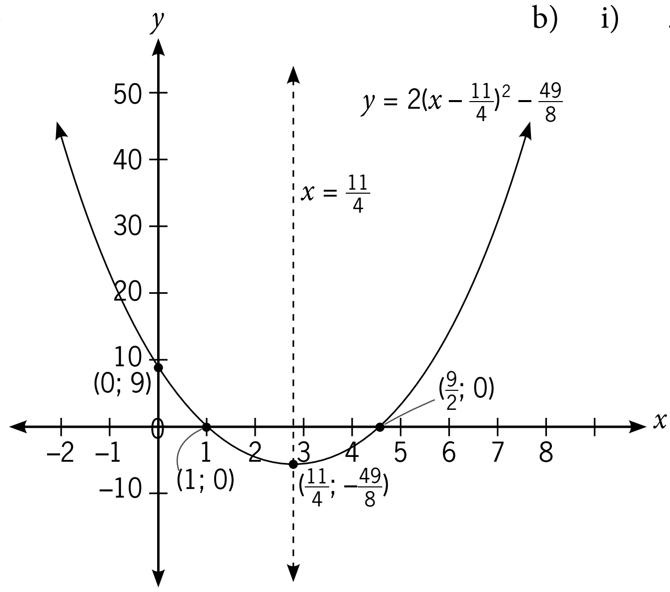
ii) $(-\infty; \frac{69}{100}]$

iii) $(-\infty; \infty)$

iv) Non-function

v) Continuous

9. a) i)



ii) $(-\infty; \infty)$;

iii) $\left[-\frac{49}{8}; \infty\right)$

iv) Function

v) Cotinuous

b) i) $x = 2\left(y - \frac{11}{4}\right)^2 - \frac{49}{8}$

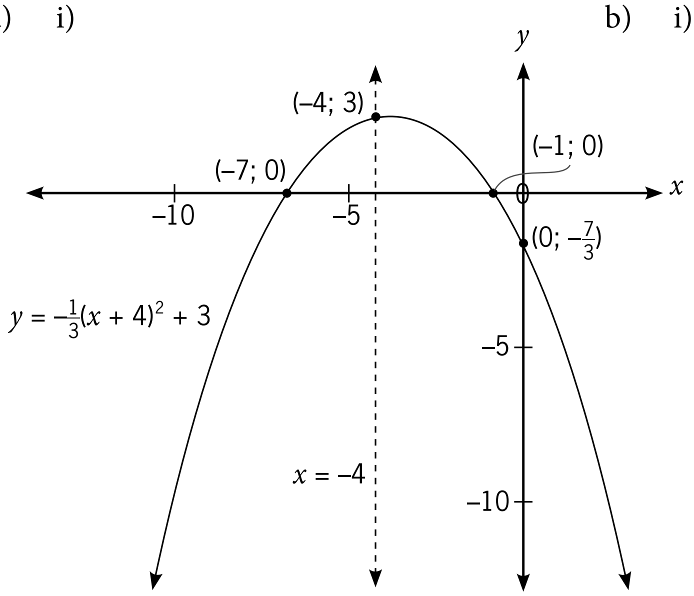
ii) $\left[-\frac{49}{8}; \infty\right)$;

iii) $(-\infty; \infty)$

iv) Non-function

v) Continuous

10. a) i)



ii) $(-\infty; \infty)$

iii) $(-\infty; 3]$

iv) Function

v) Continuous

b) i) $x = -\frac{1}{3}(y + 4)^2 + 3$

ii) $(-\infty; 3]$

iii) $(-\infty; \infty)$

iv) Non-function

v) Continuous

Summative assessment: Module 3 — SB page 203

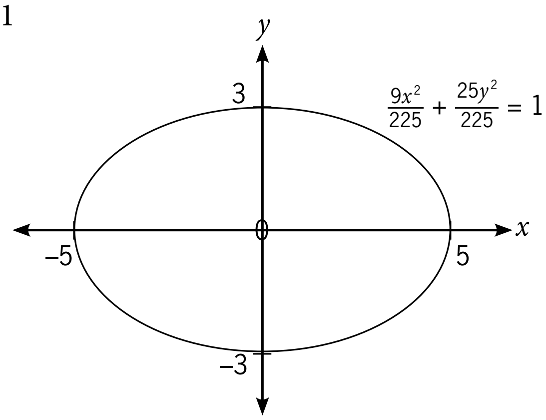
1. 1.1 Domain: $(-\infty; \infty)$; Range: $\left[\frac{95}{24}; \infty\right)$ (4)

1.2 $x = 6y^2 - 11y + 9$ (2)

1.3 Continuous (1)

1.4 Non-function (1)

2. 2.1



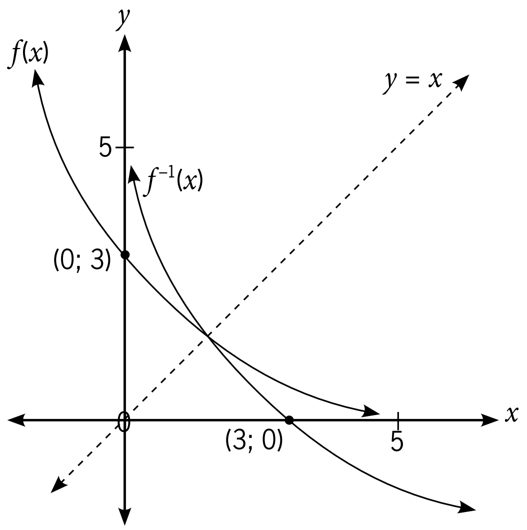
Shape ✓
Labelling ✓
x and y ✓

(3)

2.2 Both x-axis and y-axis. (1)

3. 3.1 $y = \log_{\frac{1}{2}}\left(\frac{x}{3}\right)$ (3)

3.2



(5)

3.3 $f^{-1}(f(x)) = \log_{\frac{1}{2}}\left(\frac{3(\frac{1}{2})^x}{3}\right)$

$= \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^x$

$= \log_{\frac{1}{2}}\frac{1}{2}$

$= x.1$

$f^{-1}(f(x)) = x$

$f(f^{-1}(x)) = 3\left(\frac{1}{2}\right)^{(\log_{\frac{1}{2}}(-\frac{x}{3}))}$

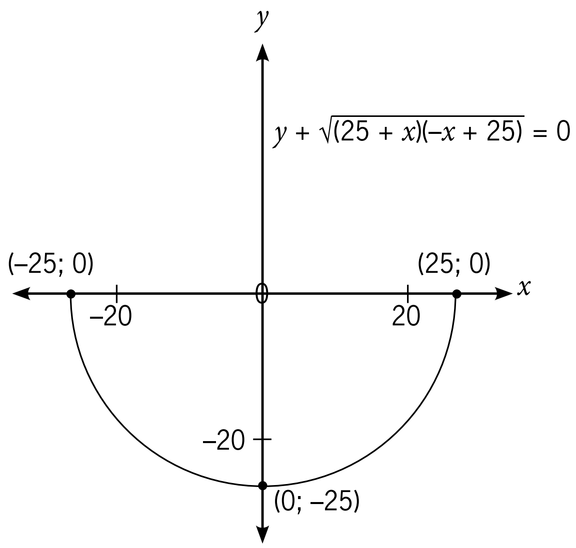
$= 3.\frac{x}{3}$

$f(f^{-1}(x)) = x$ (4)

Since $f^{-1}(f(x)) = f(f^{-1}(x))$, therefore $f(x) = -3\left(\frac{1}{2}\right)^x$ and $f^{-1}(x) = \log_{\frac{1}{2}}\left(\frac{x}{3}\right)$ are inverses of each other in the line $y = x$.

4. 4.1 [–25; 25]; [–25; 0] (4)

4.2



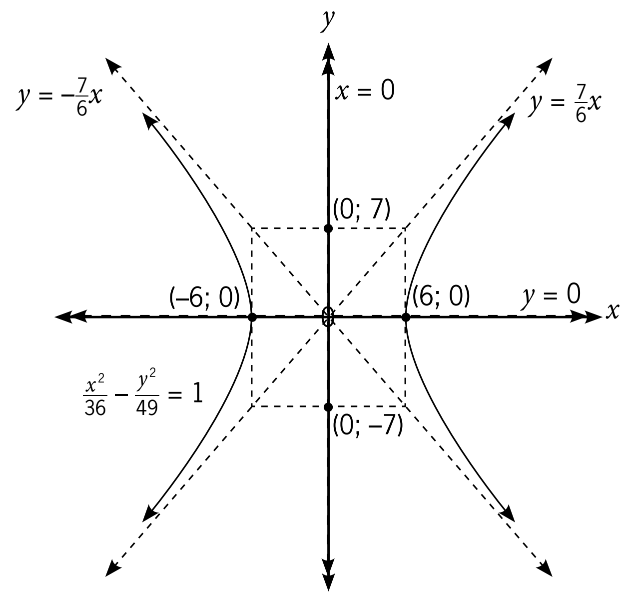
(5)

4.3 $$y + \sqrt{(25 + x)(-x + 25)} = 0$$
$$y + \sqrt{(25 + (-x))(-(-x) + 25)} = 0$$
$$y + \sqrt{(25 - x)(x + 25)} = 0$$
$$y + \sqrt{(25 + x)(-x + 25)} = 0$$

Since the equation is equivalent, the graph is symmetrical about the y-axis.

The tests for symmetry about the line x-axis, the line $y = x$ and the line $y = -x$ does not yield an equivalent equation, therefore are not lines of symmetry. (3)

5. 5.1

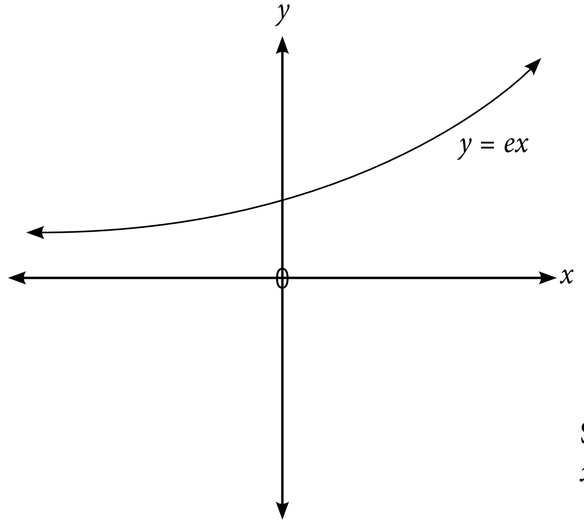


x-intercept: (–6; 0); (6; 0)

y-intercept: (0; –7); (0; 7) (6)

5.2 $y = -\frac{7}{6}x$; $y = \frac{7}{6}x$ (3)

6. 6.1 (3)



Shape ✓
x and y ✓

6.2 Range = $\{0 < y < \infty\}$ ✓ (1)

6.3 $x = e^y$ ✓ (1)

TOTAL: [50]

MODULE

4 *Trigonometry*

After they have completed this module, students should be able to:

- calculate special triangles pertaining to the four quadrants;
- apply the notion of compound angles such as $\sin(a \pm b)$, $\cos(a \pm b)$ and $\tan(a \pm b)$;
- apply the complementary angles specifically to trigonometric identities;
- apply factorisation of different types of trigonometric equations including using identities;
- derive the following identities:
  - compound angles
  - double angles
  - half angles;
- derive the co-ratios $\sin(90° \pm \theta)$, $\cos(90° \pm \theta)$ and $\tan(90° \pm \theta)$;
- use the square, invert and quotient identities and furthermore solve trigonometric equations, simplify trigonometric expressions and prove trigonometric identities; and
- draw neat sketch graphs of the following functions/relations:
  - $y = a \sin(bx + c) + d$ for $-\pi \leq x \leq \pi$
  - $y = a \cos(bx + c) + d$ for $-\pi \leq x \leq \pi$
  - $y = a \tan(bx + c) + d$ for $-\pi \leq x \leq \pi$
  - $y = \text{cosec } x$ for $-\pi \leq x \leq \pi$
  - $y = \sec x$ for $-\pi \leq x \leq \pi$
  - $y = \cot x$ for $-\pi \leq x \leq \pi$.

Introduction

Trigonometry deals with the study of triangles and is used when precise distances, lengths, heights and angles need to be calculated. The trigonometric skills in this module will require students' logical thinking skills.

Students need the following pre-knowledge to successfully complete this module.

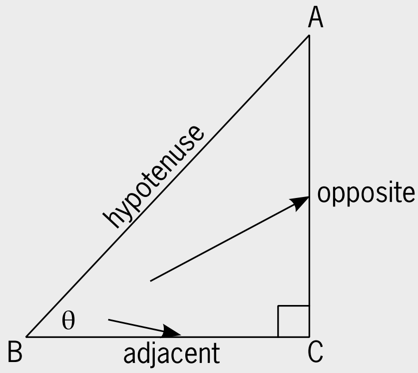
Pre-knowledge

- **Trigonometric ratios**

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\quad$ $\text{cosec}\,\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$

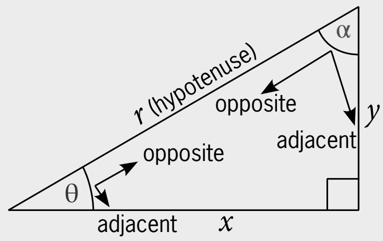
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\quad$ $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\quad$ $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$



- **Definition of the trigonometric ratios**

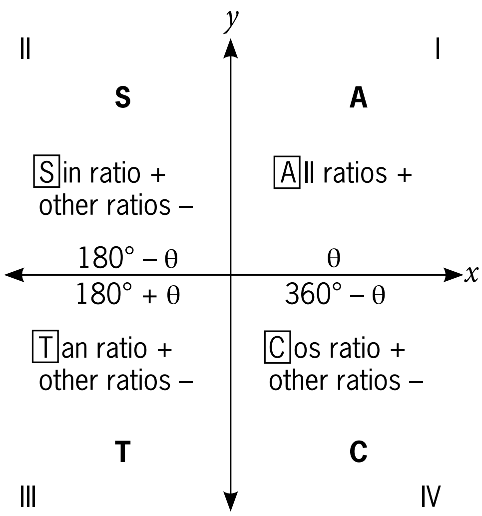
$\sin\theta = \frac{y}{r}$	$\sin\alpha = \frac{x}{r}$	$\text{cosec}\,\theta = \frac{r}{y}$	$\text{cosec}\,\alpha = \frac{r}{x}$
$\cos\theta = \frac{x}{r}$	$\cos\alpha = \frac{y}{r}$	$\sec\theta = \frac{r}{x}$	$\sec\alpha = \frac{r}{y}$
$\tan\theta = \frac{y}{x}$	$\tan\alpha = \frac{x}{y}$	$\cot\theta = \frac{x}{y}$	$\cot\alpha = \frac{y}{x}$



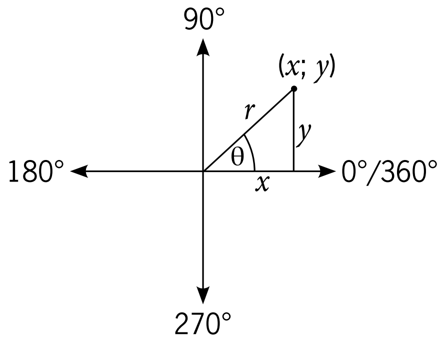
- **Theorem of Pythagoras:** $x^2 + y^2 = r^2$

- **Signs of trigonometric functions:** CAST diagram

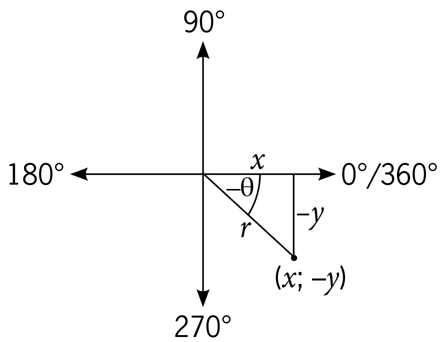
Note: θ is the reference angle.



Positive angle:

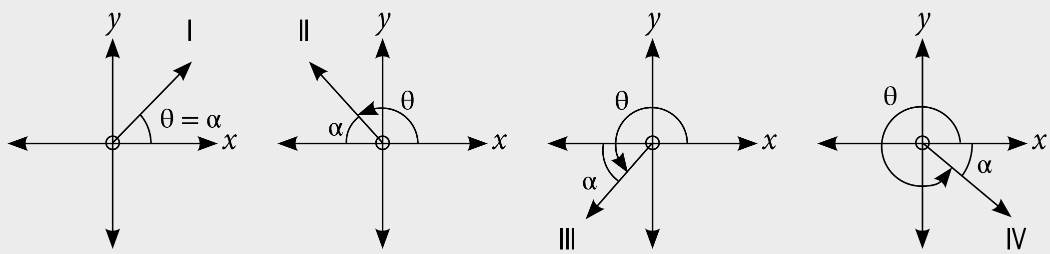


Negative angle:



Quadrant	Positive functions	Negative functions
I	all	none
II	sin, cosec	cos, sec, tan, cot
III	tan, cot	sin, cosec, cos, sec
IV	cos, sec	sin, cosec, tan, cot

- **Reference angle**

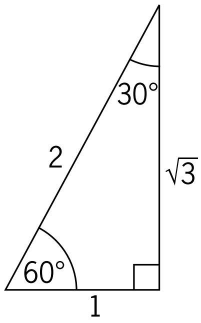


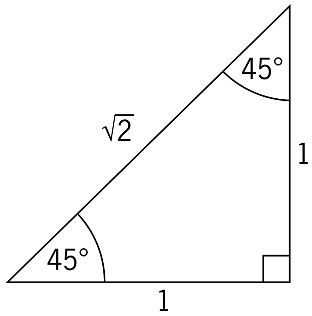
$\theta = 180° - \alpha$ $\theta = 180° + \alpha$ $\theta = 360° - \alpha$

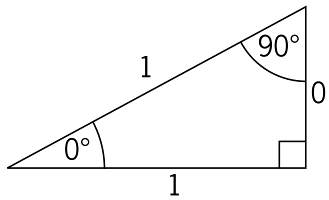
- **Trigonometric identities**

Reciprocal identities/ratios		
$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
Quotient identities		
$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	
Square/Pythagorean identities		
$\sin^2\theta + \cos^2\theta = 1$	$\tan^2\theta + 1 = \sec^2\theta$	$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

- **Special angles**

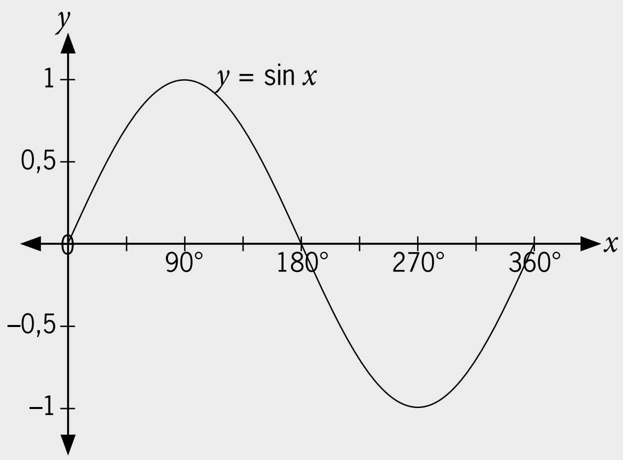




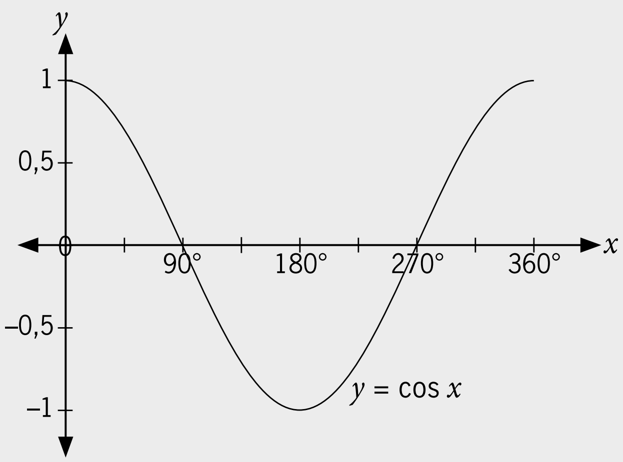


$\sin 30° = \frac{1}{2}$	$\sin 45° = \frac{1}{\sqrt{2}}$	$\sin 0° = \frac{0}{1} = 0$
$\sin 60° = \frac{\sqrt{3}}{2}$	$\cos 45° = \frac{1}{\sqrt{2}}$	$\sin 90° = \frac{1}{1} = 1$
$\cos 30° = \frac{\sqrt{3}}{2}$	$\tan 45° = \frac{1}{1} = 1$	$\cos 0° = \frac{1}{1} = 1$
$\cos 60° = \frac{1}{2}$		$\cos 90° = \frac{0}{1} = 0$
$\tan 30° = \frac{1}{\sqrt{3}}$		$\tan 0° = \frac{0}{1} = 0$
$\tan 60° = \frac{\sqrt{3}}{1}$		$\tan 90° = \frac{1}{0} = \infty$

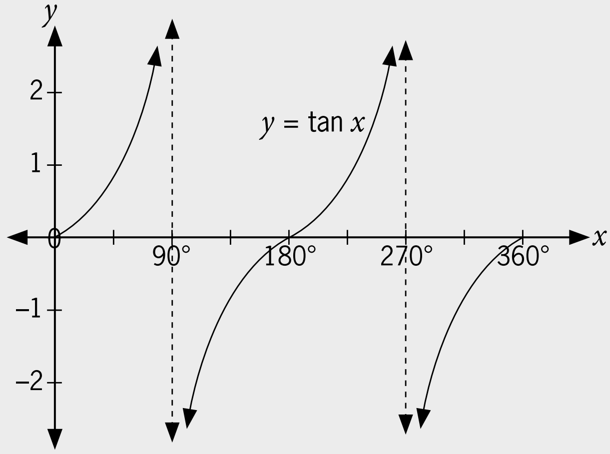
- **Standard trigonometric graphs**



$y = \sin x$	
Amplitude	1
Range	$-1 \leq y \leq 1$
Frequency	1
Period	360°
Domain	$0° \leq x \leq 360°$
Turning points	(90°; 1); (270°; –1)
***x*-intercepts**	(0°; 0); (180°; 0); (360°; 0)



$y = \cos x$	
Amplitude	1
Range	$-1 \leq y \leq 1$
Frequency	1
Period	360°
Domain	$0° \leq x \leq 360°$
Turning points	(0°; 1); (180°; –1); (360°; 1)
***x*-intercepts**	(90°; 0); (270°; 0)



$y = \tan x$	
Amplitude	undefined
Range	$y \in \mathbb{R}$
Frequency	2
Period	180°
Domain	$0° \leq x \leq 360°$, $x \neq 90°, x \neq 270°$
Asymptotes	$x = 90°$; $x = 270°$
x-intercepts	(0°; 0); (180°; 0); (360°; 0)

Reciprocal ratios

$\sin x = \frac{1}{\text{cosec } x}$; $\cos x = \frac{1}{\sec x}$; $\tan x = \frac{1}{\cot x}$

Quotient identities

$\tan x = \frac{\sin x}{\cos x}$; $\cot x = \frac{\cos x}{\sin x}$

Square identities

$\sin^2 x + \cos^2 x = 1$; $1 + \tan^2 x = \sec^2 x$; $1 + \cot^2 x = \text{cosec}^2 x$

Double-angle identities (sum/difference of two angles)

$\sin(a \pm b) = \sin a.\cos b \pm \cos a.\sin b$

$\cos(a \pm b) = \cos a.\cos b \mp \sin a.\sin b$

$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a.\tan b}$

Double-angle identities

$\sin 2x = 2 \sin x.\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$; $\cos 2x = 1 - 2 \sin^2 x$; $\cos 2x = 2 \cos^2 x - 1$

$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

and

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$; $\sin^2 x = \frac{1}{2}\left(1 - \cos 2x\right)$

Half-angle identities

$\sin \frac{1}{2}x = \sqrt{\frac{(1 - \cos x)}{2}}$ $\cos \frac{1}{2}x = \sqrt{\frac{1}{2}(1 + \cos x)}$;

$\tan \frac{1}{2}x = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$; $\tan \frac{1}{2}x = \frac{1 - \cos x}{\sin x}$; $\tan \frac{1}{2}x = \frac{\sin x}{1 + \cos x}$

Co-ratios (co-functions)		
$\sin(90° - x) = \cos x$	$\cos(90° - x) = \sin x$	$\tan(90° - x) = \cot x$
$\sin(90° + x) = \cos x$	$\cos(90° + x) = -\sin x$	$\tan(90° + x) = -\cot x$
$\text{cosec}(90° - x) = \sec x$	$\sec(90° - x) = \text{cosec}\, x$	$\cot(90° - x) = \tan x$
$\text{cosec}(90° + x) = \sec x$	$\sec(90° + x) = -\text{cosec}\, x$	$\cot(90° + x) = -\tan x$

Factorisation

- Common factor, for example
  $\sin\theta.\cos\theta + \sin\theta = \sin\theta(\cos\theta + 1)$
- Difference between squares, for example
  $\cos^2\theta - \sin^2\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$
- Trinomials, for example
  $\cos^2\theta + 2\cos\theta + 1 = (\cos\theta + 1)(\cos\theta + 1) = (\cos\theta + 1)^2$
- Sum and difference of two cubes, for example
  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- Grouping, for example
  $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$
- Fractions: find common denominators and equivalent numerators, for example
  $\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\sin^2 x.\cos^2 x}$
- Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The basic graphs for the domain of $0° \leq x \leq 360°$ for:

- $y = a\sin x + d$
- $y = a\cos x + d$
- $y = a\tan x + d$

where

- the a-value determines the shape and amplitude of the graph;
- the d-value shifts the graph vertically up or down.

Activity 4.1 SB page 216

1\. 1.1 sin 140°
$= \sin(180° - 40°)$
$= \sin 40°$

1.2 tan(–85°)
$= -\tan 85°$

1.3 sec 265°
$= \sec(180° + 85°)$
$= -\sec 85°$

1.4 $\cos\left(\frac{67}{36}\pi\right)$
$= \cos\left(2\pi - \frac{5}{36}\pi\right)$
$= \cos\left(\frac{5}{36}\pi\right)$

1.5 $\cot\left(-\frac{25}{18}\pi\right)$
$= -\cot\left(\frac{25}{18}\pi\right)$
$= -\cot\left(\pi + \frac{7}{18}\pi\right)$
$= -\cot\left(\frac{7}{18}\pi\right)$

1.6 cosec(–465°)
$= \operatorname{cosec}(-105° - 360°)$
$= \operatorname{cosec}(-105° - (1)360°)$
$= \operatorname{cosec}(-105°)$
$= -\operatorname{cosec} 105°$
$= -\operatorname{cosec}(180° - 75°)$
$= -\operatorname{cosec} 75°$

1.7 cot(−315°)
$= -\cot 315°$
$= -\cot(360° - 45°)$
$= \cot 45°$

1.8 $\sec\left(-\frac{25}{4}\pi\right)$
$= \sec\left(-\frac{1}{4}\pi - 6\pi\right)$
$= \sec\left(-\frac{1}{4}\pi - 3.2\pi\right)$
$= \sec\left(-\frac{1}{4}\pi\right)$
$= \sec\left(\frac{1}{4}\pi\right)$

2\. 2.1 tan(–225°)
$= -\tan 225°$
$= -\tan(180° + 45°)$
$= -\tan 45°$
$= -1$

2.2 sin 240°
$= \sin(180° + 60°)$
$= -\sin 60°$
$= -\frac{\sqrt{3}}{2}$

2.3 cos 120°
$= \cos(180° - 60°)$
$= -\cos 60°$
$= -\frac{1}{2}$

2.4 cosec 330°
$= \frac{1}{\sin 330°}$
$= \frac{1}{\sin(360° - 30°)}$
$= \frac{1}{-\sin 30°}$
$= \frac{1}{-\frac{1}{2}}$
$= -2$

2.5 $\cot\left(-\frac{\pi}{4}\right)$

$= -\cot\frac{\pi}{4}$

$= -1$

2.6 $\sec\left(-\frac{7}{4}\pi\right)$

$= \frac{1}{\cos\left(-\frac{7}{4}\pi\right)}$

$= \frac{1}{\cos\left(\frac{7}{4}\pi\right)}$

$= \frac{1}{\cos\left(2\pi - \frac{1}{4}\pi\right)}$

$= \frac{1}{\cos\left(\frac{1}{4}\pi\right)}$

$= \frac{1}{\frac{1}{\sqrt{2}}}$

$= \sqrt{2}$

2.7 $\sin 150°$

$= \sin(180° - 30°)$

$= \sin 30°$

$= \frac{1}{2}$

2.8 $\cot(-30°)$

$= \frac{1}{\tan(-30°)}$

$= \frac{1}{-\tan 30°}$

$= \frac{1}{-\frac{1}{\sqrt{3}}}$

$= -\sqrt{3}$

2.9 $\text{cosec}(-120°)$

$= \frac{1}{\sin(-120°)}$

$= \frac{1}{-\sin 120°}$

$= \frac{1}{-\sin(180° - 60°)}$

$= \frac{1}{-\sin 60°}$

$= \frac{1}{-\frac{\sqrt{3}}{2}}$

$= -\frac{2}{\sqrt{3}}$

3. $3\cos 150° - \sin(-45°) + 2\tan 420° + \sec^2(-120°)$

$= 3\cos(180° - 30°) - [-\sin 45°] + 2\tan(420° - 360°) + \sec^2(180° - 60°)$

$= -3\cos 30° + \sin 45° + 2\tan 60° - \sec^2 60°$

$= -3\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{\sqrt{2}} + 2\sqrt{3} - (2)^2$

$= -\frac{3\sqrt{3}}{2} + \frac{1}{\sqrt{2}} + 2\sqrt{3} - 4$

Activity 4.2 SB page 221

1. 1.1 $\sin(\alpha - 30°) = \sin \alpha\cos 30° - \sin 30°\cos \alpha$

   1.2 $\cos\left(\frac{\pi}{4} + 2\lambda\right) = \cos\left(\frac{\pi}{4}\right)\cos 2\lambda - \sin\left(\frac{\pi}{4}\right)\sin 2\lambda$

   1.3 $\tan(\rho - 60°) = \frac{\tan \rho - \tan 60°}{1 + \tan \rho.\tan 60°}$

   1.4 $\text{cosec}(105° + \omega) = \frac{1}{\sin(105° + \omega)} = \frac{1}{\sin 105° \cos \omega + \sin \omega \cos 105°}$

   1.5 $\sec(4\theta - 15°) = \frac{1}{\cos(4\theta - 15°)} = \frac{1}{\cos 4\theta \cos 15° + \sin 4\theta \sin 15°}$

   1.6 $\cot\left(75° + \frac{\beta}{3}\right) = \frac{1}{\tan\left(75° + \frac{\beta}{3}\right)} = \frac{1}{\frac{\tan 75° + \tan\frac{\beta}{3}}{1 - \tan 75° \tan\frac{\beta}{3}}} = \frac{1 - \tan 75° \tan\frac{\beta}{3}}{\tan 75° + \tan\frac{\beta}{3}}$

2. 2.1 $\cos 40° \cos 35° - \sin 40° \sin 35° = \cos(40° + 35°)$

   2.2 $\sin 4x \cos 55° + \sin 55° \cos 4x = \sin(4x + 55°)$

   2.3 $\frac{\tan 2\theta - \tan 3\rho}{1 + \tan 2\theta \tan 3\rho} = \tan(2\theta - 3\rho)$

   2.4 $\frac{1}{\sin 3\alpha \cos 2\beta - \cos 3\alpha \sin 2\beta} = \frac{1}{\sin(3\alpha - 2\beta)} = \text{cosec }(3\alpha - 2\beta)$

   2.5 $\frac{1 - \tan \lambda \tan\frac{\pi}{6}}{\tan \lambda + \tan\frac{\pi}{6}} = \left(\frac{\tan \lambda + \tan\frac{\pi}{6}}{1 - \tan \lambda \tan\frac{\pi}{6}}\right)^{-1}$

   $= \left(\tan\left(\lambda + \frac{\pi}{6}\right)\right)^{-1}$

   $= \frac{1}{\tan\left(\lambda + \frac{\pi}{6}\right)}$

   $= \cot\left(\lambda + \frac{\pi}{6}\right)$

   2.6 $(\cos 8x \cos 2x + \sin 8x \sin 2x)^{-1} = (\cos(8x - 2x))^{-1}$

   $= (\cos 6x)^{-1}$

   $= \frac{1}{\cos 6x}$

   $= \sec 6x$

3. 3.1 $\tan 75° = \tan(30° + 45°)$

   $= \frac{\tan 30° + \tan 45°}{1 - \tan 30° \tan 45°}$

   $= \frac{\frac{1}{\sqrt{3}} + \frac{1}{1}}{1 - \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{1}\right)}$

   $= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}}$

   $= \frac{1+\sqrt{3}}{\sqrt{3} - 1}$

   $= \frac{1 + \sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$

   $= \frac{4 + 2\sqrt{3}}{2}$

   $= 2 + \sqrt{3}$

3.2 $\sin 50° \cos 55° + \sin 55° \cos 50°$

$= \sin(50° + 55°)$

$= \sin 105°$

$= \sin(45° + 60°)$

$= \sin 45° \cos 60° + \sin 60° \cos 45°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1 + \sqrt{3}}{2\sqrt{2}}$

$= \frac{1 + \sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2} + \sqrt{6}}{4}$

3.3 $\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \sin 30° \cos 45°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right).\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \left(\frac{1}{2\sqrt{2}}\right)$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

$= \frac{\sqrt{2}(\sqrt{3} - 1)}{4}$

3.4 $\frac{\tan 63° - \tan 48°}{1 + \tan 63° \tan 48°}$

$= \tan(63° - 48°)$

$= \tan 15°$

$= \tan(45° - 30°)$

$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$= \frac{\left(\frac{1}{1}\right) - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{1}\right)\left(\frac{1}{\sqrt{3}}\right)}$

$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$

$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$

$= \frac{3 - 1}{3 + 2\sqrt{3} + 1}$

$= \frac{2}{4 + 2\sqrt{3}}$

$= \frac{1}{2 + \sqrt{3}}$

3.5 $\dfrac{1}{\sin 33°\sin 18° + \cos 33°\cos 18°} = \dfrac{1}{\cos 33°\cos 18° + \sin 33°\sin 18°}$

$= \dfrac{1}{\cos(33° - 18°)}$

$= \dfrac{1}{\cos 15°}$

$= \dfrac{1}{\cos(45° - 30°)}$

$= \dfrac{1}{\cos 45°\cos 30° + \sin 45°\sin 30°}$

$= \dfrac{1}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)}$

$= \dfrac{1}{\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}}$

$= \dfrac{1}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

$= \dfrac{2\sqrt{2}}{\sqrt{3}+1}$

$= \dfrac{2\sqrt{2}}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{2\sqrt{6} - 2\sqrt{2}}{2}$

$= \sqrt{6} - \sqrt{2}$

$= \sqrt{2}(\sqrt{3} - 1)$

3.6 $\sin(-105°)$

$= -\sin 105°$

$= -\sin(60° + 45°)$

$= -(\sin 60°\cos 45° + \sin 45°\cos 60°)$

$= -\left[\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)\right]$

$= -\left[\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right]$

$= -\dfrac{\sqrt{3}}{2\sqrt{2}} - \dfrac{1}{2\sqrt{2}}$

$= \dfrac{-\sqrt{3} - 1}{2\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{-\sqrt{6} - \sqrt{2}}{4}$ or $\dfrac{-(\sqrt{6} + \sqrt{2})}{4}$

4. 4.1 To prove: $\sin(x - 180°) = -\sin x$

LHS $= \sin(x - 180°)$

$= \sin x\cos 180° - \cos x\sin 180°$

$= \sin x(-1) - \cos x(0)$

$= -\sin x$

$\therefore$ LHS = RHS

4.2 To prove: $\cos(-15°) = \frac{\sqrt{6} + \sqrt{2}}{4}$

$$\begin{aligned} \text{LHS} &= \cos(-15°) \\ &= \cos(30° - 45°) \\ &= \cos 30° \cos 45° + \sin 30° \sin 45° \\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) \\ &= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3} + 1}{2\sqrt{2}} \\ &= \frac{\sqrt{3} + 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

$\therefore$ LHS = RHS

4.3 To prove: $\cos 71° + \cos 49° = \cos 11°$

$$\begin{aligned} \text{LHS} &= \cos 71° + \cos 49° \\ &= \cos(60° + 11°) + \cos(60° - 11°) \\ &= (\cos 60° \cos 11° - \sin 60° \sin 11°) + (\cos 60° \cos 11° + \sin 60° \sin 11°) \\ &= \cos 60° \cos 11° - \sin 60° \sin 11° + \cos 60° \cos 11° + \sin 60° \sin 11° \\ &= 2 \cos 60° \cos 11° \\ &= 2\left(\frac{1}{2}\right)\cos 11° \\ &= \cos 11° \end{aligned}$$

$\therefore$ LHS = RHS

4.4 To prove: $\sin 33° - \sin 57° = -\sqrt{2}\sin 12°$

$$\begin{aligned} \text{LHS} &= \sin 33° - \sin 57° \\ &= \sin(45° - 12°) - \sin(45° + 12°) \\ &= (\sin 45° \cos 12° - \sin 12° \cos 45°) - (\sin 45° \cos 12° + \sin 12° \cos 45°) \\ &= \sin 45° \cos 12° - \sin 12° \cos 45° - \sin 45° \cos 12° - \sin 12° \cos 45° \\ &= -2 \sin 12° \cos 45° \\ &= -2 \sin 12°\left(\frac{1}{\sqrt{2}}\right) \\ &= -\frac{2}{\sqrt{2}} \sin 12° \\ &= -\sqrt{2} \sin 12° \end{aligned}$$

$\therefore$ LHS = RHS

Activity 4.3 SB page 225

1. 1.1 To prove: $\sec(90° - x) = \text{cosec } x$

$$\begin{aligned}\text{LHS} &= \sec(90° - x)\\ &= \frac{1}{\cos(90° - x)}\\ &= \frac{1}{\cos 90° \cos x + \sin 90° \sin x}\\ &= \frac{1}{(0)\cos x + (1)\sin x}\\ &= \frac{1}{\sin x}\\ &= \text{cosec } x\end{aligned}$$

$\therefore$ LHS = RHS

1.2 To prove: $\tan\left(\frac{\pi}{2} - x\right) = \cot x$

$$\begin{aligned}\text{LHS} &= \tan\left(\frac{\pi}{2} - x\right)\\ &= \frac{\sin\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right)}\\ &= \frac{\sin\left(\frac{\pi}{2}\right)\cos x - \sin x \cos\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right)\cos x + \sin\left(\frac{\pi}{2}\right)\sin x}\\ &= \frac{(1)\cos x - \sin x(0)}{(0)\cos x + (1)\sin x}\\ &= \frac{\cos x}{\sin x}\\ &= \cot x\end{aligned}$$

$\therefore$ LHS = RHS

1.3 To prove: $\cot\left(\frac{\pi}{2} + x\right) = -\tan x$

$$\begin{aligned}\text{LHS} &= \cot\left(\frac{\pi}{2} + x\right)\\ &= \frac{\cos\left(\frac{\pi}{2} + x\right)}{\sin\left(\frac{\pi}{2} + x\right)}\\ &= \frac{\cos\left(\frac{\pi}{2}\right)\cos x - \sin\left(\frac{\pi}{2}\right)\sin x}{\sin\left(\frac{\pi}{2}\right)\cos x + \sin x \cos\left(\frac{\pi}{2}\right)}\\ &= \frac{(0)\cos x - (1)\sin x}{(1)\cos x + \sin x(0)}\\ &= \frac{-\sin x}{\cos x}\\ &= -\tan x\end{aligned}$$

$\therefore$ LHS = RHS

1.4 To prove: $\text{cosec}(90° + x) = \sec x$

$$\begin{aligned}\text{LHS} &= \text{cosec }(90° + x)\\ &= \frac{1}{\sin(90° + x)}\\ &= \frac{1}{\sin 90° \cos x + \sin x \cos 90°}\\ &= \frac{1}{(1)\cos x + \sin x(0)}\\ &= \frac{1}{\cos x}\\ &= \sec x\end{aligned}$$

$\therefore$ LHS = RHS

2. 2.1 $\sin 78° = \sin(90° - 12°)$
$= \cos 12°$

2.2 $\tan 51° = \tan(90° - 39°)$
$= \cot 39°$

2.3 $\sec 27° = \sec(90° - 63°)$
$= \text{cosec } 63°$

3. 3.1 $\dfrac{\sin(90° - x)\sec(360° - x)}{\cot(90° - x)\sin(180° - x)\,\text{cosec}(180° + x)}$

$$\begin{aligned}&= \frac{\cos x \sec x}{\tan x \sin x(-\text{cosec } x)}\\ &= \frac{\left(\frac{\cos x}{1}\right)\left(\frac{1}{\cos x}\right)}{-\left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin x}{1}\right)\left(\frac{1}{\sin x}\right)}\\ &= -\frac{1}{\tan x}\\ &= -\cot x\end{aligned}$$

3.2 $\dfrac{\sin^2(180° + \theta) + \cos(180° - \theta)\operatorname{cosec}(90° - \theta)}{\tan(180° - \theta)\tan(90° - \theta)}$

$= \dfrac{\sin^2\theta - \cos\theta\sec\theta}{-\tan\theta\cot\theta}$

$= \dfrac{\sin^2\theta - 1}{-1}$

$= \dfrac{-(1 - \sin^2\theta)}{-1}$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

4. 4.1 To prove: $\dfrac{\sin(90° - \theta)\cos(360° - \theta)}{\cos(90° + \theta)\sin(180° + \theta) - 1} = -1$

$\text{LHS} = \dfrac{\sin(90° - \theta)\cos(360° - \theta)}{\cos(90° + \theta)\sin(180° + \theta) - 1}$

$= \dfrac{\cos\theta\cos\theta}{(-\sin\theta)(-\sin\theta) - 1}$

$= \dfrac{\cos^2\theta}{-(1 - \sin^2\theta)}$

$= \dfrac{\cos^2\theta}{-\cos^2\theta}$

$= -1$

$\therefore \text{LHS} = \text{RHS}$

4.2 To prove: $\dfrac{\tan 315°\cos 390°\cos(-\theta)\cos(90° - \theta)}{\cos(360° - \theta)\sin 300°\sin(180° - \theta)} = 1$

$\text{LHS} = \dfrac{\tan 315°\cos 390°\cos(-\theta)\cos(90° - \theta)}{\cos(360° - \theta)\sin 300°\sin(180° - \theta)}$

$= \dfrac{\tan(360° - 45°)\cos 30°\cos(-\theta)\cos(90° - \theta)}{\cos(360° - \theta)\sin(360° - 60°)\sin(180° - \theta)}$

$= \dfrac{-\tan 45°\cos 30°\cos\theta\sin\theta}{\cos\theta(-\sin 60°)\sin\theta}$

$= \dfrac{-\frac{\sqrt{3}}{2}\cos\theta\sin\theta}{\cos\theta\left(-\frac{\sqrt{3}}{2}\right)\sin\theta}$

$= 1$

$\therefore \text{LHS} = \text{RHS}$

Activity 4.4 SB page 235

1. $\sin 2x = 2 \sin x \cos x$

$\therefore \operatorname{cosec} 2x = \frac{1}{\sin 2x}$

$= \frac{1}{2 \sin x \cos x}$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2 \cos^2 x - 1$

$= 1 - 2 \sin^2 x$

$\therefore \sec 2x = \frac{1}{\cos 2x}$

$= \frac{1}{\cos^2 x - \sin^2 x}$

$= \frac{1}{2 \cos^2 x - 1}$

$= \frac{1}{1 - 2 \sin^2 x}$

$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

$\therefore \cot 2x = \frac{1}{\tan 2x}$

$= \frac{1}{\frac{2 \tan x}{1 - \tan^2 x}}$

$= \frac{1 - \tan^2 x}{2 \tan x}$

2. 2.1 $\sin 2\theta = \sin(\theta + \theta)$

$= \sin \theta \cos \theta + \cos \theta \sin \theta$

$= 2 \sin \theta \cos \theta$

2.2 $\sin 2(60°) = 2 \sin 60° \cos 60°$

$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$= 2\left(\frac{\sqrt{3}}{4}\right)$

$= \frac{\sqrt{3}}{2}$

3. 3.1 $\cos 2\alpha = \cos(\alpha + \alpha)$

$= \cos \alpha \cos \alpha - \sin \alpha \sin \alpha$

$= \cos^2 \alpha - \sin^2 \alpha$

$= (1 - \sin^2 \alpha) - \sin^2 \alpha$

$= 1 - 2 \sin^2 \alpha$

3.2 $\cos 150° = \cos 2(75°)$

$= 1 - 2\sin^2 75°$

$= 1 - 2(\sin 75°)^2$

$= 1 - 2(\sin(30° + 45°))^2$

$= 1 - 2(\sin 30° \cos 45° + \sin 45° \cos 30°)^2$

$= 1 - 2\left[\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)\right]^2$

$= 1 - 2\left[\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}\right]^2$

$= 1 - 2\left[\frac{1 + \sqrt{3}}{2\sqrt{2}}\right]^2$

$= 1 - 2\left[\frac{1 + 2\sqrt{3} + 3}{8}\right]$

$= 1 - 2\left[\frac{4 + 2\sqrt{3}}{8}\right]$

$= 1 + \frac{-8 - 4\sqrt{3}}{8}$

$= \frac{8 - 8 - 4\sqrt{3}}{8}$

$= \frac{-4\sqrt{3}}{8}$

$= -\frac{\sqrt{3}}{2}$

3.3 $\cos(90° + \alpha) = \cos 90° \cos \alpha - \sin 90° \sin \alpha$

$= (0)\cos \alpha - (1)\sin \alpha$

$= -\sin \alpha$

3.4 $\cos 150° = \cos(90° + 60°)$

$= -\sin 60°$

$= -\frac{\sqrt{3}}{2}$

4. 4.1 $\sin 37{,}5° \cos 37{,}5° = \frac{1}{2}(2 \sin 37{,}5° \cos 37{,}5°)$

$= \frac{1}{2} \sin[2(37{,}5°)]$

$= \frac{1}{2} \sin 75°$

$= \frac{1}{2} \sin (30° + 45°)$

$= \frac{1}{2}[\sin 30° \cos 45° + \sin 45° \cos 30°]$

$= \frac{1}{2}\left[\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \frac{1}{2}\left[\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}\right]$

$= \frac{1}{2}\left[\frac{1 + \sqrt{3}}{2\sqrt{2}}\right]$

$= \frac{1 + \sqrt{3}}{4\sqrt{2}}$

$= \frac{1 + \sqrt{3}}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2} + \sqrt{6}}{8}$

$= \frac{\sqrt{2}(1 + \sqrt{3})}{8}$

4.2 $\frac{1 - \tan^2\left(\frac{\pi}{12}\right)}{2 \tan\left(\frac{\pi}{12}\right)} = \left(\frac{2 \tan\left(\frac{\pi}{12}\right)}{1 - \tan^2\left(\frac{\pi}{12}\right)}\right)^{-1}$

$= \left(\tan\left[2\left(\frac{\pi}{12}\right)\right]\right)^{-1}$

$= \left(\tan\left(\frac{\pi}{6}\right)\right)^{-1}$

$= \left(\frac{1}{\sqrt{3}}\right)^{-1}$

$= \sqrt{3}$

4.3 $\cos^2 7{,}5° - \sin^2 7{,}5°$

$= \cos 2(7{,}5°)$

$= \cos 15°$

$= \cos(45° - 30°)$

$= \cos 45° \cos 30° + \sin 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{\sqrt{1}}{2\sqrt{2}}$

$= \frac{\sqrt{3} + 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}(\sqrt{3} + 1)}{4}$

$= \frac{\sqrt{6} + \sqrt{2}}{4}$

4.4 $\left(\frac{1 - \tan^2 22{,}5°}{2 \tan 22{,}5°}\right)^{-1} = \frac{2 \tan 22{,}5°}{1 - \tan^2 22{,}5°}$

$= \tan[2(22{,}5°)]$

$= \tan 45°$

$= 1$

5. $\tan^2 x = \dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{\frac{1 - \cos 2x}{2}}{\frac{1 + \cos 2x}{2}}$

$= \dfrac{1 - \cos 2x}{1 + \cos 2x}$

6. 6.1 $\cos^2 15°$

$= \dfrac{1 + \cos 2(15°)}{2}$

$= \dfrac{1 + \cos 30°}{2}$

$= \dfrac{1 + \frac{\sqrt{3}}{2}}{2}$

$= \dfrac{2 + \sqrt{3}}{4}$

6.2 $\sin^4 22{,}5°$

$= (\sin^2 22{,}5°)^2$

$= \left(\dfrac{1 - \cos [2(22{,}5°)]}{2}\right)^2$

$= \left(\dfrac{1 - \cos 45°}{2}\right)^2$

$= \left(\dfrac{1 - \frac{1}{\sqrt{2}}}{2}\right)^2$

$= \left(\dfrac{\sqrt{2} - 1}{2\sqrt{2}}\right)^2$

$= \dfrac{3 - 2\sqrt{2}}{8}$

6.3 $\sin^2 15° \cos^4 15°$

$= (\sin^2 15° \cos^2 15°) \cos^2 15°$

$= (\sin 15° \cos 15°)^2 \cos^2 15°$

$= \left(\frac{1}{2} \sin [2(15°)]\right)^2 \left(\dfrac{1 + \cos [2(15°)]}{2}\right)$

$= \left(\frac{1}{2} \sin 30°\right)^2 \left(\dfrac{1 + \cos 30°}{2}\right)$

$= \left(\frac{1}{2}\left(\frac{1}{2}\right)\right)^2 \left(\dfrac{1 + \frac{\sqrt{3}}{2}}{2}\right)$

$= \dfrac{1}{16}\left(\dfrac{2 + \sqrt{3}}{4}\right)$

$= \dfrac{2 + \sqrt{3}}{64}$

7. 7.1 $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$= \dfrac{1 - \cos 2x}{2}$

$\therefore \sin x = \sqrt{\dfrac{1 - \cos 2x}{2}}$

$\sin \frac{x}{2} = \sqrt{\dfrac{1 - \cos x}{2}}$

7.2 $\cos^2 x = \frac{1}{2}\left(1 + \cos 2x\right)$

$= \dfrac{1 + \cos 2x}{2}$

$\therefore \cos x = \sqrt{\dfrac{1 + \cos 2x}{2}}$

$\cos \frac{x}{2} = \sqrt{\dfrac{1 + \cos x}{2}}$

7.3 $\tan 2x = \tan(x + x)$

$= \dfrac{\tan x + \tan x}{1 - \tan x \tan x}$

$= \dfrac{2 \tan x}{1 - \tan^2 x}$

8. $\sin\left(\frac{\pi}{12}\right) = \sqrt{\frac{1 - \cos\left[2\left(\frac{\pi}{12}\right)\right]}{2}}$

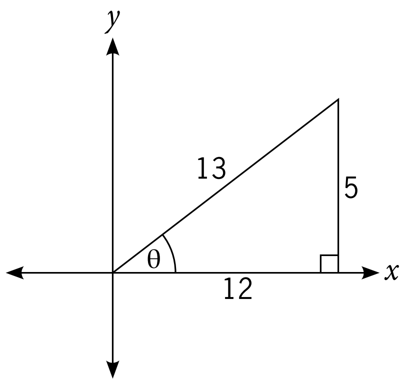
$= \sqrt{\frac{1 - \cos\left(\frac{\pi}{6}\right)}{2}}$

$= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{2 - \sqrt{3}}{4}}$

$= \frac{\sqrt{2 - \sqrt{3}}}{2}$

9.



9.1 $\sin 2\theta = 2 \sin \theta \cos \theta$

$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)$

$= \frac{120}{169}$

9.2 $\cos \frac{1}{2}\theta = \sqrt{\frac{1 + \cos \theta}{2}}$

$= \sqrt{\frac{1 + \frac{12}{13}}{2}}$

$= \sqrt{\frac{\frac{13 + 12}{13}}{2}}$

$= \sqrt{\frac{\frac{25}{13}}{2}}$

$= \frac{5}{\sqrt{13}\sqrt{2}}$

$= \frac{5}{\sqrt{26}}$

9.3 $\cos 4\theta = 1 - 2 \sin^2 2\theta$

$= 1 - 2\left(\frac{120}{169}\right)^2$

$= 1 - 2\left(\frac{120^2}{169^2}\right)$

$= -0{,}008368$

Activity 4.5 SB page 241

1. $\sin(90° - x) = 2\cos x + \frac{1}{2}$

$\cos x - 2\cos x - \frac{1}{2} = 0$

$\cos x - 2\cos x = \frac{1}{2}$

$-\cos x = \frac{1}{2}$

$\cos x = -\frac{1}{2}$

$\therefore x = 120°$ and $x = 240°$

2.

$$4\cos^2 x - 3 = -2\sin^2 x$$
$$4(1 - \sin^2 x) - 3 = -2\sin^2 x$$
$$4 - 4\sin^2 x - 3 = -2\sin^2 x$$
$$-4\sin^2 x + 1 = -2\sin^2 x$$
$$-2\sin^2 x = -1$$
$$\sin^2 x = \frac{1}{2}$$
$$\sin x = \pm \frac{1}{\sqrt{2}}$$

$\sin x = \oplus \frac{1}{\sqrt{2}}$

sin+; 1st

$x = x_{ref}$

$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$x = \frac{\pi}{4}$

sin+; 2nd

$x = \pi - x_{ref}$

$= \pi - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$x = \frac{3\pi}{4}$

$\sin x = \ominus \frac{1}{\sqrt{2}}$

sin–; 3rd

$x = \pi + x_{ref}$

$= \pi + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$x = \frac{5\pi}{4}$

sin–; 4th

$x = 2\pi - x_{ref}$

$= 2\pi - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$x = \frac{7\pi}{4}$

$0 \le x \le 2\pi$, therefore $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, $x = \frac{5\pi}{4}$ and $x = \frac{7\pi}{4}$.

3.

$$\tfrac{1}{2}\cos x = \cos 2x$$

$$\tfrac{1}{2}\cos x = 2\cos^2 x - 1$$

$$-2\cos^2 x + \tfrac{1}{2}\cos x + 1 = 0$$

$$4\cos^2 x - \cos x - 2 = 0$$

$$\cos x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-2)}}{2(4)}$$

$$\cos x = \frac{1 \pm \sqrt{33}}{8}$$

$$\cos x = \frac{1 + \sqrt{33}}{8} \qquad \cos x = \frac{1 - \sqrt{33}}{8}$$

$$\cos x = \oplus\, 0{,}843 \qquad \cos x = \ominus\, 0{,}593$$

cos+; 1st | cos+; 4th | cos−; 2nd | cos−; 3rd

$x = x_{ref}$, $= \cos^{-1}(0{,}843)$, $x = 32{,}534°$

$x = 360° - x_{ref}$, $= 360° - \cos^{-1}(0{,}843)$, $x = 327{,}466°$

$x = 180° - x_{ref}$, $= 180° - \cos^{-1}(0{,}593)$, $x = 126{,}375°$

$x = 180° + x_{ref}$, $= 180° + \cos^{-1}(0{,}593)$, $x = 233{,}625°$

$0° \leq x \leq 360°$, therefore $x = 32{,}534°$, $x = 126{,}375°$, $x = 233{,}625°$ and $x = 327{,}466°$.

4.

$$\tan^2 x - \frac{1}{\sin(90° - x)} = 1$$

$$(\sec^2 x - 1) - \frac{1}{\cos x} = 1$$

$$\sec^2 x - 1 - \frac{1}{\cos x} = 1$$

$$\frac{1}{\cos^2 x} - 1 - \frac{1}{\cos x} = 1$$

$$-2 - \frac{1}{\cos x} + \frac{1}{\cos^2 x} = 0$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$2\cos x - 1 = 0 \qquad \cos x + 1 = 0$$

$$2\cos x = +1 \qquad \cos x = \ominus 1$$

$$\cos x = \oplus \tfrac{1}{2}$$

cos+; 1st | cos+; 3rd | cos−; 2nd | cos−; 4th

$x = x_{ref}$, $= \cos^{-1}\left(\tfrac{1}{2}\right)$, $x = 60°$ and $x = -300°$

$x = 360° - x_{ref}$, $= 360° - \cos^{-1}\left(\tfrac{1}{2}\right)$, $x = 300°$ and $x = -60°$

$x = 180° - x_{ref}$, $= 180° - \cos^{-1}(1)$, $x = 180°$ and $x = -180°$

$x = 180° + x_{ref}$, $= 180° + \cos^{-1}(1)$, $x = 180°$ and $x = -180°$

$-90° \leq x \leq 180°$, therefore $x = -60°$, $x = 60°$ and $x = 180°$

5.
$$3\cot^2 x - 4\,\text{cosec}\,x = 1$$
$$3(\text{cosec}^2 x - 1) - 4\,\text{cosec}\,x = 1$$
$$3\,\text{cosec}^2 x - 3 - 4\,\text{cosec}\,x = 1$$
$$-4 - 4\,\text{cosec}\,x + 3\,\text{cosec}^2 x = 0$$
$$-4 - \frac{4}{\sin x} + \frac{3}{\sin^2 x} = 0$$
$$4\sin^2 x + 4\sin x - 3 = 0$$
$$(2\sin x - 1)(2\sin x + 3) = 0$$

$2\sin x - 1 = 0$

$2\sin x = +1$

$\sin x = \oplus\frac{1}{2}$

sin+; 1st

$x = x_{ref}$

$= \sin^{-1}\left(\frac{1}{2}\right)$

$x = 30°$

sin+; 2nd

$x = 180° - x_{ref}$

$= 180° - \sin^{-1}\left(\frac{1}{2}\right)$

$x = 150°$

$2\sin x + 3 = 0$

$2\sin x = -3$

$\sin x \neq -\frac{3}{2}$

$0° \leq x \leq 180°$, therefore $x = 30°$ and $x = 150°$.

6. $\frac{\tan 2x + \tan 30°}{1 - \tan 2x \tan 30°} = -\cot x$

$\tan(2x + 30°) = -\cot x$

$\tan(2x + 30°) = \tan(90° + x)$

$2x + 30° = 90° + x$

$x = 60°$

$0° \leq x \leq 180°$, therefore $x = 60°$

7.

$$\frac{1 - \cos 2x}{1 + \cos 2x} - 2\left(\frac{\sin x}{\cos x}\right) = 4$$

$$\tan^2 x - 2 \tan x = 4$$

$$\tan^2 x - 2 \tan x - 4 = 0$$

$$\tan x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{2 \pm 2\sqrt{5}}{2}$$

$$\tan x = 1 \pm \sqrt{5}$$

$\tan x = 1 + \sqrt{5}$

$\tan x = \oplus 3{,}236$

tan+; 1st

$x = x_{ref}$

$= \tan^{-1}(3{,}236)$

$x = 1{,}271$

and $x = -5{,}012$

tan+; 3rd

$x = \pi + x_{ref}$

$= \pi + \tan^{-1}(3{,}236)$

$x = 4{,}413$

and $x = -1{,}871$

$\tan x = 1 - \sqrt{5}$

$\tan x = \ominus 1{,}236$

tan–; 2nd

$x = \pi - x_{ref}$

$= \pi - \tan^{-1}(1{,}236)$

$x = 2{,}251$

and $x = -4{,}032$

tan–; 4th

$x = 2\pi - x_{ref}$

$= 2\pi - \tan^{-1}(1{,}236)$

$x = 5{,}393$

and $x = -0{,}891$

$-\frac{\pi}{2} \leq x \leq \pi$, therefore $x = -0{,}891$, $x = 1{,}271$ and $x = 2{,}251$

8.

$$7 \cos^2 x - 1 = -\tfrac{1}{2} \sin 2x$$

$$6 \cos^2 x - 1 + \cos^2 x = -\tfrac{1}{2}(2 \sin x.\cos x)$$

$$6 \cos^2 x - (1 - \cos^2 x) = -\sin x.\cos x$$

$$6 \cos^2 x - \sin^2 x = -\sin x.\cos x$$

$$6 - \frac{\sin^2 x}{\cos^2 x} = -\frac{\sin x}{\cos x}$$

$$6 - \tan^2 x = -\tan x$$

$$-\tan^2 x + \tan x + 6 = 0$$

$$\tan^2 x - \tan x - 6 = 0$$

$$(\tan x - 3)(\tan x + 2) = 0$$

$\tan x - 3 = 0$

$\tan x = \oplus\ 3$

tan+; 1st

$x = x_{ref}$

$= \tan^{-1}(3)$

$x = 71{,}656°$

and $x = -288{,}435°$

tan+; 3rd

$x = 180° + x_{ref}$

$= 180° + \tan^{-1}(3)$

$x = 251{,}565°$

and $x = -108{,}435°$

$\tan x + 2 = 0$

$\tan x = \ominus\ 2$

tan–; 2nd

$x = 180° - x_{ref}$

$= 180° - \tan^{-1}(2)$

$x = 116{,}656°$

and $x = -243{,}435°$

tan–; 4th

$x = 360° - x_{ref}$

$= 360° - \tan^{-1}(2)$

$x = 296{,}565°$

and $x = -63{,}435°$

$-180° \leq x \leq 180°$, therefore $x = -108{,}435°$, $x = -63{,}435°$, $x = 71{,}565°$ and $x = 116{,}565°$

Activity 4.6 SB page 246

1. $(\cos x - \sin x)^2$
$= \cos^2 x - 2 \sin x \cos x + \sin^2 x$
$= (\cos^2 x + \sin^2 x) - 2 \sin x \cos x$
$= 1 - \sin 2x$

2. $\frac{1 - \tan^2 x}{1 + \tan^2 x}$
$= \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}}$
$= \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}$
$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x} \times \frac{\cos^2 x}{\cos^2 x + \sin^2 x}$
$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$
$= \frac{\cos 2x}{1}$
$= \cos 2x$

3. $\frac{\sin 2x}{1 - \cos 2x}$
$= \frac{2 \sin x \cos x}{1 - (1 - 2 \sin^2 x)}$
$= \frac{2 \sin x \cos x}{1 - 1 + 2 \sin^2 x}$
$= \frac{2 \sin x \cos x}{2 \sin^2 x}$
$= \frac{\cos x}{\sin x}$
$= \cot x$

4. $\frac{\cos 2x}{\sin x - \cos x}$
$= \frac{\cos^2 x - \sin^2 x}{\sin x - \cos x}$
$= \frac{(\cos x + \sin x)(\cos x - \sin x)}{-(\cos x - \sin x)}$
$= -(\cos x + \sin x)$
$= -\cos x - \sin x$

5. $\frac{\tan x}{\sin 2x - \tan x}$
$= \frac{\frac{\sin x}{\cos x}}{2 \sin x \cos x - \frac{\sin x}{\cos x}}$
$= \frac{\sin x}{2 \sin x \cos^2 x - \sin x}$
$= \frac{\sin x}{\sin x(2 \cos^2 x - 1)}$
$= \frac{\sin x}{\sin x \cos 2x}$
$= \frac{1}{\cos 2x}$
$= \sec 2x$

6. $\frac{\cot x}{\operatorname{cosec} x - 1} + \frac{\cot x}{\operatorname{cosec} x + 1}$
$= \frac{\cot x(\operatorname{cosec} x + 1) + \cot x(\operatorname{cosec} x - 1)}{(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)}$
$= \frac{\cot x \operatorname{cosec} x + \cot x + \cot x \operatorname{cosec} x - \cot x}{\operatorname{cosec}^2 x - 1}$
$= \frac{2 \operatorname{cosec} x \cot x}{\cot^2 x}$
$= \frac{2 \operatorname{cosec} x}{\cot x}$
$= \frac{2\left(\frac{1}{\sin x}\right)}{\frac{\cos x}{\sin x}}$
$= \frac{2}{\sin x} \times \frac{\sin x}{\cos x}$
$= 2\frac{1}{\cos x}$
$= 2 \sec x$

7. $\dfrac{\sin^3 x - \cos^3 x}{\cos x - \sin x}$

$= \dfrac{(\sin x - \cos x)(\sin 2x + \sin x \cos x + \cos^2 x)}{-(\sin x - \cos x)}$

$= -(\sin^2 x + \sin x \cos x + \cos^2 x)$

$= -[(\sin^2 x + \cos^2 x) + \sin x \cos x]$

$= -\left[1 + \frac{1}{2}\sin 2x\right]$

$= -1 - \frac{1}{2}\sin 2x$

8. $\dfrac{\sin 4x}{\sin x} - \dfrac{\cos 4x}{\cos x}$

$= \dfrac{2 \sin 2x \cos 2x}{\sin x} - \dfrac{2\cos^2 2x - 1}{\cos x}$

$= \dfrac{2(2 \sin x \cos x)\cos 2x}{\sin x} - \dfrac{2(2\cos^2 x - 1)^2 - 1}{\cos x}$

$= \dfrac{4 \sin x \cos x \cos 2x}{\sin x} - \dfrac{2(4\cos^4 x - 4\cos^2 x + 1) - 1}{\cos x}$

$= 4\cos x \cos 2x - \dfrac{8\cos^4 x - 8\cos^2 x + 2 - 1}{\cos x}$

$= 4\cos x \cos 2x - \dfrac{8\cos^4 x - 8\cos^2 x + 1}{\cos x}$

$= \dfrac{4\cos^2 x \cos 2x - (8\cos^4 x - 8\cos^2 x + 1)}{\cos x}$

$= \dfrac{4\cos^2 x(2\cos^2 x - 1) - (8\cos^4 x - 8\cos^2 x + 1)}{\cos x}$

$= \dfrac{8\cos^4 x - 4\cos^2 x - 8\cos^4 x + 8\cos^2 x - 1}{\cos x}$

$= \dfrac{4\cos^2 x - 1}{\cos x}$

$= 4\cos x - \dfrac{1}{\cos x}$

$= 4\cos x - \sec x$

9. $\dfrac{\cos 3x + \cos x}{\sin 3x + \sin x}$

$= \dfrac{\cos 2x \cos x - \sin 2x \sin x + \cos x}{\sin 2x \cos x + \sin x \cos 2x + \sin x}$

$= \dfrac{(2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x + \cos x}{(2 \sin x \cos x)\cos x + \sin x(2\cos^2 x - 1) + \sin x}$

$= \dfrac{2\cos^3 x - \cos x - 2\sin^2 x \cos x + \cos x}{2\sin x \cos^2 x + 2\sin x \cos^2 x - \sin x + \sin x}$

$= \dfrac{2\cos^3 x - 2\sin^2 x \cos x}{4 \sin x \cos^2 x}$

$= \dfrac{2\cos^3 x - 2(1 - \cos^2 x)\cos x}{4\sin x \cos^2 x}$

$= \dfrac{2\cos^3 x - 2\cos x + 2\cos^3 x}{4\sin x\cos^2 x}$

$= \dfrac{4\cos^3 x - 2\cos x}{4\sin x \cos^2 x}$

$= \dfrac{4\cos^3 x}{4\sin x\cos^2 x} - \dfrac{2\cos x}{4\sin x \cos^2 x}$

$= \dfrac{\cos x}{\sin x} - \dfrac{1}{2\sin x\cos x}$

$= \dfrac{\cos x}{\sin x} - \dfrac{1}{\sin 2x}$

$= \cot x - \operatorname{cosec} 2x$

10. $\dfrac{1 - \left(\frac{1 + \cos 2x}{2}\right) + \cos(90° - 2x)}{2\cos x + \sin x}$

$= \dfrac{1 - \cos^2 x + \sin 2x}{2\cos x + \sin x}$

$= \dfrac{(1 - \cos^2 x) + 2\sin x \cos x}{2\cos x + \sin x}$

$= \dfrac{\sin^2 x + 2\sin x\cos x}{2\cos x + \sin x}$

$= \dfrac{\sin x(2\cos x + \sin x)}{2\cos x + \sin x}$

$= \sin x$

Activity 4.7 SB page 251

1. $\frac{1 + \cos^2\theta}{1 - \sin^2\theta} = \tan^2\theta$

$$\text{LHS} = \frac{1 - \cos^2\theta}{1 - \sin^2\theta}$$
$$= \frac{\sin^2\theta}{\cos^2\theta}$$
$$= \tan^2\theta = \text{RHS}$$

2. $\frac{\tan\theta}{1 + \tan^2\theta} = \sin\theta\cos\theta$

$$\text{LHS} = \frac{\tan\theta}{1 + \tan^2\theta}$$
$$= \frac{\tan\theta}{\sec^2\theta}$$
$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}$$
$$= \frac{\sin\theta}{\cos\theta} \times \frac{\cos^2\theta}{1}$$
$$= \sin\theta\cos\theta = \text{RHS}$$

3. $(\sqrt{2}\cos x + 1)(\sqrt{2}\cos x - 1) = \cos 2x$

$$\text{LHS} = [\sqrt{2}\cos x + 1][\sqrt{2}\cos x - 1]$$
$$= 2\cos^2 x - \sqrt{2}\cos x + \sqrt{2}\cos x - 1$$
$$= 2\cos^2 x - 1$$
$$= \cos 2x$$
$$\therefore \text{LHS} = \text{RHS}$$

4. $2\operatorname{cosec} 2x = \tan x + \cot x$

$$\text{RHS} = \tan x + \cot x$$
$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$
$$= \frac{\sin x\sin x + \cos x\cos x}{\sin x\cos x}$$
$$= \frac{\sin 2x + \cos^2 x}{\sin x\cos x}$$
$$= \frac{1}{\frac{1}{2}\sin 2x}$$
$$= 2\operatorname{cosec} 2x$$
$$\therefore \text{RHS} = \text{LHS}$$

5. $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$

$$\text{LHS} = \frac{\cot x - \tan x}{\cot x + \tan x}$$
$$= \frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}$$
$$= \frac{\frac{\cos^2 x - \sin^2 x}{\sin x\cos x}}{\frac{\cos^2 x + \sin^2 x}{\sin x\cos x}}$$
$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$$
$$= \frac{\cos 2x}{1}$$
$$= \cos 2x$$
$$\therefore \text{LHS} = \text{RHS}$$

6. $\frac{2 - \sec^2\frac{x}{2}}{\sec^2\frac{x}{2}} = \cos x$

$$\text{LHS} = \frac{2 - \sec^2\frac{x}{2}}{\sec^2\frac{x}{2}}$$
$$= \frac{2}{\sec^2\frac{x}{2}} - \frac{\sec^2\frac{x}{2}}{\sec^2\frac{x}{2}}$$
$$= \frac{2}{\frac{1}{\cos^2\frac{x}{2}}} - 1$$
$$= 2\cos^2\frac{x}{2} - 1$$
$$= \cos x$$
$$\therefore \text{LHS} = \text{RHS}$$

7. $2\tan x = \dfrac{\sin(x-y)+\sin(x+y)}{\cos x\cos y}$

$$\text{RHS} = \frac{\sin(x-y)+\sin(x+y)}{\cos x\cos y}$$
$$= \frac{(\sin x\cos y - \sin y\cos x) + (\sin x\cos y + \sin y\cos x)}{\cos x\cos y}$$
$$= \frac{\sin x\cos y - \sin y\cos x + \sin x\cos y + \sin y\cos x}{\cos x\cos y}$$
$$= \frac{2\sin x\cos y}{\cos x\cos y}$$
$$= \frac{2\sin x}{\cos x}$$
$$= 2\tan x$$

$\therefore$ RHS = LHS

8. $\sin 4x = 4\sin x\cos^3 x - 4\sin^3 x\cos x$

$$\text{RHS} = 4\sin x\cos^3 x - 4\sin^3 x\cos x$$
$$= (4\cos^2 x - 4\sin^2 x)\sin x\cos x$$
$$= 4(\cos^2 x - \sin^2 x)\sin x\cos x$$
$$= 2(\cos^2 x - \sin^2 x)\,2\sin x\cos x$$
$$= 2\cos 2x\sin 2x$$
$$= 2\sin 2x\cos 2x$$
$$= \sin 4x$$

$\therefore$ RHS = LHS

9. $1 - \tan^2 x = \dfrac{2\cos 2x}{1+\cos 2x}$

$$\text{RHS} = \frac{2\cos 2x}{1+\cos 2x}$$
$$= \frac{2(\cos^2 x - \sin^2 x)}{1 + (2\cos^2 x - 1)}$$
$$= \frac{2(\cos^2 x - \sin^2 x)}{2\cos^2 x}$$
$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x}$$
$$= \frac{\cos^2 x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$$
$$= 1 - \tan^2 x$$

$\therefore$ RHS = LHS

10. $1 - \cos 2x = \tan x\sin 2x$

$$\text{RHS} = \tan x\sin 2x$$
$$= \frac{\sin x}{\cos x}\left(\frac{2\sin x\cos x}{1}\right)$$
$$= 2\sin^2 x$$
$$= 2\left(\frac{1-\cos 2x}{2}\right)$$
$$= 1 - \cos 2x$$

$\therefore$ RHS = LHS

11. $\frac{1 - \tan x}{1 + \tan x} = \frac{\cos 2x}{1 + \sin 2x}$

$$\text{LHS} = \frac{1 - \tan x}{1 + \tan x}$$

$$= \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}}$$

$$= \frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}}$$

$$= \frac{\cos x - \sin x}{\cos x + \sin x}$$

$$= \frac{\cos x - \sin x}{\cos x + \sin x} \times \frac{\cos x + \sin x}{\cos x + \sin x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + 2\sin x \cos x + \sin^2 x}$$

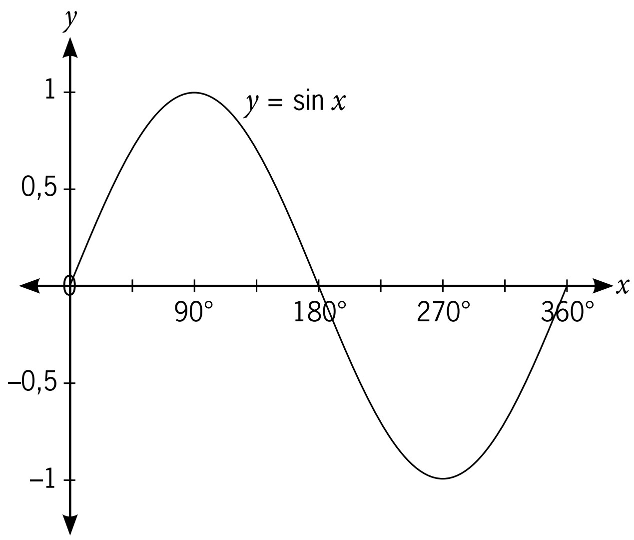
$$= \frac{\cos^2 x - \sin^2 x}{(\sin^2 x + \cos^2 x) + 2\sin x \cos x}$$

$$= \frac{\cos 2x}{1 + \sin 2x}$$

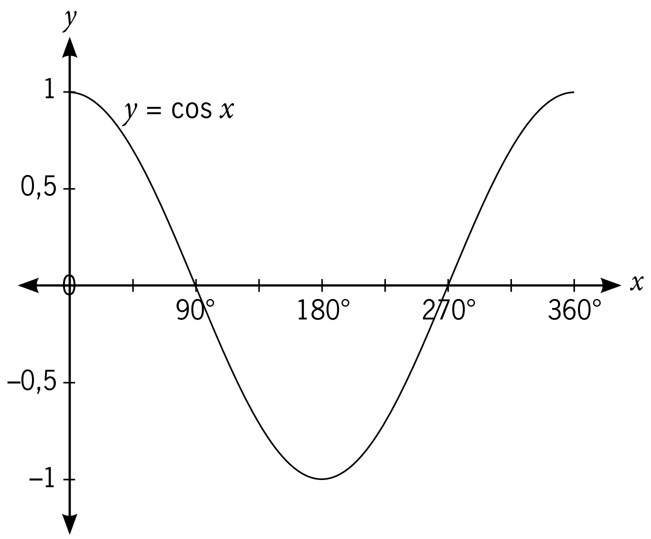
$\therefore$ LHS = RHS

Activity 4.8 SB page 268

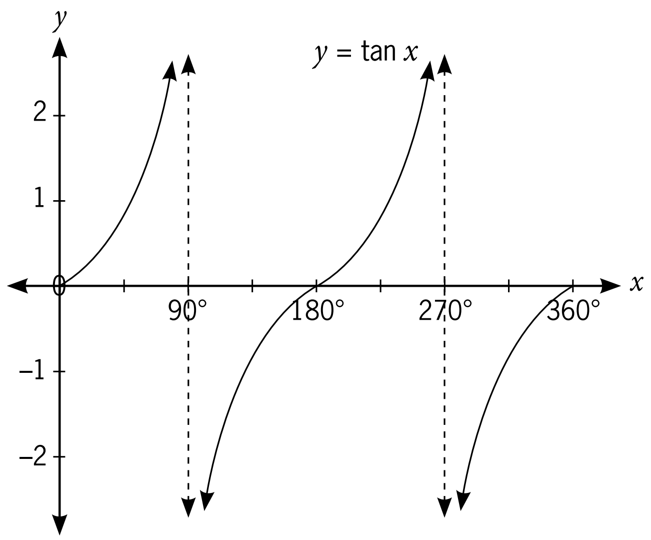
1.



$y = \sin x$	
1.1 Amplitude	1
1.2 Frequency	1
1.3 Period	360°
1.4 Domain	$0° \leq x \leq 360°$
1.5 Range	$-1 \leq y \leq 1$
1.6 Turning points	(90°; 1); (270°; –1)
1.7 *x*-intercepts	(0°; 0); (180°; 0); (360°; 0)
1.8 Asymptotes	None



$y = \cos x$	
1.1 Amplitude	1
1.2 Frequency	1
1.3 Period	360°
1.4 Domain	$0° \leq x \leq 360°$
1.5 Range	$-1 \leq y \leq 1$
1.6 Turning points	(0°; 1); (180°; –1); (360°; 1)
1.7 *x*-intercepts	(90°; 0); (270°; 0)
1.8 Asymptotes	None

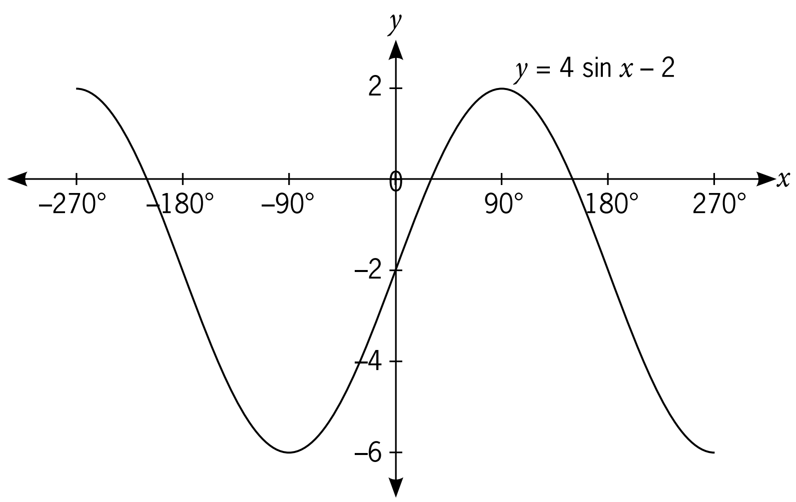


$y = \tan x$	
1.1 Amplitude	Undefined
1.2 Frequency	2
1.3 Period	180°
1.4 Domain	$0° \leq x \leq 360°$; $x \neq 90°$; $x \neq 270°$
1.5 Range	$y \in R$
1.6 Turning points	None
1.7 *x*-intercepts	(0°; 0); (180°; 0); (360°; 0)
1.8 Asymptotes	$x = 90°$; $x = 270°$

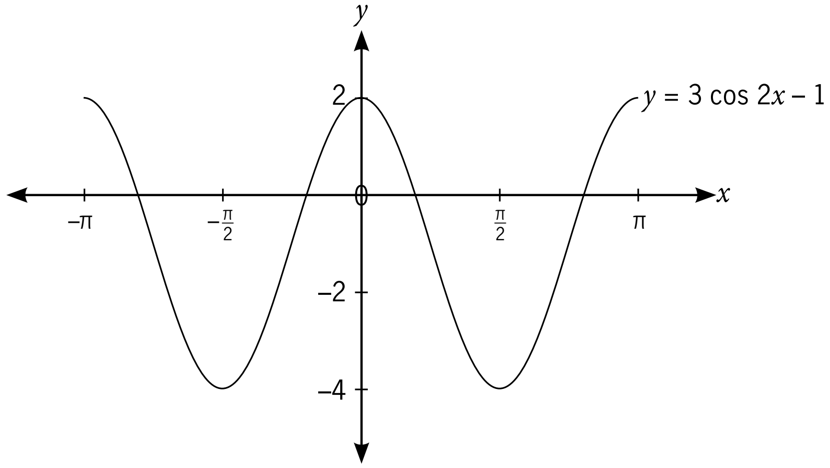
2.

Function		Amplitude	Frequency	Period	Horizontal shift	Vertical shift
2.1	$y = 4 \sin x - 2$ $-270° \leq x \leq 270°$	4	1	360°	No shift	Shift down 2
2.2	$y = 3 \cos 2x - 1$ $-\pi \leq x \leq \pi$	3	2	π	No shift	Shift down 1
2.3	$y = \tan \frac{x}{2}$ $-\pi \leq x \leq \pi$	Undefined	1	2π	No shift	No shift
2.4	$y = 2 \cos(x - 45°)$ $0° \leq x \leq 360°$	2	1	360°	Shift 45° right	No shift
2.5	$y = 2 \tan x - 4$ $-180° \leq x \leq 180°$	Undefined	2	180°	No shift	Shift down 4
2.6	$y = 4 \cos 2(x + 90°)$ $-90° \leq x \leq 90°$	4	2	180°	Shift 90° left	No shift
2.7	$y = \tan\left(x - \frac{\pi}{4}\right) + 1$ $-\frac{\pi}{2} \leq x \leq \pi$	Undefined	2	π	Shift $\frac{\pi}{4}$ right	Shift up 1
2.8	$y = 3 \sin\left(\frac{2}{3}x - 30°\right) + 1$ $-360° \leq x \leq 360°$	3	$\frac{2}{3}$	540°	Shift 45° right	Shift up 1
2.9	$y = -3 \tan(2x + 60°)$ $0° \leq x \leq 360°$	Undefined	4	90°	Shift 30° left	No shift
2.10	$y = -8 \sin\left(x - \frac{\pi}{12}\right)$ $-\frac{3\pi}{2} \leq x \leq \frac{3\pi}{2}$	8	1	2π	Shift $\frac{\pi}{12}$ right	No shift
2.11	$y = -\cos\left(\frac{x}{2} - \frac{\pi}{18}\right) + 3$ $-2\pi \leq x \leq 2\pi$	1	$\frac{1}{2}$	4π	Shift $\frac{\pi}{9}$ right	Shift up 3

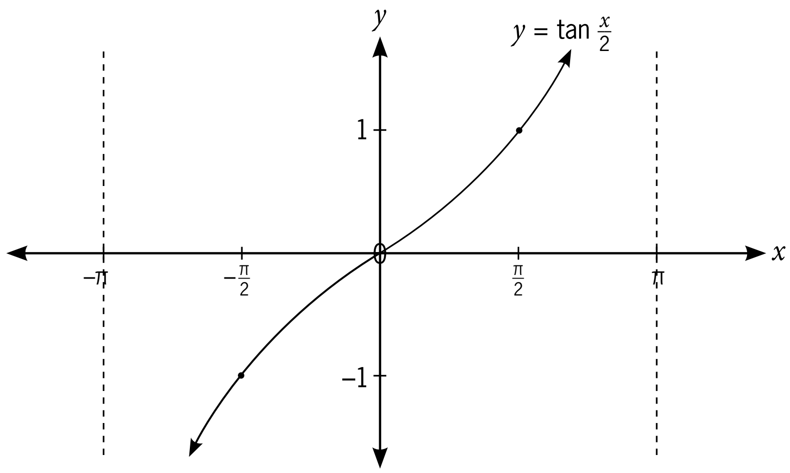
3. 3.1 $y = 4 \sin x - 2, -270° \leq x \leq 270°$



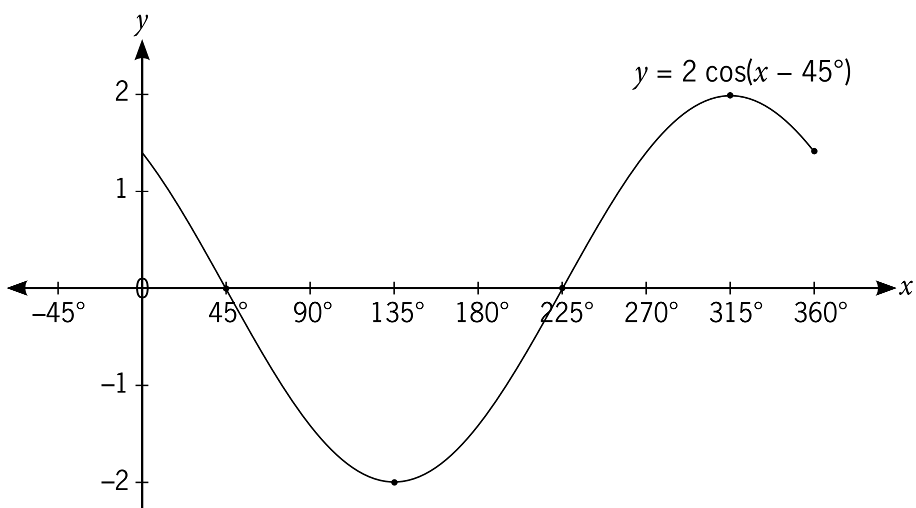
3.2 $y = 3\cos 2x - 1, -\pi \leq x \leq \pi$



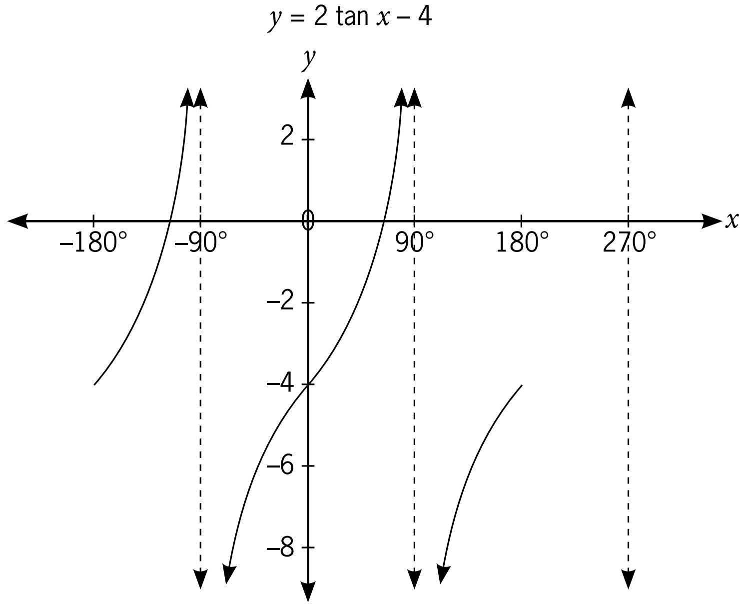
3.3 $y = \tan\frac{x}{2}; -\pi \leq x \leq \pi$



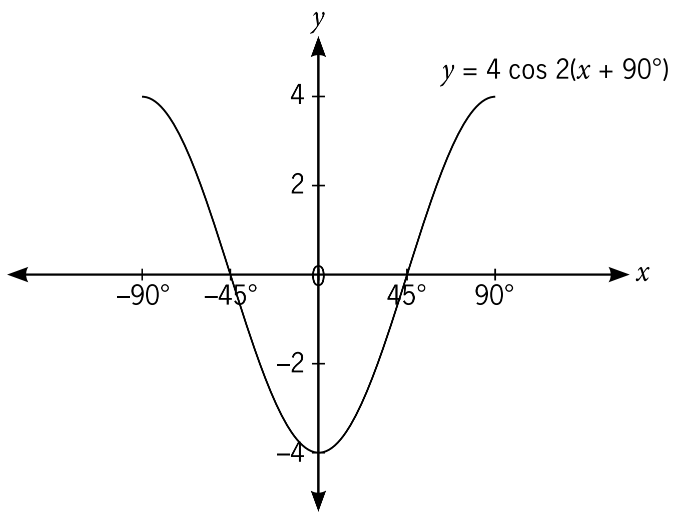
3.4 $y = 2\cos(x - 45°); 0° \leq x \leq 360°$



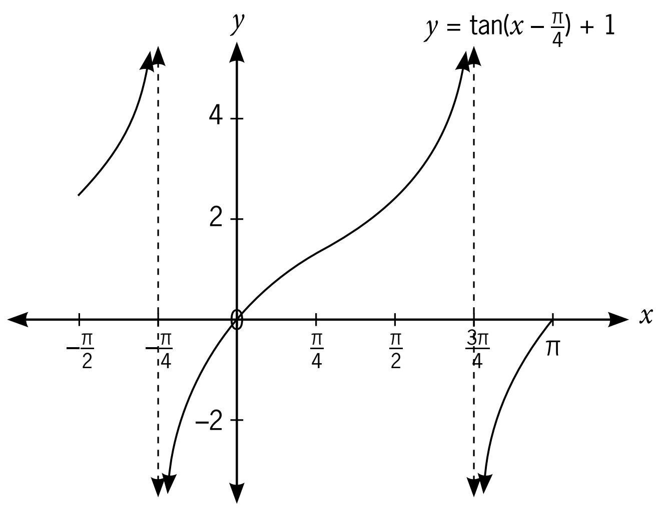
3.5 $y = 2\tan x - 4$, $-180° \leq x \leq 180°$



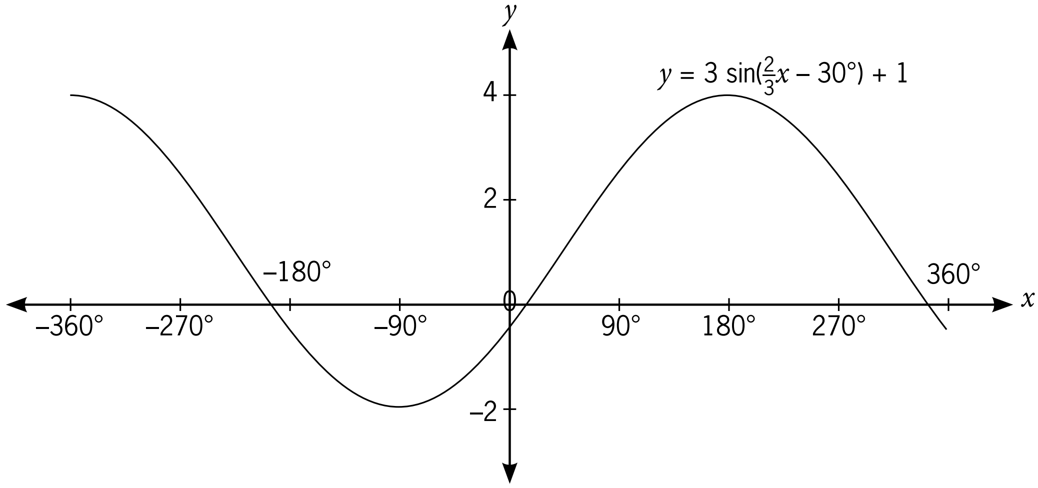
3.6 $y = 4\cos 2(x + 90°)$, $-90° \leq x \leq 90°$



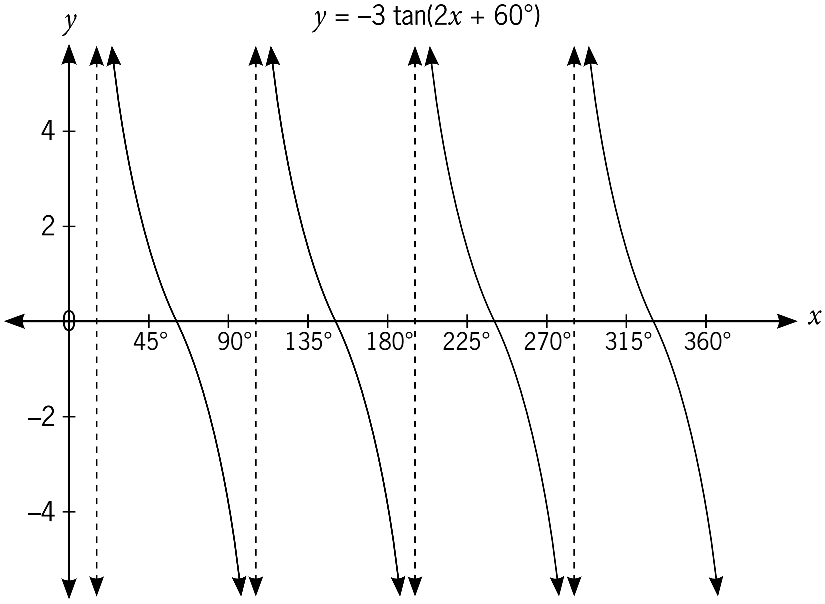
3.7 $y = \tan\left(x - \frac{\pi}{4}\right) + 1$, $-\frac{\pi}{2} \leq x \leq \pi$



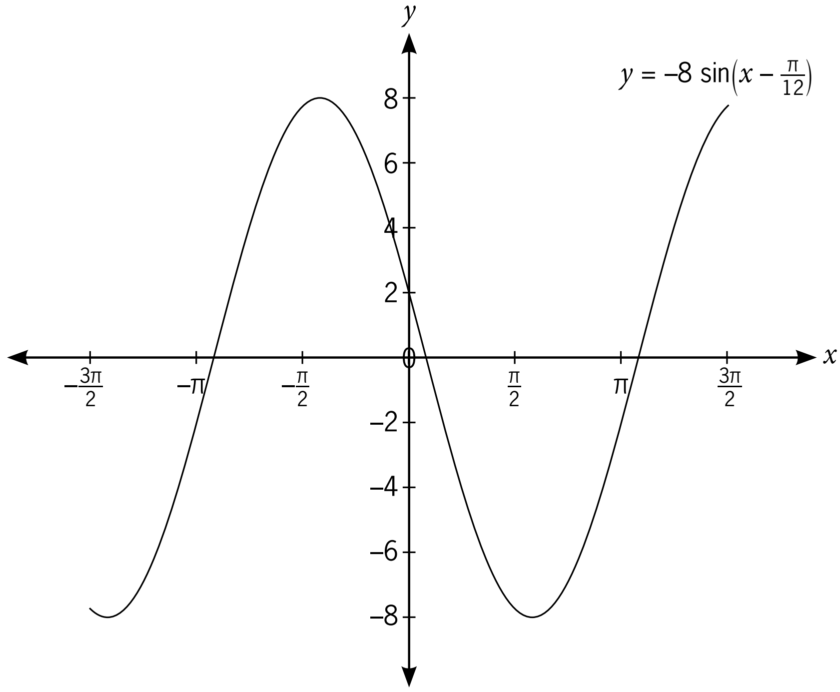
3.8 $y = 3\sin\left(\frac{2}{3}x - 30°\right) + 1, -360° \leq x \leq 360°$



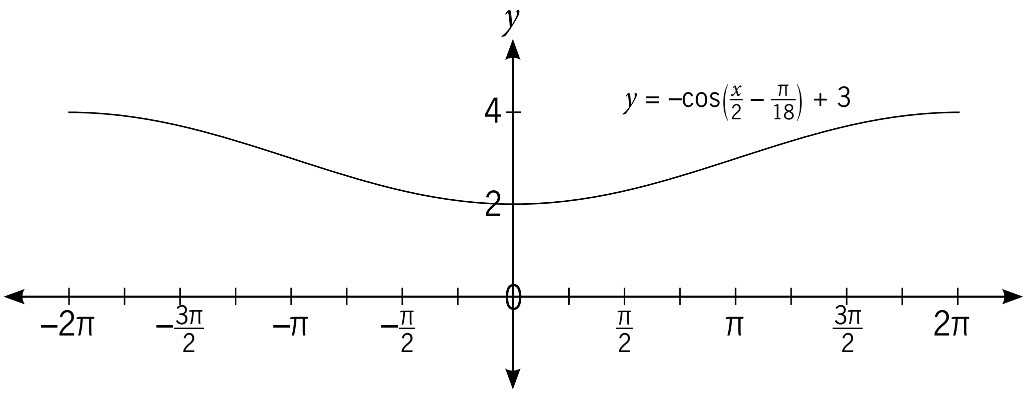
3.9 $y = -3\tan(2x + 60°), 0° \leq x \leq 360°$



3.10 $y = -8\sin\left(x - \frac{\pi}{12}\right), -\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$



3.11 $y = -\cos\left(\frac{x}{2} - \frac{\pi}{18}\right) + 3, -2\pi \le x \le 2\pi$

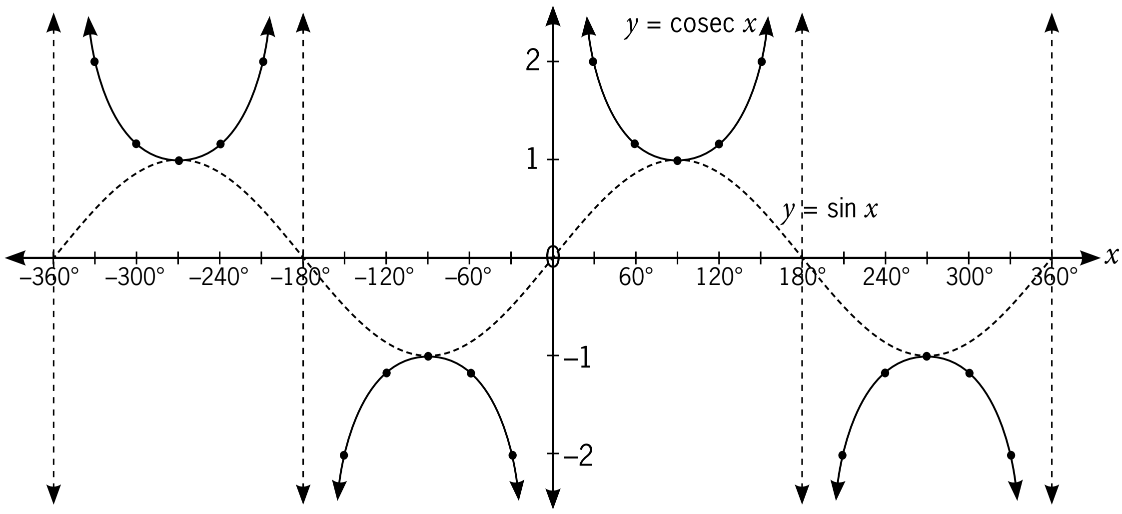


4.

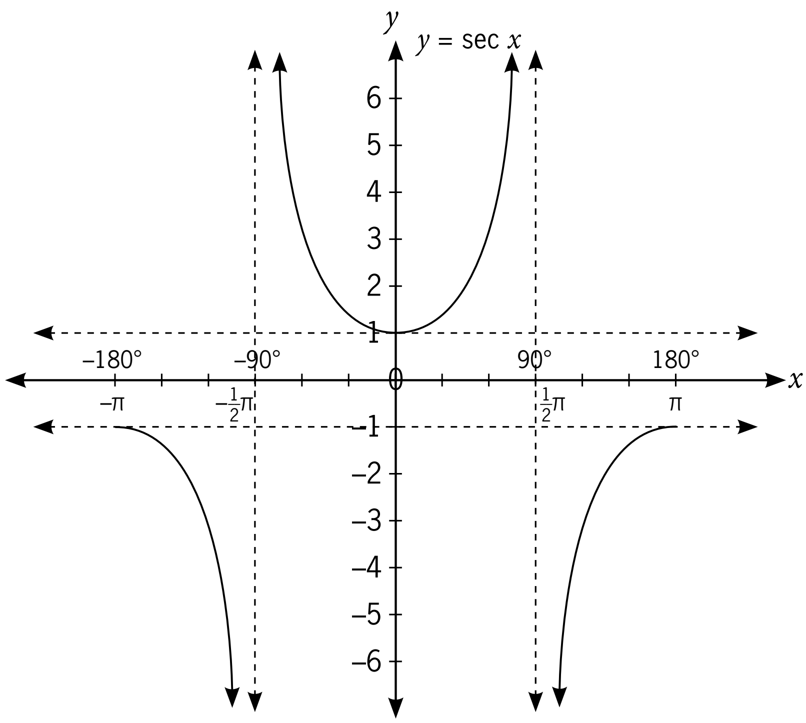
Function		**Range**	**Function/ Non-function**	**Continuous/ Discontinuous**
4.1	$y = 4\sin x - 2$ $-270° \leq x \leq 270°$	$-6 \leq y \leq 2$	Function	Continuous
4.2	$y = 3\cos 2x - 1$ $-\pi \leq x \leq \pi$	$-4 \leq y \leq 2$	Function	Continuous
4.3	$y = \tan\frac{x}{2}$ $-\pi \leq x \leq \pi$	$y \in R$	Function	Discontinuous
4.4	$y = 2\cos(x - 45°)$ $0° \leq x \leq 360°$	$-2 \leq y \leq 2$	Function	Continuous
4.5	$y = 2\tan x - 4$ $-180° \leq x \leq 180°$	$y \in R$	Function	Discontinuous
4.6	$y = 4\cos 2(x + 90°)$ $-90° \leq x \leq 90°$	$-4 \leq y \leq 4$	Function	Continuous
4.7	$y = \tan\left(x - \frac{\pi}{4}\right) + 1$ $-\frac{\pi}{2} \leq x \leq \pi$	$y \in R$	Function	Discontinuous
4.8	$y = 3\sin\left(\frac{2}{3}x - 30°\right) + 1$ $-360° \leq x \leq 360°$	$-2 \leq y \leq 4$	Function	Continuous
4.9	$y = -3\tan(2x + 60°)$ $0° \leq x \leq 360°$	$y \in R$	Function	Discontinuous
4.10	$y = -8\sin\left(x - \frac{\pi}{12}\right)$ $-\frac{3\pi}{2} \leq x \leq \frac{3\pi}{2}$	$-8 \leq y \leq 8$	Function	Continuous
4.11	$y = -\cos\left(\frac{x}{2} - \frac{\pi}{18}\right) + 3$ $-2\pi \leq x \leq 2\pi$	$2 \leq y \leq 4$	Function	Continuous

Activity 4.9 SB page 273

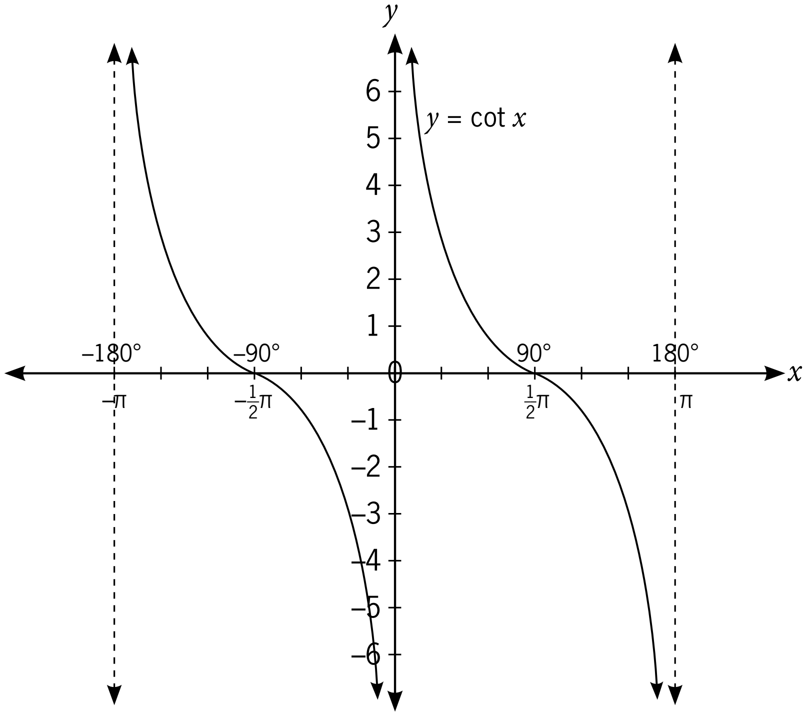
1.



2.



3.



Summative assessment: Module 4 SB page 274

1. 1.1 To prove: $\cos 105° = \frac{\sqrt{2} - \sqrt{6}}{4}$

$$\begin{aligned} \text{LHS} &= \cos 105° \\ &= \cos(45° + 60°) \\ &= \cos 45° \cos 60° - \sin 45° \sin 60° \\ &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} \\ &= \frac{1 - \sqrt{3}}{2\sqrt{2}} \\ &= \frac{1 - \sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

$\therefore$ LHS = RHS (4)

1.2 To prove: $\sin\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\sin x - \cos x)$

$$\begin{aligned} \text{LHS} &= \sin\left(x - \frac{\pi}{4}\right) \\ &= \sin x \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \cos x \\ &= \sin x \left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right) \cos x \\ &= \frac{1}{\sqrt{2}}(\sin x - \cos x) \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}(\sin x - \cos x) \\ &= \frac{\sqrt{2}}{2}(\sin x - \cos x) \end{aligned}$$

$\therefore$ LHS = RHS (4)

1 3 To prove: $\cos(x + y)\cos(x - y) = (\cos x \cos y)^2 - (\sin x \sin y)^2$

$$\begin{aligned} \text{LHS} &= \cos(x + y)\cos(x - y) \\ &= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y) \\ &= \cos^2 x \cos^2 y + \sin x \cos x \sin y \cos y - \sin x \cos x \sin y \cos y - \sin^2 x \sin^2 y \\ &= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y \\ &= (\cos x \cos y)^2 - (\sin x \sin y)^2 \end{aligned}$$

$\therefore$ LHS = RHS (4)

1.4 To prove: $\frac{\cot x - \tan x}{\cot^2 x + \tan^2 x} = \frac{1}{2}\sin 2x$

$$\text{LHS} = \frac{\cot x - \tan x}{\cot^2 x + \tan^2 x}$$

$$= \frac{\cot x - \tan x}{(\cot x + \tan x)(\cot x - \tan x)}$$

$$= \frac{1}{\cot x + \tan x}$$

$$= \frac{1}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}$$

$$= \frac{1}{\frac{\cos 2x + \sin^2 x}{\sin x \cos x}}$$

$$= \frac{1}{\frac{1}{\sin x \cos x}}$$

$$= \sin x \cos x$$

$$= \frac{1}{2}\sin 2x$$

$\therefore$ LHS = RHS (4)

2. $\sin 61° + \sin 29° = \sqrt{2}\cos 16°$

$\text{LHS} = \sin 61° + \sin 29°$

$= \sin(45° + 16°) + \sin(45° - 16°)$

$= (\sin 45° \cos 16° + \sin 16° \cos 45°) + (\sin 45° \cos 16° - \sin 16° \cos 45°)$

$= \sin 45° \cos 16° + \sin 16° \cos 45° + \sin 45° \cos 16° - \sin 16° \cos 45°$

$= 2 \sin 45° \cos 16°$

$= 2\left(\frac{1}{\sqrt{2}}\right)\cos 16°$

$= \frac{2}{\sqrt{2}}\cos 16°$

$= \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}\cos 16°$

$= \frac{2\sqrt{2}}{2}\cos 16°$

$= \sqrt{2}\cos 16°$

$\therefore$ LHS = RHS (4)

3. $\cos(x + y) = \cos x.\cos y - \sin x.\sin y$

$$= \left(-\frac{2}{\sqrt{5}}\right)\left(-\frac{3}{\sqrt{10}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{10}}\right)$$

$$= \frac{6}{\sqrt{50}} - \frac{1}{\sqrt{50}}$$

$$= \frac{5}{\sqrt{50}}$$

$$= \frac{5}{5\sqrt{2}}$$

$= \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ (4)

4. 4.1 $\cos 2x = 1 - 2\sin^2 x \qquad \therefore \sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\cos 2x = 2\cos^2 x - 1 \qquad \therefore \cos^2 x = \frac{1}{2}(1 + \cos 2x)$

But $\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$

$= \frac{\frac{1}{2}(1 - \cos 2x)}{\frac{1}{2}(1 + \cos 2x)}$

$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$ (4)

4.2 $\tan^2\left(\frac{\pi}{12}\right) = \frac{1 - \cos[2(\frac{\pi}{12})]}{1 + \cos[2(\frac{\pi}{12})]}$

$= \frac{1 - \cos\left(\frac{\pi}{6}\right)}{1 + \cos\left(\frac{\pi}{6}\right)}$

$= \frac{1 - \left(\frac{\sqrt{3}}{2}\right)}{1 + \left(\frac{\sqrt{3}}{2}\right)}$

$= \frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}}$

$\tan^2\left(\frac{\pi}{12}\right) = \frac{2 - \sqrt{3}}{2 + \sqrt{3}}$ (4)

5. 5.1 $\operatorname{cosec} x + \cot x = \sqrt{3}$

$\operatorname{cosec} x = \sqrt{3} - \cot x$

$(\operatorname{cosec} x)^2 = (\sqrt{3} - \cot x)^2$

$\operatorname{cosec}^2 x = 3 - 2\sqrt{3}\cot x + \cot^2 x$

$\operatorname{cosec}^2 x = 3 - 2\sqrt{3}\cot x + (\operatorname{cosec}^2 x - 1)$

$\operatorname{cosec}^2 x = 3 - 2\sqrt{3}\cot x + \operatorname{cosec}^2 x - 1$

$0 = 2 - 2\sqrt{3}\cot x$

$2\sqrt{3}\cot x = 2$

$\cot x = \frac{1}{\sqrt{3}}$

$\therefore \tan x = \oplus\sqrt{3}$

tan +; 1st | tan +; 3rd

$x = x_{ref}$	$x = 180° + x_{ref}$
$= \tan^{-1}(\sqrt{3})$	$= 180° + \tan^{-1}(\sqrt{3})$
$x = 60°$	$x = 240°$

Verify the answers, $\operatorname{cosec} 60° + \cot 60° = \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \sqrt{3}$

$\operatorname{cosec} 240° + \cot 240° = -\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} \neq \sqrt{3}$

$0° \leq x \leq 360°$, therefore $x = 60°$ (6)

5.2 $\cos\left(x + \frac{\pi}{3}\right) = \sin\left(x - \frac{\pi}{18}\right)$

$\cos\left(x + \frac{\pi}{3}\right) = \cos\left[\frac{\pi}{2} - \left(x - \frac{\pi}{18}\right)\right]$

$x + \frac{\pi}{3} = \frac{\pi}{2} - \left(x - \frac{\pi}{18}\right)$

$x + \frac{\pi}{3} = \frac{\pi}{2} - x + \frac{\pi}{18}$

$x + \frac{\pi}{3} = -x + \frac{5\pi}{9}$

$2x = \frac{2\pi}{9}$

$x = \frac{\pi}{9}$ (4)

$0 \leq x \leq \frac{\pi}{2}$, therefore $x = \frac{\pi}{9}$

6. $4 \tan \theta + 3 \sec \theta - 12 \sin \theta - 9 = 0$

$4\left(\frac{\sin \theta}{\cos \theta}\right) + 3\left(\frac{1}{\cos \theta}\right) - 12 \sin \theta - 9 = 0$

$4 \sin \theta + 3 - 12 \sin \theta \cos \theta - 9 \cos \theta = 0$

$(4 \sin \theta + 3) - 3 \cos \theta(4 \sin \theta + 3) = 0$

$(4 \sin \theta + 3)(1 - 3 \cos \theta) = 0$

$\sin \theta = -\frac{3}{4}$ or $\cos \theta = \frac{1}{3}$

$\therefore \theta = 228{,}59°; \theta = 311{,}41°; \theta = 70{,}53°$ or $\theta = 289{,}47°$ (7)

7. $\cos 2A = 1 - 2 \sin^2 A$

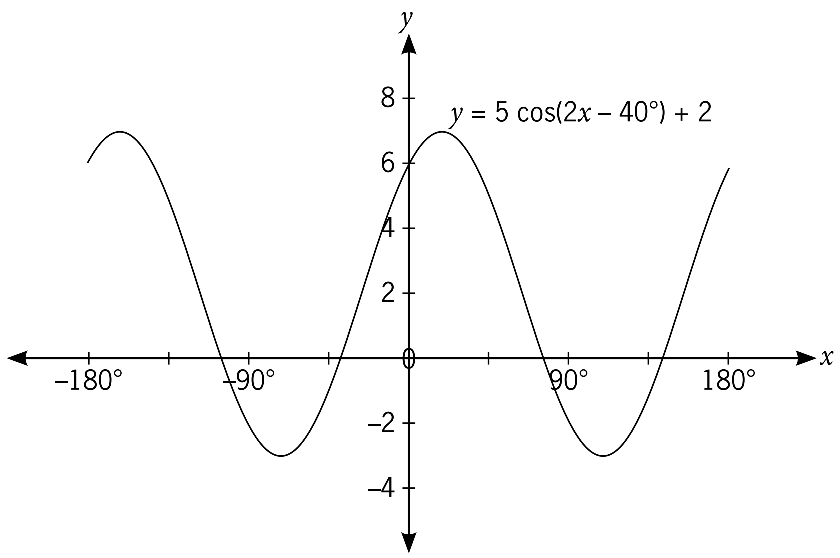
$2 \sin^2 A = 1 - \cos 2A$

$2 \sin^2 \frac{A}{2} = 1 - \cos A$

$\sin^2 \frac{A}{2} = \frac{1}{2}(1 - \cos A)$

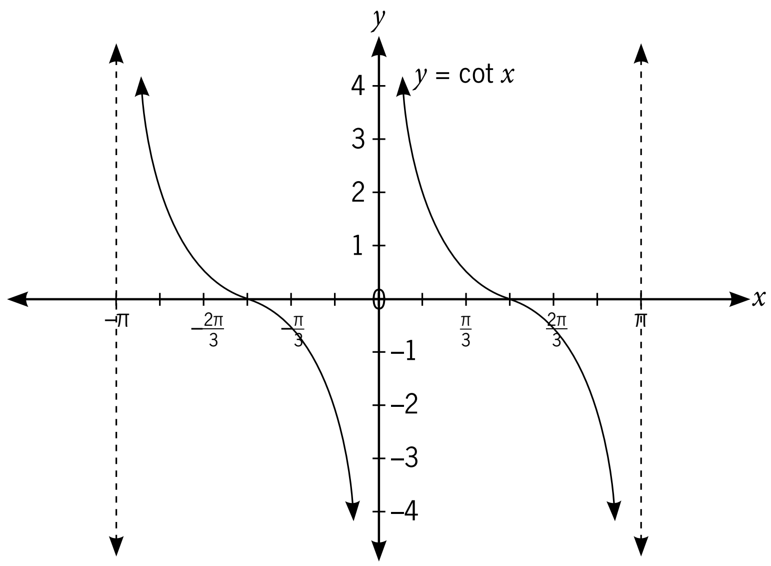
$\sin \frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}}$ (3)

8. 8.1 $y = 5 \cos(2x - 40°) + 2, -180° \leq x \leq 180°$



(4)

8.2 $y = \cot x, -\pi \leq x \leq \pi$



(4)

TOTAL: [60]

MODULE

5 Differential calculus

After they have completed this module, students should be able to:

- calculate limits that are indeterminate by making use of algebraic expressions (the theorem of L'Hôspital may not be applied);
- use the binomial theorem in general terms;
- apply the binomial theorem with rational indices to expand a simple binomial to four terms;
- define differentiation as a rate of change and derive the expression $\lim\limits_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$ or $\lim\limits_{h \to 0} \frac{f(x + h) - f(x)}{h}$ from first principles with the aid of a sketch as an introduction to differentiation; $f(x)$ may only be in one of the following forms: $f(x) = ax^n + b$ with $ax^n + bx^{n-1} + cx^{n-2} + \ldots$ and n a positive integer (exams will be limited to $n \in \{\mathbb{N}; n < 4\}$);
- determine $\frac{dy}{dx}$ of the following standard forms:
  - $y = k$
  - $y = kx^n$
  - $y = ka^x$
  - $y = ke^x$
  - $y = k \ln x$
  - $y = k \log_a x$
  - $y = k \sin x$
  - $y = k \cos x$
  - $y = k \tan x$
  - $y = k \cot x$
  - $y = k \sec x$
  - $y = k \operatorname{cosec} x$;

- apply the chain rule to determine the first derivatives of ka^{nx}, ke^{nx}, $k \log_a nx$, $k \log_e nx$, $k \sin(bx)$, $k \cos(bx)$, $k \tan(bx)$, $k \cot(bx)$, $k \sec(bx)$ and $k \operatorname{cosec}(bx)$;
- apply the product and quotient rules for differentiation of differentiate simple products and quotients (combinations of chain, product and quotient rules may not be asked);
- determine the second derivatives of trigonometric functions, algebraic terms and polynomials to determine maximum and minimum turning points and points of inflection;
- draw neat sketch graphs of $y = ax^3 + bx^2 + cx + d$, where a, b, c and d are integers; and
- sketch graphs indicating maximum and minimum values derived above.

Introduction

The word *calculus* describes a system of rules or reasoning used to do certain types of calculations. It was developed independently by Sir Isaac Newton and Gottfried Leibniz towards the end of the 17th century.

In calculus students will compare quantities that vary in a **nonlinear** way. Calculus is generally used in science and engineering. It is the mathematics of rates of change. Many concepts that they learnt about, such as velocity, acceleration and current in a circuit, do not behave in a simple linear way. Quantities continuously change, so you need calculus to interpret all the changes.

Differential calculus is a subfield of calculus and is the **study of the rates at which quantities change**. The **derivative** of a function at a chosen input value describes the rate of change of the function near that specific input value. Finding a derivative is called **differentiation**.

The two main branches of calculus are **differentiation** and **integration**.

Students need the following pre-knowledge to successfully complete this module.

Pre-knowledge

- Sketching graphs (Module 3).
- Division by 0, for example $\frac{4}{0}$, $\frac{k}{0}$ and $\frac{0}{0}$ is undefined.
- $\frac{0}{k} = 0$, $\frac{0}{3} = 0$, and so on

- Factorisation:

Common factor	$3x^2y - 6x$ $= 3x(xy - 2)$
Difference of two squares	$x^2 - 9$ $= (x - 3)(x + 3)$
Quadratic trinomial	$x^2 - 6x + 8$ $= (x - 2)(x - 4)$
Sum of two cubes	$a^3 + b^3$ $= (a + b)(a^2 - ab + b^2)$
Difference of two cubes	$a^3 - b^3$ $= (a - b)(a^2 + ab + b^2)$

- $x^0 = 1$
- FOIL: $(x + h)(x + h) = x^2 + xh + xh + h^2 = x^2 + 2xh + h^2$
- Factorising third-degree polynomials of the form $f(x) = ax^3 + bx^2 + cx + d$:

Step 1: Find one factor by using the factor theorem.

- If $f(a) = 0$, then we know that $(x - a)$ is a factor.
- We need the value for a so that $f(a) = 0$, because then $(x - a)$ will be a factor of $f(x)$.
- Find a factor of $f(x)$. Consider factors of the last (constant) term first.

Example

If $f(x) = x^3 - 7x^2 - 10x + 16$

↓

Possible factors of 16 are:
± 1; ± 2; ± 4; ± 8; ± 16

Test for factors:

Let $f(-1)$ $= (-1)^3 - 7(-1)^2 - 10(-1) + 16$ $= -1 - 7 + 10 + 16$ $= 18$ $\therefore 18 \neq 0$ $\therefore (x + 1)$ is NOT a factor because the remainder $= f(-1) \neq 0$	Let $f(1)$ $= (1)^3 - 7(1)^2 - 10(1) + 16$ $= 1 - 7 - 10 + 16$ $= 0$ $\therefore (x - 1)$ is a factor because the remainder $= f(1) = 0$

Step 2: Use inspection or long division to determine the values of a, b and c.

$f(x) = ax^3 + bx^2 + cx + d$
$= (x \pm k)(ax^2 + bx + c)$

Using the inspection method:

- Divide $f(x)$ by $(x - a)$ through inspection.
- If $f(x) = x^3 - 7x^2 - 10x + 16$ and $(x - 1)$ is a factor as proven in Step 1, then $x^3 - 7x^2 - 10x + 16 = (x - 1)(ax^2 + bx + c)$.
- Use inspection to find ax and c.

  first terms

  $(x - 1)(ax^2 + \ldots)$

  Multiply the first terms of each bracket and compare the answer to the term x^3 in the given polynomial: $f(x) = x^3 - 7x^2 - 10x + 16$

  You see that:

  $x.ax^2 = ax^3$

  $\therefore x.1x^2 = 1x^3$

  Therefore $a = 1$

  last terms

  $(x - 1)(ax^2 + bx + c)$

  Multiply the last terms of each bracket and compare the answer with the last (constant) term of $f(x)$, which is 16. You see that:

  $(-1)(c) = 16$

  $\therefore c = -16$
- Substitute $a = 1$ and $c = -16$ in $(x - 1)(ax^2 + bx + c)$.

  $\therefore (x - 1)(x^2 + bx - 16)$
- Find b using the coefficients of x.

  $\therefore x^3 - 7x^2 - 10x + 16 = (x - 1)(x^2 + bx - 16)$

  $-x^2$
  +
  bx^2

  $\therefore -x^2 + bx^2 = -7x^2$ • $(-1)(x^2) + (x)(bx) = -7x^2$

  $bx^2 = -7x^2 + x^2$ • Solve for b

  $bx^2 = -6x^2$

  $\therefore b = -6$
- Substitute $b = -6$ in $(x - 1)(x^2 + bx - 16)$, $f(x) = (x - 1)(x^2 - 6x - 16)$

Step 3: Factorise the quadratic expression completely.

$f(x) = x^3 - 7x^2 - 10x + 16$
$= (x - 1)(x^2 - 6x - 16)$ • $b = -6$
$= (x - 1)(x - 8)(x + 2)$ • Factorise $(x^2 - 6x - 16)$

Exponential laws:

$a^x \times a^y = a^{x+y}$

$a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}$

$(a^x)^y = a^{xy}$

$a^0 = 1$

$a^{-x} = \frac{1}{a^x}$

$(ab)^x = a^x b^x$

$a^{\frac{x}{y}} = (a^x)^{\frac{1}{y}} = \sqrt[y]{a^x}$

Factorisation:

$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

Useful definitions:

Definition		Example
1	$\frac{x^n}{a} = \frac{1}{a}x^n$	$\frac{x^2}{3} = \frac{1}{3}x^2$
2	$\frac{a}{x^n} = ax^{-n}$	$\frac{3}{x^2} = 3x^{-2}$
3	$\frac{ax^n}{b} = \frac{a}{b}x^n$	$\frac{2x^4}{3} = \frac{2}{3}x^4$
4	$\frac{a}{bx^n} = \frac{a}{b}x^{-n}$	$\frac{2}{3x^4} = \frac{2}{3}x^{-4}$
5	$\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$\sqrt[5]{2^3} = 2^{\frac{3}{5}}$
6	$\sqrt{ax} = \sqrt{a}\,x^{\frac{1}{2}}$	$\sqrt{2x^3} = \sqrt{2}\,x^{\frac{3}{2}}$
7	$\frac{a}{\sqrt[n]{x^m}} = \frac{a}{x^{\frac{m}{n}}} = ax^{-\frac{m}{n}}$	$\frac{2}{\sqrt[3]{x^2}} = \frac{2}{x^{\frac{2}{3}}} = 2x^{-\frac{2}{3}}$

Logarithmic laws
$\log_a x + \log_a y = \log_a xy$
$\log_a x - \log_a y = \log_a \frac{x}{ay}$
$\log_a x^k = k \log_a x$
$\log_a a = 1$
$\log_a 1 = 0$
$\log_a b = \frac{\log b}{\log a}$

Logarithmic form and exponential form
$\log_b N = a$ (logarithmic form)
↕
$N = b^a$ (exponential form)

Note

- $\ln e = \log_e e = 1$
- $\log_e x = \ln x$ with $e = 2{,}71828$
- $\log x = \log_{10} x$ (base = 10)

- Factor theorem

Activity 5.1 SB page 286

1. $\lim_{x \to 5} (x^3)$
   $= (5)^3$
   $= 125$

2. $\lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x}{1 - \sin x}$
   $= \frac{\cos^2 \frac{\pi}{2}}{1 - \sin\frac{\pi}{2}}$
   $= \frac{0}{1 - 1}$
   $= \frac{0}{0}$
   = no limit

3. $\lim_{x \to -4} (x^2 + 4x - 3)$
   $= (-4)^2 + 4(-4) - 3$
   $= 16 - 16 - 3$
   $= -3$

4. $\lim_{x \to 3} \left(\frac{x^2 + 2}{x^2 + 4x - 6}\right)$
   $= \frac{(3)^2 + 2}{(3)^2 + 4(3) - 6}$
   $= \frac{9 + 2}{9 + 12 - 6}$
   $= \frac{11}{15}$

5. $\lim_{x\to 1}\left(\frac{4x-9}{\sqrt{3x+1}}\right)$

$= \frac{4(1)-9}{\sqrt{3(1)+1}}$

$= \frac{4-9}{\sqrt{3+1}}$

$= \frac{-5}{\sqrt{4}}$

$= \frac{-5}{2}$

$= -2\frac{1}{2}$

6. $\lim_{x\to \frac{1}{2}}\left(\frac{2x^2-9x+4}{4x^2+12x-7}\right)$

$= \frac{2\left(\frac{1}{2}\right)^2-9\left(\frac{1}{2}\right)+4}{4\left(\frac{1}{2}\right)^2+12\left(\frac{1}{2}\right)-7}$

$= \frac{2\left(\frac{1}{4}\right)-\frac{9}{2}+4}{4\left(\frac{1}{4}\right)+6-7}$

$= \frac{\frac{1}{2}-\frac{9}{2}+4}{1+6-7}$

$= \frac{0}{0}$

$\lim_{x\to \frac{1}{2}}\left(\frac{2x^2-9x+4}{4x^2+12x-7}\right)$

$= \lim_{x\to \frac{1}{2}}\left[\frac{(2x-1)(x-4)}{(2x-1)(2x+7)}\right]$

$= \lim_{x\to \frac{1}{2}}\left[\frac{x-4}{2x+7}\right]$

$= \frac{\left(\frac{1}{2}\right)-4}{2\left(\frac{1}{2}\right)+7}$

$= \frac{\frac{1}{2}-4}{1+7}$

$= \frac{-\frac{7}{2}}{8}$

$= -\frac{7}{16}$

7. $\lim_{x\to -3}\left(\frac{x^4-81}{x^2+2x-3}\right)$

$= \frac{(-3)^4-81}{(-3)^2+2(-3)-3}$

$= \frac{81-81}{9-6-3}$

$= \frac{0}{0}$

$\lim_{x\to -3}\left(\frac{x^4-81}{x^2+2x-3}\right)$

$= \lim_{x\to -3}\left[\frac{(x^2+9)(x^2-9)}{(x-1)(x+3)}\right]$

$= \lim_{x\to -3}\left[\frac{(x^2+9)(x+3)(x-3)}{(x-1)(x+3)}\right]$

$= \lim_{x\to -3}\left[\frac{(x^2+9)(x-3)}{(x-1)}\right]$

$= \frac{((-3)^2+9)((-3)-3)}{((-3)-1)}$

$= \frac{(9+9)(-3-3)}{(-3-1)}$

$= \frac{(18)(-6)}{(-4)}$

$= \frac{-108}{-4}$

$= 27$

8. $\lim_{x\to 5}\left(\frac{\sqrt{5}-\sqrt{x}}{x-5}\right)$

$= \frac{\sqrt{5}-\sqrt{(5)}}{(5)-5}$

$= \frac{\sqrt{5}-\sqrt{5}}{5-5}$

$= \frac{0}{0}$

$\lim_{x\to 5}\left(\frac{\sqrt{5}-\sqrt{x}}{x-5}\right)$

$= \lim_{x\to 5}\left[\frac{-(\sqrt{x}-\sqrt{5})}{x-5}\right]$

$= \lim_{x\to 5}\left[\frac{-(\sqrt{x}-\sqrt{5})}{(\sqrt{x}+\sqrt{5})(\sqrt{x}-\sqrt{5})}\right]$

$= \lim_{x\to 5}\left[\frac{-1}{\sqrt{x}+\sqrt{5}}\right]$

$= \frac{-1}{\sqrt{(5)}+\sqrt{5}}$

$= \frac{-1}{\sqrt{5}+\sqrt{5}}$

$= -\frac{1}{2\sqrt{5}}$

9. $\lim_{x\to -1}\left(\frac{3x^2+2x-1}{1+x^3}\right)$

$=\frac{3(-1)^2+2(-1)-1}{1+(-1)^3}$

$=\frac{3(1)-2-1}{1+(-1)^3}$

$=\frac{3-2-1}{1+(-1)}$

$=\frac{3-2-1}{1-1}$

$=\frac{0}{0}$

$\lim_{x\to -1}\left(\frac{3x^2+2x-1}{1+x^3}\right)$

$=\lim_{x\to -1}\left(\frac{3x^2+2x-1}{x^3+1}\right)$

$=\lim_{x\to -1}\left[\frac{(3x-1)(x+1)}{(x+1)(x^2-x+1)}\right]$

$=\lim_{x\to -1}\left[\frac{3x-1}{x^2-x+1}\right]$

$=\frac{3(-1)-1}{(-1)^2-(-1)+1}$

$=\frac{-3-1}{(1)+1+1}$

$=\frac{-4}{1+1+1}$

$=-\frac{4}{3}$

$=-1\frac{1}{3}$

10. $\lim_{x\to 2}\left(\frac{2x-4}{6-x-x^2}\right)$

$=\frac{2(2)-4}{6-(2)-(2)^2}$

$=\frac{4-4}{6-2-(4)}$

$=\frac{0}{6-2-4}$

$=\frac{0}{0}$

$\lim_{x\to 2}\left(\frac{2x-4}{6-x-x^2}\right)$

$=\lim_{x\to 2}\left[\frac{2(x-2)}{-(x^2+x-6)}\right]$

$=\lim_{x\to 2}\left[\frac{2(x-2)}{-(x+3)(x-2)}\right]$

$=\lim_{x\to 2}\left[\frac{2}{-(x+3)}\right]$

$=\frac{2}{-((2)+3)}$

$=\frac{2}{-(2+3)}$

$=\frac{2}{-(5)}$

$=\frac{2}{-5}$

$=-\frac{2}{5}$

11. $\lim_{x\to 6}\left(\frac{x^2-36}{x^3-216}\right)$

$=\frac{(6)^2-36}{(6)^3-216}$

$=\frac{36-36}{216-216}$

$=\frac{0}{0}$

$\lim_{x\to 6}\left(\frac{x^2-36}{x^3-216}\right)$

$=\lim_{x\to 6}\left[\frac{(x+6)(x-6)}{(x-6)(x^2+6x+36)}\right]$

$=\lim_{x\to 6}\left[\frac{x+6}{x^2+6x+36}\right]$

$=\frac{(6)+6}{(6)^2+6(6)+36}$

$=\frac{6+6}{36+36+36}$

$=\frac{12}{108}$

$=\frac{1}{9}$

12. $\lim_{x\to 2}\frac{x-2}{\sqrt{2x-2}-\sqrt{x}}$

$=\lim_{x\to 2}\frac{x-2}{\sqrt{2x-2}-\sqrt{x}}\times\frac{\sqrt{2x-2}+\sqrt{x}}{\sqrt{2x-2}+\sqrt{x}}$

$=\lim_{x\to 2}\frac{(x-2)(\sqrt{2x-2}+\sqrt{x})}{(2x-2)-x}$

$=\lim_{x\to 2}\frac{(x-2)(\sqrt{2x-2}+\sqrt{x})}{x-2}$

$=\lim_{x\to 2}\sqrt{2x-2}+\sqrt{x}$

$=\sqrt{2}+\sqrt{2}$

$=2\sqrt{2}$

13. $\lim_{x\to -2}\left(\dfrac{\frac{1}{x}+\frac{1}{2}}{2+x}\right)$

$= \dfrac{\frac{1}{(-2)}+\frac{1}{2}}{2+(-2)}$

$= \dfrac{-\frac{1}{2}+\frac{1}{2}}{2-2}$

$= \dfrac{0}{0}$

$\lim_{x\to -2}\left(\dfrac{\frac{1}{x}+\frac{1}{2}}{2+x}\right)$

$= \lim_{x\to -2}\left(\dfrac{\frac{2+x}{2x}}{2+x}\right)$

$= \lim_{x\to -2}\left(\dfrac{1}{2x}\right)$

$= \dfrac{1}{2(-2)}$

$= \dfrac{1}{-4}$

$= -\dfrac{1}{4}$

14. $\lim_{x\to 4}\left(\dfrac{x^2-16}{2-\sqrt{x}}\right)$

$= \dfrac{(4)^2-16}{2-\sqrt{(4)}}$

$= \dfrac{16-16}{2-\sqrt{4}}$

$= \dfrac{0}{2-2}$

$= \dfrac{0}{0}$

$\lim_{x\to 4}\left(\dfrac{x^2-16}{2-\sqrt{x}}\right)$

$= \lim_{x\to 4}\left[\dfrac{(x+4)(x-4)}{-(\sqrt{x}-2)}\right]$

$= \lim_{x\to 4}\left[\dfrac{(x+4)(\sqrt{x}+2)(\sqrt{x}-2)}{-(\sqrt{x}-2)}\right]$

$= \lim_{x\to 4}\left[-(x+4)(\sqrt{x}+2)\right]$

$= -((4)+4)(\sqrt{(4)}+2)$

$= -(4+4)(\sqrt{4}+2)$

$= -(8)(2+2)$

$= -(8)(4)$

$= -32$

15. $\lim_{x\to \infty}\left(\dfrac{3x+2}{2x+3}\right)$

$= \dfrac{3(\infty)+2}{2(\infty)+3}$

$= \dfrac{\infty}{\infty}$

$\lim_{x\to \infty}\left(\dfrac{3x+2}{3x+3}\right)$

$= \lim_{x\to \infty}\left(\dfrac{\frac{3x}{x}+\frac{2}{x}}{\frac{2x}{x}+\frac{3}{x}}\right)$

$= \lim_{x\to \infty}\left(\dfrac{3+\frac{2}{x}}{2+\frac{3}{x}}\right)$

$= \dfrac{3+\frac{2}{\infty}}{2+\frac{3}{\infty}}$

$= \dfrac{3+0}{2+0}$

$= \dfrac{3}{2}$

$= 1\frac{1}{2}$

16. $\lim_{x\to \infty}\left(\dfrac{6x^2-4x+2}{3x^2+5x-8}\right)$

$= \dfrac{6(\infty)^2-4(\infty)+2}{3(\infty)^2+5(\infty)-8}$

$= \dfrac{\infty}{\infty}$

$\lim_{x\to \infty}\left(\dfrac{6x^2-4x+2}{3x^2+5x-8}\right)$

$= \lim_{x\to \infty}\left(\dfrac{\frac{6x^2}{x^2}-\frac{4x}{x^2}+\frac{2}{x^2}}{\frac{3x^2}{x^2}+\frac{5x}{x^2}-\frac{8}{x^2}}\right)$

$= \lim_{x\to \infty}\left(\dfrac{6-\frac{4}{x}+\frac{2}{x^2}}{3+\frac{5}{x}-\frac{8}{x^2}}\right)$

$= \dfrac{6-\frac{4}{(\infty)}+\frac{2}{(\infty)^2}}{3+\frac{5}{(\infty)}-\frac{8}{(\infty)^2}}$

$= \dfrac{6-0+0}{3+0-0}$

$= \dfrac{6}{3}$

$= 2$

17. $\lim_{x\to\infty}\left(\frac{x^2-2x+2}{x^3+2}\right)$

$=\frac{(\infty)^2-2(\infty)+2}{(\infty)^3+2}$

$=\frac{\infty}{\infty}$

$\lim_{x\to\infty}\left(\frac{x^2-2x+2}{x^3+2}\right)$

$=\lim_{x\to\infty}\left(\frac{\frac{x^2}{x^3}-\frac{2x}{x^3}+\frac{2}{x^3}}{\frac{x^3}{x^3}+\frac{2}{x^3}}\right)$

$=\lim_{x\to\infty}\left(\frac{\frac{1}{x}-\frac{2}{x^2}+\frac{2}{x^3}}{1+\frac{2}{x^3}}\right)$

$=\frac{\frac{1}{(\infty)}-\frac{2}{(\infty)^2}+\frac{2}{(\infty)^3}}{1+\frac{2}{(\infty)^3}}$

$=\frac{0-0+0}{1+0}$

$=\frac{0}{1}$

$=0$

18. $\lim_{x\to\infty}\left(\frac{3x^5-9x+2}{2x^2+5x+1}\right)$

$=\frac{3(\infty)^5-9(\infty)+2}{2(\infty)^2+5(\infty)+1}$

$=\frac{\infty}{\infty}$

$\lim_{x\to\infty}\left(\frac{3x^5-9x+2}{2x^2+5x+1}\right)$

$=\lim_{x\to\infty}\left(\frac{\frac{3x^5}{x^5}-\frac{9x}{x^5}+\frac{2}{x^5}}{\frac{2x^2}{x^5}+\frac{5x}{x^5}+\frac{1}{x^5}}\right)$

$=\lim_{x\to\infty}\left(\frac{3-\frac{9}{x^4}+\frac{2}{x^5}}{\frac{2}{x^3}+\frac{5}{x^4}+\frac{1}{x^5}}\right)$

$=\frac{3-\frac{9}{(\infty)^4}+\frac{2}{(\infty)^5}}{\frac{2}{(\infty)^3}+\frac{5}{(\infty)^4}+\frac{1}{(\infty)^5}}$

$=\frac{3-0+0}{0+0+0}$

$=\frac{3}{0}$

= no limit

19. $\lim_{x\to\infty}\left(\frac{\sqrt{1+4x^2}}{2-3x}\right)$

$=\frac{\sqrt{1+4(\infty)^2}}{2-3(\infty)}$

$=\frac{\infty}{-\infty}$

$=-\frac{\infty}{\infty}$

$\lim_{x\to\infty}\left(\frac{\sqrt{1+4x^2}}{2-3x}\right)$

$=\lim_{x\to\infty}\left(\frac{\sqrt{x^2\left(\frac{1}{x^2}+4\right)}}{2-3x}\right)$

$=\lim_{x\to\infty}\left(\frac{x\sqrt{\frac{1}{x^2}+4}}{2-3x}\right)$

$=\lim_{x\to\infty}\left(\frac{\frac{x\sqrt{\frac{1}{x^2}+4}}{x}}{\frac{2}{x}-\frac{3x}{x}}\right)$

$=\lim_{x\to\infty}\left(\frac{\sqrt{\frac{1}{x^2}+4}}{\frac{2}{x}-3}\right)$

$=\frac{\sqrt{\frac{1}{(\infty)^2}+4}}{\frac{2}{(\infty)}-3}$

$=\frac{\sqrt{0+4}}{0-3}$

$=\frac{\sqrt{4}}{-3}$

$=\frac{2}{-3}$

$=-\frac{2}{3}$

20. $\lim_{x\to 0}\left(\frac{(2+x)^3-8}{x}\right)$

$=\frac{(2+(0))^3-8}{(0)}$

$=\frac{(2)^3-8}{0}$

$=\frac{8-8}{0}$

$=\frac{0}{0}$

$\lim_{x\to 0}\left(\frac{(2+x)^3-8}{x}\right)$

$=\lim_{x\to 0}\left[\frac{((2+x)-2)((2+x)^2+2(2+x)+4)}{x}\right]$

$=\lim_{x\to 0}\left[\frac{(2+x-2)((2+x)^2+2(2+x)+4)}{x}\right]$

$=\lim_{x\to 0}\left[\frac{x((2+x)^2+2(2+x)+4)}{x}\right]$

$=\lim_{x\to 0}\left[(2+x)^2+2(2+x)+4\right]$

$=(2+(0))^2+2(2+(0))+4$

$=(2+0)^2+2(2+0)+4$

$=(2)^2+2(2)+4$

$=4+4+4$

$=12$

Activity 5.2 SB page 292

1. 1.1 The fourth term is T_4,

$\therefore\ T_4 = T_{r+1}$

$4 = r + 1$

$r = 3;\ n = 16$

Thus,

$$T_{3+1} = \frac{n!}{r!(n-r)!}x^{n-r}h^r$$

$$T_4 = \frac{16!}{3!(16-3)!}(x)^{13}(5)^3$$

$$T_4 = 70\ 000x^{13}$$

1.2 The eighth term is T_8,

$\therefore\ T_8 = T_{r+1}$

$8 = r + 1$

$r = 7;\ n = 14$

$$T_{7+1} = \frac{14!}{7!(14-7)!}(\sqrt{2})^7(-x)^7$$

$$T_8 = -27\ 456\sqrt{2}x^7$$

1.3 The seventh term is T_7,

$\therefore\ T_7 = T_{r+1}$

$7 = r + 1$

$r = 6;\ n = 20$

$$T_{6+1} = \frac{n!}{r!(n-r)!}x^{n-r}h^r$$

$$T_7 = \frac{20!}{6!(20-6)!}(k^2)^{14}(5\ell)^6$$

$$T_7 = 605\ 625\ 000k^{28}\ell^6$$

1.4 The nineth term is T_9,

$\therefore\ T_9 = T_{r+1}$

$9 = r + 1$

$r = 8;\ n = 11$

$$T_{8+1} = \frac{n!}{r!(n-r)!}x^{n-r}h^r$$

$$T_9 = \frac{11!}{8!(11-8)!}(x^3)^3\left(-\frac{3}{x}\right)^8$$

$$T_9 = 1\ 082\ 565x$$

2. 2.1 $(x+2)^7$

$$= \frac{(x)^7(2)^0}{0!} + \frac{7(x)^6(2)^1}{1!} + \frac{7.6(x)^5(2)^2}{2!} + \frac{7.6.5(x)^4(2)^3}{3!} + \dots$$

$$= \frac{x^7.1}{1} + \frac{7.x^6.2}{1} + \frac{7.6.x^5.4}{2} + \frac{7.6.5.x^4.8}{6} + \dots$$

$$= x^7 + 14x^6 + 84x^5 + 280x^4 + \dots$$

2.2 $\left(1 - \frac{x}{4}\right)^{15}$

$$= \left[(1) + \left(-\frac{x}{4}\right)\right]^{15}$$

$$= \frac{(1)^{15}\left(-\frac{x}{4}\right)^0}{0!} + \frac{15(1)^{14}\left(-\frac{x}{4}\right)^1}{1!} + \frac{15.14(1)^{13}\left(-\frac{x}{4}\right)^2}{2!} + \frac{15.14.13(1)^{12}\left(-\frac{x}{4}\right)^3}{3!} + \dots$$

$$= \frac{1.1}{1} + \frac{15.1.-\frac{x}{4}}{1} + \frac{15.14.1.\frac{x^2}{16}}{2} + \frac{15.14.13.1.-\frac{x^3}{64}}{6} + \dots$$

$$= 1 - \frac{15x}{4} + \frac{105x^2}{16} - \frac{455x^3}{64} + \dots$$

2.3 $\frac{1}{(1+x)^5}$

$$= (1+x)^{-5}$$

$$= \frac{(1)^{-5}(x)^0}{0!} + \frac{(-5)(1)^{-6}(x)^1}{1!} + \frac{(-5)(-6)(1)^{-7}(x)^2}{2!} + \frac{(-5)(-6)(-7)(1)^{-8}(x)^3}{3!} + \dots$$

$$= \frac{11}{1} + \frac{(-5)1x}{1} + \frac{(-5)(-6)1x^2}{2} + \frac{(-5)(-6)(-7)1x^3}{6} + \dots$$

$$= 1 - 5x + 15x^2 - 35x^3 + \dots$$

2.4 $\left(x - \frac{1}{5}\right)^{-12}$

$$= \left[(x) + \left(-\frac{1}{5}\right)\right]^{-12}$$

$$= \frac{(x)^{-12}\left(-\frac{1}{5}\right)^0}{0!} + \frac{(-12)(x)^{-13}\left(-\frac{1}{5}\right)^1}{1!} + \frac{(-12)(-13)(x)^{-14}\left(-\frac{1}{5}\right)^2}{2!} + \frac{(-12)(-13)(-14)(x)^{-15}\left(-\frac{1}{5}\right)^3}{3!} + \dots$$

$$= \frac{x^{-12}1}{1} + \frac{(-12)(x^{-13})\left(-\frac{1}{5}\right)}{1} + \frac{(-12)(-13)(x^{-14})\left(\frac{1}{25}\right)}{2} + \frac{(-12)(-13)(-14)(x^{-15})\left(-\frac{1}{125}\right)}{6} + \dots$$

$$= \frac{1}{x^{12}} + \frac{12}{5x^{13}} + \frac{78}{25x^{14}} + \frac{364}{125x^{15}} + \dots$$

2.5 $\left(\frac{x}{3} + y\right)^{10}$

$$= \frac{\left(\frac{x}{3}\right)^{10}(y)^0}{0!} + \frac{10\left(\frac{x}{3}\right)^9(y)^1}{1!} + \frac{10.9\left(\frac{x}{3}\right)^8(y)^2}{2!} + \frac{10.9.8\left(\frac{x}{3}\right)^7(y)^3}{3!} + \dots$$

$$= \frac{\frac{x^{10}}{59\,049}1}{1} + \frac{10\frac{x^9}{19\,683}y}{1} + \frac{10.9\frac{x^8}{6\,561}y^2}{2} + \frac{10.9.8\frac{x^7}{2\,187}y^3}{6} + \dots$$

$$= \frac{x^{10}}{59\,049} + \frac{10x^9y}{19\,683} + \frac{5x^8y^2}{729} + \frac{40x^7y^3}{729} + \dots$$

2.6 $(4x + 3)^6$

$= \frac{(4x)^6.(3)^0}{0!} + \frac{6(4x)^5(3)^1}{1!} + \frac{6.5(4x)^4(3)^2}{2!} + \frac{6.5.4(4x)^3(3)^3}{3!} + \ldots$

$= \frac{(4x)^6 1}{1} + \frac{6(4x)^5.3}{1} + \frac{6.5(4x)^4.9}{2} + \frac{6.5.4(4x)^3.27}{6} + \ldots$

$= 4\,096x^6 + 18\,432x^5 + 34\,560x^4 + 34\,560x^3 + \ldots$

Sum of the first four binomial coefficients

$= 4\,096 + 18\,432 + 34\,560 + 34\,560$

$= 91\,648$

2.7 $(x - 2y)^9$

$= [(x) + (-2y)]^9$

$= \frac{(x)^9(-2y)^0}{0!} + \frac{9(x)^8(-2y)^1}{1!} + \frac{9.8(x)^7(-2y)^2}{2!} + \frac{9.8.7(x)^6(-2y)^3}{3!} + \ldots$

$= \frac{x^9.1}{1} + \frac{9.x^8(-2y)}{1} + \frac{9.8x^7 4y^2}{2} + \frac{9.8.7x^6.-8y^3}{6} + \ldots$

$= x^9 - 18x^8y + 144x^7y^2 - 672x^6y^3 + \ldots$

Sum of the first four binomial coefficients

$= 1 - 18 + 144 - 672$

$= -545$

2.8 $(2a - 3b)^{-8}$

$= [(2a) + (-3b)]^{-8}$

$= \frac{(2a)^{-8}(-3b)^0}{0!} + \frac{(-8)(2a)^{-9}(-3b)^1}{1!} + \frac{(-8)(-9)(2a)^{-10}(-3b)^2}{2!} + \frac{(-8)(-9)(-10)(2a)^{-11}(-3b)^3}{3!} + \ldots$

$= \frac{\frac{1}{256a^8}.1}{1} + \frac{(-8)\frac{1}{512a^9}(-3b)}{1} + \frac{(-8)(-9)\frac{1}{1\,024a^{10}}9b^2}{2} + \frac{(-8)(-9)(-10)\frac{1}{2\,048a^{11}}(-27b^3)}{6} + \ldots$

$= \frac{1}{256a^8} + \frac{3b}{64a^9} + \frac{81b^2}{256a^{10}} + \frac{405b^3}{256a^{11}} + \ldots$

2.9 $(x + 7)^{\frac{3}{4}}$

$= \frac{(x)^{\frac{3}{4}}(7)^0}{0!} + \frac{\frac{3}{4}(x)^{-\frac{1}{4}}(7)^1}{1!} + \frac{\frac{3}{4}\left(-\frac{1}{4}\right)(x)^{-\frac{5}{4}}(7)^2}{2!} + \frac{\frac{3}{4}\left(-\frac{1}{4}\right)\left(-\frac{5}{4}\right)(x)^{-\frac{9}{4}}(7)^3}{3!} + \ldots$

$= \frac{x^{\frac{3}{4}}.1}{1} + \frac{\frac{3}{4}x^{-\frac{1}{4}}7}{1} + \frac{\frac{3}{4}\left(-\frac{1}{4}\right)x^{-\frac{5}{4}}49}{2} + \frac{\frac{3}{4}\left(-\frac{1}{4}\right)\left(-\frac{5}{4}\right)x^{-\frac{9}{4}}343}{6} + \ldots$

$= x^{\frac{3}{4}} + \frac{21}{4}x^{-\frac{1}{4}} - \frac{147}{32}x^{-\frac{5}{4}} + \frac{1\,715}{128}x^{-\frac{9}{4}} + \ldots$

$= x^{\frac{3}{4}} + \frac{21}{4x^{\frac{1}{4}}} - \frac{147}{32x^{\frac{5}{4}}} + \frac{1\,715}{128x^{\frac{9}{4}}} + \ldots$

$= x^{\frac{3}{4}} + \frac{21}{4x^{\frac{1}{4}}} - \frac{147}{32x^1.x^{\frac{1}{4}}} + \frac{1\,715}{128x^2.x^{\frac{1}{4}}} + \ldots$

$= \sqrt[4]{x^3} + \frac{21}{4\sqrt[4]{x}} - \frac{147}{32x\sqrt[4]{x}} + \frac{1\,715}{128x^2\sqrt[4]{x}} + \ldots$

2.10 $\sqrt{\left(\frac{x}{5}-6\right)^9}$

$$= \left[\left(\frac{x}{5}-6\right)^9\right]^{\frac{1}{2}}$$

$$= \left(\frac{x}{5}-6\right)^{\frac{9}{2}}$$

$$= \left[\left(\frac{x}{5}\right)+(-6)\right]^{\frac{9}{2}}$$

$$= \frac{\left(\frac{x}{5}\right)^{\frac{9}{2}}(-6)^0}{0!} + \frac{\frac{9}{2}\left(\frac{x}{5}\right)^{\frac{7}{2}}(-6)^1}{1!} + \frac{\frac{9}{2}\frac{7}{2}\left(\frac{x}{5}\right)^{\frac{5}{2}}(-6)^2}{2!} + \frac{\frac{9}{2}\frac{7}{2}\frac{5}{2}\left(\frac{x}{5}\right)^{\frac{3}{2}}(-6)^3}{3!} + \ldots$$

$$= \frac{\left(\frac{x}{5}\right)^{\frac{9}{2}}.1}{1} + \frac{\frac{9}{2}\left(\frac{x}{5}\right)^{\frac{7}{2}}(-6)}{1} + \frac{\frac{9}{2}\frac{7}{2}\left(\frac{x}{5}\right)^{\frac{5}{2}}36}{2} + \frac{\frac{9}{2}\frac{7}{2}\frac{5}{2}\left(\frac{x}{5}\right)^{\frac{3}{2}}(-216)}{6} + \ldots$$

$$= \left(\frac{x}{5}\right)^{\frac{9}{2}} - 27\left(\frac{x}{5}\right)^{\frac{7}{2}} + \frac{567}{2}\left(\frac{x}{5}\right)^{\frac{5}{2}} - \frac{2835}{2}\left(\frac{x}{5}\right)^{\frac{3}{2}} + \ldots$$

$$= \left(\frac{x}{5}\right)^4\left(\frac{x}{5}\right)^{\frac{1}{2}} - 27\left(\frac{x}{5}\right)^3\left(\frac{x}{5}\right)^{\frac{1}{2}} + \frac{567}{2}\left(\frac{x}{5}\right)^2\left(\frac{x}{5}\right)^{\frac{1}{2}} - \frac{2835}{2}\left(\frac{x}{5}\right)^1\left(\frac{x}{5}\right)^{\frac{1}{2}} + \ldots$$

$$= \frac{x^4\sqrt{x}}{625\sqrt{5}} - \frac{27x^3\sqrt{x}}{125\sqrt{5}} + \frac{567x^2\sqrt{x}}{50\sqrt{5}} - \frac{567x\sqrt{x}}{2\sqrt{5}} + \ldots$$

$$= \frac{\sqrt{5}x^4\sqrt{x}}{3\,125} - \frac{27\sqrt{5}x^3\sqrt{x}}{625} + \frac{567\sqrt{5}x^2\sqrt{x}}{250} - \frac{567\sqrt{5}x\sqrt{x}}{10} \ldots$$

2.11 $\left(4r+\frac{S}{2}\right)^{-\frac{8}{5}}$

$$= \frac{(4r)^{-\frac{8}{5}}\left(\frac{S}{2}\right)^0}{0!} + \frac{\left(-\frac{8}{5}\right)(4r)^{-\frac{13}{5}}\left(\frac{S}{2}\right)^1}{1!} + \frac{\left(-\frac{8}{5}\right)\left(-\frac{13}{5}\right)(4r)^{-\frac{18}{5}}\left(\frac{S}{2}\right)^2}{2!} + \frac{\left(-\frac{8}{5}\right)\left(-\frac{13}{5}\right)\left(-\frac{18}{5}\right)(4r)^{-\frac{23}{5}}\left(\frac{S}{2}\right)^3}{3!} + \ldots$$

$$= \frac{(4r)^{-\frac{8}{5}}.1}{1} + \frac{\left(-\frac{8}{5}\right)(4r)^{-\frac{13}{5}}\frac{S}{2}}{1} + \frac{\left(-\frac{8}{5}\right)\left(-\frac{13}{5}\right)(4r)^{-\frac{18}{5}}\frac{S^2}{4}}{2} + \frac{\left(-\frac{8}{5}\right)\left(-\frac{13}{5}\right)\left(-\frac{18}{5}\right)(4r)^{-\frac{23}{5}}\frac{S^3}{8}}{6} + \ldots$$

$$= (4r)^{-\frac{8}{5}} - \frac{4S}{5}(4r)^{-\frac{13}{5}} + \frac{13S^2}{25}(4r)^{-\frac{18}{5}} - \frac{39S^3}{125}(4r)^{-\frac{23}{5}} + \ldots$$

$$= \frac{1}{(4r)^{\frac{8}{5}}} - \frac{4S}{5}.\frac{1}{(4r)^{\frac{13}{5}}} + \frac{13S^2}{25}.\frac{1}{(4r)^{\frac{18}{5}}} - \frac{39S^3}{125}.\frac{1}{(4r)^{\frac{23}{5}}} + \ldots$$

$$= \frac{1}{(4r)^1(4r)^{\frac{3}{5}}} - \frac{4S}{5}.\frac{1}{(4r)^2(4r)^{\frac{3}{5}}} + \frac{13S^2}{25}.\frac{1}{(4r)^3(4r)^{\frac{3}{5}}} - \frac{39S^3}{125}.\frac{1}{(4r)^4(4r)^{\frac{3}{5}}} + \ldots$$

$$= \frac{1}{4r\sqrt[5]{(4r)^3}} - \frac{S}{20r^2\sqrt[5]{(4r)^3}} + \frac{13S^2}{1\,600r^3\sqrt[5]{(4r)^3}} - \frac{39S^3}{32\,000r^4\sqrt[5]{(4r)^3}} + \ldots$$

2.12 $\dfrac{1}{\sqrt[3]{3x - \frac{1}{x^2}}}$

$= \dfrac{1}{\left(3x - \frac{1}{x^2}\right)^{\frac{1}{3}}}$

$= \left(3x - \dfrac{1}{x^2}\right)^{-\frac{1}{3}}$

$= \left[(3x) + \left(-\dfrac{1}{x^2}\right)\right]^{-\frac{1}{3}}$

$= \dfrac{(3x)^{-\frac{1}{3}}\left(-\frac{1}{x^2}\right)^0}{0!} + \dfrac{\left(-\frac{1}{3}\right)(3x)^{-\frac{4}{3}}\left(-\frac{1}{x^2}\right)^1}{1!} + \dfrac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)(3x)^{-\frac{7}{3}}\left(-\frac{1}{x^2}\right)^2}{2!} + \dfrac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)(3x)^{-\frac{10}{3}}\left(-\frac{1}{x^2}\right)^3}{3!} + \ldots$

$= \dfrac{(3x)^{-\frac{1}{3}}.1}{1} + \dfrac{\left(-\frac{1}{3}\right)(3x)^{-\frac{4}{3}}\left(-\frac{1}{x^2}\right)}{1} + \dfrac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)(3x)^{-\frac{7}{3}}\frac{1}{x^4}}{2} + \dfrac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)(3x)^{-\frac{10}{3}}\left(-\frac{1}{x^6}\right)}{6} + \ldots$

$= (3x)^{-\frac{1}{3}} + \dfrac{1}{3x^2}.(3x)^{-\frac{4}{3}} + \dfrac{2}{9x^4}.(3x)^{-\frac{7}{3}} + \dfrac{14}{81x^6}.(3x)^{-\frac{10}{3}} + \ldots$

$= \dfrac{1}{(3x)^{\frac{1}{3}}} + \dfrac{1}{3x^2}.\dfrac{1}{(3x)^{\frac{4}{3}}} + \dfrac{2}{9x^4}.\dfrac{1}{(3x)^{\frac{7}{3}}} + \dfrac{14}{81x^6}.\dfrac{1}{(3x)^{\frac{10}{3}}} + \ldots$

$= \dfrac{1}{(3x)^{\frac{1}{3}}} + \dfrac{1}{3x^2}.\dfrac{1}{3x.(3x)^{\frac{1}{3}}} + \dfrac{2}{9x^4}.\dfrac{1}{(3x)^2(3x)^{\frac{1}{3}}} + \dfrac{14}{81x^6}.\dfrac{1}{(3x)^3(3x)^{\frac{1}{3}}} + \ldots$

$= \dfrac{1}{\sqrt[3]{3x}} + \dfrac{1}{9x^3\sqrt[3]{3x}} + \dfrac{2}{81x^6\sqrt[3]{3x}} + \dfrac{14}{2\,187x^9\sqrt[3]{3x}}$

Activity 5.3 — SB page 300

1.
$$f(x) = 5x - 3$$
$$f(x + h) = 5(x + h) - 3$$
$$f(x + h) = 5x + 5h - 3$$
$$f(x + h) - f(x) = (5x + 5h - 3) - (5x - 3)$$
$$= 5x + 5h - 3 - 5x + 3$$
$$f(x + h) - f(x) = 5h$$
$$\frac{f(x + h) - f(x)}{h} = \frac{5h}{h}$$
$$\frac{f(x + h) - f(x)}{h} = 5$$
$$f'(x) = \lim_{h\to 0}\frac{f(x + h) - f(x)}{h}$$
$$= \lim_{h\to 0}(5)$$
$$f'(x) = 5$$

2.
$$y = 2x^2$$
$$\therefore f(x) = 2x^2$$
$$f(x+h) = 2(x+h)^2$$
$$= 2(x+h)(x+h)$$
$$= 2(x^2 + 2xh + h^2)$$
$$f(x+h) = 2x^2 + 4xh + 2h^2$$
$$f(x+h) - f(x) = (2x^2 + 4xh + 2h^2) - (2x^2)$$
$$= 2x^2 + 4xh + 2h^2 - 2x^2$$
$$f(x+h) - f(x) = 4xh + 2h^2$$
$$\frac{f(x+h) - f(x)}{h} = \frac{4xh + 2h^2}{h}$$
$$= \frac{h(4x + 2h)}{h}$$
$$\frac{f(x+h) - f(x)}{h} = 4x + 2h$$
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} (4x + 2h)$$
$$= 4x + 2(0)$$
$$f'(x) = 4x$$
$$\therefore \frac{dy}{dx} = 4x$$

3.
$$y = 4$$
$$f(x) = 4x^0$$
$$f(x+h) = 4(x+h)^0$$
$$= 4$$
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{4 - 4}{h}$$
$$= \lim_{h \to 0} \frac{0}{h}$$
$$= \lim_{h \to 0} 0$$
$$\therefore f'(x) = 0$$

4.
$$f(x) = 9x^2 - 4$$
$$f(x+h) = 9(x+h)^2 - 4$$
$$= 9(x+h)(x+h) - 4$$
$$= 9(x^2 + 2xh + h^2) - 4$$
$$f(x+h) = 9x^2 + 18xh + 9h^2 - 4$$
$$f(x+h) - f(x) = (9x^2 + 18xh + 9h^2 - 4) - (9x^2 - 4)$$
$$= 9x^2 + 18xh + 9h^2 - 4 - 9x^2 + 4$$
$$f(x+h) - f(x) = 18xh + 9h^2$$

$$\frac{f(x+h)-f(x)}{h} = \frac{18xh+9h^2}{h}$$

$$= \frac{h(18x+9h)}{h}$$

$$\frac{f(x+h)-f(x)}{h} = 18x+9h$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}(18x+9h)$$

$$= 18x+9(0)$$

$$f'(x) = 18x$$

5.

$$y = -3x^2+7$$

$$\therefore f(x) = -3x^2+7$$

$$f(x+h) = -3(x+h)^2+7$$

$$= -3(x+h)(x+h)+7$$

$$= -3(x^2+2xh+h^2)+7$$

$$f(x+h) = -3x^2-6xh-3h^2+7$$

$$f(x+h)-f(x) = (-3x^2-6xh-3h^2+7)-(-3x^2+7)$$

$$= -3x^2-6xh-3h^2+7+3x^2-7$$

$$f(x+h)-f(x) = -6xh-3h^2$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-6xh-3h^2}{h}$$

$$= \frac{h(-6x-3h)}{h}$$

$$\frac{f(x+h)-f(x)}{h} = -6x-3h$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}(-6x-3h)$$

$$= -6x-3(0)$$

$$f'(x) = -6x$$

$$\therefore \frac{dy}{dx} = -6x$$

6.

$$f(x) = x^2-6x$$

$$f(x+h) = (x+h)^2-6(x+h)$$

$$= (x+h)(x+h)-6(x+h)$$

$$f(x+h) = x^2+2xh+h^2-6x-6h$$

$$f(x+h)-f(x) = (x^2+2xh+h^2-6x-6h)-(x^2-6x)$$

$$= x^2+2xh+h^2-6x-6h-x^2+6x$$

$$f(x+h)-f(x) = 2xh-6h+h^2$$

$$\frac{f(x+h)-f(x)}{h} = \frac{2xh-6h+h^2}{h}$$

$$= \frac{h(2x-6+h)}{h}$$

$$\frac{f(x+h)-f(x)}{h} = 2x - 6 + h$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h \to 0} (2x - 6 + h)$$

$$= 2x - 6 + (0)$$

$$f'(x) = 2x - 6$$

7.

$$y = 2x^2 + 3x - 8$$

$$\therefore f(x) = 2x^2 + 3x - 8$$

$$f(x+h) = 2(x+h)^2 + 3(x+h) - 8$$

$$= 2(x+h)(x+h) + 3(x+h) - 8$$

$$= 2(x^2 + 2xh + h^2) + 3(x+h) - 8$$

$$f(x+h) = 2x^2 + 4xh + 2h^2 + 3x + 3h - 8$$

$$f(x+h) - f(x) = (2x^2 + 4xh + 2h^2 + 3x + 3h - 8) - (2x^2 + 3x - 8)$$

$$= 2x^2 + 4xh + 2h^2 + 3x + 3h - 8 - 2x^2 - 3x + 8$$

$$f(x+h) - f(x) = 4xh + 3h + 2h^2$$

$$\frac{f(x+h)-f(x)}{h} = \frac{4xh + 3h + 2h^2}{h}$$

$$= \frac{h(4x + 3 + 2h)}{h}$$

$$\frac{f(x+h)-f(x)}{h} = 4x + 3 + 2h$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h \to 0} (4x + 3 + 2h)$$

$$= 4x + 3 + 2(0)$$

$$f'(x) = 4x + 3$$

$$\therefore \frac{dy}{dx} = 4x + 3$$

8.

$$f(x) = x^3$$

$$f(x+h) = (x+h)^3$$

$$= \frac{(x)^3(h)^0}{0!} + \frac{3(x)^2(h)^1}{1!} + \frac{3.2(x)^1(h)^2}{2!} + \frac{3.2.1(x)^0.(h)^3}{3!}$$

$$= \frac{x^3.1}{1} + \frac{3.x^2.h}{1} + \frac{3.2.x^1.h^2}{2} + \frac{3.2.1.1.h^3}{6}$$

$$f(x+h) = x^3 + 3x^2h + 3xh^2 + h^3$$

$$f(x+h) - f(x) = (x^3 + 3x^2h + 3xh^2 + h^3) - (x^3)$$

$$= x^3 + 3x^2h + 3xh^2 + h^3 - x^3$$

$$f(x+h) - f(x) = 3x^2h + 3xh^2 + h^3$$

$$\frac{f(x+h)-f(x)}{h} = \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \frac{h(3x^2 + 3xh + h^2)}{h}$$

$$\frac{f(x+h)-f(x)}{h} = 3x^2 + 3xh + h^2$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} (3x^2 + 3xh + h^2)$$

$$= 3x^2 + 3x(0) + (0)^2$$

$$f'(x) = 3x^2$$

9. $$y = 5x^3 - 4$$

$$\therefore f(x) = 5x^3 - 4$$

$$f(x+h) = 5(x+h)^3 - 4$$

$$= 5\left[\frac{(x)^3(h)^0}{0!} + \frac{3(x)^2(h)^1}{1!} + \frac{3.2(x)^1(h)^2}{2!} + \frac{3.2.1(x)^0(h)^3}{3!}\right] - 4$$

$$= 5\left[\frac{x^3.1}{1} + \frac{3.x^2.h}{1} + \frac{3.2.x.h^2}{2} + \frac{3.2.1.1.h^3}{6}\right] - 4$$

$$= 5[x^3 + 3x^2h + 3xh^2 + h^3] - 4$$

$$f(x+h) = 5x^3 + 15x^2h + 15xh^2 + 5h^3 - 4$$

$$f(x+h) - f(x) = (5x^3 + 15x^2h + 15xh^2 + 5h^3 - 4) - (5x^3 - 4)$$

$$= 5x^3 + 15x^2h + 15xh^2 + 5h^3 - 4 - 5x^3 + 4$$

$$f(x+h) - f(x) = 15x^2h + 15xh^2 + 5h^3$$

$$\frac{f(x+h)-f(x)}{h} = \frac{15x^2h + 15xh^2 + 5h^3}{h}$$

$$= \frac{h(15x^2 + 15xh + 5h^2)}{h}$$

$$\frac{f(x+h)-f(x)}{h} = 15x^2 + 15xh + 5h^2$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} (15x^2 + 15xh + 5h^2)$$

$$= 15x^2 + 15x(0) + 5(0)^2$$

$$f'(x) = 15x^2$$

$$\therefore \frac{dy}{dx} = 15x^2$$

10. $$f(x) = -2x^3 + 4x$$

$$f(x+h) = -2(x+h)^3 + 4(x+h)$$

$$= -2\left[\frac{(x)^3(h)^0}{0!} + \frac{3(x)^2(h)^1}{1!} + \frac{3.2(x)^1(h)^2}{2!} + \frac{3.2.1(x)^0(h)^3}{3!}\right] + 4(x+h)$$

$$= -2\left[\frac{x^3.1}{1} + \frac{3.x^2.h}{1} + \frac{3.2.x.h^2}{2} + \frac{3.2.1.1.h^3}{6}\right] + 4(x+h)$$

$$= -2[x^3 + 3x^2h + 3xh^2 + h^3] + 4(x+h)$$

$$f(x+h) = -2x^3 - 6x^2h - 6xh^2 - 2h^3 + 4x + 4h$$

$$f(x+h) - f(x) = (-2x^3 - 6x^2h - 6xh^2 - 2h^3 + 4x + 4h) - (-2x^3 + 4x)$$

$$= -2x^3 - 6x^2h - 6xh^2 - 2h^3 + 4x + 4h + 2x^3 - 4x$$

$$f(x+h) - f(x) = -6x^2h + 4h - 6xh^2 - 2h^3$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-6x^2h + 4h - 6xh^2 - 2h^3}{h}$$

$$= \frac{h(-6x^2 + 4 - 6xh - 2h^2)}{h}$$

$$\frac{f(x+h)-f(x)}{h} = -6x^2 + 4 - 6xh - 2h^2$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} (-6x^2 + 4 - 6xh - 2h^2)$$

$$= -6x^2 + 4 - 6x(0) - 2(0)^2$$

$$f'(x) = -6x^2 + 4$$

11. $y = -6x^4$

$$\therefore f(x) = -6x^4$$

$$f(x+h) = -6(x+h)^4$$

$$= -6\left[\frac{(x)^4(h)^0}{0!} + \frac{4(x)^3(h)^1}{1!} + \frac{4.3(x)^2(h)^2}{2!} + \frac{4.3.2(x)^1(h)^3}{3!} + \frac{4.3.2.1(x)^0(h)^4}{4!}\right]$$

$$= -6\left[\frac{x^4.1}{1} + \frac{4.x^3.h}{1} + \frac{4.3.x^2.h^2}{2} + \frac{4.3.2.x.h^3}{6} + \frac{4.3.2.1.1.h^4}{24}\right]$$

$$= -6[x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4]$$

$$f(x+h) = -6x^4 - 24x^3h - 36x^2h^2 - 24xh^3 - 6h^4$$

$$f(x+h) - f(x) = (-6x^4 - 24x^3h - 36x^2h^2 - 24xh^3 - 6h^4) - (-6x^4)$$

$$= -6x^4 - 24x^3h - 36x^2h^2 - 24xh^3 - 6h^4 + 6x^4$$

$$f(x+h) - f(x) = -24x^3h - 36x^2h^2 - 24xh^3 - 6h^4$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-24x^3h - 36x^2h^2 - 24xh^3 - 6h^4}{h}$$

$$= \frac{h(-24x^3 - 36x^2h - 24xh^2 - 6h^3)}{h}$$

$$\frac{f(x+h)-f(x)}{h} = -24x^3 - 36x^2h - 24xh^2 - 6h^3$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} (-24x^3 - 36x^2h - 24xh^2 - 6h^3)$$

$$= -24x^3 - 36x^2(0) - 24x(0) - 6(0)^3$$

$$f'(x) = -24x^3$$

$$\therefore \frac{dy}{dx} = -24x^3$$

12.
$$y = \frac{9}{4\sqrt[4]{x}}$$
$$f(x) = \frac{9}{4}x^{-\frac{1}{4}}$$
$$f(x+h) = \frac{9}{4}(x+h)^{-\frac{1}{4}}$$
$$= \frac{9}{4}\left[x^{-\frac{1}{4}} + \left(-\frac{1}{4}\right)x^{-\frac{5}{4}}h + \frac{\left(-\frac{1}{4}\right)\left(-\frac{5}{4}\right)}{2!}x^{-\frac{9}{4}}h^2 + \frac{\left(-\frac{1}{4}\right)\left(-\frac{5}{4}\right)\left(-\frac{9}{4}\right)}{3!}x^{-\frac{13}{4}}h^3 + \ldots\right]$$
$$= \frac{9}{4}\left[x^{-\frac{1}{4}} - \frac{1}{4}x^{-\frac{5}{4}}h + \frac{5}{32}x^{-\frac{9}{4}}h^2 - \frac{15}{128}x^{-\frac{13}{4}}h^3 + \ldots\right]$$
$$= \frac{9}{4}x^{-\frac{1}{4}} - \frac{9}{16}x^{-\frac{5}{4}}h + \frac{45}{128}x^{-\frac{9}{4}}h^2 - \frac{135}{512}x^{-\frac{13}{4}}h^3 + \ldots$$
$$\lim_{h\to 0}\frac{\frac{9}{4}x^{-\frac{1}{4}} - \frac{9}{16}x^{-\frac{5}{4}}h + \frac{45}{128}x^{-\frac{9}{4}}h^2 - \frac{135}{512}x^{-\frac{13}{4}}h^3 + \ldots \frac{9}{4}x^{-\frac{1}{4}}}{h}$$
$$= -\frac{9}{16}x^{-\frac{5}{4}}$$
$$= -\frac{9}{16\sqrt[4]{x^5}} \text{ or } -\frac{0{,}563}{\sqrt[4]{x^5}}$$

Activity 5.4 SB page 306

1. 1.1
$$y = 3$$
$$\therefore \frac{dy}{dx} = \frac{d}{dx}(3)$$
$$= 0$$

1.2
$$y = -x^8$$
$$\therefore \frac{dy}{dx} = \frac{d}{dx}(-x^8)$$
$$= -\frac{d}{dx}(x^8)$$
$$= -8(x^{8-1})$$
$$= -8x^7$$

1.3
$$f(x) = 4x^2$$
$$f'(x) = 8x$$

1.4
$$f(x) = \frac{x}{5}$$
$$= \frac{1}{5}x$$
$$f'(x) = \frac{1}{5}$$

1.5
$$y = \frac{5}{x^4}$$
$$= 5x^{-4}$$
$$f'(x) = -20x^{-5}$$
$$= \frac{-20}{x^5}$$

1.6
$$f(x) = a^2x$$
$$f'(x) = a^2$$

1.7
$$f(x) = \frac{1}{2}x^{-8}$$
$$f'(x) = (-8)\frac{1}{2}x^{-9}$$
$$= -4x^{-9}$$
$$= -\frac{4}{x^9}$$

1.8
$$f(x) = -3\sqrt{x}$$
$$= -3x^{\frac{1}{2}}$$
$$f'(x) = -\frac{3}{2}x^{-\frac{1}{2}}$$
$$= -\frac{3}{2\sqrt{x}}$$

1.9 $f(x) = \frac{2}{\sqrt[4]{x}}$

$= \frac{2}{x^{\frac{1}{4}}}$

$= 2x^{-\frac{1}{4}}$

$f'(x) = -\frac{1}{4}(2)x^{-\frac{5}{4}}$

$= -\frac{1}{2}x^{-\frac{5}{4}}$

$= -\frac{1}{2\sqrt[4]{x^5}}$

1.10 $y = mn^2a^3$

$\frac{dy}{dx} = \frac{d}{dx}(mn^2a^3)$

$= 0$

1.11 $y = 2\pi^2$

$\frac{dy}{dx} = \frac{d}{dx}(2\pi^2)$

$= 0$

2\. 2.1 $y = \frac{6}{x^2}$

$= 6x^{-2}$

$\frac{dy}{dx} = -12x^{-3}$

$= -\frac{12}{x^3}$

2.2 $y = -\frac{1}{4x^3}$

$= -\frac{1}{4}x^{-3}$

$\frac{dy}{dx} = \frac{3}{4}x^{-4}$

$= \frac{3}{4x^4}$

2.3 $y = -2\sqrt[3]{x}$

$= -2x^{\frac{1}{3}}$

$\frac{dy}{dx} = -\frac{2}{3}x^{-\frac{2}{3}}$

$= -\frac{2}{3\sqrt[3]{x^2}}$

2.4 $y = \sqrt{5x}$

$= \sqrt{5}\,x^{\frac{1}{2}}$

$\frac{dy}{dx} = \sqrt{5}\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$

$= \frac{\sqrt{5}}{2\sqrt{x}}$

2.5 $y = \frac{x^2}{\sqrt{x}}$

$= x^{2-\frac{1}{2}}$

$= x^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

$= \frac{3}{2}\sqrt{x}$

$= \frac{3\sqrt{x}}{2}$

2.6 $y = a^2b^2x^2$

$\frac{dy}{dx} = 2a^2b^2x$

2.7 $y = \frac{\pi x^3}{6}$

$= \frac{\pi}{6}x^3$

$\frac{dy}{dx} = 3 \times \frac{\pi}{6}x^2$

$= \frac{\pi}{2}x^2$ or $\frac{\pi x^2}{2}$

2.8 $y = \frac{1}{ax^{-3}}$

$= \frac{1}{a}x^3$

$\frac{dy}{dx} = 3\left(\frac{1}{a}\right)x^2$

$= \frac{3}{a}x^2$

$= \frac{3x^2}{a}$ or $\frac{3}{a}x^2$

2.9 $y = -\frac{1}{3\sqrt{x^3}}$

$= -\frac{1}{3}x^{-\frac{3}{2}}$

$\frac{dy}{dx} = -\frac{3}{2}\left(-\frac{1}{3}\right)x^{-\frac{5}{2}}$

$= \frac{1}{2\sqrt{x^5}}$

3\. 3.1 $y = -x^3 - 3x^2 + 2x - 8$

$\frac{dy}{dx} = \frac{d}{dx}(-x^3 - 3x^2 + 2x - 8)$

$= -\frac{d}{dx}x^3 - 3\frac{d}{dx}x^2 + 2\frac{d}{dx}x - \frac{d}{dx}8$

$= -(3x^2) - 3(2x) + 2(1) - (0)$

$= -3x^2 - 6x + 2$

3.2 $y = x^2\left(x - \frac{1}{x}\right)^2 + 5$

$= x^2\left(x - \frac{1}{x}\right)\left(x - \frac{1}{x}\right) + 5$

$= x^2\left(x^2 - 1 - 1 + \frac{1}{x^2}\right) + 5$

$= x^2\left(x^2 - 2 + \frac{1}{x^2}\right) + 5$

$= x^4 - 2x^2 + 1 + 5$

$y = x^4 - 2x^2 + 6$

$\frac{dy}{dx} = \frac{d}{dx}(x^4) - \frac{d}{dx}(2x^2) + \frac{d}{dx}(6)$

$= \frac{d}{dx}x^4 - 2\frac{d}{dx}x^2 + \frac{d}{dx}6$

$= 4x^3 - 2(2x) + 0$

$\frac{dy}{dx} = 4x^3 - 4x$

3.3 $y = \frac{(x^2 + 1)(x - 2)}{x^2}$

$= \frac{x^3 - 2x^2 + x - 2}{x^2}$

$= \frac{x^3}{x^2} - \frac{2x^2}{x^2} + \frac{x}{x^2} - \frac{2}{x^2}$

$= x - 2 + \frac{1}{x} - \frac{2}{x^2}$

$y = x - 2 + x^{-1} - 2x^{-2}$

$\frac{dy}{dx} = \frac{d}{dx}(x) - \frac{d}{dx}(2) + \frac{d}{dx}(x^{-1}) - \frac{d}{dx}(2x^{-2})$

$= \frac{d}{dx}x - \frac{d}{dx}2 + \frac{d}{dx}x^{-1} - 2\frac{d}{dx}x^{-2}$

$= 1 - 0 - x^{-2} - 2(-2.x^{-3})$

$= 1 - x^{-2} + 4.x^{-3}$

$\frac{dy}{dx} = 1 - \frac{1}{x^2} + \frac{4}{x^3}$

3.4 $y = \frac{\frac{1}{3}x^3 - 9}{2x - 6}$

$= \frac{\frac{1}{3}(x^3 - 27)}{2(x - 3)}$

$= \frac{\frac{1}{3}(x - 3)(x^2 + 3x + 9)}{2(x - 3)}$

$= \frac{1}{6}(x^2 + 3x + 9)$

$y = \frac{1}{6}x^2 + \frac{1}{2}x + \frac{3}{2}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{6}x^2\right) + \frac{d}{dx}\left(\frac{1}{2}x\right) + \frac{d}{dx}\left(\frac{3}{2}\right)$

$= \frac{1}{6}\frac{d}{dx}x^2 + \frac{1}{2}\frac{d}{dx}x + \frac{d}{dx}\frac{3}{2}$

$= \frac{1}{6}(2x) + \frac{1}{2}(1) + 0$

$\frac{dy}{dx} = \frac{1}{3}x + \frac{1}{2}$

Activity 5.5 SB page 314

1. $y = \frac{1}{2}e^{4x}$

$\frac{dy}{dx} = 2e^{4x}$

2. $y = 3e^{-x}$

$\frac{dy}{dx} = -3e^{-x}$

3. $y = 3 \ln x$

$\frac{dy}{dx} = \frac{3}{x}$

4. $y = \log_4 x$

$\frac{dy}{dx} = \frac{1}{x \ln 4}$

5. $y = \log_3 x^2$

$\frac{dy}{dx} = \frac{2}{x \ln 3}$

6. $y = -5 \log_2 \sqrt{x}$

$= -5 \log_2 x^{\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{5}{2}.\frac{1}{x \ln 2}$

$= \frac{-5}{2x \ln 2}$

7. $y = 2(4^x)$

$\frac{dy}{dx} = 2.4^x \ln 4$

8. $y = -4(2^{-3x})$

$\frac{dy}{dx} = -3.-4.2^{-3x} \ln 2$

$= 12.2^{-3x} \ln 2$

9. $f(x) = \sqrt[3]{125^x}$

$= \left[(5^3)^x\right]^{\frac{1}{3}}$

$= \left[5^{3x}\right]^{\frac{1}{3}}$

$f(x) = 5^x$

$f'(x) = \frac{d}{dx}5^x$

$f'(x) = 5^x \ln 5$

10. $f(x) = e^{\ln 4 + x}$

$= e^{\ln 4}.e^x$

$f(x) = 4.e^x$

$f'(x) = \frac{d}{dx}(4.e^x)$

$= 4\frac{d}{dx}e^x$

$f'(x) = 4e^x$

11. $f(x) = -5 \ln x^2$

$= -5(2) \ln x$

$f(x) = -10 \ln x$

$f'(x) = \frac{d}{dx}(-10 \ln x)$

$= -10\frac{d}{dx} \ln x$

$= -10.\frac{1}{x}$

$f'(x) = -\frac{10}{x}$

12. $f(x) = 4 \log\sqrt{x^7}$

$= 4 \log(x^7)^{\frac{1}{2}}$

$= 4 \log x^{\frac{7}{2}}$

$= 4\left(\frac{7}{2}\right) \log x$

$f(x) = 14 \log x$

$f'(x) = \frac{d}{dx}(14 \log x)$

$= 14\frac{d}{dx} \log x$

$= 14.\frac{1}{x \ln 10}$

$f'(x) = \frac{14}{x \ln 10}$

13. $y = \frac{1}{8}\ln\left(\frac{1}{\sqrt{x^4}}\right)$

$= \frac{1}{8}\ln\left(\frac{1}{(x^4)^{\frac{1}{2}}}\right)$

$= \frac{1}{8}\ln\left(\frac{1}{x^2}\right)$

$= \frac{1}{8}\ln x^{-2}$

$= \frac{1}{8}(-2)\ln x$

$y = -\frac{1}{4}\ln x$

$\frac{dy}{dx} = \frac{d}{dx}\left(-\frac{1}{4}\ln x\right)$

$= -\frac{1}{4}\frac{d}{dx}\ln x$

$= -\frac{1}{4}.\frac{1}{x}$

$\frac{dy}{dx} = -\frac{1}{4x}$

14. $y = 6\log_{\frac{1}{3}}x$

$\frac{dy}{dx} = \frac{d}{dx}\left(6\log_{\frac{1}{3}}x\right)$

$= 6\frac{d}{dx}\log_{\frac{1}{3}}x$

$= 6.\frac{1}{x\ln\frac{1}{3}}$

$= \frac{6}{x\ln\frac{1}{3}}$

15. $f(x) = -3\log xy$

$= -3\log x - 3\log y$

$\therefore f'(x) = -\frac{3}{x}$

16. $f(x) = -2\ln 3x^2$

$= -2(\ln 3 + \ln x^2)$

$= -2\ln 3 - 2\ln x^2$

$= -2\ln 3 - 4\ln x$

$\therefore f'(x) = -\frac{4}{x}$

17. $f(x) = 2\log_5 4x^3$

$= 2\log_5 4 + 2\log_5 x^3$

$= 2\log_5 4 + 6\log_5 x$

$\therefore f'(x) = \frac{6}{x\ln 5}$

18. $y = -3\log_4 \frac{1}{2}x^5$

$= -3\left(\log_4 \frac{1}{2} + \log_4 x^5\right)$

$= -3\log_4 \frac{1}{2} - 3\log_4 x^5$

$= -3\log_4 \frac{1}{2} - 15\log_4 x$

$\therefore \frac{dy}{dx} = \frac{15}{4\ln x}$

Activity 5.6 SB page 319

1. 1.1 $y = \sin 5x$

$\frac{dy}{dx} = \frac{d}{dx}(\sin 5x)$

$= \cos 5x.\frac{d}{dx}5x$

$= \cos 5x.5$

$= 5\cos 5x$

1.2 $y = -3\cos x$

$\frac{dy}{dx} = \frac{d}{dx}(-3\cos x)$

$= -3\frac{d}{dx}(\cos x)$

$= -3(-\sin x)$

$= 3\sin x$

1.3 $y = 4 \tan 4x$

$$\frac{dy}{dx} = \frac{d}{dx}(4 \tan 4x)$$
$$= 4\frac{d}{dx}(\tan 4x)$$
$$= 4 \sec^2 4x.\frac{d}{dx}4x$$
$$= 4 \sec^2 4x.4$$
$$= 16 \sec^2 4x$$

1.4 $y = -a^2 \operatorname{cosec} x$

$$\frac{dy}{dx} = \frac{d}{dx}(-a^2 \operatorname{cosec} x)$$
$$= -a^2 \frac{d}{dx}(\operatorname{cosec} x)$$
$$= -a^2(-\cot x \operatorname{cosec} x)$$
$$= a^2 \cot x \operatorname{cosec} x$$

1.5 $y = \frac{1}{2}\sec \pi x$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}\sec \pi x\right)$$
$$= \frac{1}{2}\frac{d}{dx}(\sec \pi x)$$
$$= \frac{1}{2}\sec \pi x \tan \pi x.\frac{d}{dx}(\pi x)$$
$$= \frac{\pi}{2}\sec \pi x \tan \pi x$$

1.6 $y = \frac{1}{3}\cot 3x$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}\cot 3x\right)$$
$$= \frac{1}{3}\frac{d}{dx}(\cot 3x)$$
$$= \frac{1}{3}.-\operatorname{cosec}^2 3x.\frac{d}{dx}(3x)$$
$$= \frac{1}{3}.-\operatorname{cosec}^2 3x.3$$
$$= -\operatorname{cosec}^2 3x$$

1.7 $y = \sin 2\theta$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(\sin 2\theta)$$
$$= \cos 2\theta.\frac{d}{d\theta}(2\theta)$$
$$= 2 \cos 2\theta$$

1.8 $y = \frac{2}{\sin x}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2}{\sin x}\right)$$
$$= \frac{d}{dx}(2 \operatorname{cosec} x)$$
$$= 2\frac{d}{dx}(\operatorname{cosec} x)$$
$$= 2(-\cot x \operatorname{cosec} x)$$
$$= -2 \cot x \operatorname{cosec} x$$

1.9 $y = 4 \cos \pi x$

$$\frac{dy}{dx} = \frac{d}{dx}(4 \cos \pi x)$$
$$= 4.\frac{d}{dx}(4 \cos \pi x)$$
$$= 4(-\sin \pi x).\frac{d}{dx}(\pi x)$$
$$= 4(-\sin \pi x)\pi$$
$$= -4\pi \sin \pi x$$

1.10 $f(x) = \frac{2 \cos x}{\sin 2x}$

$$= \frac{2 \cos x}{2 \sin x \cos x}$$
$$= \frac{1}{\sin x}$$
$$f(x) = \operatorname{cosec} x$$
$$f'(x) = \frac{d}{dx} \operatorname{cosec} x$$
$$f'(x) = -\operatorname{cosec} x \cot x$$

1.11 $f(x) = \frac{1}{\cos^4 x - \sin^4 x}$

$= \frac{1}{(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)}$

$= \frac{1}{1.\cos x}$

$= \frac{1}{\cos x}$

$= \sec x$

$f'(x) = \frac{d}{dx}\sec x$

$= \sec x \tan x$

1.12 $y = 10 \sin \frac{x}{2}.\cos \frac{x}{2}$

$= 5\left(2 \sin \frac{x}{2}.\cos \frac{x}{2}\right)$

$y = 5 \sin x$

$\frac{dy}{dx} = \frac{d}{dx}(5 \sin x)$

$= 5\frac{d}{dx} \sin x$

$\frac{dy}{dx} = 5 \cos x$

1.13 $f(x) = \frac{\cos x}{\sqrt{1 - \cos^2 x}}$

$= \frac{\cos x}{\sin x}$

$f(x) = \cot x$

$f'(x) = \frac{d}{dx} \cot x$

$f'(x) = -\text{cosec}^2 x$

1.14 $y = \sqrt{4 + 4 \tan^2 x}$

$= \sqrt{4(1 + \tan^2 x)}$

$= \sqrt{4}.\sqrt{1 + \tan^2 x}$

$= 2\sqrt{\sec^2 x}$

$y = 2 \sec x$

$\frac{dy}{dx} = \frac{d}{dx}(2 \sec x)$

$= 2\frac{d}{dx} \sec x$

$\frac{dy}{dx} = 2 \sec x \tan x$

1.15 $f(x) = \frac{1}{\sqrt{3 \text{cosec}^2 x - 3}}$

$= \frac{1}{\sqrt{3(\text{cosec}^2 x - 1)}}$

$= \frac{1}{\sqrt{3}.\sqrt{\text{cosec}^2 x - 1}}$

$= \frac{1}{\sqrt{3}.\sqrt{\cot^2 x}}$

$= \frac{1}{\sqrt{3} \cot x}$

$= \frac{1}{\sqrt{3}}.\frac{1}{\cot x}$

$f'(x) = \frac{1}{\sqrt{3}} \tan x$

$f'(x) = \frac{d}{dx}\left(\frac{1}{\sqrt{3}} \tan x\right)$

$= \frac{1}{\sqrt{3}}\frac{d}{dx} \tan x$

$f'(x) = \frac{1}{\sqrt{3}} \sec^2 x$

2. 2.1 $y = 3\cot x + \frac{10}{e^x} - 4\sqrt[3]{x} - 8$

$= 3\cot x + 10.e^{-x} - 4.x^{\frac{1}{3}} - 8$

$\frac{dy}{dx} = \frac{d}{dx}(3\cot x) + \frac{d}{dx}(10.e^{-x}) - \frac{d}{dx}\left(4.x^{\frac{1}{3}}\right) - \frac{d}{dx}(8)$

$= 3\frac{d}{dx}\cot x + 10\frac{d}{dx}e^{-x} - 4\frac{d}{dx}x^{\frac{1}{3}} - \frac{d}{dx}8$

$= 3(-\text{cosec}^2 x) + 10(e^{-x}.-1) - 4\left(\frac{1}{3}.x^{-\frac{2}{3}}\right) - 0$

$= -3\,\text{cosec}^2 x - \frac{10}{e^x} - \frac{4}{3\sqrt[3]{x^2}}$

2.2 $y = 7\sin 2x - 3^{-x} + \frac{1}{2}\log\sqrt{x} - \frac{5}{2}x^4$

$= 7\sin 2x - 3^{-x} + \frac{1}{2}\log x^{\frac{1}{2}} - \frac{5}{2}x^4$

$= 7\sin 2x - 3^{-x} + \frac{1}{2}\left(\frac{1}{2}\right)\log x - \frac{5}{2}x^4$

$y = 7\sin 2x - 3^{-x} + \frac{1}{4}\log x - \frac{5}{2}x^4$

$\frac{dy}{dx} = \frac{d}{dx}(7\sin 2x) - \frac{d}{dx}(3^{-x}) + \frac{d}{dx}\left(\frac{1}{4}\log x\right) - \frac{d}{dx}\left(\frac{5}{2}x^4\right)$

$= 7\frac{d}{dx}\sin 2x - \frac{d}{dx}3^{-x} + \frac{1}{4}\frac{d}{dx}\log x - \frac{5}{2}\frac{d}{dx}x^4$

$= 7(\cos 2x.2) - (3^{-x}\ln 3.-1) + \frac{1}{4}\left(\frac{1}{x\ln 10}\right) - \frac{5}{2}(4x^3)$

$\frac{dy}{dx} = 14\cos 2x + \frac{\ln 3}{3^x} + \frac{1}{4x\ln 10} - 10x^3$

2.3 $y = \frac{1}{4}\sec x - \sqrt{2^{8x}} - \sqrt{3}\ln 2x + 10^{\log 2}$

$= \frac{1}{4}\sec x - (2^{8x})^{\frac{1}{2}} - \sqrt{3}\ln 2x + 2$

$y = \frac{1}{4}\sec x - 2^{4x} - \sqrt{3}\ln 2x + 2$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{4}\sec x\right) - \frac{d}{dx}(2^{4x}) - \frac{d}{dx}(\sqrt{3}\ln 2x) + \frac{d}{dx}(2)$

$= \frac{1}{4}\frac{d}{dx}\sec x - \frac{d}{dx}2^{4x} - \sqrt{3}\frac{d}{dx}\ln 2x + \frac{d}{dx}2$

$= \frac{1}{4}(\sec x\tan x) - (2^{4x}\ln 2.4) - \sqrt{3}\left(\frac{1}{2x}.2\right) + 0$

$\frac{dy}{dx} = \frac{1}{4}\sec x\tan x - 4.2^{4x}\ln 2 - \frac{\sqrt{3}}{x}$

2.4 $y = -\frac{1}{2}\cos 4x + 11\ln x + \frac{5}{x^2} - e^{x-\ln 3}$

$= -\frac{1}{2}\cos 4x + 11\ln x + 5.x^{-2} - e^x.e^{-\ln 3}$

$= -\frac{1}{2}\cos 4x + 11\ln x + 5.x^{-2} - e^x.e^{\ln 3^{-1}}$

$= -\frac{1}{2}\cos 4x + 11\ln x + 5.x^{-2} - e^x.e^{\ln\frac{1}{3}}$

$= -\frac{1}{2}\cos 4x + 11\ln x + 5.x^{-2} - e^x.\frac{1}{3}$

$y = -\frac{1}{2}\cos 4x + 11\ln x + 5.x^{-2} - \frac{1}{3}.e^x$

$\frac{dy}{dx} = \frac{d}{dx}\left(-\frac{1}{2}\cos 4x\right) + \frac{d}{dx}(11\ln x) + \frac{d}{dx}(5x^{-2}) - \frac{d}{dx}\left(\frac{1}{3}.e^x\right)$

$= -\frac{1}{2}\frac{d}{dx}\cos 4x + 11\frac{d}{dx}\ln x + 5\frac{d}{dx}x^{-2} - \frac{1}{3}\frac{d}{dx}e^x$

$= -\frac{1}{2}(-\sin 4x.4) + 11\left(\frac{1}{x}\right) + 5(-2.x^{-3}) - \frac{1}{3}(e^x)$

$\frac{dy}{dx} = 2\sin 4x + \frac{11}{x} - \frac{10}{x^3} - \frac{1}{3}e^x$

2.5 $y = -\frac{12}{\sin x} - 3e^{\sqrt{9x^2}} + \ln\frac{1}{x^5} + 13$

$= -12\operatorname{cosec} x - 3e^{3x} + \ln x^{-5} + 13$

$y = -12\operatorname{cosec} x - 3e^{3x} - 5\ln x + 13$

$\frac{dy}{dx} = \frac{d}{dx}(-12\operatorname{cosec} x) - \frac{d}{dx}(3e^{3x}) - \frac{d}{dx}(5\ln x) + \frac{d}{dx}(13)$

$= -12\frac{d}{dx}\operatorname{cosec} x - 3\frac{d}{dx}e^{3x} - 5\frac{d}{dx}\ln x + \frac{d}{dx}13$

$= -12(-\operatorname{cosec} x\cot x) - 3(e^{3x}.3) - 5\left(\frac{1}{x}\right) + 0$

$\frac{dy}{dx} = 12\operatorname{cosec} x\cot x - 9e^{3x} - \frac{5}{x}$

2.6 $y = -2 \tan x + \frac{\log_5 x}{4} - \frac{1}{x^2\sqrt{x}} + \frac{6^x}{\ln 6}$

$= -2 \tan x + \frac{1}{4}\log_5 x - \frac{1}{x^2.x^{\frac{1}{2}}} + \frac{1}{\ln 6}.6^x$

$= -2 \tan x + \frac{1}{4}\log_5 x - \frac{1}{x^{\frac{5}{2}}} + \frac{1}{\ln 6}.6^x$

$y = -2 \tan x + \frac{1}{4}\log_5 x - x^{-\frac{5}{2}} + \frac{1}{\ln 6}.6^x$

$\frac{dy}{dx} = \frac{d}{dx}(-2 \tan x) + \frac{d}{dx}\left(\frac{1}{4}\log_5 x\right) - \frac{d}{dx}\left(x^{-\frac{5}{2}}\right) + \frac{d}{dx}\left(\frac{1}{\ln 6}.6^x\right)$

$= -2\frac{d}{dx}\tan x + \frac{1}{4}\frac{d}{dx}\log_5 x - \frac{d}{dx}x^{-\frac{5}{2}} + \frac{1}{\ln 6}\frac{d}{dx}6^x$

$= -2(\sec^2 x) + \frac{1}{4}\left(\frac{1}{x \ln 5}\right) - \left(-\frac{5}{2}.x^{-\frac{7}{2}}\right) + \frac{1}{\ln 6}(6^x \ln 6)$

$= -2 \sec^2 x + \frac{1}{4x \ln 5} + \frac{5}{2}.\frac{1}{x^{\frac{7}{2}}} + 6^x$

$= -2 \sec^2 x + \frac{1}{4x \ln 5} + \frac{5}{2}.\frac{1}{x^3.x^{\frac{1}{2}}} + 6^x$

$\frac{dy}{dx} = -2 \sec^2 x + \frac{1}{4x \ln 5} + \frac{5}{2x^3\sqrt{x}} + 6^x$

Activity 5.7 — SB page 328

1.

Function	Rewrite as $y = f[g(x)]$	$u = g(x)$	$\frac{du}{dx} = g'(x)$	$y = f(u)$	$\frac{dy}{du} = f'(u)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \left(\frac{1}{3}\right)^{-\frac{x}{10}}$	$y = 3^{\frac{x}{10}}$	$u = \frac{x}{10}$	$\frac{du}{dx} = \frac{1}{10}$	$y = 3^u$	$\frac{dy}{du} = 3^u \ln 3$	$\frac{dy}{dx} = \frac{3^{\frac{x}{10}} \ln 3}{10}$
$y = \frac{7}{e^{3x}}$	$y = 7.e^{-3x}$	$u = -3x$	$\frac{du}{dx} = -3$	$y = 7.e^u$	$\frac{dy}{du} = 7.e^u$	$\frac{dy}{dx} = -21.e^{-3x}$
$y = \sqrt{e^{5x}}$	$y = e^{\frac{5x}{2}}$	$u = \frac{5x}{2}$	$\frac{du}{dx} = \frac{5}{2}$	$y = e^u$	$\frac{dy}{du} = e^u$	$\frac{dy}{dx} = \frac{5}{2}.e^{\frac{5x}{2}}$
$y = e^{\ln 2+9x}$	$y = 2.e^{9x}$	$u = 9x$	$\frac{du}{dx} = 9$	$y = 2.e^u$	$\frac{dy}{du} = 2.e^u$	$\frac{dy}{dx} = 18.e^{9x}$
$y = 6 \log\left(\frac{x}{8}\right)$	$y = 6 \log\left(\frac{x}{8}\right)$	$u = \frac{x}{8}$	$\frac{du}{dx} = \frac{1}{8}$	$y = 6 \log u$	$\frac{dy}{du} = \frac{6}{u \ln 10}$	$\frac{dy}{dx} = \frac{48}{x \ln 10}$
$y = \ln(\sqrt{4x})$	$y = \frac{1}{2} \ln 4x$	$u = 4x$	$\frac{du}{dx} = 4$	$y = \frac{1}{2} \ln u$	$\frac{dy}{du} = \frac{1}{2u}$	$\frac{dy}{dx} = \frac{1}{2x}$
$y = \cot\left(\frac{1}{2x^{-1}}\right)$	$y = \cot\left(\frac{x}{2}\right)$	$u = \frac{x}{2}$	$\frac{du}{dx} = \frac{1}{2}$	$y = \cot u$	$\frac{dy}{du} = -\text{cosec}^2 u$	$\frac{dy}{dx} = -\frac{1}{2} \text{cosec}^2\left(\frac{x}{2}\right)$
$y = 4 \cos\left(\frac{\pi}{2} + \frac{3x}{4}\right)$	$y = -4 \sin \frac{3x}{4}$	$u = \frac{3x}{4}$	$\frac{du}{dx} = \frac{3}{4}$	$y = -4 \sin u$	$\frac{dy}{du} = -4 \cos u$	$\frac{dy}{dx} = -3 \cos \frac{3x}{4}$
$y = \frac{1}{\sqrt{2}}\left(\cos\left(\pi - \sqrt{3x^2}\right)\right)^{-1}$	$y = -\frac{1}{\sqrt{2}} \sec(\sqrt{3}x)$	$u = \sqrt{3}x$	$\frac{du}{dx} = \sqrt{3}$	$y = -\frac{1}{\sqrt{2}} \sec u$	$\frac{dy}{du} = -\frac{1}{\sqrt{2}} \sec u \tan u$	$\frac{dy}{dx} = -\frac{\sqrt{6} \sec(\sqrt{3}x) \tan(\sqrt{3}x)}{2}$
$y = 3 \cos(\pi + 6x)$	$y = -3 \cos 6x$	$u = 6x$	$\frac{du}{dx} = 6$	$y = -3 \cos u$	$\frac{dy}{du} = 3 \sin u$	$\frac{dy}{dx} = 18 \sin 6x$

2\.

2.1 $y = 3\ln(\cos x)$

Let $u = \cos x$ then $y = 3\ln u$

Therefore,

$\frac{du}{dx} = -\sin x \qquad \frac{dy}{du} = 3.\frac{1}{u}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 3.\frac{1}{u} \times -\sin x$

$= -\frac{3\sin x}{u}$

$= -\frac{3\sin x}{\cos x}$

$\frac{dy}{dx} = -3\tan x$

2.2 $y = 8.e^{\log x}$

Let $u = \log x$ then $y = 8.e^{u}$

Therefore,

$\frac{du}{dx} = \frac{1}{x\ln 10} \qquad \frac{dy}{du} = 8.e^{u}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 8.e^{u} \times \frac{1}{x\ln 10}$

$= \frac{8e^{u}}{x\ln 10}$

$\frac{dy}{dx} = \frac{8e^{\log x}}{x\ln 10}$

2.3 $y = 4\cot(7x + 2)$

Let $u = 7x + 2$ then $y = 4\cot u$

Therefore,

$\frac{du}{dx} = 7 \qquad \frac{dy}{du} = 4.-\text{cosec}^2 u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 4.-\text{cosec}^2 u \times 7$

$= -28\,\text{cosec}^2 u$

$\frac{dy}{dx} = -28\,\text{cosec}^2(7x + 2)$

2.4 $y = \sec(4x - 2)$

Let $u = 4x - 2$ then $y = \sec u$

Therefore,

$\frac{du}{dx} = 4 \qquad \frac{dy}{du} = \sec u.\tan u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \sec u.\tan u \times 4$

$= 4\sec u.\tan u$

$\frac{dy}{dx} = 4\sec(4x - 2)\tan(4x - 2)$

2.5 $y = (2x - 1)^4$

Let $u = 2x - 1$ then $y = u^4$

Therefore,

$\frac{du}{dx} = 2 \qquad \frac{dy}{du} = 4u^3$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 4u^3 \times 2$

$= 8u^3$

$\frac{dy}{dx} = 8(2x - 1)^3$

2.6 $y = \sqrt{x^2 + x - 1}$

$y = (x^2 + x - 1)^{\frac{1}{2}}$

Let $u = x^2 + x - 1$ then $y = u^{\frac{1}{2}}$

Therefore,

$\frac{du}{dx} = 2x + 1 \qquad \frac{dy}{dx} = \frac{1}{2}u^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{2}u^{-\frac{1}{2}} \times (2x + 1)$

$= \frac{2x + 1}{2\sqrt{u}}$

$\frac{dy}{dx} = \frac{2x + 1}{2\sqrt{x^2 + x - 1}}$

2.7 $y = \sqrt[3]{x^2 - 1}$

$y = (x^2 - 1)^{\frac{1}{3}}$

Let $u = x^2 - 1$ then $y = u^{\frac{1}{3}}$

Therefore,

$\frac{du}{dx} = 2x$ $\frac{dy}{du} = \frac{1}{3}u^{-\frac{2}{3}}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{3}u^{-\frac{2}{3}} \times 2x$

$= \frac{2x}{3\sqrt[3]{u^2}}$

$\frac{dy}{dx} = \frac{2x}{3\sqrt[3]{(x^2 - 1)^2}}$

2.8 $y = \sqrt[4]{\tan x}$

$y = (\tan x)^{\frac{1}{4}}$

Let $u = \tan x$ then $y = u^{\frac{1}{4}}$

Therefore,

$\frac{du}{dx} = \sec^2 x$ $\frac{dy}{du} = \frac{1}{4}u^{-\frac{3}{4}}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{4}u^{-\frac{3}{4}} \times \sec^2 x$

$= \frac{\sec^2 x}{4\sqrt[4]{u^3}}$

$\frac{dy}{dx} = \frac{\sec^2 x}{4\sqrt[4]{\tan^3 x}}$

Activity 5.8 SB page 331

1. $y = \left(x^2 + 3x\right)\left(2x - 2\right)$

$\frac{dy}{dx} = (x^2 + 3x)(2) + (2x - 2)(2x + 3)$

$= 2x^2 + 6x + \left(4x^2 - 4x + 6x - 6\right)$

$= 2x^2 + 6x + 4x^2 + 2x - 6$

$= 6x^2 + 8x - 6$

2. $y = x^5.3^x$

Let $f(x) = x^5$ and $g(x) = 3^x$

Thus $f'(x) = 5x^4$ and $g'(x) = 3^x \ln 3$

$\frac{dy}{dx} = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$

$= (x^5)(3^x \ln 3) + (3^x)(5x^4)$

$\frac{dy}{dx} = x^4.3^x(x \ln 3 + 5)$

3. $y = e^x.\log x$

Let $f(x) = e^x$ and $g(x) = \log x$

Thus $f'(x) = e^x$ and $g'(x) = \frac{1}{x \ln 10}$

$\frac{dy}{dx} = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$

$= (e^x)\left(\frac{1}{x \ln 10}\right) + (\log x)(e^x)$

$\frac{dy}{dx} = e^x\left(\frac{1}{x \ln 10} + \log x\right)$

4. $y = \ln x \sin x$

$\text{Let } f(x) = \ln x \text{ and } g(x) = \sin x$

$\text{Thus } f'(x) = \frac{1}{x} \text{ and } g'(x) = \cos x$

$\frac{dy}{dx} = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$

$= (\ln x)(\cos x) + (\sin x)\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = \cos x \ln x + \frac{\sin x}{x}$

5. $s = t^3 \cos t$

$\text{Let } f(t) = t^3 \text{ and } g(t) = \cos t$

$\text{Thus } f'(t) = 3t^2 \text{ and } g'(x) = -\sin t$

$\frac{ds}{dt} = f(t)\frac{d}{dt}g(t) + g(t)\frac{d}{dt}f(t)$

$= (t^3)(-\sin t) + (\cos t)(3t^2)$

$= t^2(-t \sin t + 3 \cos t)$

6. $y = \tan x.e^x$

$\text{Let } f(x) = \tan x \text{ and } g(x) = e^x$

$\text{Thus } f'(x) = \sec^2 x \text{ and } g'(x) = e^x$

$\frac{dy}{dx} = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$

$= (\tan x)(e^x) + (e^x)(\sec^2 x)$

$\frac{dy}{dx} = e^x(\tan x + \sec^2 x)$

7. $y = (\ln x - 1).\sqrt{x}$

$y = (\ln x - 1).x^{\frac{1}{2}}$

$\text{Let } f(x) = \ln x - 1 \text{ and } g(x) = x^{\frac{1}{2}}$

$\text{Thus } f'(x) = \frac{1}{x} \text{ and } g'(x) = \frac{1}{2}.x^{-\frac{1}{2}}$

$\frac{dy}{dx} = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$

$= (\ln x - 1)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) + \left(x^{\frac{1}{2}}\right)\left(\frac{1}{x}\right)$

$= \frac{\ln x - 1}{2\sqrt{x}} + \frac{x^{\frac{1}{2}}}{x}$

$= \frac{\sqrt{x}(\ln x - 1)}{2x} + \frac{\sqrt{x}}{x}$

$= \frac{\sqrt{x}(\ln x - 1) + 2\sqrt{x}}{2x}$

$= \frac{\sqrt{x} \ln x - \sqrt{x} + 2\sqrt{x}}{2x}$

$= \frac{\sqrt{x} \ln x + \sqrt{x}}{2x}$

$\frac{dy}{dx} = \frac{\sqrt{x}(\ln x + 1)}{2x}$

8. $x = \left(2 + \frac{1}{x^4}\right)\log_6 t$

$= (2 + x^{-4})\log_6 t$

Let $f(t) = 2 + x^{-4}$ and $g(t) = \log_6 t$

Thus $f'(t) = -4x^{-5}$ and $g'(t) = \frac{1}{t \ln 6}$

$\frac{dx}{dt} = f(t)\frac{d}{dt}g(t) + g(t)\frac{d}{dt}f(t)$

$= (2 + x^{-4})\left(\frac{1}{t \ln 6}\right) + (\log_6 t)(-4.x^{-5})$

$= \frac{2}{t \ln 6} + \frac{1}{tx^4 \ln 6} - \frac{4 \log_6 t}{x^5}$

9. $y = 4^x(\cot x - x^2)$

Let $f(x) = 4^x$ and $g(x) = \cot x - x^2$

Thus $f'(x) = 4^x \ln 4$ and $g'(x) = -\text{cosec}^2 x - 2x$

$\frac{dy}{dx} = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$

$= (4^x)(-\text{cosec}^2 x - 2x) + (\cot x - x^2)(4^x \ln 4)$

$\frac{dy}{dx} = 4^x(-\text{cosec}^2 x - 2x + \cot x \ln 4 - x^2 \ln 4)$

10. $y = (\sqrt{x^3} - \sec x)\,\text{cosec}\, x$

$y = (x^{\frac{3}{2}} - \sec x)\,\text{cosec}\, x$

Let $f(x) = x^{\frac{3}{2}} - \sec x$ and $g(x) = \text{cosec}\, x$

Thus $f'(x) = \frac{3}{2}.x^{\frac{1}{2}} - \sec x \tan x$ and $g'(x) = -\,\text{cosec}\, x \cot x$

$\frac{dy}{dx} = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$

$= (x^{\frac{3}{2}} - \sec x)(-\text{cosec}\, x.\cot x) + (\text{cosec}\, x)\left(\frac{3}{2}x^{\frac{1}{2}} - \sec x \tan x\right)$

$= -x^{\frac{3}{2}}\text{cosec}\, x \cot x + \text{cosec}^2 x + \frac{3}{2}x^{\frac{1}{2}}\text{cosec}\, x - \sec^2 x$

$\frac{dy}{dx} = -x\sqrt{x}\,\text{cosec}\, x \cot x + \text{cosec}^2 x + \frac{3\sqrt{x}\,\text{cosec}\, x}{2} - \sec^2 x$

Activity 5.9 SB page 333

1. $y = \frac{1 - x^4}{\sqrt{x}}$

$\frac{dy}{dx} = \frac{x^{\frac{1}{2}}(-4x^3) - (1 - x^4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(x^{\frac{1}{2}}\right)^2}$

$= \frac{-4x^{\frac{7}{2}} - \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{\frac{7}{2}}}{x}$

$= \frac{-\frac{7}{2}x^{\frac{7}{2}} - \frac{1}{2}x^{-\frac{1}{2}}}{x}$

$= -\frac{7}{2}x^{\frac{5}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

$= -\frac{7}{2}\sqrt{x^5} - \frac{1}{2\sqrt{x^3}}$

2. $$y = \frac{x^7}{\ln x}$$

Let $f(x) = x^7$ and $g(x) = \ln x$

Thus $f'(x) = 7x^6$ and $g'(x) = \frac{1}{x}$

$$\frac{dy}{dx} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$

$$= \frac{(\ln x)(7x^6) - (x^7)\left(\frac{1}{x}\right)}{(\ln x)^2}$$

$$= \frac{7x^6 \ln x - x^6}{\ln^2 x}$$

$$\frac{dy}{dx} = \frac{x^6(7 \ln x - 1)}{\ln^2 x}$$

3. $$y = \frac{4 \cos x}{10^x}$$

Let $f(x) = 4 \cos x$ and $g(x) = 10^x$

Thus $f'(x) = -4 \sin x$ and $g'(x) = 10^x \ln 10$

$$\frac{dy}{dx} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$

$$= \frac{(10^x)(-4 \sin x) - (4 \cos x)(10^x \ln 10)}{(10^x)^2}$$

$$= \frac{4.10^x(- \sin x - \cos x.\ln 10)}{(10^x)^2}$$

$$\frac{dy}{dx} = \frac{4(- \sin x - \cos x \ln 10)}{10^x}$$

4. $$x = \frac{\log_3 r}{r^5}$$

Let $f(r) = \log_3 r$ and $g(r) = r^5$

Thus $f'(r) = \frac{1}{r \ln 3}$ and $g'(r) = 5r^4$

$$\frac{dx}{dr} = \frac{g(r)\frac{d}{dr}f(r) - f(r)\frac{d}{dr}g(r)}{(g(r))^2}$$

$$= \frac{r^5\left(\frac{1}{r\ln 3}\right) - (\log_3 r)(5r^4)}{(r^5)^2}$$

$$= \frac{\frac{r^4}{\ln 3} - 5r^4 \log_3 r}{r^{10}}$$

$$= \frac{r^4\left(\frac{1}{\ln 3} - 5 \log_3 r\right)}{r^{10}}$$

$$= \frac{\frac{1}{\ln 3} - 5 \log_3 r}{r^6}$$

5.
$$y = (2 \ln x)^{-1}$$
$$y = \frac{1}{2 \ln x}$$
Let $f(x) = 1$ and $g(x) = 2 \ln x$

Thus $f'(x) = 0$ and $g'(x) = \frac{2}{x}$

$$\frac{dy}{dx} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$
$$= \frac{(2 \ln x)(0) - (1)\left(\frac{2}{x}\right)}{(2 \ln x)^2}$$
$$= \frac{-\frac{2}{x}}{(2 \ln x)^2}$$
$$\frac{dy}{dx} = -\frac{1}{2x \ln^2 x}$$

6.
$$r = \operatorname{cosec} \theta$$
$$= \frac{1}{\sin \theta}$$
Let $f(\theta) = 1$ and $g(x) = \sin \theta$

Thus $f'(\theta) = 0$ and $g'(\theta) = \cos \theta$

$$\frac{dr}{d\theta} = \frac{g(\theta)\frac{d}{d\theta}f(\theta) - f(\theta)\frac{d}{d\theta}g(\theta)}{(g(\theta))^2}$$
$$= \frac{(\sin \theta)(0) - (1)(\cos \theta)}{(\sin \theta)^2}$$
$$= \frac{-\cos \theta}{\sin^2 \theta}$$
$$= -\frac{1}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta}$$
$$= -\operatorname{cosec} \theta \cot \theta$$

7.
$$y = \frac{e^x}{x^3 + 1}$$
Let $f(x) = e^x$ and $g(x) = x^3 + 1$

Thus $f'(x) = e^x$ and $g'(x) = 3x^2$

$$\frac{dy}{dx} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$
$$= \frac{(x^3 + 1)(e^x) - (e^x)(3x^2)}{(x^3 + 1)^2}$$
$$\frac{dy}{dx} = \frac{e^x(x^3 - 3x^2 + 1)}{(x^3 + 1)^2}$$

8. $$y = \frac{4 - x^2}{2^x}$$

Let $f(x) = 4 - x^2$ and $g(x) = 2^x$

Thus $f'(x) = -2x$ and $g'(x) = 2^x \ln 2$

$$\frac{dy}{dx} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$

$$= \frac{(2^x)(-2x) - (4 - x^2)(2^x \ln 2)}{(2^x)^2}$$

$$= \frac{2^x(x^2 \ln 2 - 2x - 4 \ln 2)}{(2^x)^2}$$

$$\frac{dy}{dx} = \frac{x^2 \ln 2 - 2x - 4 \ln 2}{2^x}$$

9. $$s = \frac{\sqrt[3]{t}}{\sec t - t}$$

$$= \frac{t^{\frac{1}{3}}}{\sec t - t}$$

Let $f(t) = t^{\frac{1}{3}}$ and $g(t) = \sec t - t$

Thus $f'(t) = \frac{1}{3}.t^{-\frac{2}{3}}$ and $g'(t) = \sec t.\tan t - 1$

$$\frac{ds}{dt} = \frac{g(t)\frac{d}{dt}f(t) - f(t)\frac{d}{dt}g(t)}{(g(t))^2}$$

$$= \frac{(\sec t - t)\left(\frac{1}{3}t^{-\frac{2}{3}}\right) - \left(t^{\frac{1}{3}}\right)(\sec t \tan t - 1)}{(\sec t - t)^2}$$

$$= \frac{\frac{1}{3}t^{-\frac{2}{3}}\sec t - \frac{1}{3}t^{\frac{1}{3}} - t^{\frac{1}{3}}\sec t \tan t + t^{\frac{1}{3}}}{(\sec t - t)^2}$$

$$= \frac{\frac{1}{3}t^{-\frac{2}{3}}\sec t - t^{\frac{1}{3}}\sec t \tan t + \frac{2}{3}t^{\frac{1}{3}}}{(\sec t - t)^2}$$

$$= \frac{\frac{1}{3}t^{\frac{1}{3}}(t^{-1}\sec t - 3 \sec t \tan t + 2)}{(\sec t - t)^2}$$

$$= \frac{t^{\frac{1}{3}}(\sec t - 3t \sec t \tan t + 2t)}{3t(\sec t - t)^2}$$

$$= \frac{\sqrt[3]{t}(\sec t - 3t \sec t \tan t + 2t)}{3t(\sec t - t)^2}$$

10. $y = \frac{e^x + \sin x}{\log_4 x}$

Let $f(x) = e^x + \sin x$ and $g(x) = \log_4 x$

Thus $f'(x) = e^x + \cos x$ and $g'(x) = \frac{1}{x \ln 4}$

$$\frac{dy}{dx} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$

$$= \frac{(\log_4 x)(e^x + \cos x) - (e^x + \sin x)\left(\frac{1}{x \ln 4}\right)}{(\log_4 x)^2}$$

$$= \frac{e^x \log_4 x + \cos x \log_4 x - \frac{e^x}{x \ln 4} - \frac{\sin x}{x \ln 4}}{(\log_4 x)^2}$$

$$\frac{dy}{dx} = \frac{x e^x \log_4 x \ln 4 + x \cos x \log_4 x \ln 4 - e^x - \sin x}{x \ln 4 (\log_4 x)^2}$$

Activity 5.10 — SB page 338

1. $y = -2x^3 + 5x^2 + 3x - 12$

$$\frac{dy}{dx} = -6x^2 + 10x + 3$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-6x^2 + 10x + 3)$$

$$= \frac{d}{dx}(-6x^2) + \frac{d}{dx}(10x) + \frac{d}{dx}(3)$$

$$= -6\frac{d}{dx}x^2 + 10\frac{d}{dx}x + \frac{d}{dx}3$$

$$= -6(2x) + 10(1) + 0$$

$$\frac{d^2y}{dx^2} = -12x + 10$$

2. $y = x^2\left(x - \frac{1}{x}\right)^2 + 5$

$$\frac{dy}{dx} = 4x^3 - 4x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(4x^3 - 4x)$$

$$= \frac{d}{dx}(4x^3) - \frac{d}{dx}(4x)$$

$$= 4\frac{d}{dx}x^3 - 4\frac{d}{dx}x$$

$$= 4(3x^2) - 4(1)$$

$$\frac{d^2y}{dx^2} = 12x^2 - 4$$

3. $y = \frac{(x^2 + 1)(x - 2)}{x^2}$

$$\frac{dy}{dx} = 1 - \frac{1}{x^2} + \frac{4}{x^3}$$

$$\frac{dy}{dx} = 1 - x^{-2} + 4x^{-3}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(1 - x^{-2} + 4x^{-3})$$

$$= \frac{d}{dx}(1) - \frac{d}{dx}(x^{-2}) + \frac{d}{dx}(4x^{-3})$$

$$= \frac{d}{dx}1 - \frac{d}{dx}x^{-2} + 4\frac{d}{dx}x^{-3}$$

$$= 0 - (-2x^{-3}) + 4(-3x^{-4})$$

$$\frac{d^2y}{dx^2} = \frac{2}{x^3} - \frac{12}{x^4}$$

4. $s = \frac{\frac{1}{3}t^3 - 9}{2t - 6}$

$$\frac{ds}{dt} = \frac{1}{3}t + \frac{1}{2}$$

$$\therefore \frac{d^2s}{dt^2} = \frac{d}{dt}\left(\frac{1}{3}t + \frac{1}{2}\right)$$

$$= \frac{d}{dt}\left(\frac{1}{3}t\right) + \frac{d}{dt}\left(\frac{1}{2}\right)$$

$$= \frac{1}{3}\frac{d}{dt}t + \frac{d}{dt}\frac{1}{2}$$

$$= \frac{1}{3}(1) + 0$$

$$\frac{d^2s}{dt^2} = \frac{1}{3}$$

5. $y = 3 \cot x + \frac{10}{e^{\frac{x}{2}}} - 4\sqrt[3]{x} - 8$

$\frac{dy}{dx} = -3 \operatorname{cosec}^2 x - \frac{5}{e^{\frac{x}{2}}} - \frac{4}{3\sqrt[3]{x^2}}$

$\frac{dy}{dx} = -3(\operatorname{cosec} x)^2 - 5e^{-\frac{x}{2}} - \frac{4}{3}x^{-\frac{2}{3}}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(-3(\operatorname{cosec} x)^2 - 5e^{-\frac{x}{2}} - \frac{4}{3}x^{-\frac{2}{3}}\right)$

$= \frac{d}{dx}(-3(\operatorname{cosec} x)^2) - \frac{d}{dx}\left(5e^{-\frac{x}{2}}\right) - \frac{d}{dx}\left(\frac{4}{3}x^{-\frac{2}{3}}\right)$

$= -3\frac{d}{dx}((\operatorname{cosec} x)^2) - 5\frac{d}{dx}\left(e^{-\frac{x}{2}}\right) - \frac{4}{3}\frac{d}{dx}x^{-\frac{2}{3}}$

$= -3(2 \operatorname{cosec} x.-\operatorname{cosec} x \cot x) - 5\left(e^{-\frac{x}{2}}.-\frac{1}{2}\right) - \frac{4}{3}\left(-\frac{2}{3}x^{-\frac{5}{3}}\right)$

$\frac{d^2y}{dx^2} = 6 \operatorname{cosec}^2 x \cot x + \frac{5}{2e^{\frac{x}{2}}} + \frac{8}{9x\sqrt[3]{x^5}}$

6. $y = 7 \sin 2x - 3^{-x} + \frac{1}{2}\log\sqrt{x} - \frac{5}{2}x^4$

$\frac{dy}{dx} = 14 \cos 2x + \frac{\ln 3}{3^x} + \frac{1}{4x \ln 10} - 10x^3$

$\frac{dy}{dx} = 14 \cos 2x + \ln 3.3^{-x} + \frac{1}{4 \ln 10}.x^{-1} - 10x^3$

$\frac{d^2y}{dx^2} = \frac{d}{dx}(14 \cos 2x + \ln 3.3^{-x} + \frac{1}{4 \ln 10}.x^{-1} - 10x^3)$

$= \frac{d}{dx}(14 \cos 2x) + \frac{d}{dx}(\ln 3.3^{-x}) + \frac{d}{dx}\left(\frac{1}{4 \ln 10}.x^{-1}\right) - \frac{d}{dx}(10x^3)$

$= 14\frac{d}{dx}(\cos(2x)) + \ln 3\frac{d}{dx}(3^{-x}) + \frac{1}{4 \ln 10}\frac{d}{dx}x^{-1} - 10\frac{d}{dx}x^3$

$= 14(-\sin 2x.2) + \ln 3(3^{-x} \ln 3.-1) + \frac{1}{4 \ln 10}(-1x^{-2}) - 10(3x^2)$

$\frac{d^2y}{dx^2} = -28 \sin 2x - \frac{(\ln 3)^2}{3^x} - \frac{1}{4x^2 \ln 10} - 30x^2$

7. $y = \frac{1}{4} \sec x - \sqrt{2^{8x}} - \sqrt{3} \ln 2x + 10^{\log 2}$

$\frac{dy}{dx} = \frac{1}{4} \sec x \tan x - 4.2^{4x} \ln 2 - \frac{\sqrt{3}}{x}$

$\frac{dy}{dx} = \frac{1}{4} \sec x \tan x - 4.2^{4x} \ln 2 - \sqrt{3}x^{-1}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{4} \sec x \tan x - 4.2^{4x} \ln 2 - \sqrt{3}x^{-1}\right)$

$= \frac{d}{dx}\left(\frac{1}{4} \sec x \tan x\right) - \frac{d}{dx}(4.2^{4x} \ln 2) - \frac{d}{dx}(\sqrt{3}x^{-1})$

$= \frac{1}{4}\frac{d}{dx} \sec x \tan x - 4 \ln 2\frac{d}{dx}2^{4x} - \sqrt{3}\frac{d}{dx}x^{-1}$

$= \frac{1}{4}[\sec x\frac{d}{dx} \tan x + \tan x \frac{d}{dx} \sec x] - 4 \ln 2\frac{d}{dx}(2^{4x}) - \sqrt{3}\frac{d}{dx}x^{-1}$

$= \frac{1}{4}[\sec x \sec^2 x + \tan x \sec x \tan x] - 4 \ln 2(2^{4x} \ln 2.4) - \sqrt{3}(-1.x^{-2})$

$= \frac{1}{4}[\sec^3 x + \sec x \tan^2 x] - 16.2^{4x}(\ln 2)^2 + \frac{\sqrt{3}}{x^2}$

$$= \frac{1}{4}[\sec^3 x + \sec x\,(\sec^2 x - 1)] - 16.2^{4x}(\ln 2)^2 + \frac{\sqrt{3}}{x^2}$$

$$= \frac{1}{4}[\sec^3 x + \sec^3 x - \sec x] - 16.2^{4x}(\ln 2)^2 + \frac{\sqrt{3}}{x^2}$$

$$= \frac{1}{4}[2\sec^3 x - \sec x] - 16.2^{4x}(\ln 2)^2 + \frac{\sqrt{3}}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}\sec^3 x - \frac{1}{4}\sec x - 16.2^{4x}(\ln 2)^2 + \frac{\sqrt{3}}{x^2}$$

8. $y = -\frac{1}{2}\cos 4x + 11\ln x + \frac{5}{x^2} - e^{x - \ln 3}$

$$\frac{dy}{dx} = 2\sin 4x + \frac{11}{x} - \frac{10}{x^3} - \frac{1}{3}e^x$$

$$\frac{dy}{dx} = 2\sin 4x + 11x^{-1} - 10x^{-3} - \frac{1}{3}e^x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(2\sin 4x + 11x^{-1} - 10x^{-3} - \frac{1}{3}e^x)$$

$$= \frac{d}{dx}(2\sin 4x) + \frac{d}{dx}(11x^{-1}) - \frac{d}{dx}(10x^{-3}) - \frac{d}{dx}(\frac{1}{3}e^x)$$

$$= 2\frac{d}{dx}\sin 4x + 11\frac{d}{dx}x^{-1} - 10\frac{d}{dx}x^{-3} - \frac{1}{3}\frac{d}{dx}e^x$$

$$= 2(\cos 4x4) + 11(-1x^{-2}) - 10(-3x^{-4}) - \frac{1}{3}(e^x)$$

$$\frac{d^2y}{dx^2} = 8\cos 4x - \frac{11}{x^2} + \frac{30}{x^4} - \frac{1}{3}e^x$$

9. $y = -\frac{12}{\sin x} - 3e^{\sqrt{9x^2}} + \ln\frac{1}{x^5} + 13$

$$\frac{dy}{dx} = 12\,\text{cosec}\,x\cot x - 9e^{3x} - \frac{5}{x}$$

$$\frac{dy}{dx} = 12\,\text{cosec}\,x\cot x - 9e^{3x} - 5.x^{-1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(12\,\text{cosec}\,x\cot x - 9e^{3x} - 5.x^{-1})$$

$$= \frac{d}{dx}(12\,\text{cosec}\,x\cot x) - \frac{d}{dx}(9e^{3x}) - \frac{d}{dx}(5.x^{-1})$$

$$= 12\frac{d}{dx}\text{cosec}\,x\cot x - 9\frac{d}{dx}e^{3x} - 5\frac{d}{dx}x^{-1}$$

$$= 12[\text{cosec}\,x\frac{d}{dx}\cot x + \cot x\frac{d}{dx}\text{cosec}\,x] - 9\frac{d}{dx}e^{3x} - 5\frac{d}{dx}x^{-1}$$

$$= 12[\text{cosec}\,x.-\text{cosec}^2 x + \cot x.-\text{cosec}\,x\cot x] - 9(e^{3x}.3) - 5(-1x^{-2})$$

$$= 12[-\text{cosec}^3 x - \text{cosec}\,x\cot^2 x] - 27e^{3x} + \frac{5}{x^2}$$

$$= 12[-\text{cosec}^3 x - \text{cosec}\,x(\text{cosec}^2 x - 1] - 27e^{3x} + \frac{5}{x^2}$$

$$= 12[-\text{cosec}^3 x - \text{cosec}^3 x + \text{cosec}\,x] - 27e^{3x} + \frac{5}{x^2}$$

$$= 12[-2\,\text{cosec}^3 x + \text{cosec}\,x] - 27e^{3x} + \frac{5}{x^2}$$

$$\frac{d^2y}{dx^2} = -24\,\text{cosec}^3 x + 12\,\text{cosec}\,x - 27e^{3x} + \frac{5}{x^2}$$

10. $y = -2\tan x + \frac{\log_5 x}{4} - \frac{1}{x^2\sqrt{x}} + \frac{6^x}{\ln 6}$

$\frac{dy}{dx} = -2\sec^2 x + \frac{1}{4x\ln 5} + \frac{5}{2x^3\sqrt{x}} + 6^x$

$\frac{dy}{dx} = -2(\sec x)^2 + \frac{1}{4\ln 5}.x^{-1} + \frac{5}{2}.x^{-\frac{7}{2}} + 6^x$

$\frac{d^2y}{dx^2} = \frac{d}{dx}(-2(\sec x)^2 + \frac{1}{4\ln 5}.x^{-1} + \frac{5}{2}.x^{-\frac{7}{2}} + 6^x)$

$= \frac{d}{dx}(-2(\sec x)^2) + \frac{d}{dx}\left(\frac{1}{4\ln 5}.x^{-1}\right) + \frac{d}{dx}\left(\frac{5}{2}.x^{-\frac{7}{2}}\right) + \frac{d}{dx}(6^x)$

$= -2\frac{d}{dx}(\sec x)^2 + \frac{1}{4\ln 5}\frac{d}{dx}x^{-1} + \frac{5}{2}\frac{d}{dx}x^{-\frac{7}{2}} + \frac{d}{dx}6^x$

$= -2(2\sec x\sec x\tan x) + \frac{1}{4\ln 5}(-1x^{-2}) + \frac{5}{2}\left(-\frac{7}{2}x^{-\frac{9}{2}}\right) + 6^x\ln 6$

$\frac{d^2y}{dx^2} = -4\sec^2 x\tan x - \frac{1}{4x^2\ln 5} - \frac{35}{4x^4\sqrt{x}} + 6^x\ln 6$

Activity 5.11 — SB page 360

1. 1.1 a) $y = x^3 + 3x^2$

$\frac{dy}{dx} = 3x^2 + 6x$

$0 = 3x^2 + 6x$

$0 = 3x(x + 2)$

$\therefore\ x = 0$ or $x = -2$

$\therefore\ y = 0$ or $y = (-2)^3 + 3(-2)^2$

$\therefore\ y = 4$

Thus, the turning points of $y = x^3 + 3x^2$ are (0; 0) and (–2; 4)

b) Nature of turning points:

$\frac{d^2y}{dx^2} = 6x + 6$

For $x = 0$:

$\frac{d^2y}{dx^2} = 6(0) + 6 = 6 > 0 \therefore$ Minimum (0; 0)

For $x = -2$:

$\frac{d^2y}{dx^2} = 6(-2) + 6 = -6 < 0 \therefore$ Maximum (–2; 4)

c) $\frac{d^2y}{dx^2} = 0$

$6x + 6 = 0$

$6x = -6$

$x = -1$

$\therefore\ y = (-1)^3 + 3(-1)^2$

$= -1 + 3$

$= 2$

Point of inflection: (–1; 2)

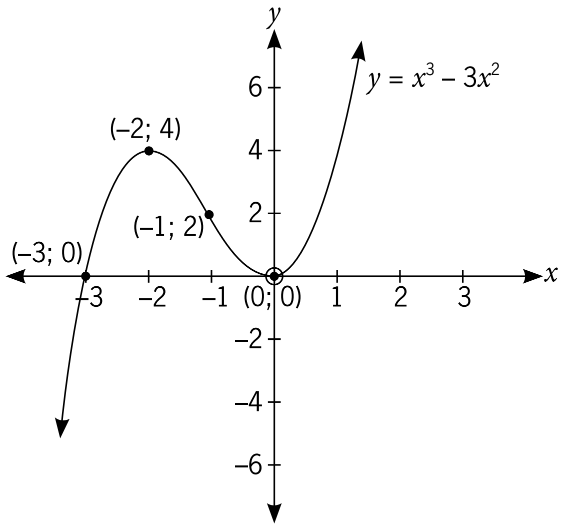
d) x-intercept: $y = 0$

$0 = x^2(x + 3)$

$\therefore\ x = 0$ or $x = -3$

y-intercept: $x = 0$

$\therefore\ y = 0$



1.2 a) $y = x^3 - 4x^2 + 4x$

$\frac{dy}{dx} = 3x^2 - 8x + 4$

$0 = 3x^2 - 8x + 4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore\ x = \frac{2}{3}$ or $x = 2$

$\therefore\ y = 1{,}19$ or $y = 0$

Thus, the turning points of $y = x^3\ 44x^2 + 4x$ are $\left(\frac{2}{3}; 1{,}19\right)$ and $(2; 0)$

b) $\frac{d^2y}{dx^2} = 6x - 8$

For $x = \frac{2}{3}$:

$\frac{d^2y}{dx^2} = 6\left(\frac{2}{3}\right) - 8 = -4 < 0 \therefore$ Maximum $\left(\frac{2}{3}; 1{,}19\right)$

For $x = 2$:

$\frac{d^2y}{dx^2} = 6(2) - 8 = 4 > 0 \therefore$ Minimum $(2; 0)$

c) $\frac{d^2y}{dx^2} = 6x - 8 = 0$

$6x = 8$

$x = \frac{4}{3}$

$\therefore\ y = \left(\frac{4}{3}\right)^3 - 4\left(\frac{4}{3}\right)^2 + 4\left(\frac{4}{3}\right)$

$= \frac{64}{27} - 4\left(\frac{16}{9}\right) + \frac{16}{3}$

$= 0{,}593$

Point of inflection: $\left(\frac{4}{3}; 0{,}593\right)$

d) x-intercepts: $y = 0$

$0 = x^3 - 4x^2 + 4x$

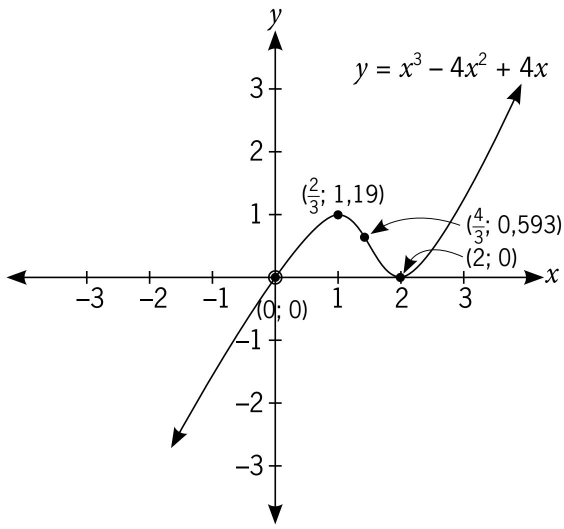
$0 = x(x^2 - 4x + 4)$

$0 = x(x - 2)(x - 2)$

$\therefore\ x = 0$ or $x = 2$

y-intercept: $x = 0$

$\therefore\ y = 0$



1.3 $y = x^3 - x^2 - 3x + 3$

a) $y = x^3 - x^2 - 3x + 3$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 - x^2 - 3x + 3)$$

$$= \frac{d}{dx}x^3 - \frac{d}{dx}x^2 - \frac{d}{dx}3x + \frac{d}{dx}3$$

$$= \frac{d}{dx}x^3 - \frac{d}{dx}x^2 - 3\frac{d}{dx}x + \frac{d}{dx}3$$

$$= 3x^2 - 2x - 3.1 + 0$$

$$\frac{dy}{dx} = 3x^2 - 2x - 3$$

But $\frac{dy}{dx} = 0$ at the turning points,

$3x^2 - 2x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-3)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{40}}{6}$$

$x = \frac{2 + \sqrt{40}}{6}$ and $x = \frac{2 - \sqrt{40}}{6}$

$x = 1{,}387$ $\quad$ $x = -0{,}721$

Substitute $x = -0{,}721$ and $x = 1{,}387$ into $y = x^3 - x^2 - 3x + 3$,

$y = x^3 - x^2 - 3x + 3$

$= (-0{,}721)^3 - (-0{,}721)^2 - 3(-0{,}721) + 3$

$y = 4{,}268$

$y = x^3 - x^2 - 3x + 3$

$= (1{,}387)^3 - (1{,}387)^2 - 3(1{,}387) + 3$

$y = -0{,}417$

Thus, the turning points of $y = x^3 - x^2 - 3x + 3$ are $(-0{,}721; 4{,}268)$ and $(1{,}387; -0{,}417)$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2 - 2x - 3)$$

$$= \frac{d}{dx}3x^2 - \frac{d}{dx}2x - \frac{d}{dx}3$$

$$= 3\frac{d}{dx}x^2 - 2\frac{d}{dx}x - \frac{d}{dx}3$$

$$= 3.2x - 2.1 - 0$$

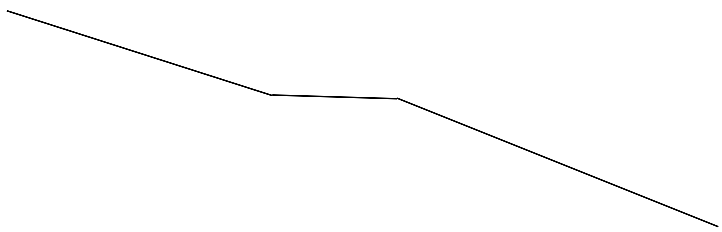
$$\frac{d^2y}{dx^2} = 6x - 2$$

b)

Stationary point	(–0,721; 4,268)	$\left(\frac{1}{3}; 1\frac{25}{27}\right)$	(1,387; –0.417)
Sign of $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = -6,325 < 0$	$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = +6,325 > 0$
Conclusion	Maximum	Test fails	Minimum

↓

$x = \frac{1}{3}$

Interval	$-0,721 < x < \frac{1}{3}$	$\frac{1}{3} < x < 1,387$
Test value	$x = 0$	$x = \frac{2}{3}$
Sign of $\frac{dy}{dx}$	$\frac{dy}{dx} = -3 < 0$	$\frac{dy}{dx} = -3 < 0$
Conclusion	Decreasing	Decreasing
Inflection point		

c) But $\frac{d^2y}{dx^2} = 0$ at the point of inflection,

$6x - 2 = 0$

$6x = 2$

$x = \frac{1}{3}$

Substitute $x = \frac{1}{3}$ into $y = x^3 - x^2 - 3x + 3$,

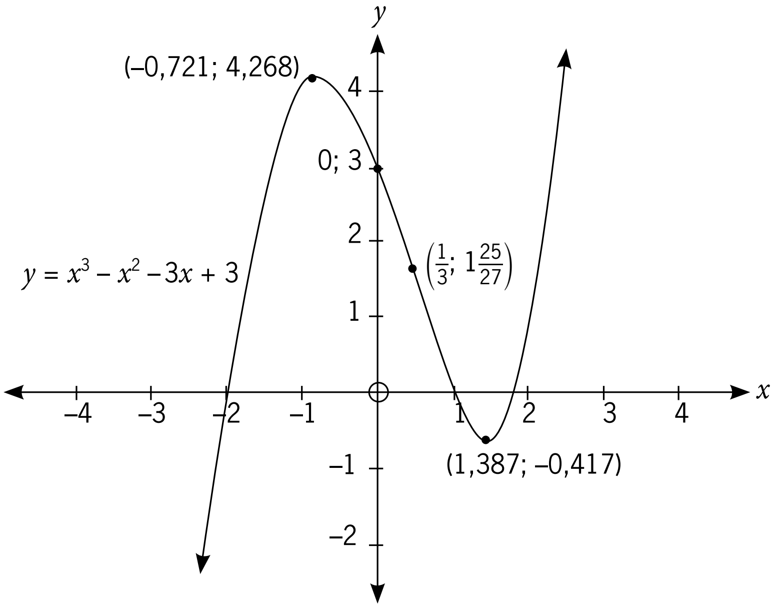
$y = x^3 - x^2 - 3x + 3$

$= \left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 3\left(\frac{1}{3}\right) + 3$

$y = 1\frac{25}{27}$

Thus, the point of inflection of $y = x^3 - x^2 - 3x + 3$ is $\left(\frac{1}{3}; 1\frac{25}{27}\right)$.

d)



1.4 $y = -x^3 - 6x^2 - 3x + 5$

a) $y = -x^3 - 6x^2 - 3x + 5$

$$\frac{dy}{dx} = \frac{d}{dx}(-x^3 - 6x^2 - 3x + 5)$$

$$= \frac{d}{dx}(-x^3) - \frac{d}{dx}6x^2 - \frac{d}{dx}3x + \frac{d}{dx}5$$

$$= -\frac{d}{dx}x^3 - 6\frac{d}{dx}x^2 - 3\frac{d}{dx}x + \frac{d}{dx}5$$

$$= -(3x^2) - 6.2x - 3.1 + 0$$

$$\frac{dy}{dx} = -3x^2 - 12x - 3$$

But $\frac{dy}{dx} = 0$ at the turning points,

$$-3x^2 - 12x - 3 = 0$$

$$x^2 + 4x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{12}}{2}$$

$x = \frac{-4 + \sqrt{12}}{2}$ and $x = \frac{-4 - \sqrt{12}}{2}$

$x = -0{,}268$ $\qquad$ $x = -3{,}732$

Substitute $x = -3{,}732$ and $x = -0{,}268$ into $y = -x^3 - 6x^2 - 3x + 5$,

$$y = -x^3 - 6x^2 - 3x + 5$$
$$= -(-3{,}732)^3 - 6(-3{,}732)^2 - 3(-3{,}732) + 5$$
$$y = -15{,}392$$
$$y = -x^3 - 6x^2 - 3x + 5$$
$$= -(-0{,}268)^3 - 6(-0{,}268)^2 - 3(-0{,}268) + 5$$
$$y = 5{,}392$$

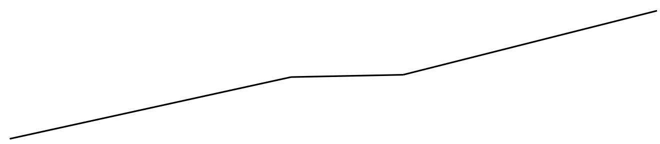
Thus, the turning points of $y = -x^3 - 6x^2 - 3x + 5$ are $(-3{,}732; -15{,}392)$ and $(-0{,}268; 5{,}392)$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-3x^2 - 12x - 3)$$
$$= \frac{d}{dx}(-3x^2) - \frac{d}{dx}12x - \frac{d}{dx}3$$
$$= -3\frac{d}{dx}x^2 - 12\frac{d}{dx}x - \frac{d}{dx}3$$
$$= -3.2x - 12.1 - 0$$
$$\frac{d^2y}{dx^2} = -6x - 12$$

b)

Stationary point	$(-3{,}732; -15{,}392)$	$(-2; -5)$	$(-0{,}268; 5{,}392)$
Sign of $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = +10{,}392 > 0$	$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = -10{,}392 < 0$
Conclusion	Minimum	Test fails	Maximum

$x = -2$

Interval	$-3{,}732 < x < -2$	$-2 < x < -0{,}268$
Test value	$x = -3$	$x = -1$
Sign of $\frac{dy}{dx}$	$\frac{dy}{dx} = +6 > 0$	$\frac{dy}{dx} = +6 > 0$
Conclusion	Increasing	Increasing
Inflection point		

c) But $\frac{d^2y}{dx^2} = 0$ at the point of inflection,

$-6x - 12 = 0$

$-6x = 12$

$x = -2$

Substitute $x = -2$ into $y = -x^3 - 6x^2 - 3x + 5$,

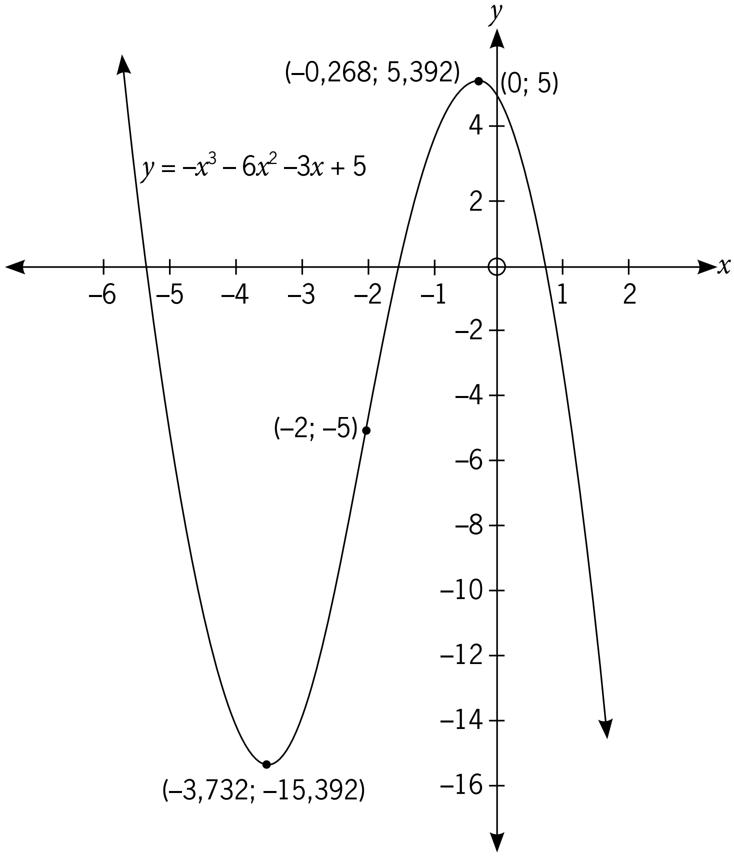
$y = -x^3 - 6x^2 - 3x + 5$

$= -(-2)^3 - 6(-2)^2 - 3(-2) + 5$

$y = -5$

Thus, the point of inflection of $y = -x^3 - 6x^2 - 3x + 5$ is $(-2; -5)$.

d)



1.5 $y = 2x^3 + x^2 - 4x - 3$

a) $y = 2x^3 + x^2 - 4x - 3$

$\frac{dy}{dx} = \frac{d}{dx}(2x^3 + x^2 - 4x - 3)$

$= \frac{d}{dx}2x^3 + \frac{d}{dx}x^2 - \frac{d}{dx}4x + \frac{d}{dx}3$

$= 2\frac{d}{dx}x^3 + \frac{d}{dx}x^2 - 4\frac{d}{dx}x + \frac{d}{dx}3$

$= 2.3x^2 + 2x - 4.1 - 0$

$\frac{dy}{dx} = 6x^2 + 2x - 4$

But $\frac{dy}{dx} = 0$ at the turning points,

$$6x^2 + 2x - 4 = 0$$

$$3x^2 + x - 2 = 0$$

$$(3x - 2)(x + 1) = 0$$

$$3x - 2 = 0 \quad \text{and} \quad x + 1 = 0$$

$$3x = 2 \qquad x = -1$$

$$x = \frac{2}{3}$$

Substitute $x = -1$ and $x = \frac{2}{3}$ into $y = 2x^3 + x^2 - 4x - 3$,

$$y = 2x^3 + x^2 - 4x - 3$$

$$= 2(-1)^3 + (-1)^2 - 4(-1) - 3$$

$$y = 0$$

$$y = 2x^3 + x^2 - 4x - 3$$

$$= 2\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) - 3$$

$$y = -4\frac{17}{27}$$

Thus, the turning points of $y = 2x^3 + x^2 - 4x - 3$ are $(-1; 0)$ and $\left(\frac{2}{3}; -4\frac{17}{27}\right)$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(6x^2 + 2x - 4)$$

$$= \frac{d}{dx}6x^2 + \frac{d}{dx}2x - \frac{d}{dx}4$$

$$= 6\frac{d}{dx}x^2 + 2\frac{d}{dx}x - \frac{d}{dx}4$$

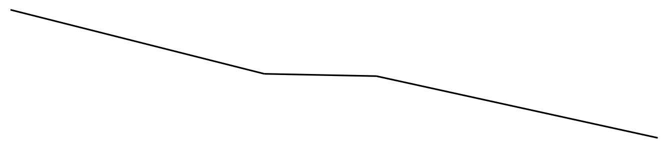
$$= 6.2x + 2.1 - 0$$

$$\frac{d^2y}{dx^2} = 12x + 2$$

b)

Stationary point	$(-1; 0)$	$\left(-\frac{1}{6}; -2\frac{17}{54}\right)$	$\left(\frac{2}{3}; -4\frac{17}{27}\right)$
Sign of $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = -10 < 0$	$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = +10 > 0$
Conclusion	Maximum	Test fails	Minimum

$x = -\frac{1}{6}$

Interval	$-1 < x < -\frac{1}{6}$	$-\frac{1}{6} < x < \frac{2}{3}$
Test value	$x = -\frac{1}{2}$	$x = \frac{1}{6}$
Sign of $\frac{dy}{dx}$	$\frac{dy}{dx} = -3\frac{1}{2} < 0$	$\frac{dy}{dx} = -3\frac{1}{2} < 0$
Conclusion	Decreasing	Decreasing
Inflection point		

c) But $\frac{d^2y}{dx^2} = 0$ at the point of inflection,

$12x + 2 = 0$

$12x = -2$

$x = -\frac{1}{6}$

Substitute $x = -\frac{1}{6}$ into $y = 2x^3 + x^2 - 4x - 3$,

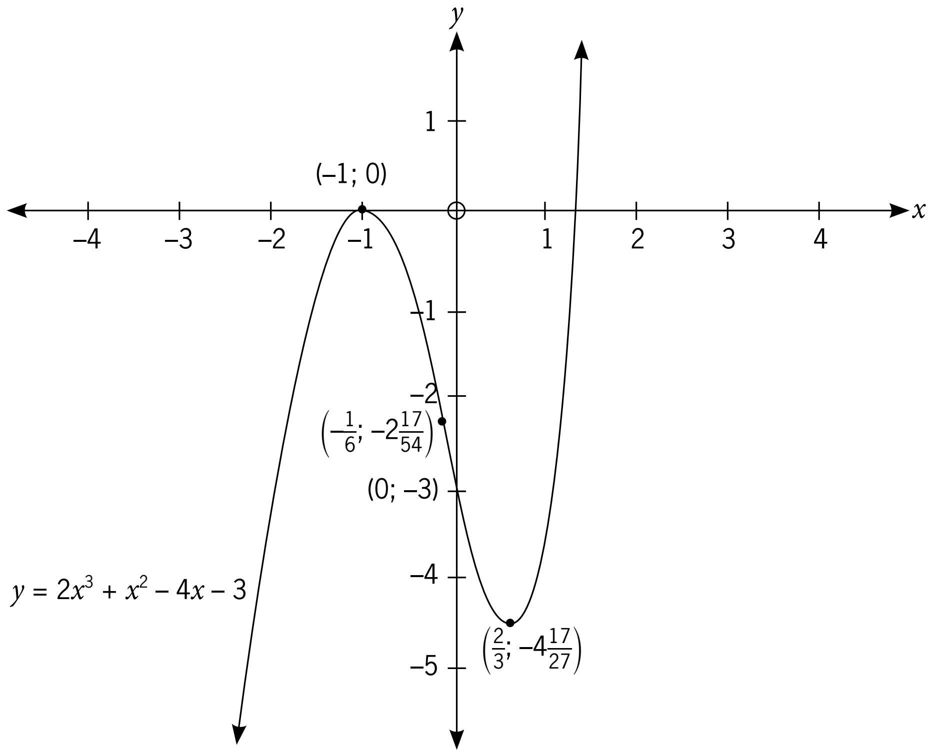
$y = 2x^3 + x^2 - 4x - 3$

$= 2\left(-\frac{1}{6}\right)^3 + \left(-\frac{1}{6}\right)^2 - 4\left(-\frac{1}{6}\right) - 3$

$y = -2\frac{17}{54}$

Thus, the point of inflection of $y = 2x^3 + x^2 - 4x - 3$ is $\left(-\frac{1}{6}; -2\frac{17}{54}\right)$.

d)



1.6 $y = -3x^3 + 11x^2 - 8x - 4$

a) $y = -3x^3 + 11x^2 - 8x - 4$

$$\frac{dy}{dx} = \frac{d}{dx}(-3x^3 + 11x^2 - 8x - 4)$$
$$= \frac{d}{dx}(-3x^3) + \frac{d}{dx}11x^2 - \frac{d}{dx}8x - \frac{d}{dx}4$$
$$= -3\frac{d}{dx}x^3 + 11\frac{d}{dx}x^2 - 8\frac{d}{dx}x - \frac{d}{dx}4$$
$$= -3.3x^2 + 11.2x - 8.1 - 0$$
$$\frac{dy}{dx} = -9x^2 + 22x - 8$$

But $\frac{dy}{dx} = 0$ at the turning points,

$$-9x^2 + 22x - 8 = 0$$
$$9x^2 - 22x + 8 = 0$$
$$(9x - 4)(x - 2) = 0$$

$9x - 4 = 0$ and $x - 2 = 0$

$9x = 4$ $x = 2$

$x = \frac{4}{9}$

Substitute $x = \frac{4}{9}$ and $x = 2$ into $y = -3x^3 + 11x^2 - 8x - 4$,

$$y = -3x^3 + 11x^2 - 8x - 4$$

$$= -3\left(\frac{4}{9}\right)^3 + 11\left(\frac{4}{9}\right)^2 - 8\left(\frac{4}{9}\right) - 4$$

$$y = -5\frac{157}{243}$$

$$y = -3x^3 + 11x^2 - 8x - 4$$

$$= -3(2)^3 + 11(2)^2 - 8(2) - 4$$

$$y = 0$$

Thus, the turning points of $y = -3x^3 + 11x^2 - 8x - 4$ are $\left(\frac{4}{9}; -5\frac{157}{243}\right)$ and $(2; 0)$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-9x^2 + 22x - 8)$$

$$= \frac{d}{dx}(-9x^2) + \frac{d}{dx}22x - \frac{d}{dx}8$$

$$= -9\frac{d}{dx}x^2 + 22\frac{d}{dx}x - \frac{d}{dx}8$$

$$= -9.2x + 22.1 - 0$$

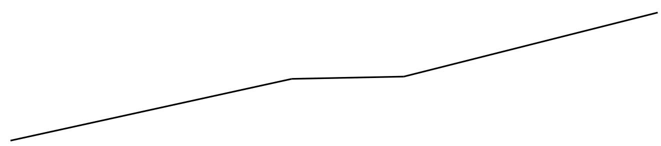
$$\frac{d^2y}{dx^2} = -18x + 22$$

b)

Stationary point	$\left(\frac{4}{9}; -5\frac{157}{243}\right)$	$\left(1\frac{2}{9}; -2\frac{200}{243}\right)$	$(2; 0)$
Sign of $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = +14 > 0$	$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = -14 < 0$
Conclusion	Minimum	Test fails	Maximum

↓

$x = 1\frac{2}{9}$

Interval	$\frac{4}{9} < x < 1\frac{2}{9}$	$1\frac{2}{9} < x < 2$
Test value	$x = 1$	$x = 1\frac{4}{9}$
Sign of $\frac{dy}{dx}$	$\frac{dy}{dx} = +5 > 0$	$\frac{dy}{dx} = +5 > 0$
Conclusion	Increasing	Increasing
Inflection point		

c) But $\frac{d^2y}{dx^2} = 0$ at the point of inflection,

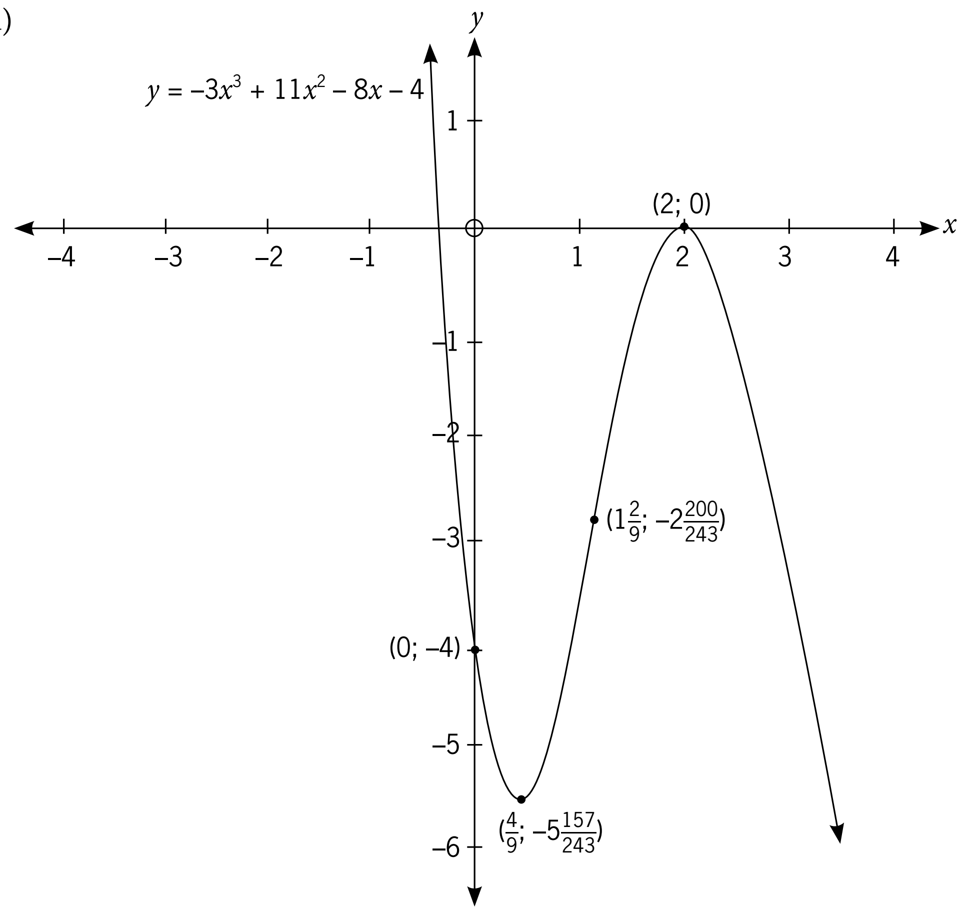
$$-18x + 22 = 0$$
$$-18x = -22$$
$$x = \frac{11}{9}$$
$$x = 1\tfrac{2}{9}$$

Substitute $x = \frac{11}{9}$ into $y = -3x^3 + 11x^2 - 8x - 4$,

$$y = -3x^3 + 11x^2 - 8x - 4$$
$$= -3\left(\frac{11}{9}\right)^3 + 11\left(\frac{11}{9}\right)^2 - 8\left(\frac{11}{9}\right) - 4$$
$$y = -2\tfrac{200}{243}$$

Thus, the point of inflection of $y = -3x^3 + 11x^2 - 8x - 4$ is $\left(1\tfrac{2}{9}; -2\tfrac{200}{243}\right)$.

d)



1.7 $y = x^3 - x^2 - 8x + 12$

a) $y = x^3 - x^2 - 8x + 12$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 - x^2 - 8x + 12)$$

$$= \frac{d}{dx}x^3 - \frac{d}{dx}x^2 - \frac{d}{dx}8x + \frac{d}{dx}12$$

$$= \frac{d}{dx}x^3 - \frac{d}{dx}x^2 - 8\frac{d}{dx}x + \frac{d}{dx}12$$

$$= 3x^2 - 2x - 8.1 + 0$$

$$\frac{dy}{dx} = 3x^2 - 2x - 8$$

But $\frac{dy}{dx} = 0$ at the turning points,

$$3x^2 - 2x - 8 = 0$$

$$(3x + 4)(x - 2) = 0$$

$3x + 4 = 0$ and $x - 2 = 0$

$3x = -4$ $\quad$ $x = 2$

$x = -\frac{4}{3}$

$x = -1\frac{1}{3}$

Substitute $x = -\frac{4}{3}$ and $x = 2$ into $y = x^3 - x^2 - 8x + 12$,

$$y = x^3 - x^2 - 8x + 12$$

$$= \left(-\frac{4}{3}\right)^3 - \left(-\frac{4}{3}\right)^2 - 8\left(-\frac{4}{3}\right) + 12$$

$$y = 18\frac{14}{27}$$

$$y = x^3 - x^2 - 8x + 12$$

$$= (2)^3 - (2)^2 - 8(2) + 12$$

$$y = 0$$

Thus, the turning points of $y = x^3 - x^2 - 8x + 12$ are $\left(-1\frac{1}{3}; 18\frac{14}{27}\right)$ and $(2; 0)$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2 - 2x - 8)$$

$$= \frac{d}{dx}3x^2 - \frac{d}{dx}2x - \frac{d}{dx}8$$

$$= 3\frac{d}{dx}x^2 - 2\frac{d}{dx}x - \frac{d}{dx}8$$

$$= 3.2x - 2.1 - 0$$

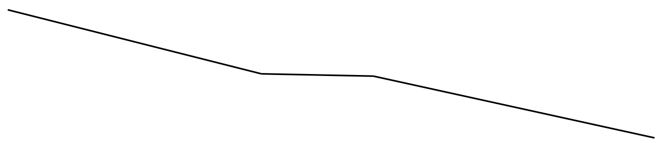
$$\frac{d^2y}{dx^2} = 6x - 2$$

b)

Stationary point	$\left(-1\frac{1}{3}; 18\frac{14}{27}\right)$	$\left(\frac{1}{3}; 9\frac{7}{27}\right)$	$(2; 0)$
Sign of $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = -10 < 0$	$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = +10 > 0$
Conclusion	Maximum	Test fails	Minimum

↓

$x = \frac{1}{3}$

Interval	$-1\frac{1}{3} < x < \frac{1}{3}$	$\frac{1}{3} < x < 2$
Test value	$x = -\frac{1}{3}$	$x = 1$
Sign of $\frac{dy}{dx}$	$\frac{dy}{dx} = -7 < 0$	$\frac{dy}{dx} = -7 < 0$
Conclusion	Decreasing	Decreasing
Inflection point		

c) But $\frac{d^2y}{dx^2} = 0$ at the point of inflection,

$6x - 2 = 0$

$6x = 2$

$x = \frac{1}{3}$

Substitute $x = \frac{1}{3}$ into $y = x^3 - x^2 - 8x + 12$,

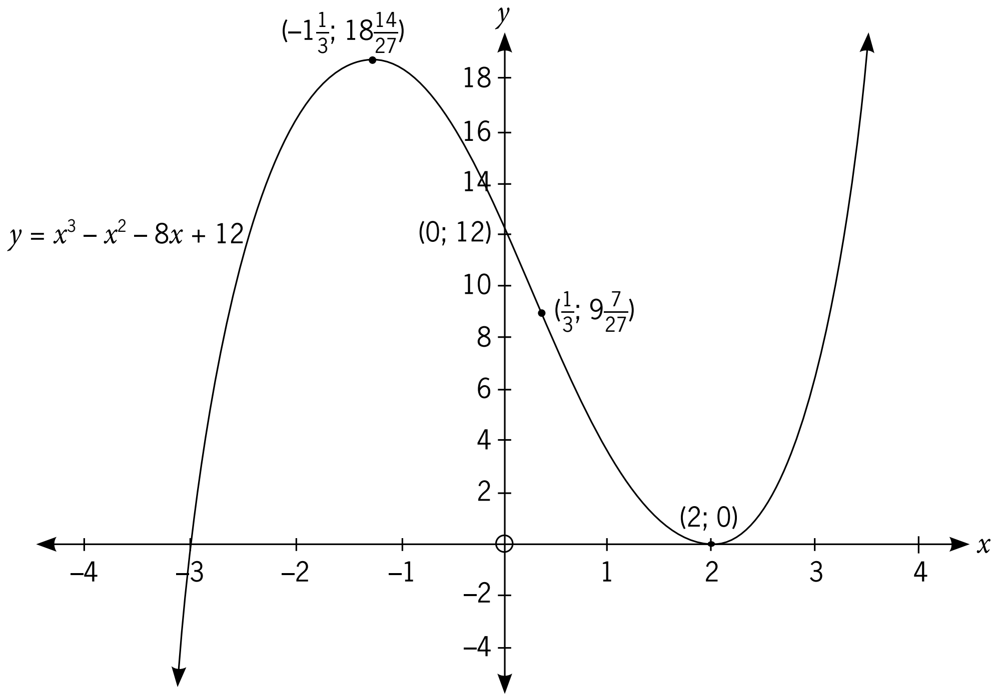
$y = x^3 - x^2 - 8x + 12$

$= \left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 8\left(\frac{1}{3}\right) + 12$

$y = 9\frac{7}{27}$

Thus, the point of inflection of $y = x^3 - x^2 - 8x + 12$ is $\left(\frac{1}{3}; 9\frac{7}{27}\right)$.

d)



1.8 $y = -x^3 - 2x^2 + 15x + 36$

a) $y = -x^3 - 2x^2 + 15x + 36$

$$\frac{dy}{dx} = \frac{d}{dx}(-x^3 - 2x^2 + 15x + 36)$$
$$= \frac{d}{dx}(-x^3) - \frac{d}{dx}2x^2 + \frac{d}{dx}15x + \frac{d}{dx}36$$
$$= -\frac{d}{dx}x^3 - 2\frac{d}{dx}x^2 + 15\frac{d}{dx}x + \frac{d}{dx}36$$
$$= -(3x^2) - 2.2x + 15.1 + 0$$
$$\frac{dy}{dx} = -3x^2 - 4x + 15$$

But $\frac{dy}{dx} = 0$ at the turning points,

$$-3x^2 - 4x + 15 = 0$$
$$3x^2 + 4x - 15 = 0$$
$$(3x - 5)(x + 3) = 0$$

$3x - 5 = 0$ and $x + 3 = 0$

$3x = 5$ $\quad$ $x = -3$

$x = \frac{5}{3}$

$x = 1\frac{2}{3}$

Substitute $x = -3$ and $x = \frac{5}{3}$ into $y = -x^3 - 2x^2 + 15x + 36$,

$$y = -x^3 - 2x^2 + 15x + 36$$
$$= -(-3)^3 - 2(-3)^2 + 15(-3) + 36$$
$$y = 0$$
$$y = -x^3 - 2x^2 + 15x + 36$$
$$= -\left(\frac{5}{3}\right)^3 - 2\left(\frac{5}{3}\right)^2 + 15\left(\frac{5}{3}\right) + 36$$
$$y = 50\frac{22}{27}$$

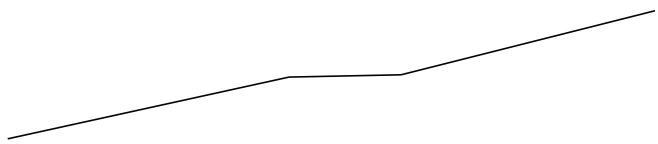
Thus, the turning points of $y = -x^3 - 2x^2 + 15x + 36$ are $(-3; 0)$ and $\left(1\frac{2}{3}; 50\frac{22}{27}\right)$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-3x^2 - 4x + 15)$$
$$= \frac{d}{dx}(-3x^2) - \frac{d}{dx}4x + \frac{d}{dx}15$$
$$= -3\frac{d}{dx}x^2 - 4\frac{d}{dx}x + \frac{d}{dx}15$$
$$= -3.2x - 4.1 + 0$$
$$\frac{d^2y}{dx^2} = -6x - 4$$

b)

Stationary point	$(-3; 0)$	$\left(-\frac{2}{3}; 25\frac{11}{27}\right)$	$\left(1\frac{2}{3}; 50\frac{22}{27}\right)$
Sign of $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = +14 > 0$	$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = -14 < 0$
Conclusion	Minimum	Test fails	Maximum

$x = -\frac{2}{3}$

Interval	$-3 < x < -\frac{2}{3}$	$-\frac{2}{3} < x < 1\frac{2}{3}$
Test value	$x = -1$	$x = -\frac{1}{3}$
Sign of $\frac{dy}{dx}$	$\frac{dy}{dx} = +16 > 0$	$\frac{dy}{dx} = +16 > 0$
Conclusion	Increasing	Increasing
Inflection point		

c) But $\frac{d^2y}{dx^2} = 0$ at the point of inflection,

$-6x - 4 = 0$

$-6x = 4$

$x = -\frac{2}{3}$

Substitute $x = -\frac{2}{3}$ into $y = -x^3 - 2x^2 + 15x + 36$,

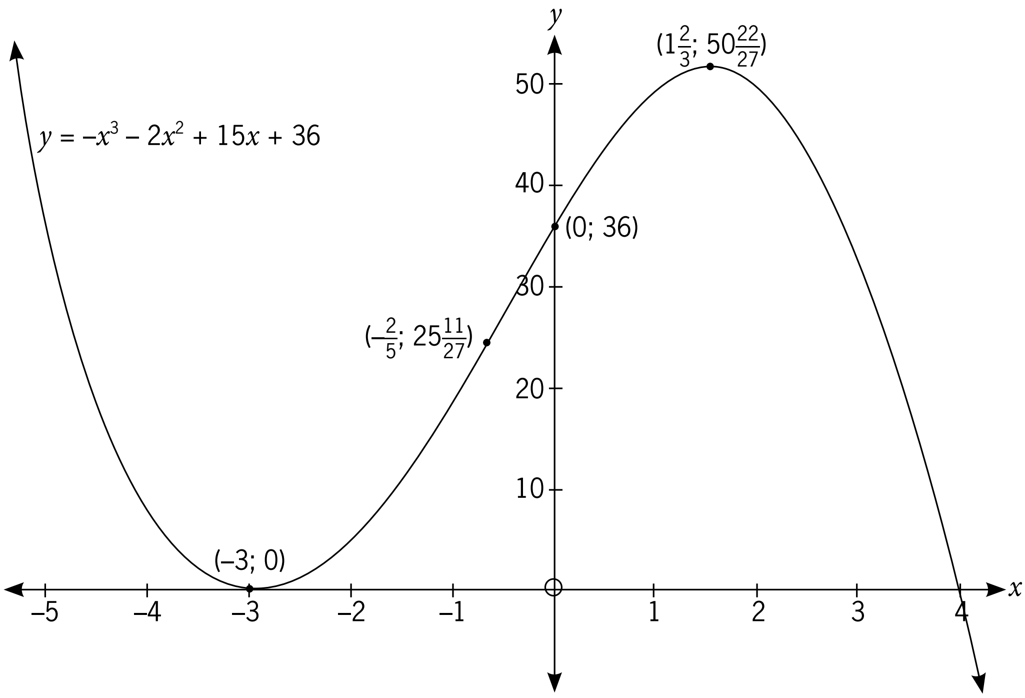
$y = -x^3 - 2x^2 + 15x + 36$

$= -\left(-\frac{2}{3}\right)^3 - 2\left(-\frac{2}{3}\right)^2 + 15\left(-\frac{2}{3}\right) + 36$

$y = 25\frac{11}{27}$

Thus, the point of inflection of $y = -x^3 - 2x^2 + 15x + 36$ is $\left(-\frac{2}{3}; 25\frac{11}{27}\right)$.

d)



1.9 $y = 4x^3 - 8x^2 - 7x + 17$

a) $y = 4x^3 - 8x^2 - 7x + 17$

$\frac{dy}{dx} = \frac{d}{dx}(4x^3 - 8x^2 - 7x + 17)$

$= \frac{d}{dx}4x^3 - \frac{d}{dx}8x^2 - \frac{d}{dx}7x + \frac{d}{dx}17$

$= 4\frac{d}{dx}x^3 - 8\frac{d}{dx}x^2 - 7\frac{d}{dx}x + \frac{d}{dx}17$

$= 4.3x^2 - 8.2x - 7.1 + 0$

$\frac{dy}{dx} = 12x^2 - 16x - 7$

But $\frac{dy}{dx} = 0$ at the turning points,

$12x^2 - 16x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-16) \pm \sqrt{(-16)^2 - 4(12)(-7)}}{2(12)}$$

$$= \frac{16 \pm \sqrt{592}}{24}$$

$$x = \frac{16 + \sqrt{592}}{24} \quad \text{and} \quad x = \frac{16 - \sqrt{592}}{24}$$

$$x = 1{,}681 \qquad\qquad x = -0{,}347$$

Substitute $x = -0{,}347$ and $x = 1{,}681$ into $y = 4x^3 - 8x^2 - 7x + 17$,

$y = 4x^3 - 8x^2 - 7x + 17$

$= 4(-0{,}347)^3 - 8(-0{,}347)^2 - 7(-0{,}347) + 17$

$y = 18{,}299$

$y = 4x^3 - 8x^2 - 7x + 17$

$= 4(1{,}681)^3 - 8(1{,}681)^2 - 7(1{,}681) + 17$

$y = 1{,}627$

Thus, the turning points of $y = 4x^3 - 8x^2 - 7x + 17$ are $(-0{,}347; 18{,}299)$ and $(1{,}681; 1{,}627)$.

$\frac{d^2y}{dx^2} = \frac{d}{dx}(12x^2 - 16x - 7)$

$= \frac{d}{dx}12x^2 - \frac{d}{dx}16x - \frac{d}{dx}7$

$= 12\frac{d}{dx}x^2 - 16\frac{d}{dx}x - \frac{d}{dx}7$

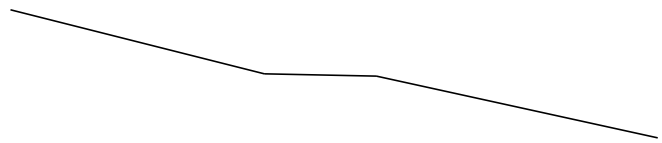
$= 12.2x - 16.1 - 0$

$\frac{d^2y}{dx^2} = 24x - 16$

b)

Stationary point	$(-0{,}347;\ 18{,}299)$	$\left(\frac{2}{3};\ 9\frac{26}{27}\right)$	$(1{,}681;\ 1{,}627)$
Sign of $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = -24{,}331 < 0$	$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = +24{,}331 > 0$
Conclusion	Maximum	Test fails	Minimum

$x = \frac{2}{3}$

Interval	$-0{,}347 < x < \frac{2}{3}$	$\frac{2}{3} < x < 1{,}681$
Test value	$x = \frac{1}{3}$	$x = 1$
Sign of $\frac{dy}{dx}$	$\frac{dy}{dx} = -11 < 0$	$\frac{dy}{dx} = -11 < 0$
Conclusion	Decreasing	Decreasing
Inflection point		

c) But $\frac{d^2y}{dx^2} = 0$ at the point of inflection,

$24x - 16 = 0$

$24x = 16$

$x = \frac{2}{3}$

Substitute $x = \frac{2}{3}$ into $y = 4x^3 - 8x^2 - 7x + 17$,

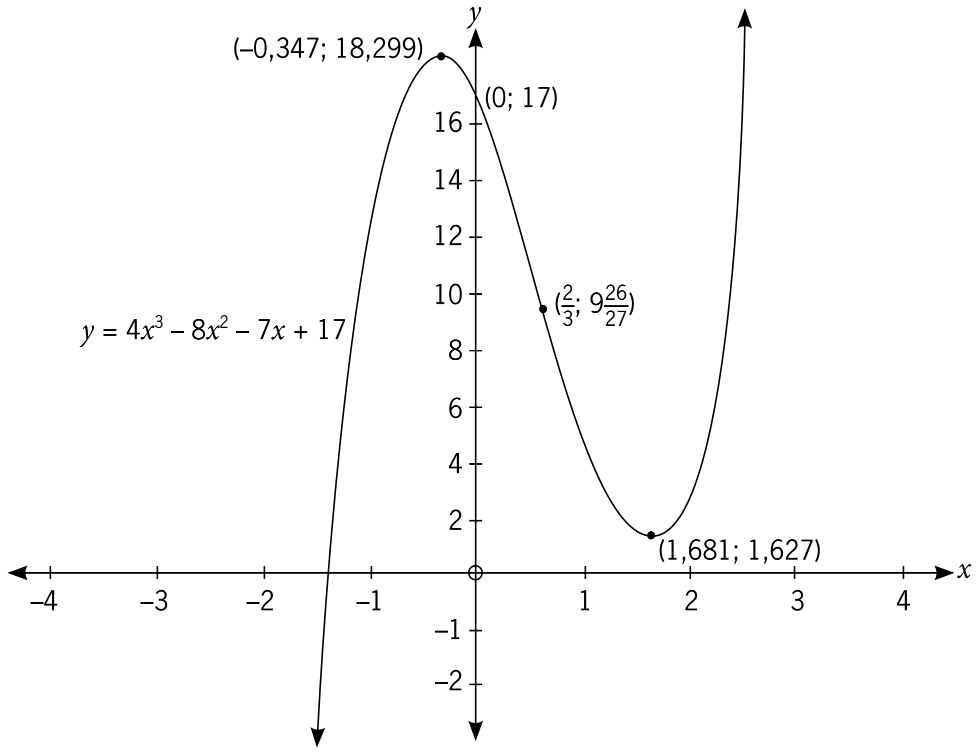
$y = 4x^3 - 8x^2 - 7x + 17$

$= 4\left(\frac{2}{3}\right)^3 - 8\left(\frac{2}{3}\right)^2 - 7\left(\frac{2}{3}\right) + 17$

$y = 9\frac{26}{27}$

Thus, the point of inflection of $y = 4x^3 - 8x^2 - 7x + 17$ is $\left(\frac{2}{3};\ 9\frac{26}{27}\right)$.

d)



1.10 $y = -5x^3 + 9x^2 - 2x + 8$

a) $y = -5x^3 + 9x^2 - 2x + 8$

$$\frac{dy}{dx} = \frac{d}{dx}(-5x^3 + 9x^2 - 2x + 8)$$

$$= \frac{d}{dx}(-5x^3) + \frac{d}{dx}9x^2 - \frac{d}{dx}2x + \frac{d}{dx}8$$

$$= -5\frac{d}{dx}x^3 + 9\frac{d}{dx}x^2 - 2\frac{d}{dx}x + \frac{d}{dx}8$$

$$= -5.3x^2 + 9.2x - 2.1 + 0$$

$$\frac{dy}{dx} = -15x^2 + 18x - 2$$

But $\frac{dy}{dx} = 0$ at the turning points,

$$-15x^2 + 18x - 2 = 0$$

$$15x^2 - 18x + 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-18) \pm \sqrt{(-18)^2 - 4(15)(2)}}{2(15)}$$

$$x = \frac{18 \pm \sqrt{204}}{30}$$

$x = \frac{18 + \sqrt{204}}{30}$ and $x = \frac{18 - \sqrt{204}}{30}$

$x = 1{,}076$ $\quad$ $x = 0{,}124$

Substitute $x = 0{,}124$ and $x = 1{,}076$ into $y = -5x^3 + 9x^2 - 2x + 8$,

$$y = -5x^3 + 9x^2 - 2x + 8$$
$$= -5(0{,}124)^3 - 9(0{,}124)^2 - 2(0{,}124) + 8$$
$$y = 7{,}881$$
$$y = -5x^3 + 9x^2 - 2x + 8$$
$$= -5(1{,}076)^3 - 9(1{,}076)^2 - 2(1{,}076) + 8$$
$$y = 10{,}039$$

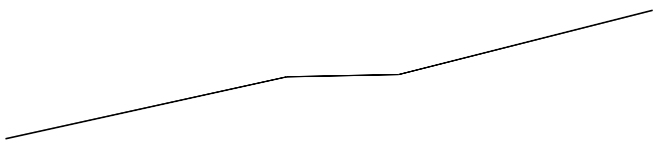
Thus, the turning points of $y = -5x^3 + 9x^2 - 2x + 8$ are (0,124; 7,881) and (1,076; 10,039).

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-15x^2 + 18x - 2)$$
$$= \frac{d}{dx}(-15x^2) + \frac{d}{dx}18x - \frac{d}{dx}2$$
$$= -15\frac{d}{dx}x^2 + 18\frac{d}{dx}x - \frac{d}{dx}2$$
$$= -15.2x + 18.1 - 0$$
$$\frac{d^2y}{dx^2} = -30x + 18$$

b)

Stationary point	(0,124; 7,881)	$\left(\frac{3}{5}; 8\frac{24}{25}\right)$	(1,076; 10,039)
Sign of $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = +14{,}283 > 0$	$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = -14{,}283 < 0$
Conclusion	Minimum	Test fails	Maximum

$x = \frac{3}{5}$

Interval	$0{,}124 < x < \frac{3}{5}$	$\frac{3}{5} < x < 1{,}076$
Test value	$x = \frac{2}{5}$	$x = \frac{4}{5}$
Sign of $\frac{dy}{dx}$	$\frac{dy}{dx} = +2{,}800 > 0$	$\frac{dy}{dx} = +2{,}800 > 0$
Conclusion	Increasing	Increasing
Inflection point		

c) But $\frac{d^2y}{dx^2} = 0$ at the point of inflection,

$-30x + 18 = 0$

$-30x = -18$

$x = \frac{3}{5}$

Substitute $x = \frac{3}{5}$ into $y = -5x^3 + 9x^2 - 2x + 8$,

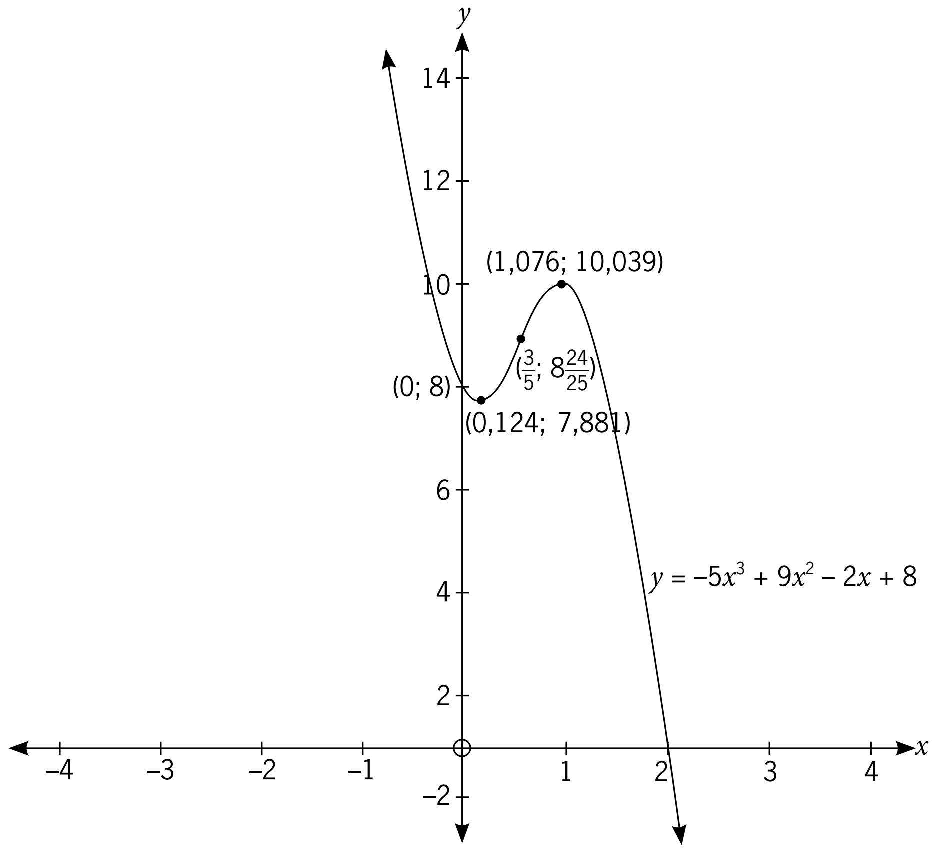
$y = -5x^3 + 9x^2 - 2x + 8$

$= -5\left(\frac{3}{5}\right)^3 + 9\left(\frac{3}{5}\right)^2 - 2\left(\frac{3}{5}\right) + 8$

$y = 8\frac{24}{25}$

Thus, the point of inflection of $y = -5x^3 + 9x^2 - 2x + 8$ is $\left(\frac{3}{5}; 8\frac{24}{25}\right)$.

d)



1.11 $f(x) = x^3 + 2x^2 - 10x - 30$

a) $y = x^3 + 2x^2 - 10x - 30$

$\frac{dy}{dx} = \frac{d}{dx}(x^3 + 2x^2 - 10x - 30)$

$= \frac{d}{dx}x^3 + \frac{d}{dx}2x^2 - \frac{d}{dx}10x - \frac{d}{dx}30$

$= \frac{d}{dx}x^3 + 2\frac{d}{dx}x^2 - 10\frac{d}{dx}x - \frac{d}{dx}30$

$= 3x^2 + 2.2x - 10.1 - 0$

$\frac{dy}{dx} = 3x^2 + 4x - 10$

But $\frac{dy}{dx} = 0$ at the turning points,

$3x^2 + 4x - 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(3)(-10)}}{2(3)}$$

$$x = \frac{-4 \pm \sqrt{136}}{6}$$

$$x = \frac{-4 + \sqrt{136}}{6} \quad \text{and} \quad x = \frac{-4 - \sqrt{136}}{6}$$

$$x = 1{,}277 \qquad\qquad x = -2{,}610$$

Substitute $x = -2{,}610$ and $x = 1{,}277$ into $y = x^3 + 2x^2 - 10x - 30$,

$y = x^3 + 2x^2 - 10x - 30$

$= (-2{,}610)^3 + 2(-2{,}610)^2 - 10(-2{,}610) - 30$

$y = -8{,}055$

$y = x^3 + 2x^2 - 10x - 30$

$= (1{,}277)^3 + 2(1{,}277)^2 - 10(1{,}277) - 30$

$y = -37{,}426$

Thus, the turning points of $y = x^3 + 2x^2 - 10x - 30$ are $(-2{,}610; -8{,}055)$ and $(1{,}277; -37{,}426)$.

$\frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2 + 4x - 10)$

$= \frac{d}{dx}3x^2 + \frac{d}{dx}4x - \frac{d}{dx}10$

$= 3\frac{d}{dx}x^2 + 4\frac{d}{dx}x - \frac{d}{dx}10$

$= 3.2x + 4.1 - 0$

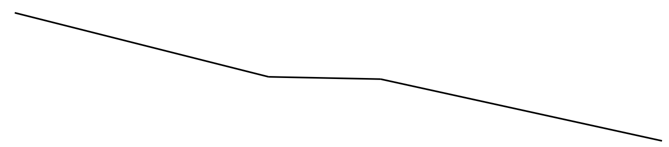
$\frac{d^2y}{dx^2} = 6x + 4$

b)

Stationary point	(−2,610; −8,055)	$\left(-\frac{2}{3}; -22\frac{20}{27}\right)$	(1,277; −37,426)
Sign of $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = -11{,}622 < 0$	$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = +11{,}622 > 0$
Conclusion	Maximum	Test fails	Minimum

↓

$x = -\frac{2}{3}$

Interval	$-2{,}610 < x < -\frac{2}{3}$	$-\frac{2}{3} < x < 1{,}277$
Test value	$x = -1$	$x = -\frac{1}{3}$
Sign of $\frac{dy}{dx}$	$\frac{dy}{dx} = -11 < 0$	$\frac{dy}{dx} = -11 < 0$
Conclusion	Decreasing	Decreasing
Inflection point		

c) But $\frac{d^2y}{dx^2} = 0$ at the point of inflection,

$6x + 4 = 0$

$6x = -4$

$x = -\frac{2}{3}$

Substitute $x = -\frac{2}{3}$ into $y = x^3 + 2x^2 - 10x - 30$,

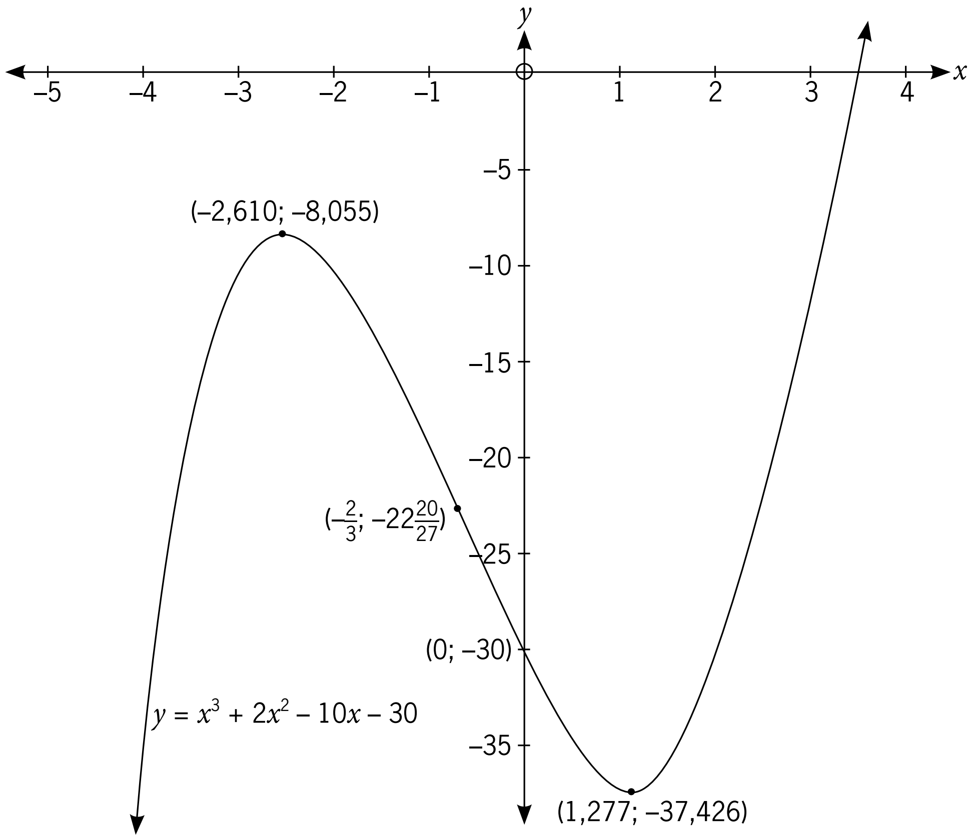
$y = x^3 + 2x^2 - 10x - 30$

$= \left(-\frac{2}{3}\right)^3 + 2\left(-\frac{2}{3}\right)^2 - 10\left(-\frac{2}{3}\right) - 30$

$y = -22\frac{20}{27}$

Thus, the point of inflection of $y = x^3 + 2x^2 - 10x - 30$ is $\left(-\frac{2}{3}; -22\frac{20}{27}\right)$.

d)



1.12 $f(x) = -x^3 - 4x^2 - 3x - 2$

a) $y = -x^3 - 4x^2 - 3x - 2$

$\frac{dy}{dx} = \frac{d}{dx}(-x^3 - 4x^2 - 3x - 2)$

$= \frac{d}{dx}(-x^3) - \frac{d}{dx}4x^2 - \frac{d}{dx}3x - \frac{d}{dx}2$

$= -\frac{d}{dx}x^3 - 4\frac{d}{dx}x^2 - 3\frac{d}{dx}x - \frac{d}{dx}2$

$= -3x^2 - 4.2x - 3.1 - 0$

$\frac{dy}{dx} = -3x^2 - 8x - 3$

But $\frac{dy}{dx} = 0$ at the turning points,

$-3x^2 - 8x - 3 = 0$

$3x^2 + 8x + 3 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(8) \pm \sqrt{(8)^2 - 4(3)(3)}}{2(3)}$

$x = \frac{-8 \pm \sqrt{28}}{6}$

$x = \frac{-8 + \sqrt{28}}{6}$ and $x = \frac{-8 - \sqrt{28}}{6}$

$x = -0,451$ $x = -2,215$

Substitute $x = -2{,}215$ and $x = -0{,}451$ into $y = -x^3 - 4x^2 - 3x - 2$

$$y = -x^3 - 4x^2 - 3x - 2$$
$$= -(-2{,}215)^3 - 4(-2{,}215)^2 - 3(-2{,}215) - 2$$
$$y = -4{,}113$$
$$y = -x^3 - 4x^2 - 3x - 2$$
$$= -(-0{,}451)^3 - 4(-0{,}451)^2 - 3(-0{,}451) - 2$$
$$y = -1{,}369$$

Thus, the turning points of $y = -x^3 - 4x^2 - 3x - 2$ are $(-2{,}215; -4{,}113)$ and $(-0{,}451; -1{,}369)$.

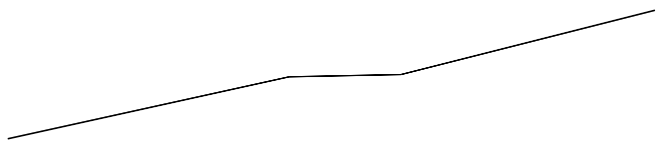
$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-3x^2 - 8x - 3)$$
$$= \frac{d}{dx}(-3x^2) - \frac{d}{dx}8x - \frac{d}{dx}3$$
$$= -3\frac{d}{dx}x^2 - 8\frac{d}{dx}x - \frac{d}{dx}3$$
$$= -3.2x - 8.1 - 0$$
$$\frac{d^2y}{dx^2} = -6x - 8$$

b)

Stationary point	$(-2{,}215; -4{,}113)$	$\left(-1\frac{1}{3}; -2\frac{20}{27}\right)$	$(-0{,}451; -1{,}369)$
Sign of $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = +5{,}292 > 0$	$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = -5{,}292 < 0$
Conclusion	Minimum	Test fails	Maximum

↓

$x = -1\frac{1}{3}$

Interval	$-2{,}215 < x < -1\frac{1}{3}$	$-1\frac{1}{3} < x < -0{,}451$
Test value	$x = -1\frac{2}{3}$	$x = -1$
Sign of $\frac{dy}{dx}$	$\frac{dy}{dx} = +2 > 0$	$\frac{dy}{dx} = +2 > 0$
Conclusion	Increasing	Increasing
Inflection point		

c) But $\frac{d^2y}{dx^2} = 0$ at the point of inflection,

$-6x - 8 = 0$

$-6x = 8$

$x = -\frac{4}{3}$

$x = -1\frac{1}{3}$

Substitute $x = -\frac{4}{3}$ into $y = -x^3 - 4x^2 - 3x - 2$,

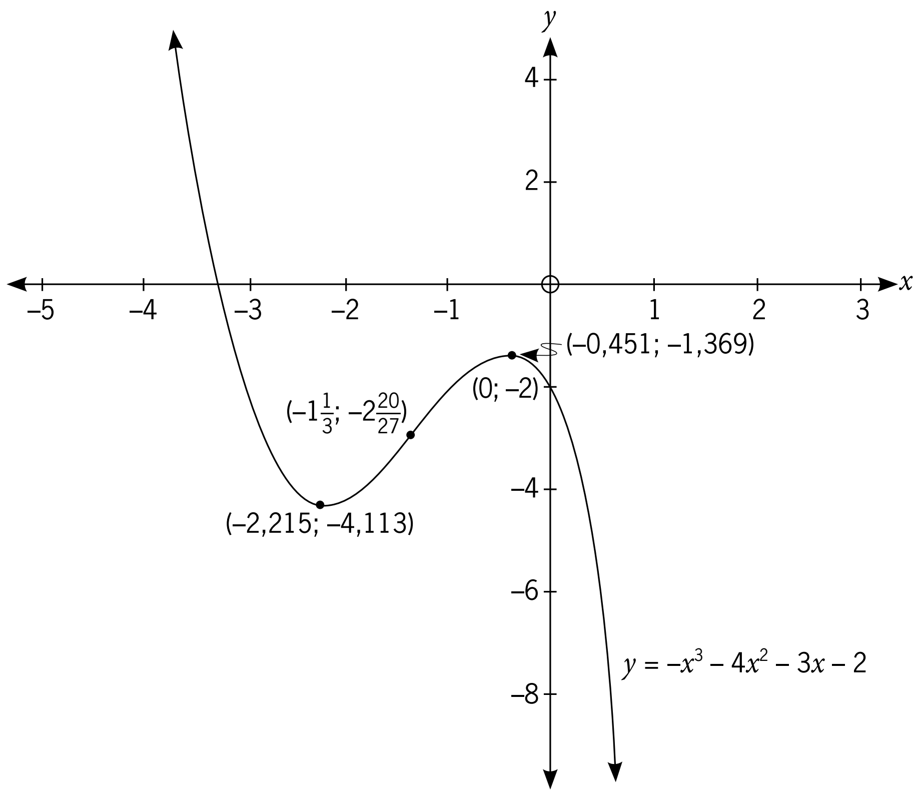
$y = -x^3 - 4x^2 - 3x - 2$

$= -\left(-\frac{4}{3}\right)^3 - 4\left(-\frac{4}{3}\right)^2 - 3\left(-\frac{4}{3}\right) - 2$

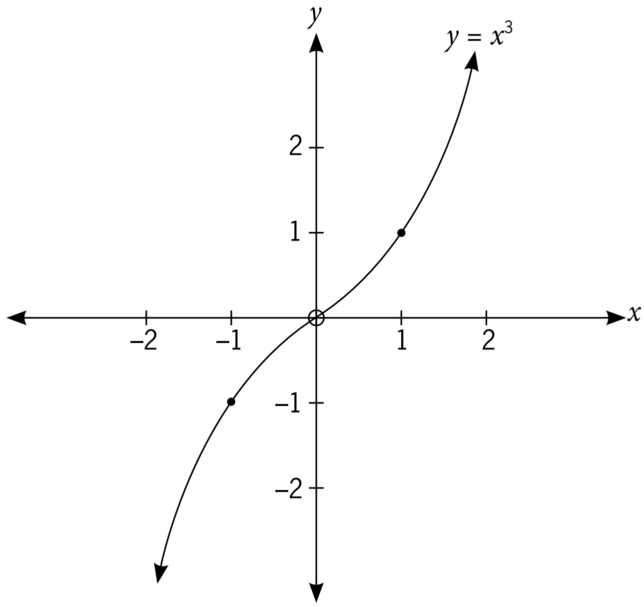
$y = -2\frac{20}{27}$

Thus, the point of inflection of $y = -x^3 - 4x^2 - 3x - 2$ is $\left(-1\frac{1}{3}; -2\frac{20}{27}\right)$.

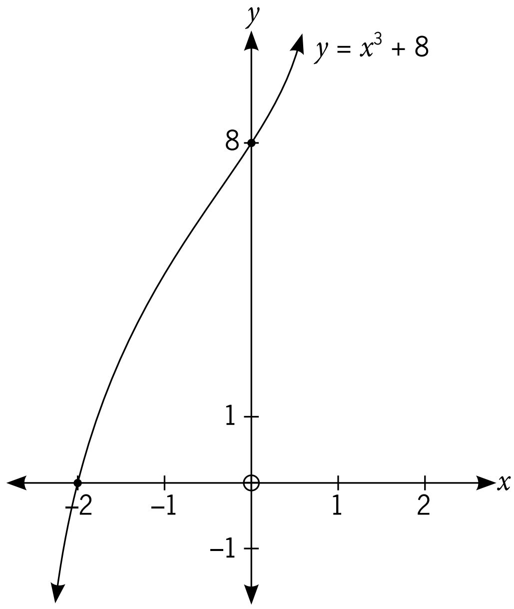
d)



2. 2.1



2.2



x-intercept: $y = 0$

$0 = x^3 + 8$

$x^3 = -8$

$x = -2$

y-intercept: $x = 0$

$y = 8$

Summative assessment: Module 5 SB page 361

1\. 1.1 $\lim\limits_{x \to 2}\left(\frac{\sqrt{2+7x}}{16-5x}\right)$

$= \frac{\sqrt{2+7(2)}}{16-5(2)}$

$= \frac{\sqrt{2+14}}{16-10}$

$= \frac{\sqrt{16}}{6}$

$= \frac{4}{6}$

$= \frac{2}{3}$ (3)

1.2 $\lim\limits_{x \to -3}\left(\frac{27+x^3}{x^2-9}\right)$

$= \frac{27+(-3)^3}{(-3)^2-9}$

$= \frac{27-27}{9-9}$

$= \frac{0}{0}$

$\lim\limits_{x \to -3}\left(\frac{27+x^3}{x^2-9}\right)$

$= \lim\limits_{x \to -3}\left(\frac{x^3+27}{x^2-9}\right)$

$= \lim\limits_{x \to -3}\left[\frac{(x+3)(x^2-3x+9)}{(x+3)(x-3)}\right]$

$= \lim\limits_{x \to -3}\left[\frac{x^2-3x+9}{x-3}\right]$

$= \frac{(-3)^2-3(-3)+9}{(-3)-3}$

$= \frac{9+9+9}{-3-3}$

$= \frac{27}{-6}$

$= -\frac{9}{2}$

$= -4\frac{1}{2}$ (3)

1.3 $\lim\limits_{x \to \infty}\left(\frac{3x^4+x^2}{x^3(5x+2)}\right)$

$= \frac{3(\infty)^4+(\infty)^2}{(\infty)^3(5(\infty)+2)}$

$= \frac{\infty}{\infty}$

$\lim\limits_{x \to \infty}\left(\frac{3x^4+x^2}{x^3(5x+2)}\right)$

$= \lim\limits_{x \to \infty}\left(\frac{3x^4+x^2}{5x^4+2x^3}\right)$

$= \lim\limits_{x \to \infty}\left(\frac{\frac{3x^4}{x^4}+\frac{x^2}{x^4}}{\frac{5x^4}{x^4}+\frac{2x^3}{x^4}}\right)$

$= \lim\limits_{x \to \infty}\left(\frac{3+\frac{1}{x^2}}{5+\frac{2}{x}}\right)$

$= \frac{3+\frac{1}{(\infty)^2}}{5+\frac{2}{(\infty)}}$

$= \frac{3+0}{5+0}$

$= \frac{3}{5}$ (3)

2. 2.1 $(2 - 3x)^{-\frac{1}{2}}$

$= [(2) + (-3x)]^{-\frac{1}{2}}$

$= \frac{(2)^{-\frac{1}{2}}(-3x)^0}{0!} + \frac{\left(-\frac{1}{2}\right)(2)^{-\frac{3}{2}}(-3x)^1}{1!} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(2)^{-\frac{5}{2}}(-3x)^2}{2!} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(2)^{-\frac{7}{2}}(-3x)^3}{3!} + \ldots$

$= \frac{(2)^{-\frac{1}{2}}1}{1} + \frac{\left(-\frac{1}{2}\right)(2)^{-\frac{3}{2}}(-3x)}{1} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(2)^{-\frac{5}{2}}(9x^2)}{2} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(2)^{-\frac{7}{2}}(-27x^3)}{6} + \ldots$

$= \frac{1}{\sqrt{2}} + \frac{3x}{4\sqrt{2}} + \frac{27x^2}{32\sqrt{2}} + \frac{135x^3}{128\sqrt{2}} + \ldots$

$= \frac{\sqrt{2}}{2} + \frac{3\sqrt{2}x}{8} + \frac{27\sqrt{2}x^2}{64} + \frac{135\sqrt{2}x^3}{256} + \ldots$ (4)

2.2 There are 13 terms in the expansion.

Thus, the middle term is T_7.

The seventh term is T_7,

$\therefore T_7 = T_{r+1}$

$7 = r + 1$

$r = 6$

$T_{6+1} = \frac{n!}{r!(n-r)!} x^{n-r} h^r$

$T_7 = \frac{12!}{6!(12-6)!} (z^2)^6 (4y^2)^6$

$T_7 = 3\ 784\ 704 z^{12} y^{12}$ (4)

3. 3.1

$f(x) = -x^3 - 2x^2$

$f(x+h) = -(x+h)^3 - 2(x+h)^2$

$= -\left[\frac{(x)^3(h)^0}{0!} + \frac{3(x)^2(h)^1}{1!} + \frac{3.2(x)^1(h)^2}{2!} + \frac{3.2.1(x)^0(h)^3}{3!}\right] - 2(x+h)(x+h)$

$= -\left[\frac{x^3.1}{1} + \frac{3.x^2.h}{1} + \frac{3.2.x^1.h^2}{2} + \frac{3.2.1.1.h^3}{6}\right] - 2(x^2 + 2xh + h^2)$

$= -[x^3 + 3x^2h + 3xh^2 + h^3] - 2(x^2 + 2xh + h^2)$

$= -x^3 - 3x^2h - 3xh^2 - h^3 - 2x^2 - 4xh - 2h^2$

$f(x+h) = -x^3 - 2x^2 - 3x^2h - 4xh - 3xh^2 - 2h^2 - h^3$

$f(x+h) - f(x) = (-x^3 - 2x^2 - 3x^2h - 4xh - 3xh^2 - 2h^2 - h^3) - (-x^3 - 2x^2)$

$= -x^3 - 2x^2 - 3x^2h - 4xh - 3xh^2 - 2h^2 - h^3 + x^3 + 2x^2$

$f(x+h) - f(x) = -3x^2h - 4xh - 3xh^2 - 2h^2 - h^3$

$\frac{f(x+h) - f(x)}{h} = \frac{-3x^2h - 4xh - 3xh^2 - 2h^2 - h^3}{h}$

$= \frac{h(-3x^2 - 4x - 3xh - 2h - h^2)}{h}$

$\frac{f(x+h) - f(x)}{h} = -3x^2 - 4x - 3xh - 2h - h^2$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} (-3x^2 - 4x - 3xh - 2h - h^2)$

$= -3x^2 - 4x - 3x(0) - 2(0) - (0)^2$

$f'(x) = -3x^2 - 4x$ (5)

3.2 $f(x) = -x^3 - 2x^2$

$f'(x) = \frac{d}{dx}(-x^3) - \frac{d}{dx}(2x^2)$

$= -\frac{d}{dx}x^3 - 2\frac{d}{dx}x^2$

$= -(3x^2) - 2(2x)$

$f'(x) = -3x^2 - 4x$ (2)

4. 4.1 $y = -5 \sin\left(\frac{\pi}{2} - \frac{x}{5}\right)$

$y = -5 \cos\frac{x}{5}$

Let $u = \frac{x}{5}$ then $y = -5 \cos u$

Therefore,

$\frac{du}{dx} = \frac{1}{5}$ $\frac{dy}{du} -5.-\sin u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= -5.-\sin u \times \frac{1}{5}$

$= \frac{5 \sin u}{5}$

$= \frac{5 \sin \frac{x}{5}}{5}$

$\frac{dy}{dx} = \sin \frac{x}{5}$ (4)

4.2 $y = e^x.\sin x$

Let $f(x) = e^x$ and $g(x) = \sin x$

Thus $f'(x) = e^x$ and $g'(x) = \cos x$

$\frac{dy}{dx} = f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$

$= (e^x)(\cos x) + (\sin x)(e^x)$

$\frac{dy}{dx} = e^x (\cos x + \sin x)$ (4)

4.3 $y = \frac{\ln x}{\sqrt{x}}$

Let $f(x) = \ln x$ and $g(x) = x^{\frac{1}{2}}$

Thus $f'(x) = \frac{1}{x}$ and $g'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$

$$= \frac{\left(x^{\frac{1}{2}}\right)\left(\frac{1}{x}\right) - (\ln x)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{\left(x^{\frac{1}{2}}\right)^2}$$

$$= \frac{x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}\ln x}{x}$$

$$= \frac{x^{-\frac{1}{2}}\left(1 - \frac{1}{2}\ln x\right)}{x}$$

$$= \frac{1 - \frac{1}{2}\ln x}{x\sqrt{x}}$$

$$= \frac{\sqrt{x}\left(1 - \frac{1}{2}\ln x\right)}{x^2}$$

$$\frac{dy}{dx} = \frac{\sqrt{x}(2 - \ln x)}{2x^2}$$ (4)

5. 5.1 $y = \tan x + \log_3 x - 4e^{-x} - \frac{1}{x^5}$

$$y = \tan x + \log_3 x - 4e^{-x} - x^{-5}$$

$$\frac{dy}{dx} = \frac{d}{dx}(\tan x) + \frac{d}{dx}(\log_3 x) - \frac{d}{dx}(4e^{-x}) - \frac{d}{dx}(x^{-5})$$

$$= \frac{d}{dx}\tan x + \frac{d}{dx}\log_3 x - 4\frac{d}{dx}e^{-x} - \frac{d}{dx}x^{-5}$$

$$= \sec^2 x + \frac{1}{x\ln 3} - 4(e^{-x}.-1) - (-5x^{-6})$$

$$\frac{dy}{dx} = \sec^2 x + \frac{1}{x\ln 3} + \frac{4}{e^x} + \frac{5}{x^6}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\sec^2 x) + \frac{d}{dx}\left(\frac{1}{x\ln 3}\right) + \frac{d}{dx}\left(\frac{4}{e^x}\right) + \frac{d}{dx}\left(\frac{5}{x^6}\right)$$

$$= \frac{d}{dx}(\sec^2 x) + \frac{d}{dx}\left(\frac{1}{\ln 3}.x^{-1}\right) + \frac{d}{dx}(4e^{-x}) + \frac{d}{dx}(5x^{-6})$$

$$= \frac{d}{dx}(\sec x)^2 + \frac{1}{\ln 3}\frac{d}{dx}x^{-1} + 4\frac{d}{dx}e^{-x} + 5\frac{d}{dx}x^{-6}$$

$$= (2\sec x \sec x \tan x) + \frac{1}{\ln 3}(-1x^{-2}) + 4(e^{-x}.-1) + 5(-6x^{-7})$$

$$\frac{d^2y}{dx^2} = 2\sec^2 x \tan x - \frac{1}{x^2\ln 3} - \frac{4}{e^x} - \frac{30}{x^7}$$ (6)

5.2 $y = \ln\left(\frac{x}{6}\right) - \sqrt{e^x} + \text{cosec } x - 2^{\log_2 x^2}$

$= \ln\left(\frac{x}{6}\right) - (e^x)^{\frac{1}{2}} + \text{cosec } x - x^2$

$y = \ln\left(\frac{x}{6}\right) - e^{\frac{x}{2}} + \text{cosec } x - x^2$

$\frac{dy}{dx} = \frac{d}{dx}\left(\ln\left(\frac{x}{6}\right)\right) - \frac{d}{dx}\left(e^{\frac{x}{2}}\right) + \frac{d}{dx}(\text{cosec } x) - \frac{d}{dx}(x^2)$

$= \frac{d}{dx}\left(\ln\left(\frac{x}{6}\right)\right) - \frac{d}{dx}e^{\frac{x}{2}} + \frac{d}{dx}\text{cosec } x - \frac{d}{dx}x^2$

$= \left(\frac{1}{\frac{x}{6}}.\frac{1}{6}\right) - \left(e^{\frac{x}{2}}.\frac{1}{2}\right) + (-\text{cosec } x \cot x) - 2x$

$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{2}e^{\frac{x}{2}} - \text{cosec } x \cot x - 2x$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\right) - \frac{d}{dx}\left(\frac{1}{2}e^{\frac{x}{2}}\right) - \frac{d}{dx}(\text{cosec } x \cot x) - \frac{d}{dx}(2x)$

$= \frac{d}{dx}(x^{-1}) - \frac{d}{dx}\left(\frac{1}{2}e^{\frac{x}{2}}\right) - \frac{d}{dx}(\text{cosec } x \cot x) - \frac{d}{dx}(2x)$

$= \frac{d}{dx}x^{-1} - \frac{1}{2}\frac{d}{dx}e^{\frac{x}{2}} - \frac{d}{dx}\text{ cosec } x \cot x - 2\frac{d}{dx}x$

$= \frac{d}{dx}x^{-1} - \frac{1}{2}\frac{d}{dx}e^{\frac{x}{2}} - [\text{cosec } x\frac{d}{dx}\cot x + \cot x\frac{d}{dx}\text{cosec } x] - 2\frac{d}{dx}x$

$= -1.x^{-2} - \frac{1}{2}\left(e^{\frac{x}{2}}.\frac{1}{2}\right) - [\text{cosec } x.-\text{cosec}^2 x + \cot x.-\text{ cosec } x \cot x] - 2.1$

$= -\frac{1}{x^2} - \frac{1}{4}e^{\frac{x}{2}} - [-\text{cosec}^3 x - \text{cosec } x \cot^2 x] - 2$

$= -\frac{1}{x^2} - \frac{1}{4}e^{\frac{x}{2}} - [-\text{cosec}^3 x - \text{cosec } x(\text{cosec}^2 x - 1)] - 2$

$= -\frac{1}{x^2} - \frac{1}{4}e^{\frac{x}{2}} - [-\text{cosec}^3 x - \text{cosec}^3 x + \text{cosec } x] - 2$

$= -\frac{1}{x^2} - \frac{1}{4}e^{\frac{x}{2}} - [-2\text{ cosec}^3 x + \text{cosec } x] - 2$

$\frac{d^2y}{dx^2} = -\frac{1}{x^2} - \frac{1}{4}e^{\frac{x}{2}} + 2\text{ cosec}^3 x - \text{cosec } x - 2$ (6)

6. 6.1 $y = x^3 - 4x^2 + x + 6$

$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 4x^2 + x + 6)$

$= \frac{d}{dx}x^3 - \frac{d}{dx}4x^2 + \frac{d}{dx}x + \frac{d}{dx}6$

$= \frac{d}{dx}x^3 - 4\frac{d}{dx}x^2 + \frac{d}{dx}x + \frac{d}{dx}6$

$= 3x^2 - 4.2x + 1 + 0$

$\frac{dy}{dx} = 3x^2 - 8x + 1$

But $\frac{dy}{dx} = 0$ at the turning points,

$3x^2 - 8x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(1)}}{2(3)}$$

$$x = \frac{8 \pm \sqrt{52}}{6}$$

$$x = \frac{8 + \sqrt{52}}{6} \quad \text{and} \quad x = \frac{8 - \sqrt{52}}{6}$$

$$x = 2{,}535 \qquad x = 0{,}132$$

Substitute $x = 0{,}132$ and $x = 2{,}535$ into $y = x^3 - 4x^2 + x + 6$,

$y = x^3 - 4x^2 + x + 6$

$= (0{,}132)^3 - 4(0{,}132)^2 + (0{,}132) + 6$

$y = 6{,}065$

$y = x^3 - 4x^2 + x + 6$

$= (2{,}535)^3 - 4(2{,}535)^2 + (2{,}535) + 6$

$y = -0{,}879$

Thus, the turning points of $y = x^3 - 4x^2 + x + 6$ are $(0{,}132; 6{,}065)$ and $(2{,}535; -0{,}879)$. (6)

6.2 $\frac{dy}{dx} = 3x^2 - 8x + 1$

$\frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2 - 8x + 1)$

$= \frac{d}{dx}3x^2 - \frac{d}{dx}8x + \frac{d}{dx}1$

$= 3\frac{d}{dx}x^2 - 8\frac{d}{dx}x + \frac{d}{dx}1$

$= 3.2x - 8.1 + 0$

$\frac{d^2y}{dx^2} = 6x - 8$

Stationary point	$(0{,}132; 6{,}065)$	$(2{,}535; -0{,}879)$
Sign of $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = -7{,}211 < 0$	$\frac{d^2y}{dx^2} = +7{,}211 > 0$
Conclusion	Maximum	Minimum

(4)

6.3 $\frac{d^2y}{dx^2} = 6x - 8$

But $\frac{d^2y}{dx^2} = 0$ at the point of inflection,

$6x - 8 = 0$

$6x = 8$

$x = \frac{4}{3}$

$x = 1\frac{1}{3}$

Substitute $x = \frac{4}{3}$ into $y = x^3 - 4x^2 + x + 6$,

$y = x^3 - 4x^2 + x + 6$

$= \left(\frac{4}{3}\right)^3 - 4\left(\frac{4}{3}\right)^2 + \left(\frac{4}{3}\right) + 6$

$y = 2\frac{16}{27}$

Thus, the point of inflection of $y = x^3 - 4x^2 + x + 6$ is $\left(1\frac{1}{3}; 2\frac{16}{27}\right)$. (3)

6.4 x-intercept(s), $y = 0$:

$y = x^3 - 4x^2 + x + 6$

$0 = x^3 - 4x^2 + x + 6$

From the factor theorem $y = 0$ when $x = -1$. Therefore, $x + 1$ is a factor of $y = x^3 - 4x^2 + x + 6$.

Thus, the quadratic is

$x^2 - 5x + 6 = 0$

Therefore,

$0 = x^3 - 4x^2 + x + 6$

$0 = (x + 1)(x^2 - 5x + 6)$

$0 = (x + 1)(x - 2)(x - 3)$

$x + 1 = 0$ or $x - 2 = 0$ or $x - 3 = 0$

$x = -1$ $x = 2$ $x = 3$

$(-1; 0)$ $(2; 0)$ $(3; 0)$

y-intercept, $x = 0$:

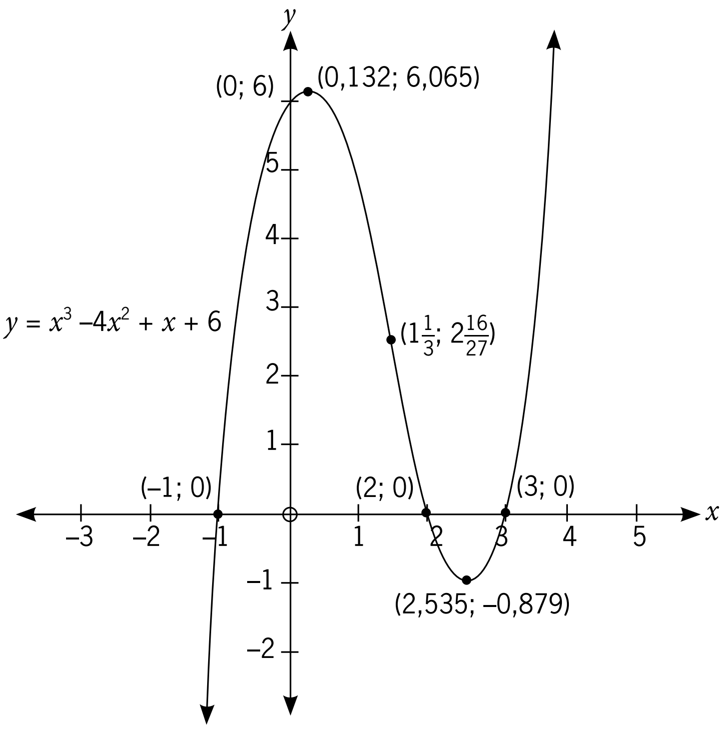
$y = x^3 - 4x^2 + x + 6$

$= (0)^3 - 4(0)^2 + (0) + 6$

$y = 6$

$(0; 6)$ (4)

6.5


y
x
(0; 6)
(0,132; 6,065)
$y = x^3 - 4x^2 + x + 6$
$(1\frac{1}{3}; 2\frac{16}{27})$
(–1; 0)
(2; 0)
(3; 0)
(2,535; –0,879)
–3
–2
–1
1
2
3
4
5
–1
–2
1
2
3
4
5

(5)

TOTAL: [70]

MODULE

6 Integral calculus

After they have completed this module, students should be able to:

- understand the concept of integration as a summation function (definite integral) and as a process of anti-differentiation (indefinite integral);
- apply standard forms of integrals as a process of anti-differentiation;
- integrate functions given on the formula sheet:
  - $k\,x^n$, n real with $n \neq -1$
  - $\frac{k}{x}$, $k\,a^{nx}$, $k\,e^{nx}$ with $a \geq 0$, $k, n \in \mathbb{R}$
  - $k\sin(bx)$ and $k\cos(bx)$ with b and $k \in \mathbb{R}$;
- integrate polynomials consisting of terms of the above forms;
- apply integration to determine the magnitude of an area included by a curve and the x-axis, or by a curve, the x-axis and the ordinates $x = a$ and $x = b$, where a and b are integers;
- using the definite integral with two limits to calculate the area bounded by the graph, the x-axis and values given to define the area; areas include areas above the x-axis, areas below the x-axis and joined areas above and below the x-axis:

  $A = \int_a^b y\,dx$ or $A_{total} = \int_a^b y\,dx + \int_c^d y\,dx$; and
- calculate the intersection points of two curves, and sketch the two graphs on the same system of axes indicating the area bounded by the two intersection points calculated and show the representative strip used to calculate the area.

Introduction

In this module students will analyse and represent mathematical and contextual situations using integrals and find areas under curves by using integration rules. Integration can also be used to find areas, volumes and central points.

Integration and differentiation are the two main branches of calculus. Integration is an important concept in mathematics and it is the **inverse process of differentiation**.

The term 'integral' may also be referred to as the anti-derivative. Integrals and derivatives have numerous applications in science and engineering, for example to calculate surface areas and to formulate physical laws of electrodynamics.

Integral calculus is the division of calculus that is concerned with:

- the inverse operation of differentiation – this will be dealt with when we solve **indefinite integrals**; and
- the summing of the values of a function over a particular range. One of the main uses is to calculate the areas of irregular shapes. This will be dealt with when we solve **definite integrals**.

To summarise: Integration is the process of finding the definite or indefinite integral of a function and it is the inverse of differentiation (anti-differentiation).

Students need the following pre-knowledge to successfully complete this module.

Pre-knowledge

- Laws of exponents

Law
$x^m \times x^n = x^{m+n}$
$x^m \div x^n = x^{m-n}$
$(x^m)^n = x^{mn}$
$(xy)^m = x^m \times y^m$
$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
$x^{\frac{m}{n}} = \sqrt[n]{x^m}$; $x > 0$; $n > 0$

Deductions/Definition
$x^{-m} = \frac{1}{x^m}$
$\frac{1}{x^{-m}} = x^m$
$\left(\frac{x}{y}\right)^{-m} = \left(\frac{y}{x}\right)^m$
$x^0 = 1$

- Logarithmic laws
- Trigonometric identities
- Standard derivatives

$f(x)$	$f'(x)$
k kx^n	0 $n.kx^{n-1}$
ka^x ka^{nx}	$ka^x \ln a$ $n.ka^{nx} \ln a$
ke^x ke^{nx}	ke^x $n.ke^{nx}$
$k \ln x$ $k \ln(nx)$	$\frac{k}{x}$ $\frac{k}{x}$
$k \log_a x$ $k \log_a(nx)$	$\frac{k}{x \ln a}$ $\frac{k}{x \ln a}$
$k \sin(nx)$ $k \cos(nx)$ $k \tan(nx)$ $k \cot(nx)$ $k \sec(nx)$ $k \operatorname{cosec}(nx)$	$nk \cos(nx)$ $-nk \sin(nx)$ $nk \sec^2(nx)$ $-nk \operatorname{cosec}^2(nx)$ $nk \sec(nx).\tan(nx)$ $-nk \operatorname{cosec}(nx).\cot(nx)$

- Rules for standard integrals

It is essential to be able to sketch and recognise the following graphs dealt with in *Module 3: Sketch graphs*.

	Graph	Equation
1.	Straight line	$y = mx + c$ or $ax + by + c = 0$
2.	Circle or semi-circles	$x^2 + y^2 = r^2$; $y = \pm\sqrt{r^2 - x^2}$; $x = \pm\sqrt{r^2 - y^2}$
3.	Rectangular hyperbola	$xy = c$ or $y = \frac{c}{x}$ or $x = \frac{c}{y}$
4.	Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
5.	Central hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
6.	Exponential graph	$y = ka^{nx}$; $y = ke^{nx}$
7.	Logarithmic graph	$y = k\log_a(nx)$; $y = k\log_e(nx)$
8.	Parabola	$y = ax^2 + bx + c$
9.	Cubic graph	$y = ax^3 + bx^2 + cx + d$
10.	Trigonometric curves	$y = a\sin(bx + c) + d$; $y = a\cos(bx + c) + d$; $y = a\tan(bx + c) + d$ $y = \text{cosec } x$; $y = \sec x$; $y = \cot x$

Activity 6.1

SB page 373

1. $\int x^5\,dx$

   $= \frac{x^6}{6} + c$

   or $\frac{1}{6}x^6 + c$

2. $\int 4x^2\,dx$

   $= \frac{4x^3}{3} + c$

   or $\frac{4}{3}x^3 + c$

3. $\int 2\,dx$

   $= 2x + c$

4. $\int da$

   $= a + c$

5. $\int \frac{2}{x^2}\,dx$

   $= \int 2x^{-2}\,dx$

   $= \frac{2x^{-1}}{-1} + c$

   $= -\frac{2}{x} + c$

6. $\int 3\,dy$

   $= 3y + c$

7. $\int 0{,}23\,dr$

   $= 0{,}23r + c$

8. $\int -3x^4\,dx$

   $= \frac{-3x^5}{5} + c$

   or $= -\frac{3}{5}x^5 + c$

9. $\int \frac{dx}{x^3}$

$= \int \frac{1}{x^3}\, dx$

$= \int x^{-3}\, dx$

$= \frac{x^{-2}}{-2} + c$

$= -\frac{1}{2x^2} + c$

10. $3\int x^3\, dx$

$= 3.\frac{x^4}{4} + c$

$= \frac{3}{4}x^4 + c$

11. $-2 \int 3x\, dx$

$= -2.\frac{3x^2}{2} + c$

$= -3x^2 + c$

12. $\int \sqrt{x^2}\, dx$

$= \int x\, dx$

$= \frac{x^2}{2} + c$

or $\frac{1}{2}x^2 + c$

13. $\int 3\sqrt{x^3}\, dx$

$= \int 3.x^{\frac{3}{2}}\, dx$

$= 3.\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + c$

$= 3.\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$

$= 3.\frac{2}{5}x^{\frac{5}{2}} + c$

$= \frac{6}{5}\sqrt{x^5} + c$

14. $\int \frac{1}{3x^2}\, dx$

$= \int \frac{1}{3}x^{-2}\, dx$

$= \frac{1}{3}.\frac{x^{-2+1}}{-2+1} + c$

$= \frac{1}{3}.\frac{x^{-1}}{-1} + c$

$= -\frac{1}{3x} + c$

15. $\int \frac{4}{x^5}\, dx$

$= \int 4x^{-5}\, dx$

$= \frac{4x^{-4}}{-4} + c$

$= -\frac{1}{x^4} + c$

16. $\int x^{\pi}\, dx$

$= \frac{x^{\pi+1}}{\pi+1} + c$

17. $\int \pi\, dx$

$= \pi x + c$

18. $\int -\frac{1}{4}t\, dt$

$= -\frac{1}{4}.\frac{t^2}{2} + c$

$= -\frac{1}{8}t^2 + c$

19. $\int a\,dx$
$= \int ax^0\,dx$
$= ax + c$

20. $\int \sqrt{5x}\,dx$
$= \int \sqrt{5}\,x^{\frac{1}{2}}\,dx$
$= \sqrt{5}.\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$
$= \sqrt{5}.\frac{2}{3}\sqrt{x^3} + c$
$= \frac{2\sqrt{5}}{3}\sqrt{x^3} + c$

Activity 6.2 SB page 375

1. $\int \frac{7}{x}\,dx$
$= 7 \ln x + c$

2. $\int 2x^{-1}\,dx$
$= 2 \ln x + c$

3. $\int \frac{1}{r}\,dr$
$= \ln r + c$

4. $\int \frac{1}{3x}\,dx$
$= \frac{1}{3} \ln x + c$

5. $4\int \frac{dx}{4x}$
$= 4.\frac{1}{4} \ln x + c$
$= \ln x + c$

6. $\int \frac{2}{a}\,da$
$= 2 \ln a + c$

7. $\int -\frac{3}{x}\,dx$
$= -3 \ln x + c$

8. $\int \frac{dx}{x}$
$= \ln x + c$

9. $\int \frac{k}{x}\,dx$
$= k \ln x + c$

10. $\int -3x^{-1}\,dx$
$= -3 \ln x + c$

Activity 6.3 SB page 378

1. $\int 5^x\,dx$
$= \frac{5^x}{\ln 5} + c$

2. $\int 4^{3x}\,dx$
$= \frac{4^{3x}}{3 \ln 4} + c$

3. $\int 7^{-x}\,dx$
$= \frac{7^{-x}}{-\ln 7} + c$
$= -\frac{1}{7^x \ln 7} + c$

4. $\int 4(2)^{3x}\,dx$
$= \frac{4.2^{3x}}{3 \ln 2} + c$

5. $\int \left(\frac{1}{2}\right)^a\,da$
$= \frac{\left(\frac{1}{2}\right)^a}{\ln \frac{1}{2}} + c$

6. $\int \frac{1}{10^{-x}}\,dx$
$= \int 10^x\,dx$
$= \frac{10^x}{\ln 10} + c$

7. $\int 5.4^{5x}\,dx$

$= \frac{5.4^{5x}}{5 \ln 4} + c$

$= \frac{4^{x}}{\ln 4} + c$

8. $\int \pi^{x}\,dx$

$= \frac{\pi^{x}}{\ln \pi} + c$

9. $\int 8.3^{-4x}\,dx$

$= \frac{8.3^{-4x}}{-4 \ln 3} + c$

$= \frac{-2.3^{-4x}}{\ln 3} + c$

$= \frac{-2}{3^{4x}\ln 3} + c$

10. $3\int 2^{-5x}\,dx$

$= \frac{3.2^{-5x}}{-5 \ln 2} + c$

$= \frac{3}{-5.2^{5x}\ln 2} + c$

11. $\int \left(\frac{1}{5}\right)^{2x} dx$

$= \frac{\left(\frac{1}{5}\right)^{2x}}{2 \ln\left(\frac{1}{5}\right)} + c$

12. $\int 2^{4t}\,dt$

$= \frac{2^{4t}}{4 \ln 2} + c$

13. $\int e^{-x}\,dx$

$= \frac{e^{-x}}{-1} + c$

$= -\frac{1}{e^{x}} + c$

14. $\int 2e^{7x}\,dx$

$= 2\int e^{7x}\,dx$

$= 2\int \left(e^{7}\right)^{x} dx$

$= 2.\frac{e^{7x}}{7 \ln e} + c$

$= 2.\frac{e^{7x}}{7.1} + c$

$= \frac{2}{7}e^{7x} + c$

15. $\int e^{x+\ln 3}\,dx$

$= \int e^{x}.e^{\ln 3}\,dx$

$= e^{\ln 3}\int e^{x}\,dx$

$= 3\int e^{x}\,dx$

$= 3e^{x} + c$

Activity 6.4 SB page 381

1. 1.1 $\int \sin 5x\,dx$

$= -\frac{\cos 5x}{5} + c$

1.2 $\int -2 \sin \frac{x}{2}\,dx$

$= \frac{2 \cos \frac{x}{2}}{\frac{1}{2}} + c$

$= 4 \cos \frac{x}{2} + c$

1.3 $\int 3\cos 3x\,dx$

$= \frac{3\sin 3x}{3} + c$

$= \sin 3x + c$

1.4 $\int 4\cos 2\theta\,d\theta$

$= \frac{4\sin 2\theta}{2} + c$

$= 2\sin 2\theta + c$

1.5 $\int 3\sin\frac{ax}{4}\,dx$

$= -\frac{3\cos\frac{ax}{4}}{\frac{a}{4}} + c$

$= -\frac{12}{a}\cos\frac{ax}{4} + c$

1.6 $\int -2\cos\frac{1}{2}x\,dx$

$= -\frac{2\sin\frac{1}{2}x}{\frac{1}{2}} + c$

$= -4\sin\frac{1}{2}x + c$

1.7 $\int \frac{1}{\sec 4x}\,dx$

$= \int \cos 4x\,dx$

$= \frac{\sin 4x}{4} + c$

1.8 $\int 2\sin x\cos x\,dx$

$= \int \sin 2x\,dx$

$= -\frac{\cos 2x}{2} + c$

1.9 $-\int \frac{4}{\operatorname{cosec} x}\,dx$

$= -\int 4\sin x\,dx$

$= 4\cos x + c$

2. 2.1 $\int \sqrt{(1+\cos x)(1-\cos x)}\,dx$

$= \int \sqrt{1-\cos^2 x}\,dx$

$= \int \sin x\,dx$

$= -\cos x + c$

Differentiate to check:

$\frac{d}{dx}(-\cos x + c)$

$= -\frac{d}{dx}\cos x + \frac{d}{dx}c$

$= -(-\sin x) + 0$

$= \sin x$

2.2 $\int \left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)dx$

$= \int \cos x\,dx$

$= \sin x + c$

$\frac{d}{dx}(\sin x + c)$

$= \frac{d}{dx}\sin x + \frac{d}{dx}c$

$= \cos x + 0$

$= \cos x$

2.3 $\int \frac{\sec x\sin 2x}{(1-\cos 2x)(1+\cot^2 x)}\,dx$

$= \int \frac{\frac{1}{\cos x}.2\sin x\cos x}{2\sin^2 x\operatorname{cosec}^2 x}\,dx$

$= \int \frac{2\sin x}{2\sin^2 x.\frac{1}{\sin^2 x}}\,dx$

$= \int \sin x\,dx$

$= -\cos x + c$

$\frac{d}{dx}(-\cos x + c)$

$= -\frac{d}{dx}\cos x + \frac{d}{dx}c$

$= -(-\sin x) + 0$

$= \sin x$

2.4 $\int \left(\frac{\cos x + \sin x}{1 + \tan x}\right) dx$

$= \int \left(\frac{\cos x + \sin x}{1 + \frac{\sin x}{\cos x}}\right) dx$

$= \int \left(\frac{\cos x + \sin x}{\frac{\cos x + \sin x}{\cos x}}\right) dx$

$= \int \left(\frac{\cos x + \sin x}{1} \times \frac{\cos x}{\cos x + \sin x}\right) dx$

$= \int \cos x \, dx$

$= \sin x + c$

$= \frac{d}{dx}(\sin x + c)$

$= \frac{d}{dx} \sin x + \frac{d}{dx}c$

$= \cos x + 0$

$= \cos x$

Activity 6.5 SB page 389

1. 1.1 $\int \frac{e^{4x}}{7} dx$

$= \frac{1}{7}\int e^{4x} \, dx$

$u = 4x$

$du = 4 \, dx$

$\frac{1}{4} du = dx$

$\therefore \frac{1}{7}\int e^{4x} \, dx$

$= \frac{1}{7}\int e^{u}.\frac{1}{4} \, du$

$= \frac{1}{28}\int e^{u} \, du$

$= \frac{1}{28}.e^{u} + c$

$= \frac{1}{28}.e^{4x} + c$

Verify: $\frac{d}{dx}\left(\frac{1}{28}.e^{4x} + c\right)$

$= \frac{1}{28}\frac{d}{dx}.e^{4x} + \frac{d}{dx}c$

$= \frac{1}{28}.e^{4x}.4 + 0$

$= \frac{1}{7}.e^{4x}$

1.2 $\int e^{\frac{1}{2x^{-1}}} dx$

$= \int e^{\frac{x}{2}} dx$

$u = \frac{x}{2}$

$du = \frac{1}{2} dx$

$2du = dx$

$\therefore \int e^{\frac{x}{2}}$

$= \int e^{u}.2 \, du$

$= 2\int e^{u} \, du$

$= 2e^{u} + c$

$= 2e^{\frac{x}{2}} + c$

Verify: $\frac{d}{dx}\left(2.e^{\frac{x}{2}} + c\right)$

$= 2\frac{d}{dx}e^{\frac{x}{2}} + \frac{d}{dx}c$

$= 2.e^{\frac{x}{2}}.\frac{1}{2} + 0$

$= e^{\frac{x}{2}}$

1.3 $\int \frac{1}{e^{-9x+\ln 4}}\,dx$

$= \int e^{9x-\ln 4}\,dx$

$= \int e^{9x}.e^{-\ln 4}\,dx$

$= \int e^{9x}.e^{\ln 4^{-1}}\,dx$

$= \int e^{9x}.e^{\ln \frac{1}{4}}\,dx$

$= \int e^{9x}.\frac{1}{4}\,dx$

$= \frac{1}{4}\int e^{9x}\,dx$

$u = 9x$

$du = 9\,dx$

$\frac{1}{9}\,du = dx$

$\therefore \frac{1}{4}\int e^{9x}\,dx$

$= \frac{1}{4}\int e^{u}.\frac{1}{9}\,du$

$= \frac{1}{36}\int e^{u}\,du$

$= \frac{1}{36}.e^{u} + c$

$= \frac{1}{36}.e^{9x} + c$

Verify: $\frac{d}{dx}\left(\frac{1}{36}.e^{9x} + c\right)$

$= \frac{1}{36}\frac{d}{dx}.e^{9x} + \frac{d}{dx}c$

$= \frac{1}{36}.e^{9x}.9 + 0$

$= \frac{1}{4}.e^{9x}$

1.4 $\int 2.5^{3x}\,dx$

$= 2\int 5^{3x}\,dx$

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}\,du = dx$

$\therefore 2\int 5^{3x}\,dx$

$= 2\int 5^{u}.\frac{1}{3}\,du$

$= \frac{2}{3}\int 5^{u}\,du$

$= \frac{2}{3}.\frac{5^{u}}{\ln 5} + c$

$= \frac{2}{3}.\frac{5^{3x}}{\ln 5} + c$

Verify: $\frac{d}{dx}\left(\frac{2}{3}.\frac{5^{3x}}{\ln 5} + c\right)$

$= \frac{2}{3\ln 5}\frac{d}{dx}5^{3x} + \frac{d}{dx}c$

$= \frac{2}{3\ln 5}.5^{3x}\ln 5.3 + 0$

$= 2.5^{3x}$

1.5 $\frac{1}{8}\int \sin 10x\, dx$

$u = 10x$

$du = 10\, dx$

$\frac{1}{10}\, du = dx$

$\therefore \frac{1}{8}\int \sin 10x\, dx$

$= \frac{1}{8}\int \sin u . \frac{1}{10}\, du$

$= \frac{1}{80}\int \sin u\, du$

$= \frac{1}{80} . -\cos u + c$

$= -\frac{1}{80} . \cos 10x + c$

Verify: $\frac{d}{dx}\left(-\frac{1}{80} . \cos 10x + c\right)$

$= -\frac{1}{80}\frac{d}{dx}\cos 10x + \frac{d}{dx}c$

$= -\frac{1}{80} . -\sin 10x . 10 + 0$

$= \frac{1}{8}\sin 10x$

1.6 $2\int \cos \frac{7x}{9}\, dx$

$u = \frac{7x}{9}$

$du = \frac{7}{9}\, dx$

$\frac{9}{7}\, du = dx$

$\therefore 2\int \cos \frac{7x}{9}\, dx$

$= 2\int \cos u . \frac{9}{7}\, du$

$= \frac{18}{7}\int \cos u\, du$

$= \frac{18}{7} . \sin u + c$

$= \frac{18}{7} . \sin \frac{7x}{9} + c$

Verify: $\frac{d}{dx}\left(\frac{18}{7} . \sin \frac{7x}{9} + c\right)$

$= \frac{18}{7}\frac{d}{dx}\sin \frac{7x}{9} + \frac{d}{dx}c$

$= \frac{18}{7} . \cos \frac{7x}{9} . \frac{7}{9} + 0$

$= 2\cos \frac{7x}{9}$

1.7 $\int \cos\left(\frac{\pi}{2} + 3x\right) dx$

$= -\int \sin 3x\, dx$

$u = 3x$

$du = 3\, dx$

$\frac{1}{3}\, du = dx$

$\therefore -\int \sin 3x\, dx$

$= -\int \sin u . \frac{1}{3}\, du$

$= -\frac{1}{3}\int \sin u\, du$

$= -\frac{1}{3} . -\cos u + c$

$= \frac{1}{3}\cos 3x + c$

Verify: $\frac{d}{dx}\left(\frac{1}{3}\cos 3x + c\right)$

$= \frac{1}{3}\frac{d}{dx}\cos 3x + \frac{d}{dx}c$

$= \frac{1}{3} . -\sin 3x . 3 + 0$

$= -\sin 3x$

1.8 $\int(\cos^4 x - \sin^4 x)\,dx$

$= \int\left[(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)\right]dx$

$= \int[1.\cos 2x]\,dx$

$= \int \cos 2x\,dx$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$\therefore \int \cos 2x\,dx$

$= \int \cos u.\frac{1}{2}\,du$

$= \frac{1}{2}\int \cos u\,du$

$= \frac{1}{2}.\sin u + c$

$= \frac{1}{2}\sin 2x + c$

Verify: $\frac{d}{dx}\left(\frac{1}{2}\sin 2x + c\right)$

$= \frac{1}{2}\frac{d}{dx}\sin 2x + \frac{d}{dx}c$

$= \frac{1}{2}.\cos 2x.2 + 0$

$= \cos 2x$

2. 2.1 $\int \frac{8}{e^{3x}}\,dx$

$= 8\int e^{-3x}\,dx$

$u = -3x$

$du = -3\,dx$

$-\frac{1}{3}\,du = dx$

$\therefore 8\int e^{-3x}\,dx$

$= 8\int e^{u}.-\frac{1}{3}\,du$

$= -\frac{8}{3}\int e^{u}\,du$

$= -\frac{8}{3}.e^{u} + c$

$= -\frac{8}{3}.e^{-3x} + c$

Verify: $\frac{d}{dx}\left(-\frac{8}{3}.e^{-3x} + c\right)$

$= -\frac{8}{3}\frac{d}{dx}e^{-3x} + \frac{d}{dx}c$

$= -\frac{8}{3}.e^{-3x}.-3 + 0$

$= 8.e^{-3x}$

2.2 $\int \sqrt[5]{e^{-6x}}\,dx$

$= \int\left(e^{-6x}\right)^{\frac{1}{5}}dx$

$= \int e^{-\frac{6x}{5}}\,dx$

$u = -\frac{6x}{5}$

$du = -\frac{6}{5}\,dx$

$-\frac{5}{6}\,du = dx$

Verify: $\frac{d}{dx}\left(-\frac{5}{6}.e^{-\frac{6x}{5}} + c\right)$

$= -\frac{5}{6}\frac{d}{dx}e^{-\frac{6x}{5}} + \frac{d}{dx}c$

$= -\frac{5}{6}.e^{-\frac{6x}{5}}.-\frac{6}{5} + 0$

$= e^{-\frac{6x}{5}}$

$\therefore \int e^{-\frac{6x}{5}}\,dx$

$= \int e^{u}.-\frac{5}{6}\,du$

$= -\frac{5}{6}\int e^{u}.du$

$= -\frac{5}{6}.e^{u} + c$

$= -\frac{5}{6}.e^{-\frac{6x}{5}} + c$

2.3 $\int (3^{x})^{-4}\,dx$

$= \int 3^{-4x}\,dx$

$u = -4x$

$du = -4\,dx$

$-\frac{1}{4}\,du = dx$

$= \int 3^{u}.-\frac{1}{4}\,du$

$\therefore \int 3^{-4x}\,dx$

$= -\frac{1}{4}\int 3^{u}\,du$

$= -\frac{1}{4}.\frac{3^{u}}{\ln 3} + c$

$= -\frac{1}{4}.\frac{3^{-4x}}{\ln 3} + c$

Verify: $\frac{d}{dx}\left(-\frac{1}{4}.\frac{3^{-4x}}{\ln 3} + c\right)$

$= -\frac{1}{4\ln 3}\frac{d}{dx}3^{-4x} + \frac{d}{dx}c$

$= -\frac{1}{4\ln 3}.3^{-4x}\ln 3.-4 + 0$

$= 3^{-4x}$

2.4 $\int \sqrt{6^{4x-2}}\,dx$

$= \int (6^{4x-2})^{\frac{1}{2}}\,dx$

$= \int 6^{2x-1}\,dx$

$= \int 6^{2x}.6^{-1}\,dx$

$= \int 6^{2x}.\frac{1}{6}\,dx$

$= \frac{1}{6}\int 6^{2x}\,dx$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$\therefore \frac{1}{6}\int 6^{2x}\,dx$

$= \frac{1}{6}\int 6^{u}.\frac{1}{2}\,du$

$= \frac{1}{12}\int 6^{u}\,du$

$= \frac{1}{12}.\frac{6^{u}}{\ln 6} + c$

$= \frac{1}{12}.\frac{6^{2x}}{\ln 6} + c$

Verify: $\frac{d}{dx}\left(\frac{1}{12}.\frac{6^{2x}}{\ln 6} + c\right)$

$= \frac{1}{12\ln 6}\frac{d}{dx}6^{2x} + \frac{d}{dx}c$

$= \frac{1}{12\ln 6}.6^{2x}(\ln 6)(2) + 0$

$= \frac{1}{6}.6^{2x}$

2.5 $\int \sin\left(\frac{\pi}{2} - \frac{5x}{2}\right) dx$

$= \int \cos \frac{5x}{2}\, dx$

$u = \frac{5x}{2}$

$du = \frac{5}{2}\, dx$

$\frac{2}{5}\, du = dx$

$\therefore \int \cos \frac{5x}{2}\, dx$

$= \int \cos u.\frac{2}{5}\, du$

$= \frac{2}{5}\int \cos u\, du$

$= \frac{2}{5}.\sin u + c$

$= \frac{2}{5} \sin \frac{5x}{2} + c$

Verify: $\frac{d}{dx}\left(\frac{2}{5} \sin \frac{5x}{2} + c\right)$

$= \frac{2}{5}\, \frac{d}{dx} \sin \frac{5x}{2} + \frac{d}{dx}c$

$= \frac{2}{5}.\cos \frac{5x}{2}.\frac{5}{2} + 0$

$= \cos \frac{5x}{2}$

2.6 $\int \frac{\sin 2x}{\sec 2x}\, dx$

$= \int \frac{\sin 2x}{\frac{1}{\cos 2x}}\, dx$

$= \int \sin 2x.\cos 2x\, dx$

$= \int \frac{1}{2} \sin 4x\, dx$

$= \frac{1}{2}\int \sin 4x\, dx$

$u = 4x$

$du = 4\, dx$

$\frac{1}{4}\, du = dx$

$\therefore \frac{1}{2}\int \sin 4x\, dx$

$= \frac{1}{2}\int \sin u.\frac{1}{4}\, du$

$= \frac{1}{8}\int \sin u\, du$

$= \frac{1}{8}.-\cos u + c$

$= -\frac{1}{8} \cos 4x + c$

Verify: $\frac{d}{dx}\left(-\frac{1}{8} \cos 4x + c\right)$

$= -\frac{1}{8}\, \frac{d}{dx} \cos 4x + \frac{d}{dx}c$

$= -\frac{1}{8}.-\sin 4x.4 + 0$

$= \frac{1}{2} \sin 4x$

2.7 $\frac{1}{3}\int(\cos 7x\cos 2x - \sin 7x\sin 2x)\,dx$

$= \frac{1}{3}\int\cos(7x + 2x)\,dx$

$= \frac{1}{3}\int\cos 9x\,dx$

$u = 9x$

$du = 9\,dx$

$\frac{1}{9}\,du = dx$

$\therefore \frac{1}{3}\int\cos 9x\,dx$

$= \frac{1}{3}\int\cos u.\frac{1}{9}\,du$

$= \frac{1}{27}\int\cos u\,du$

$= \frac{1}{27}.\sin u + c$

$= \frac{1}{27}\sin 9x + c$

Verify: $\frac{d}{dx}\left(\frac{1}{27}\sin 9x + c\right)$

$= \frac{1}{27}\frac{d}{dx}\sin 9x + \frac{d}{dx}c$

$= \frac{1}{27}.\cos 9x.9 + 0$

$= \frac{1}{3}\cos 9x$

2.8 $\int 4\sin 2x\cos 2x\cos 4x\,dx$

$= \int 2(2\sin 2x\cos 2x)\cos 4x\,dx$

$= \int 2\sin 4x\cos 4x\,dx$

$= \int\sin 8x\,dx$

$u = 8x$

$du = 8\,dx$

$\frac{1}{8}\,du = dx$

$\therefore \int\sin 8x\,dx$

$= \int\sin u.\frac{1}{8}\,du$

$= \frac{1}{8}\int\sin u\,du$

$= \frac{1}{8}.-\cos u + c$

$= -\frac{1}{8}\cos 8x + c$

Verify: $\frac{d}{dx}\left(-\frac{1}{8}\cos 8x + c\right)$

$= -\frac{1}{8}\frac{d}{dx}\cos 8x + \frac{d}{dx}c$

$= -\frac{1}{8}.-\sin 8x.8 + 0$

$= \sin 8x$

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1\.

1.1 $\int(x^4 + 3x)\,dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} + c$

1.2 $\int(x^5 - \frac{1}{x^3} + 5)\,dx$

$= \int(x^5 - x^{-3} + 5)\,dx$

$= \frac{x^6}{6} - \frac{x^{-2}}{-2} + 5x + c$

$= \frac{1}{6}x^6 + \frac{1}{2x^2} + 5x + c$

1.3 $\int(x^2 + y^2)\,dx$

$= \frac{x^3}{3} + y^2x + c$

or $\frac{1}{3}x^3 + y^2x + c$

1.4 $\int(x^2 + y^2)\,dy$

$= x^2y + \frac{y^3}{3} + c$

or $x^2y + \frac{1}{3}y^3 + c$

1.5 $\int(x^2 + y^2)\,da$

$= x^2a + y^2a + c$

1.6 $\int\left(\frac{2}{x} - 2\sqrt{x} + \pi x\right)dx$

$= \int\left(\frac{2}{x} - 2x^{\frac{1}{2}} + \pi x\right)dx$

$= 2\ln x - \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{\pi x^2}{2} + c$

$= 2\ln x - \frac{4}{3}\sqrt{x^3} + \frac{1}{2}\pi x^2 + c$

1.7 $\int(8 - \frac{4}{x} - \frac{1}{3x} - 2^{3x})\,dx$

$= \int(8 - \frac{12 - 1}{3x} - 2^{3x})\,dx$

$= \int(8 - \frac{11}{3x} - 2^{3x})\,dx$

$= 8x - \frac{11}{3}\ln x - \frac{2^{3x}}{3\ln 2} + c$ or

$= 8x - 4\ln x - \frac{1}{3}\ln x - \frac{2^{3x}}{3\ln 2} + c$

$= 8x - \frac{11}{3}\ln 2 - \frac{2^{3x}}{3\ln 2} + c$

1.8 $\int\left(3.4^{-2x} - x^{-1} - \frac{3}{x^4}\right)dx$

$\int(3.4^{-2x} - x^{-1} - 3x^{-4})\,dx$

$= \frac{3.4^{-2x}}{-2\ln 4} - \ln x - \frac{3x^{-3}}{-3} + c$

$= \frac{3}{-2.4^{2x}\ln 4} - \ln x + \frac{1}{x^3} + c$

1.9 $\int(\sqrt{x^3} + 4x^{-1} - \frac{1}{2x^3} - a)\,dx$

$= \int(x^{\frac{3}{2}} + 4x^{-1} - \frac{1}{2}x^{-3} - a)\,dx$

$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 4\ln x - \frac{1}{2}\cdot\frac{x^{-2}}{-2} - ax + c$

$= \frac{2}{5}\sqrt{x^5} + 4\ln x + \frac{1}{4x^2} - ax + c$

1.10 $\int\left(2x^{-2} + \frac{1}{x^2} - 3 + \frac{3}{x}\right)dx$

$= \int\left(3x^{-2} - 3 + \frac{3}{x}\right)dx$ or

$= \frac{3x^{-1}}{-1} - 3x + 3\ln x + c$

$= -\frac{3}{x} - 3x + 3\ln x + c$

$\int\left(2x^{-2} + x^{-2} - 3 + \frac{3}{x}\right)dx$

$= \frac{2x^{-1}}{-1} + \frac{x^{-1}}{-1} - 3x + 3\ln x + c$

$= -\frac{2}{x} - \frac{1}{x} - 3x + 3\ln x + c$

$= -\frac{3}{x} - 3x + 3\ln x + c$

1.11 $\int(2^x - \pi^{2x} + x^\pi - c)\,dx$

$= \frac{2^x}{\ln 2} - \frac{\pi^{2x}}{2\ln\pi} + \frac{x^{\pi+1}}{\pi+1} - cx + c_1$

1.12 $\int\left[\frac{2}{\sqrt{x}} + 6x - 3.2^{4x} + \left(\frac{1}{3}\right)^{-x}\right]dx$

$= \int\left(2x^{-\frac{1}{2}} + 6x - 3.2^{4x} + \left(\frac{1}{3}\right)^{-x}\right)dx$

$= \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6x^2}{2} - \frac{3.2^{4x}}{4\ln 2} + \frac{\left(\frac{1}{3}\right)^{-x}}{-\ln\left(\frac{1}{3}\right)} + c$

$= 4\sqrt{x} + 3x^2 - \frac{3.2^{4x}}{4\ln 2} - \frac{1}{\left(\frac{1}{3}\right)^x \ln\left(\frac{1}{3}\right)} + c$

2. 2.1 $\int\frac{5x^2+2x-1}{x}\,dx$

$= \int\left(5x + 2 - \frac{1}{x}\right)dx$

$= \frac{5x^2}{2} + 2x - \ln x + c$

2.2 $\int\left(bx + \frac{1}{bx}\right)dx$

$= \frac{bx^2}{2} + \frac{1}{b}\ln x + c$

2.3 $\int(x^2+3)(x-4)\,dx$

$= \int(x^3 - 4x^2 + 3x - 12)\,dx$

$= \frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2} - 12x + c$

or $\frac{1}{4}x^4 - \frac{4}{3}x^3 + \frac{3}{2}x^2 - 12x + c$

2.4 $\int\frac{t^5+3t-1}{t^5}\,dt$

$= \int(1 + 3t^{-4} - t^{-5})\,dt$

$= t + \frac{3t^{-3}}{-3} - \frac{t^{-4}}{-4} + c$

$= t - \frac{1}{t^3} + \frac{1}{4t^4} + c$

2.5 $\int(3-\sqrt{x})^2\,dx$

$= \int(3-\sqrt{x})(3-\sqrt{x})\,dx$

$= \int(9 - 3\sqrt{x} - 3\sqrt{x} + x)\,dx$

$= \int(9 - 6x^{\frac{1}{2}} + x)\,dx$

$= 9x - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^2}{2} + c$

$= 9x - 4\sqrt{x^3} + \frac{1}{2}x^2 + c$

2.6 $\int x\left(x^2 - \frac{3}{x}\right)dx$

$= \int(x^3 - 3)\,dx$

$= \frac{x^4}{4} - 3x + c$

or $\frac{1}{4}x^4 - 3x + c$

3. 3.1 $\int\left(\frac{x^2-9x+8}{\sqrt{x}}\right)dx$

$= \int\left(\frac{x^2}{\sqrt{x}} - \frac{9x}{\sqrt{x}} + \frac{8}{\sqrt{x}}\right)dx$

$= \int\left(x^{\frac{3}{2}} - 9x^{\frac{1}{2}} + 8x^{-\frac{1}{2}}\right)dx$

$= \int x^{\frac{3}{2}}\,dx - 9\int x^{\frac{1}{2}}\,dx + 8\int x^{-\frac{1}{2}}\,dx$

$= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} - 9.\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 8.\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$

$= \frac{2}{5}x^2\sqrt{x} - 6x\sqrt{x} + 16\sqrt{x} + c$

3.2 $\int(e^{-x} - e^x)^2\,dx$

$= \int(e^{-x} - e^x)(e^{-x} - e^x)\,dx$

$= \int\left(e^{-2x} - 2e^0 + e^{2x}\right)dx$

$= \int\left(e^{-2x} - 2 + e^{2x}\right)dx$

$= \frac{e^{-2x}}{-2} - 2x + \frac{e^{2x}}{2} + c$

$= -\frac{1}{2e^{2x}} - 2x + \frac{1}{2}e^{2x} + c$

3.3 $\int\left(2e^{-4x} + 2a^{2x} + \pi x\right) dx$

$= \frac{2e^{-4x}}{-4} + \frac{2a^{2x}}{2 \ln a} + \frac{\pi x^2}{2} + c$

$= -\frac{1}{2e^{4x}} + \frac{a^{2x}}{\ln a} + \frac{1}{2}\pi x^2 + c$

3.4 $\int\left(3 \sin 2x + 4.3^{2x} - \frac{6}{x}\right) dx$

$= -\frac{3 \cos 2x}{2} + \frac{4.3^{2x}}{2 \ln 3} - 6 \ln x + c$

$= -\frac{3}{2}\cos 2x + \frac{2.3^{2x}}{\ln 3} - 6 \ln x + c$

3.5 $\int\left(x^{\pi} - \pi^{x} - \frac{x}{n} + \frac{n}{x}\right) dx$

$= \frac{x^{\pi+1}}{\pi + 1} - \frac{\pi^x}{\ln \pi} - \frac{1}{n}.\frac{x^{n_1+1}}{n_1 + 1} - n \ln x + c$

3.6 $\int\left(\frac{1 - x^2 \sin 2x}{x^2}\right) dx$

$= \int\left(\frac{1}{x^2} - \frac{x^2 \sin 2x}{x^2}\right) dx$

$= \int\left(x^{-2} - \sin 2x\right) dx$

$= \frac{x^{-1}}{-1} + \frac{\cos 2x}{2} + c$

$= -\frac{1}{x} + \frac{1}{2}\cos 2x + c$

3.7 $\int\left(3 \cos \frac{1}{2}x + b \sin ax\right) dx$

$= \frac{3 \sin \frac{1}{2}x}{\frac{1}{2}} - \frac{b \cos ax}{a} + c$

$= 6 \sin \frac{1}{2}x - \frac{b}{a}\cos ax + c$

3.8 $\int\left(\frac{\sin^2 x + \cos^2 x}{\sec x} - 3.10^{-3x}\right) dx$

$= \int\left(\frac{1}{\sec x} - 3.10^{-3x}\right) dx$

$= \int\left(\cos x - 3.10^{-3x}\right) dx$

$= \sin x - \frac{3.10^{-3x}}{-3 \ln 10} + c$

$= \sin x + \frac{10^{-3x}}{\ln 10} + c$

$= \sin x + \frac{1}{(\ln 10)\, 10^{3x}} + c$

4.1 $\int\left(4 \sin x \cos x - \frac{6}{x} - 2e^{x}\right) dx$

$= 2\int 2 \sin x \cos x\, dx - \int \frac{6}{x}\, dx - 2\int e^x\, dx$

$= 2\int \sin 2x\, dx - \int \frac{6}{x}\, dx - 2\int e^x\, dx$

$u = 2x$

$du = 2\, dx$

$\frac{1}{2}.du = dx$

$= 2\int \sin u.\frac{1}{2}\, du - \int \frac{6}{x}\, dx - 2\int e^x\, dx$

$= \int \sin u\, du - \int \frac{6}{x}\, dx - 2\int e^x\, dx$

$= -\cos u - 6 \ln x - 2.e^x + c$

$= -\cos 2x - 6 \ln x - 2e^x + c$

4.2 $\int (e^{-4x} - 9\sin^2 x)\, dx$

$= \int \left[e^{-4x} - 9\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)\right] dx$

$= \int \left[e^{-4x} - \frac{9}{2} + \frac{9}{2}\cos 2x\right] dx$

$= \int e^{-4x}\, dx - \int \frac{9}{2}\, dx + \frac{9}{2}\int \cos 2x\, dx$

$\downarrow \qquad\qquad \downarrow$

$u = -4x \qquad v = 2x$

$du = -4\, dx \qquad dv = 2\, dx$

$-\frac{1}{4}\, du = dx \qquad \frac{1}{2}\, dv = dx$

$= \int e^u - \frac{1}{4}\, du - \int \frac{9}{2}\, dx + \frac{9}{2}\int \cos v \frac{1}{2}\, dv$

$= -\frac{1}{4}\int e^u\, du - \int \frac{9}{2}\, dx + \frac{9}{4}\int \cos v\, dv$

$= -\frac{1}{4}e^u - \frac{9}{2}x + \frac{9}{4}\sin v + c$

$= -\frac{1}{4}e^{-4x} - \frac{9}{2}x + \frac{9}{4}\sin 2x + c$

Verify: $\frac{d}{dx}\left(-\frac{1}{4}e^{-4x} - \frac{9}{2}x + \frac{9}{4}\sin 2x + c\right)$

$= -\frac{1}{4}\frac{d}{dx}e^{-4x} - \frac{9}{2}\frac{d}{dx}x + \frac{9}{4}\frac{d}{dx}\sin 2x + \frac{d}{dx}c$

$= -\frac{1}{4}.e^{-4x}.-4 - \frac{9}{2}.1 + \frac{9}{4}\cos 2x.2 + 0$

$= e^{-4x} - \frac{9}{2} + \frac{9}{2}\cos 2x$

4.3 $\int \left(\frac{4\sin^3 x - 108}{2\sin x - 6}\right) dx$

$= \int \left[\frac{4(\sin^3 x - 27)}{2(\sin x - 3)}\right] dx$

$= \int \left[\frac{4(\sin x - 3)(\sin^2 x + 3\sin x + 9)}{2(\sin x - 3)}\right] dx$

$= \int [2(\sin^2 x + 3\sin x + 9]\, dx$

$= \int \left[2\left(\frac{1}{2} - \frac{1}{2}\cos 2x + 3\sin x + 9\right)\right] dx$

$= \int \left[2\left(-\frac{1}{2}\cos 2x + 3\sin x + \frac{19}{2}\right)\right] dx$

$= \int [-\cos 2x + 6\sin x + 19]\, dx$

$= -\int \cos 2x\, dx + 6\int \sin x\, dx + \int 19\, dx$

$\downarrow$

$u = 2x$

$du = 2\, dx$

$\frac{1}{2}\, du = dx$

$= -\int \cos u.\frac{1}{2}\,du + 6\int \sin x\,dx + \int 19\,dx$

$= -\frac{1}{2}\int \cos u\,du + 6\int \sin x\,dx + \int 19\,dx$

$= -\frac{1}{2}.\sin u + 6.-\cos x + 19x + c$

$= -\frac{1}{2}\sin 2x - 6\cos x + 19x + c$

Verify: $\frac{d}{dx}\left(-\frac{1}{2}\sin 2x - 6\cos x + 19x + c\right)$

$= -\frac{1}{2}\frac{d}{dx}\sin 2x - 6\frac{d}{dx}\cos x + 19\frac{d}{dx}x + \frac{d}{dx}c$

$= -\frac{1}{2}.\cos 2x.2 - 6.-\sin x + 19.1 + 0$

$= -\cos 2x + 6\sin x + 19$

Activity 6.7 SB page 400

1. 1.1 $\int_0^2 2x\,dx$

$= \left[\frac{2x^2}{2}\right]_0^2$

$= [x^2]_0^2$

$= (2)^2 - (0)^2$

$= 4$

1.2 $\int_1^3 (5x^2 - 1)\,dx$

$= \left[\frac{5x^3}{3} - x\right]_1^3$

$= \left(\frac{5(3)^3}{3} - (3)\right) - \left(\frac{5(1)^3}{3} - (1)\right)$

$= (42) - \left(\frac{2}{3}\right)$

$= 41\frac{1}{3}$

1.3 $\int_1^2 (x^3 + 2x^2 - 5)\,dx$

$= \left[\frac{x^4}{4} + \frac{2x^3}{3} - 5x\right]_1^2$

$= \left(\frac{2^4}{4} + \frac{2(2)^3}{3} - 5(2)\right) - \left(\frac{1^4}{4} + \frac{2(1)^3}{3} - 5(1)\right)$

$= \left(4 + \frac{16}{3} - 10\right) - \left(\frac{1}{4} + \frac{2}{3} - 5\right)$

$= \left(-6 + \frac{16}{3}\right) - \left(\frac{3+8}{12} - 5\right)$

$= -\frac{2}{3} + 4\frac{1}{12}$

$= \frac{-8+49}{12} = \frac{41}{12}$

$= 3\frac{5}{12}$

1.4 $\int_2^4 \left(\sqrt{x} - \frac{1}{x^3} + 4x^3\right)dx$

$= \int_2^4 (x^{\frac{1}{2}} - x^{-3} + 4x^3)\,dx$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{-2}}{-2} + \frac{4x^4}{4}\right]_2^4$

$= \left(\frac{2}{3}(4)^{\frac{3}{2}} + \frac{1}{2(4)^2} + (4)^4\right) - \left(\frac{2}{3}(2)^{\frac{3}{2}} + \frac{1}{2(2)^2} + (2)^4\right)$

$= (261{,}365) - (18{,}011)$

$= 243{,}354$

1.5 $\int_3^5 (x^{-\frac{5}{3}})\, dx$

$= \left[\frac{x^{-\frac{2}{3}}}{-\frac{2}{3}}\right]_3^5$

$= \left(-\frac{3}{2}(5)^{-\frac{2}{3}}\right) - \left(-\frac{3}{2}(3)^{-\frac{2}{3}}\right)$

$= (-0{,}513) - (-0{,}721)$

$= 0{,}208$

1.6 $\int_{0,5}^{1} \frac{1}{x}\, dx$

$= [\ln x]_{0,5}^{1}$

$= (\ln 1) - (\ln 0{,}5)$

$= 0{,}693$

1.7 $\int_0^{\frac{1}{2}} 5^{-2x}\, dx$

$= \left[\frac{5^{-2x}}{-2 \ln 5}\right]_0^{\frac{1}{2}}$

$= \left(\frac{5^{-2(\frac{1}{2})}}{-2 \ln 5}\right) - \left(\frac{5^{-2(0)}}{-2 \ln 5}\right)$

$= \left(\frac{5^{-1}}{-2 \ln 5}\right) - \left(\frac{1}{-2 \ln 5}\right)$

$= \left(\frac{1}{-10 \ln 5}\right) + \left(\frac{1}{2 \ln 5}\right) = -0{,}062 + 0{,}311$

$= 0{,}249$

1.8 $\int_1^2 (3x^{-1} + 2^x)\, dx$

$= \left[3 \ln x + \frac{2^x}{\ln 2}\right]_1^2$

$= \left(3 \ln 2 + \frac{2^2}{\ln 2}\right) - \left(3 \ln 1 + \frac{2}{\ln 2}\right)$

$= \left(3 \ln 2 + \frac{4}{\ln 2}\right) - \left(0 + \frac{2}{\ln 2}\right)$

$= 4{,}965$

1.9 $\int_{-2}^{1} (4x - 1)^2\, dx$

$= \int_{-2}^{1} (4x - 1)(4x - 1)\, dx$

$= \int_{-2}^{1} (16x^2 - 8x + 1)\, dx$

$= \left[\frac{16x^3}{3} - \frac{8x^2}{2} + x\right]_{-2}^{1}$

$= \left(\frac{16}{3}(1^3) - 4(1^2) + (1)\right) - \left(\frac{16}{3}(-2^3) - 4(-2^2) + (-2)\right)$

$= \left(\frac{16}{3} - 4 + 1\right) - \left(-\frac{128}{3} - 16 - 2\right)$

$= 2{,}333 - (-60{,}667)$

$= 62{,}9997$

$= 63$

1.10 $\int_2^3 \frac{t^2 + 2t - 1}{t^2}\, dt$

$= \int_2^3 \left(1 + \frac{2}{t} - t^{-2}\right) dt$

$= \left[t + 2 \ln t - \frac{t^{-1}}{-1}\right]_2^3$

$= \left[t + 2 \ln t + \frac{1}{t}\right]_2^3$

$= \left(3 + 2 \ln 3 + \frac{1}{3}\right) - \left(2 + 2 \ln 2 + \frac{1}{2}\right)$

$= 5{,}531 - 3{,}886$

$= 1{,}645$

1.11 $\int_0^3 \left(\frac{y}{3} - \frac{1}{3}\right) dy$

$= \left[\frac{1}{3}.\frac{y^2}{2} - \frac{1}{3}y\right]_0^3$

$= \left[\frac{y^2}{6} - \frac{1}{3}y\right]_0^3$

$= \left(\frac{3^2}{6} - \frac{1}{3}(3)\right) - \left(\frac{0^2}{6} - \frac{1}{3}(0)\right)$

$= \left(\frac{9}{6} - 1\right)$

$= 0{,}5$

1.12 $\int_2^3 \left(\frac{x + 2}{x}\right)^2 dx$

$= \int_2^3 \frac{(x + 2)(x + 2)}{x^2}\, dx$

$= \int_2^3 \left(\frac{x^2 + 4x + 4}{x^2}\right) dx$

$= \int_2^3 1 + 4x^{-1} + 4x^{-2}\, dx$

$= \left[x + 4 \ln x + 4.\frac{x^{-1}}{-1}\right]_2^3$

$= \left[x + 4 \ln x - \frac{4}{x}\right]_2^3$

$= \left(3 + 4 \ln 3 - \frac{4}{3}\right) - \left(2 + 4 \ln 2 - \frac{4}{2}\right)$

$= 6{,}061 - 2{,}773$

$= 3{,}288$

2. $w = \int_{0,2}^{0,5} \frac{20}{V}\, d\,V$

$= [20 \ln V]_{0,2}^{0,5}$

$= (20 \ln 0{,}5) - (20 \ln 0{,}2)$

$= -13{,}863 - (32{,}189)$

$= 18{,}326$

3. 3.1 $\int_{-1}^{2} \sqrt[3]{\frac{2}{x}}\, dx$

$= \int_{-1}^{2} \frac{\sqrt[3]{2}}{\sqrt[3]{x}}\, dx$

$= \sqrt[3]{2}\int_{-1}^{2} x^{-\frac{1}{3}}\, dx$

$= \sqrt[3]{2}\left[\frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}\right]_{-1}^{2}$

$= \sqrt[3]{2}\left[\frac{3}{2}x^{\frac{2}{3}}\right]_{-1}^{2}$

$= \sqrt[3]{2}\left[\left(\frac{3}{2}(2)^{\frac{2}{3}}\right) - \left(\frac{3}{2}(-1)^{\frac{2}{3}}\right)\right]$

$= 1{,}110$

3.2 $\int_{3}^{5} \frac{x^{-1}}{\sqrt{17}}\, dx$

$= \frac{1}{\sqrt{17}}\int_{3}^{5} \frac{1}{x}\, dx$

$= \frac{1}{\sqrt{17}}[\ln|x|]_{3}^{5}$

$= \frac{1}{\sqrt{17}}[(\ln 5) - (\ln 3)]$

$= 0{,}124$

3.3 $\int_{-3}^{-1} e^{x+\ln 3}\, dx$

$= \int_{-3}^{-1} e^{x}.e^{\ln 3}\, dx$

$= e^{\ln 3}\int_{-3}^{-1} e^{x}\, dx$

$= 3\int_{-3}^{-1} e^{x}\, dx$

$= 3[e^{x}]_{-3}^{-1}$

$= 3\left[\left(e^{(-1)}\right) - \left(e^{(-3)}\right)\right]$

$= 0{,}954$

3.4 $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \sqrt{(1+\cos x)(1-\cos x)}\, dx$

$= \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \sqrt{1-\cos^2 x}\, dx$

$= \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \sin x\, dx$

$= [-\cos x]_{\frac{\pi}{12}}^{\frac{\pi}{4}}$

$= \left[\left(-\cos\left(\frac{\pi}{4}\right)\right) - \left(-\cos\left(\frac{\pi}{12}\right)\right)\right]$

$= 0{,}259$

3.5 $\int_{\frac{\pi}{12}}^{\frac{\pi}{3}} \cos 3x\, dx$

$= -\int_{\frac{\pi}{12}}^{\frac{\pi}{3}} \sin 3x\, dx$

$u = 3x$

$du = 3\, dx$

$\frac{1}{3}\, du = dx$

$= -\int_{\frac{\pi}{12}}^{\frac{\pi}{3}} \sin u.\frac{1}{3}\, du$

$= -\frac{1}{3}\int_{\frac{\pi}{12}}^{\frac{\pi}{3}} \sin u\, du$

$= -\frac{1}{3}[-\cos u]_{\frac{\pi}{12}}^{\frac{\pi}{3}}$

$= -\frac{1}{3}[\,-\cos 3x]_{\frac{\pi}{12}}^{\frac{\pi}{3}}$

$= -\frac{1}{3}\left[\left(-\cos\left(3.\frac{\pi}{3}\right)\right) - \left(-\cos\left(3.\frac{\pi}{12}\right)\right)\right]$

$= -0{,}569$

3.6 $\frac{1}{3}\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} (\cos 7x.\cos 2x - \sin 7x.\sin 2x)\, dx$

$= \frac{1}{3}\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos(7x + 2x)\, dx$

$= \frac{1}{3}\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos 9x\, dx$

$u = 9x$

$du = 9\, dx$

$\frac{1}{9}\, du = dx$

$= \frac{1}{3}\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos u.\frac{1}{9}\, du$

$= \frac{1}{27}\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos u\, du$

$= \frac{1}{27}[\sin u]_{\frac{\pi}{2}}^{\frac{3\pi}{4}}$

$= \frac{1}{27}[\,\sin 9x]_{\frac{\pi}{2}}^{\frac{3\pi}{4}}$

$= \frac{1}{27}\left[\left(\sin\left(9.\frac{3\pi}{4}\right)\right) - \left(\sin\left(9.\frac{\pi}{2}\right)\right)\right]$

$= -0{,}011$

3.7 $\int_1^2 \left(\frac{1}{3x} - \frac{4}{x^2} + \frac{2}{\sqrt{x}}\right) dx$

$= \int_1^2 \left(\frac{1}{3}.\frac{1}{x} - 4.x^{-2} + 2.x^{-\frac{1}{2}}\right) dx$

$= \left[\frac{1}{3}.\ln|x| - 4.\frac{x^{-2+1}}{-2+1} + 2.\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_1^2$

$= \left[\frac{1}{3}\ln|x| + \frac{4}{x} + 4\sqrt{x}\right]_1^2$

$= \left[\left(\frac{1}{3}\ln(2) + \frac{4}{(2)} + 4\sqrt{(2)}\right) - \left(\frac{1}{3}\ln(1) + \frac{4}{(1)} + 4\sqrt{(1)}\right)\right]$

$= -0{,}112$

3.8 $\int_{\frac{\pi}{5}}^{\frac{4\pi}{5}} \left(\frac{4\sin^3 x - 108}{2\sin x - 6}\right) dx$

$= \int_{\frac{\pi}{5}}^{\frac{4\pi}{5}} \left(\frac{4[(\sin x - 3)(\sin^2 x + 3\sin x + 9)]}{2(\sin x - 3)}\right) dx$

$= \int_{\frac{\pi}{5}}^{\frac{4\pi}{5}} [2(\sin^2 x + 3\sin x + 9)]\ dx$

$= \int_{\frac{\pi}{5}}^{\frac{4\pi}{5}} \left[2\left(\frac{1}{2} - \frac{1}{2}\cos 2x + 3\sin x + 9\right)\right] dx$

$= \int_{\frac{\pi}{5}}^{\frac{4\pi}{5}} \left[2\left(-\frac{1}{2}\cos 2x + 3\sin x + \frac{19}{2}\right)\right] dx$

$= \int_{\frac{\pi}{5}}^{\frac{4\pi}{5}} [-\cos 2x + 6\sin x + 19]\ dx$

$= \left[-\frac{\sin 2x}{2} + 6.-\cos x + 19x\right]_{\frac{\pi}{5}}^{\frac{4\pi}{5}}$

$= \left[\left(-\frac{1}{2}\sin\left(2.\frac{4\pi}{5}\right) - 6\cos\left(\frac{4\pi}{5}\right) + 19\left(\frac{4\pi}{5}\right)\right) - \left(-\frac{1}{2}\sin\left(2.\frac{\pi}{5}\right) - 6\cos\left(\frac{\pi}{5}\right) + 19\left(\frac{\pi}{5}\right)\right)\right]$

$= 46{,}473$

Activity 6.8 — SB page 415

1. $\Delta A = y\Delta x$

$A = \int_0^2 y\ dx$

$= \int_0^2 (-x^2 + 2x)\ dx$

$= \left[-\frac{x^3}{3} + \frac{2x^2}{2}\right]_0^2$

$= \left[-\frac{1}{3}x^3 + x^2\right]_0^2$

$= \left(-\frac{1}{3}(2)^3 + (2)^2\right) - \left(-\frac{1}{3}(0)^3 + (0)\right)$

$= -\frac{8}{3} + 4$

$= 1{,}333$ units^2

2. $\Delta A = y\Delta x$

$A = \int_1^2 x^3\ dx$

$= \left[\frac{x^4}{4}\right]_1^2$

$= \left(\frac{2^4}{4}\right) - \left(\frac{1^4}{4}\right)$

$= 3{,}75$ units^2

3. 3.1 $y = 2x + 2$

x-intercept: $y = 0$

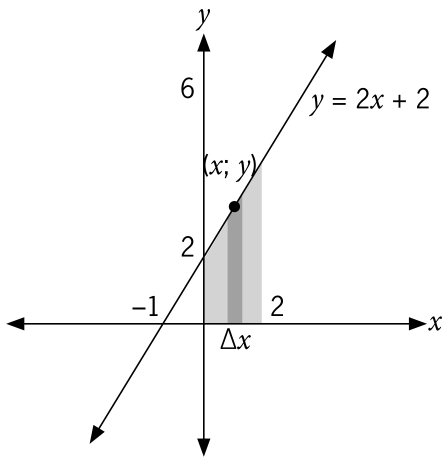
$0 = 2x + 2$

$2x = -2$

$\therefore x = -1$

y-intercept: $x = 0$

$\therefore y = 2$



3.2 $\Delta A = y\Delta x$

$A = \int_0^2 y\ dx$

$= \int_0^2 (2x + 2)\ dx$

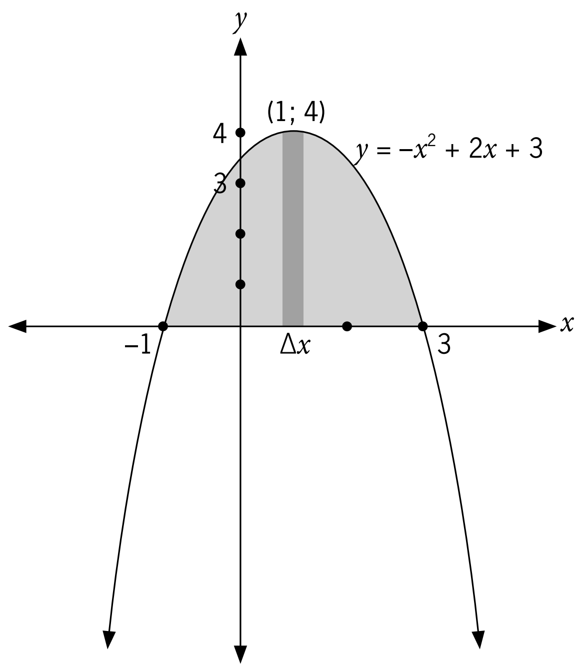
$= \left[\frac{2x^2}{2} + 2x\right]_0^2$

$= [x^2 + 2x]_0^2$

$= ((2)^2 + 2(2)) - (0 + 2(0))$

$= 8$ units^2

4. $y = -x^2 + 2x + 3$



x-intercept: $y = 0$

$0 = -x^2 + 2x + 3$

$0 = x^2 - 2x - 3$

$0 = (x - 3)(x + 1)$

$\therefore x = 3; x = -1$

y-intercept: $x = 0$

$\therefore y = 3$

$$\text{TP: } x = \frac{-b}{2a}$$
$$= \frac{-2}{2(-1)}$$
$$= 1$$

$$y = -(1)^2 + 2(1) + 3$$
$$= 4$$

$\therefore$ TP: (1; 4)

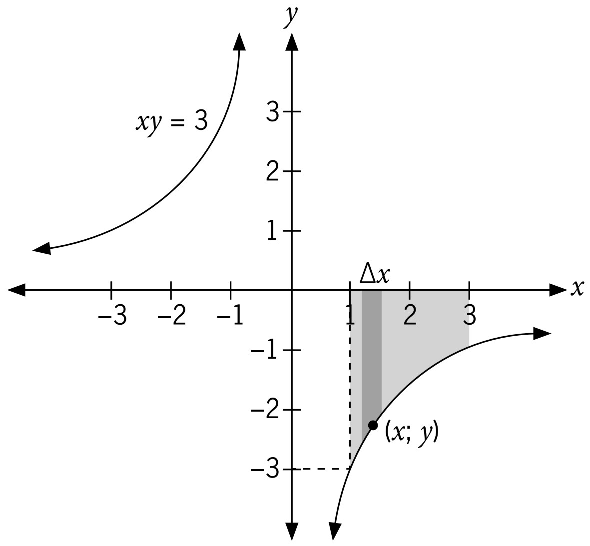
$\Delta A = y\Delta x$

$$A = \int_{-1}^{3} y\, dx$$
$$= \int_{-1}^{3} (-x^2 + 2x + 3)\, dx$$
$$= \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 3x\right]_{-1}^{3}$$
$$= \left(-\tfrac{1}{3}(3)^3 + (3)^2 + 3(3)\right) - \left(-\tfrac{1}{3}(-1)^3 + (-1)^2 + 3(-1)\right)$$
$$= (9) - \left(-1\tfrac{2}{3}\right)$$
$$= 10\tfrac{2}{3} \text{ units}^2 \text{ or } 10{,}667 \text{ units}^2$$

5. $\Delta A = y\Delta x$

$$A = \int_{0}^{1} (x^2 - x)\, dx$$
$$= \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_{0}^{1}$$
$$= \left(\frac{(1)^3}{3} - \frac{(1)^2}{2}\right) - (0)$$
$$= \left|\frac{1}{3} - \frac{1}{2}\right|$$
$$= 0{,}167 \text{ units}^2$$

6. 6.1



6.2 $\Delta A = y\Delta x$

$A = -\int_1^3 y\,dx$

$= -\int_1^3 \left(-\frac{3}{x}\right) dx$

$= -[-3\ln x]_1^3$

$= -[(-3\ln 3) - (-3\ln 1)]$

$= -[(-3{,}296) - (0)]$

$= 3{,}296 \text{ units}^2$

7. 7.1 $f(x) = x^3 - 6x^2 + 8x$

y-intercept: $x = 0$

$\therefore y = 0$

x-intercept: $y = 0$

$x^3 - 6x^2 + 8x = 0$

$x(x^2 - 6x + 8) = 0$

$x(x - 4)(x - 2) = 0$

$x = 0$ or $x = 4$ or $x = 2$

Turning points

$f'(x) = 3x^2 - 12x + 8 = 0$

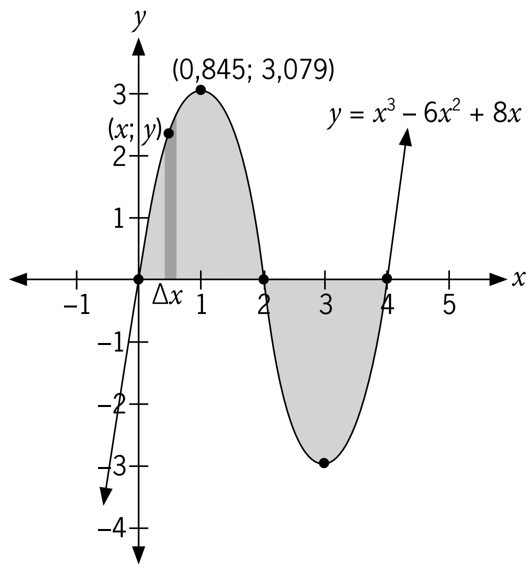
$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(8)}}{2(3)}$

$= \frac{12 \pm \sqrt{48}}{6}$

$= 3{,}155$ or $0{,}845$

$f(3{,}155) = -3{,}079$

$f(0{,}8450) = 3{,}079$



7.2 $\Delta A = y\Delta x$

$A_1 = \int_0^2 y\,dx$

$= \int_0^2 (x^3 - 6x^2 + 8x)\,dx$

$= \left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_0^2$

$= \left(\frac{(2)^4}{4} - 2(2)^3 + 4(2)^2\right) - (0)$

$= 4$ units^2

$A_2 = -\int_2^4 y\,dx$

$= -\int_2^4 (x^3 - 6x^2 + 8x)\,dx$

$= -\left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_2^4$

$= -\left[\left(\frac{1}{4}(4)^4 - 2(4)^3 + 4(4)^2\right) - \left(\frac{1}{4}(2)^4 - 2(2)^3 + 4(2)^2\right)\right]$

$= -[(0) - (4)]$

$= 4$ units^2

$\therefore$ Total area $= A_1 + A_2$

$= (4 + 4)$ units^2

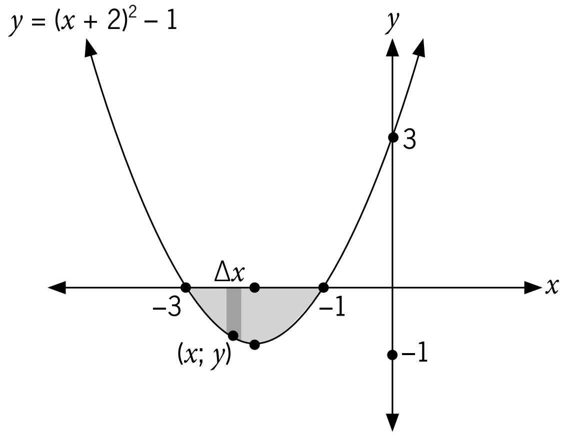
$= 8$ units^2

Alternative:

$A_1 = 4$ units$^2 \times 2$

Total A = 8 units^2

8. 8.1 $f(x) = (x + 2)^2 - 1$



x-intercept: $y = 0$

$0 = (x + 2)^2 - 1$

$0 = x^2 + 4x + 4 - 1$

$0 = x^2 + 4x + 3$

$0 = (x + 3)(x + 1)$

$\therefore x = -3;\ x = -1$

y-intercept: $x = 0$

$y = 3$

TP: $x = \frac{-b}{2a}$

$= \frac{-4}{2}$

$= -2$

$y = (-2)^2 + 4(-2) + 3$

$= 4 - 8 + 3$

$= -1$

$\therefore$ TP: $(-2; -1)$

8.2 $\Delta A = y\Delta x$

$A = -\int_{-3}^{-1}(x^2 + 4x + 3)\,dx$

$= -\left[\frac{x^3}{3} + 2x^2 + 3x\right]_{-3}^{-1}$

$= -\left[\left(\frac{1}{3}(-1)^3 + 2(-1)^2 + 3(-1)\right) - \left(\frac{1}{3}(-3)^3 + 2(-3)^2 + 3(-3)\right)\right]$

$= -\left[\left(-\frac{1}{3} + 2 - 3\right) - (-9 + 18 - 9)\right]$

$= -[(-1{,}333) - (0)]$

$= -[-1{,}333]$

$= 1{,}333 \text{ units}^2$

9. 9.1 Area $= \int_0^{\frac{\pi}{4}} 2\cos x\,dx$

$= 2\int_0^{\frac{\pi}{4}} \cos x\,dx$

$= 2[\sin x]_0^{\frac{\pi}{4}}$

$= 2\left[\sin\left(\frac{\pi}{4}\right) - \sin 0\right]$

$= 2\left[\frac{1}{\sqrt{2}} - 0\right]$

$= 1{,}414 \text{ units}^2$

9.2 Find $\frac{dy}{dx}$ to get the TP.

$\frac{d}{dx} = -2\sin x$

$y = 2\cos x$

For turning points, let $\frac{dy}{dx} = 0$:

$-2\sin x = 0$

$x = \sin^{-1}0$

$x = 0°$ or $x = 180°$

y-value when $x = 0°$:

$y = 2\cos 0°$

$= 2$

y-value when $x = 180°$:

$y = 2\cos 180°$

$= -2$

$\therefore$ Turning points: $(0°; 2)$ and $(180°; -2)$

9.3 $\frac{d^y}{dx^2} = -2 \cos x$

$= -2 \cos 0°$

$= -2$

or: $\frac{d^2y}{dx^2} = -2 \cos x$

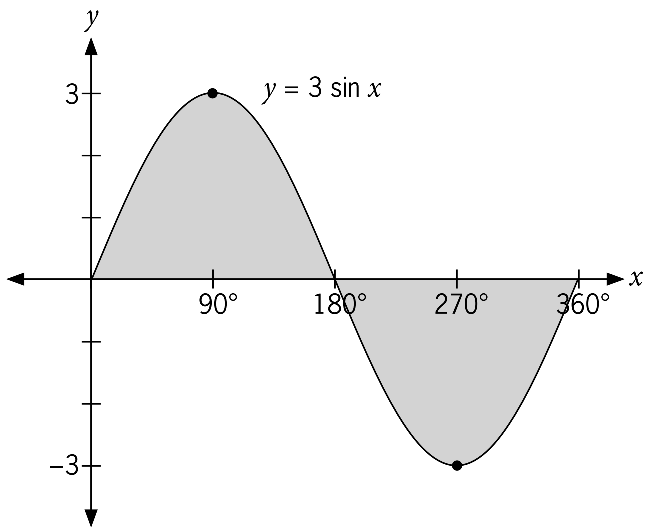
$= -2 \cos 180°$

$= 2$

Maximum: (0°; 2)

Minimum: (180°; −2)

10. 10.1 $y = 3 \sin x$ for $0° \leq x \leq 360°$



$\Delta A = y\Delta x$

$A = 2\int_{0°}^{180°} 3 \sin x \, dx$

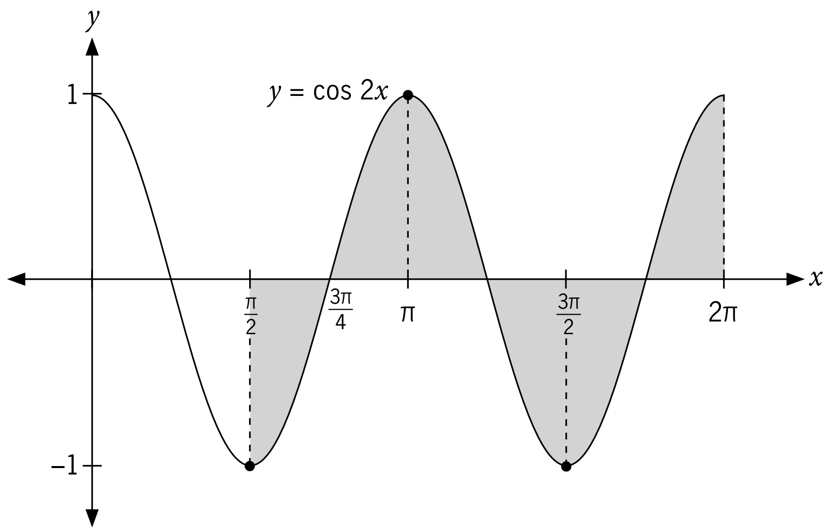
$= 2[-3 \cos x]_{0°}^{180°}$

$= -6[\cos 180° - \cos 0°]$

$= -6[\,-1 - 1]$

$= 12 \text{ units}^2$

10.2 $y = \cos 2x$, the x-axis and the ordinates $x = \frac{\pi}{2}$ and 2π



$$\Delta A = y\Delta x$$

$$A = -\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos 2x \, dx$$

$$= -\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{4}}$$

$$= -\frac{1}{2}\left[\sin 2\left(\frac{3\pi}{4}\right) - \sin 2\left(\frac{\pi}{2}\right)\right]$$

$$= -\frac{1}{2}\left[\sin\frac{3\pi}{2} - \sin \pi\right] \text{ or } = -\frac{1}{2}[\sin 270° - \sin 180°]$$

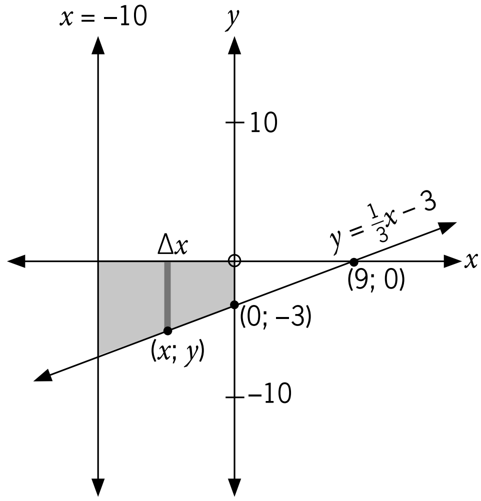
$$= -\frac{1}{2}[-1 - 0]$$

$$= \frac{1}{2}$$

$$\therefore A = \frac{1}{2} \times 6$$

$$A_T = 3 \text{ units}^2$$

11. 11.1 $y = \frac{1}{3}x - 3$



x-intercepts, $y = 0$:

$y = \frac{1}{3}x - 3$

$0 = \frac{1}{3}x - 3$

$-\frac{1}{3}x = -3$

$x = 9$

$\therefore (9; 0)$

y-intercept, $x = 0$:

$y = \frac{1}{3}x - 3$

$= \frac{1}{3}(0) - 3$

$y = -3$

$\therefore (0; -3)$

$\Delta A = |y\Delta x|$

$A = \left|\int_{-10}^{0} y\, dx\right|$ • Area below the x-axis

$= \left|\int_{-10}^{0} \left(\frac{1}{3}x - 3\right) dx\right|$

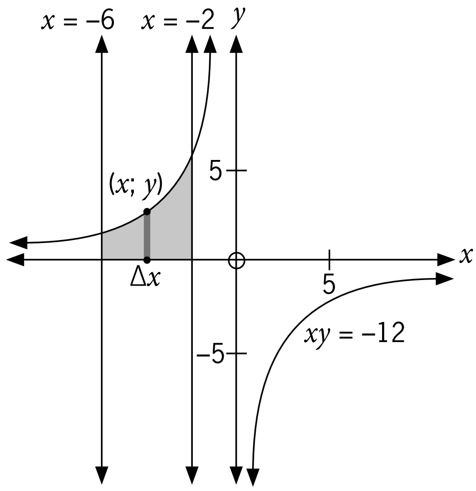
$= \left|\left[\frac{1}{6}x^2 - 3x\right]_{-10}^{0}\right|$

$= \left|\left[\frac{1}{6}(0)^2 - 3(0)\right] - \left[\frac{1}{6}(-10)^2 - 3(-10)\right]\right|$

$A = 46\frac{2}{3}$ units^2

11.2 $xy = -12$

$\therefore y = -\frac{12}{x}$



x	*y*
−7,000	1,714
−6,000	2,000
−5,000	2,400
−4,000	3,000
−3,000	4,000
−2,000	6,000
−1,000	12,000
0,000	ERROR
1,000	−12,000
2,000	−6,000
3,000	−4,000
4,000	−3,000
5,000	−2,400
6,000	−2,000
7,000	−1,714

$\Delta A = y\Delta x$

$A = \int_{-6}^{-2} y\, dx$

- Area above the x-axis

$= \int_{-6}^{-2} \left(-\frac{12}{x}\right) dx$

$= \left[-12 \ln|x|\right]_{-6}^{-2}$

$= \left[-12 \ln|(-2)|\right] - \left[-12 \ln|(-6)|\right]$

$A = 13{,}183 \text{ units}^2$

11.3. $y = x^2 + 3x - 4$

x-intercepts, $y = 0$:

$y = x^2 + 3x - 4$

$0 = x^2 + 3x - 4$

$0 = (x - 1)(x + 4)$

$x - 1 = 0$ or $x + 4 = 0$

$x = 1$ $\qquad$ $x = -4$

$\therefore (1; 0)$ $\qquad$ $\therefore (-4; 0)$

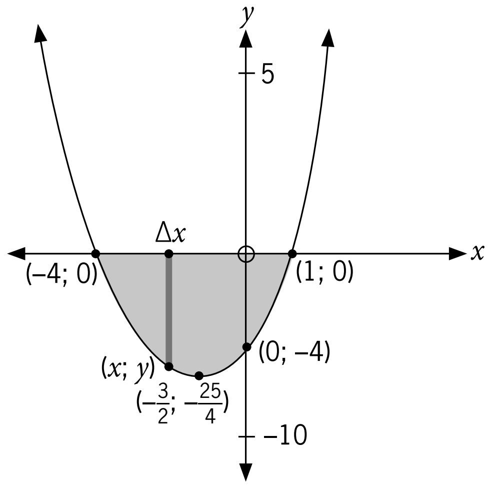
***y*-intercept, $x = 0$:**

$y = x^2 + 3x - 4$

$= (0)^2 + 3(0) - 4$

$y = -4$

$\therefore (0; -4)$



Axis of symmetry, $x = -\frac{b}{2a}$:

$x = -\frac{b}{2a}$

$= -\frac{(3)}{2(1)}$

$x = -1\frac{1}{2}$

Turning point(s):

$y = x^2 + 3x - 4$

$= \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) - 4$

$y = -6\frac{1}{4}$

$\therefore \left(-\frac{3}{2}; -6\frac{1}{4}\right)$

$\Delta A = |y\Delta x|$

$A = \left|\int_{-4}^{1} y\, dx\right|$ • Area below the x-axis or $-\int_{-4}^{1} y\, dx$

$= \left|\int_{-4}^{1} (x^2 + 3x - 4)\, dx\right|$

$= \left|\left[\frac{1}{3}x^3 + \frac{3}{2}x^2 - 4x\right]_{-4}^{1}\right|$

$= \left|\left[\frac{1}{3}(1)^3 + \frac{3}{2}(1)^2 - 4(1)\right] - \left[\frac{1}{3}(-4)^3 + \frac{3}{2}(-4)^2 - 4(-4)\right]\right|$

$A = 20\frac{5}{6}$ units^2

11.4 $y = (x - 4)^2 + 3$

***x*-intercepts, $y = 0$:**

$y = (x - 4)^2 + 3$

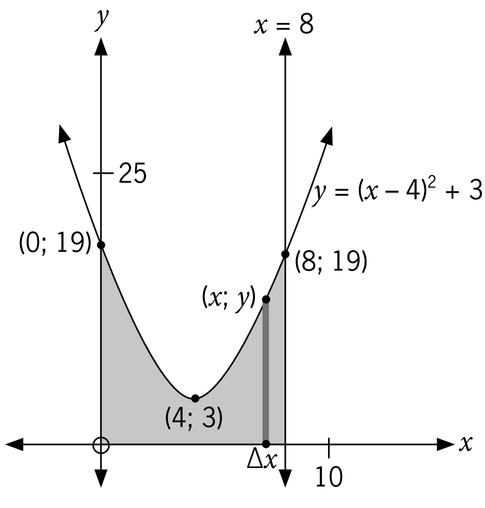
$0 = (x - 4)^2 + 3$

$-(x - 4)^2 = 3$

$(x - 4)^2 = -3$

$\sqrt{(x - 4)^2} = \sqrt{-3}$

$\therefore$ There are no real x-intercepts.



***y*-intercept, $x = 0$:**

$y = (x - 4)^2 + 3$

$= ((0) - 4)^2 + 3$

$y = 19$

$\therefore (0; 19)$

Axis of symmetry:

$y = (x - 4)^2 + 3$

$\therefore x - 4 = 0$

$x = 4$

Turning point(s):

$y = (x - 4)^2 + 3$

Since the coefficient of the bracket is positive, therefore $y = 3$ is a minimum value.

$\therefore (4; 3)$

$\Delta A = y\Delta x$

$A = \int_0^8 y\, dx$ • Area above the x-axis

$= \int_0^8 \left[(x - 4)^2 + 3\right] dx$

$= \int_0^8 \left[(x^2 - 8x + 19\right] dx$

$= \left[\frac{1}{3}x^3 - 4x^2 + 19x\right]_0^8$

$= \left[\frac{1}{3}(8)^3 - 4(8)^2 + 19(8)\right] - \left[\frac{1}{3}(0)^3 - 4(0)^2 + 19(0)\right]$

$A = 66\frac{2}{3}$ units^2

11.5 $y = x^3 + 3x^2 - 2$

***x*-intercepts, *y* = 0:**

$y = x^3 + 3x^2 - 2$

$0 = x^3 + 3x^2 - 2$

Consider the factors of the constant term.

$x = \pm 1$ and $x = \pm 2$

$f(x) = x^3 + 3x^2 - 2$

$f(-1) = (-1)^3 + 3(-1)^2 - 2$

$f(-1) = 0$

$\therefore x + 1$ is a factor

$$
\begin{array}{r|l}
 & x^2 + 2x - 2 \\
x + 1 & x^3 + 3x^2 + 0x - 2 \\
 & \underline{x^3 + 1x^2} \downarrow \\
 & 2x^2 + 0x \\
 & \underline{2x^2 + 2x} \downarrow \\
 & -2x - 2 \\
 & \underline{-2x - 2} \\
 & \underline{\quad . \quad .}
\end{array}
$$

$\therefore (x + 1)(x^2 + 2x - 2) = 0$

$x + 1 = 0$ or $x^2 + 2x - 2 = 0$

$x = -1$ $\qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore (-1; 0)$ $\qquad = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{-2 \pm 2\sqrt{3}}{2}$

$x = -1 + \sqrt{3}$ or $x = -1 - \sqrt{3}$

$x = 0{,}732$ $\qquad x = -2{,}732$

$\therefore (0{,}732; 0)$ $\qquad \therefore (-2{,}732; 0)$

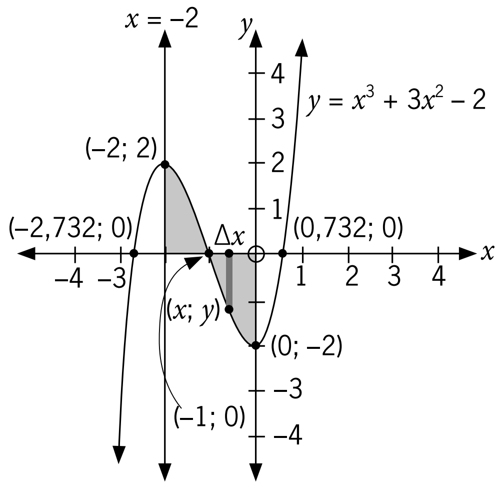
***y*-intercept, *x* = 0:**

$y = x^3 + 3x^2 - 2$

$0 = (0)^3 + 3(0)^2 - 2$

$y = -2$

$\therefore (0; -2)$



Turning point(s):

$y = x^3 + 3x^2 - 2$

$\therefore \frac{dy}{dx} = 3x^2 + 6x = 0$

$3x(x + 2) = 0$

$3x = 0$ or $x + 2 = 0$

$x = 0$ $\qquad x = -2$

Substitute $x = 0$ and $x = -2$ into $y = x^3 + 3x^2 - 2$.

$y = (0)^3 + 3(0)^2 - 2 = -2 \quad \therefore (0; -2)$

$y = (-2)^3 + 3(-2)^2 - 2 = 2 \quad \therefore (-2; 2)$

$\Delta A = |y\Delta x|$

$A = \left|\int_{-1}^{0} y\, dx\right|$

- Area below the x-axis or $A = -\int_{-1}^{0} y\, dx$

$= \left|\int_{-1}^{0} (x^3 + 3x^2 - 2)\, dx\right|$

$= \left|\left[\frac{1}{4}x^4 + x^3 - 2x\right]_{-1}^{0}\right|$

$= \left|\left[\frac{1}{4}(0)^4 + (0)^3 - 2(0)\right] - \left[\frac{1}{4}(-1)^4 + (-1)^3 - 2(-1)\right]\right|$

$A = 1\frac{1}{4}$ units^2

Therefore,

$A_{total} = 2 \times A$

$= 2 \times \frac{5}{4}$

$A_{total} = 2\frac{1}{2}$ units^2

11.6 $y = -(x + 3)(x - 1)^2$

***x*-intercepts, *y* = 0:**

$y = -(x + 3)(x - 1)^2$

$0 = -(x + 3)(x - 1)^2$

$0 = -(x + 3)$ or $0 = (x - 1)^2$

$0 = -x - 3$ $\quad$ $0 = x - 1$

$x = -3$ $\quad$ $1 = x$

$\therefore (-3; 0)$ $\quad$ $\therefore (1; 0)$

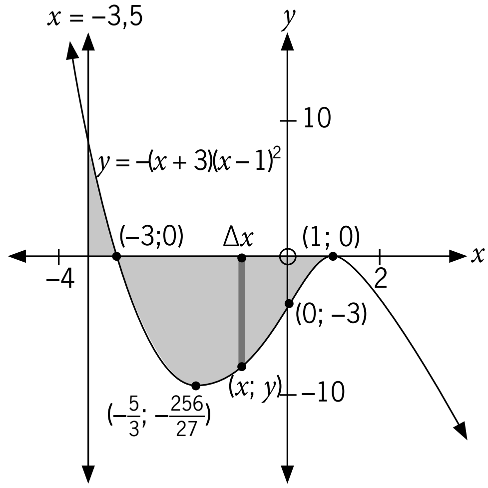
***y*-intercept, *x* = 0:**

$y = -(x + 3)(x - 1)^2$

$= -((0) + 3)((0) - 1)^2$

$y = -3$

$\therefore (0; -3)$



Turning point(s):

$y = -(x + 3)(x - 1)^2$

$y = -x^3 - x^2 + 5x - 3$

$\therefore \frac{dy}{dx} = -3x^2 - 2x + 5 = 0$

$3x^2 + 2x - 5 = 0$

$(3x + 5)(x - 1) = 0$

$3x + 5 = 0$ or $x - 1 = 0$

$3x = -5$ $\quad$ $x = 1$

$x = -1\frac{2}{3}$

Substitute $x = -1\frac{2}{3}$ and $x = 1$ into $y = -(x + 3)(x - 1)^2$.

$y = -\left(\left(-\frac{5}{3}\right) + 3\right)\left(\left(-\frac{5}{3}\right) - 1\right)^2 = -9\frac{13}{27}$ $\quad$ $\therefore \left(-1\frac{2}{3}; -9\frac{13}{27}\right)$

$y = -((1) + 3)((1) - 1)^2 = 0$ $\quad$ $\therefore (1; 0)$

Area below the x-axis,

$\Delta A = |y\Delta x|$

$$A = \left|\int_{-3}^{1} y\,dx\right|$$

$$= \left|\int_{-3}^{1} \left[-(x+3)(x-1)^2\right]\,dx\right|$$

$$= \left|\int_{-3}^{1} \left[-x^3 - x^2 + 5x - 3\right]\,dx\right|$$

$$= \left|\left[-\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 + \tfrac{5}{2}x^2 - 3x\right]_{-3}^{1}\right|$$

$$= \left|\left[-\tfrac{1}{4}(1)^4 - \tfrac{1}{3}(1)^3 + \tfrac{5}{2}(1)^2 - 3(1)\right] - \left[-\tfrac{1}{4}(-3)^4 - \tfrac{1}{3}(-3)^3 + \tfrac{5}{2}(-3)^2 - 3(-3)\right]\right|$$

$$A = 21\tfrac{1}{3} \text{ units}^2$$

Area above the x-axis,

$\Delta A = y\Delta x$

$$A = \int_{-3,5}^{-3} y\,dx$$

$$= \int_{-3,5}^{-3} \left[-(x+3)(x-1)^2\right]\,dx$$

$$= \int_{-3,5}^{-3} \left[-x^3 - x^2 + 5x - 3\right]\,dx$$

$$= \left[-\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 + \tfrac{5}{2}x^2 - 3x\right]_{-3,5}^{-3}$$

$$= \left[-\tfrac{1}{4}(-3)^4 - \tfrac{1}{3}(-3)^3 + \tfrac{5}{2}(-3)^2 - 3(-3)\right]$$

$$- \left[-\tfrac{1}{4}(-3,5)^4 - \tfrac{1}{3}(-3,5)^3 + \tfrac{5}{2}(-3,5)^2 - 3(-3,5)\right]$$

$$A = 2\tfrac{67}{192} \text{ units}^2$$

Therefore,

$$A_{total} = 21\tfrac{1}{3} + 2\tfrac{67}{192}$$

$$A_{total} = 23\tfrac{131}{192} \text{ units}^2$$

11.7 $y = x^3 - 9x^2 + 24x + 4$

x-intercepts, $y = 0$:

$y = x^3 - 9x^2 + 24x + 4$

$0 = x^3 - 9x^2 + 24x + 4$

The x-intercept will have to be computed by approximation.

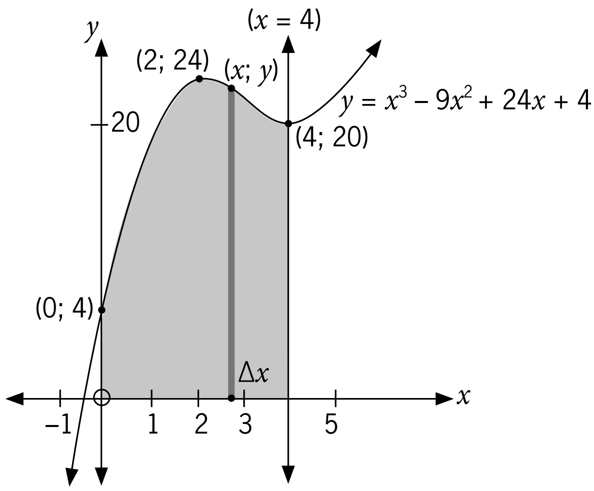
***y*-intercept, $x = 0$:**

$y = x^3 - 9x^2 + 24x + 4$

$= (0)^3 - 9(0)^2 + 24(0) + 4$

$y = 4$

$\therefore (0; 4)$



Turning point(s):

$y = x^3 - 9x^2 + 24x + 4$

$\therefore \frac{dy}{dx} = 3x^2 - 18x + 24 = 0$

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

$x - 2 = 0 \quad \text{or} \quad x - 4 = 0$

$x = 2 \qquad x = 4$

Substitute $x = 2$ and $x = 4$ into $y = x^3 - 9x^2 + 24x + 4$.

$y = (2)^3 - 9(2)^2 + 24(2) + 4 = 24 \quad \therefore (2; 24)$

$y = (4)^3 - 9(4)^2 + 24(4) + 4 = 20 \quad \therefore (4; 20)$

$\Delta A = y\Delta x$

$A = \int_0^4 y\,dx$ • Area above the x-axis

$= \int_0^4 \left[x^3 - 9x^2 + 24x + 4\right] dx$

$= \left[\frac{1}{4}x^4 - 3x^3 + 12x^2 + 4x\right]_0^4$

$= \left[\frac{1}{4}(4)^4 - 3(4)^3 + 12(4)^2 + 4(4)\right] - \left[\frac{1}{4}(0)^4 - 3(0)^3 + 12(0)^2 + 4(0)\right]$

$A = 80 \text{ units}^2$

Activity 6.9 SB page 424

1. $A = \int_{-1}^{2} \left[(-x^2 + 6) - (x^2 - 2x + 2)\right] dx$

$= \int_{-1}^{2} \left[-x^2 + 6 - x^2 + 2x - 2\right] dx$

$= \int_{-1}^{2} \left[-2x^2 + 2x + 4\right] dx$

$= \left[-\frac{2x^3}{3} + \frac{2x^2}{2} + 4x\right]_{-1}^{2}$

$= \left[-\frac{2}{3}x^3 + x^2 + 4x\right]_{-1}^{2}$

$= \left(-\frac{2}{3}(2)^3 + (2)^2 + 4(2)\right) - \left(-\frac{2}{3}(-1)^3 + (-1)^2 + 4(-1)\right)$

$= \left(-\frac{16}{3} + 4 + 8\right) - \left(\frac{2}{3} + 1 - 4\right)$

$= \left(-\frac{16}{3} + 12\right) - \left(\frac{2}{3} - 3\right)$

$= 9 \text{ units}^2$

2. $x^2 = 3x - 2$

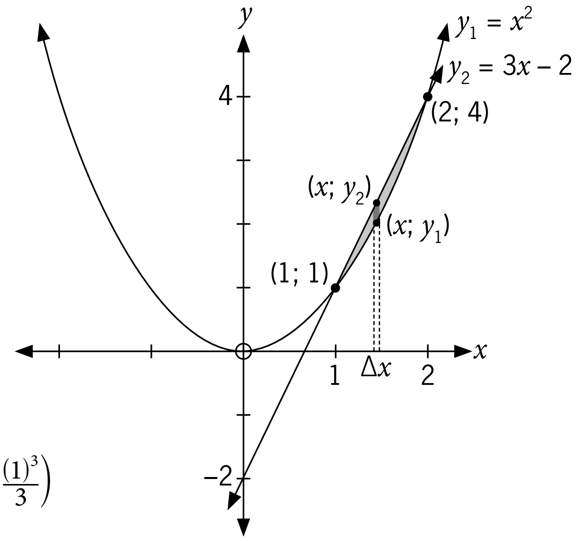
$0 = x^2 - 3x + 2$

$0 = (x - 2)(x - 1)$

$x = 2;\ x = 1$

$y = 4;\ y = 1$

$\therefore$ (2; 4) and (1; 1)



$$A = \int_1^2 (3x - 2 - x^2)\, dx$$

$$= \left[\frac{3x^2}{2} - 2x - \frac{x^3}{3}\right]_1^2$$

$$= \left(\frac{3(2)^2}{2} - 2(2) - \frac{(2)^3}{3}\right) - \left(\frac{3(1)^2}{2} - 2(1) - \frac{(1)^3}{3}\right)$$

$$= \left(6 - 4 - \frac{8}{3}\right) - \left(\frac{3}{2} - 2 - \frac{1}{3}\right)$$

$$= 0{,}167 \text{ units}^2$$

3. $-x^2 - 2x = x$

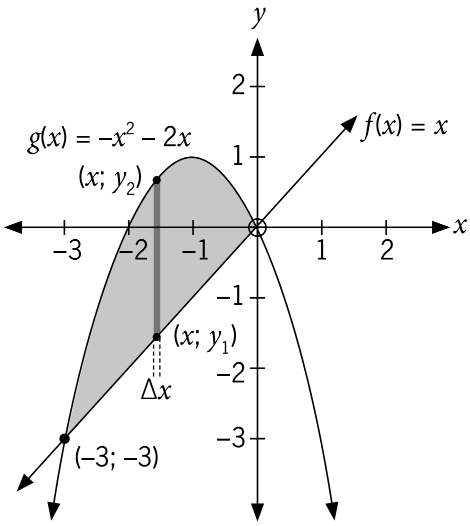
$\therefore x^2 + 3x = 0$

$x(x + 3) = 0$

$x = 0;\ x = -3$

$y = 0;\ y = -3$

$\therefore$ (0; 0) and ((− 3; −3)



$$A = \int_{-3}^{0} \left[(-x^2 - 2x) - x\right] dx$$

$$= \int_{-3}^{0} (-x^2 - 3x)\, dx$$

$$= \left[-\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-3}^{0}$$

$$= 0 - \left(-\frac{(-3)^3}{3} - \frac{3(-3)^2}{2}\right)$$

$$= 0 - \left(\frac{27}{3} - \frac{27}{2}\right)$$

$$= 4\tfrac{1}{2} \text{ or } 4{,}5 \text{ units}^2$$

4. Points of intersection:

$-\frac{1}{4}x^2 + 9 = 6 - x$

$-\frac{1}{4}x^2 + 9 - 6 + x = 0$

$-\frac{1}{4}x^2 + x + 3 = 0$

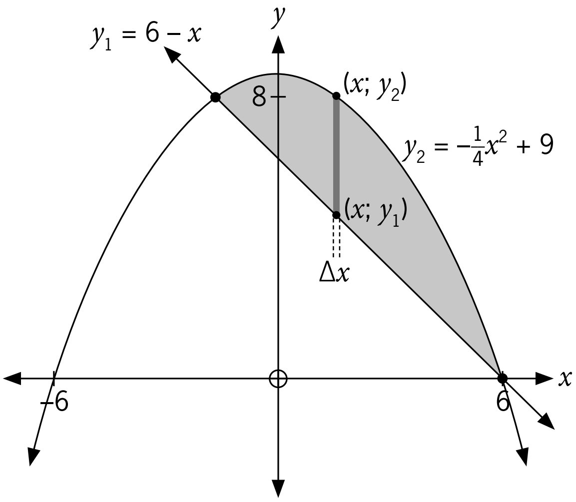
$x^2 - 4x - 12 = 0$

$(x - 6)(x + 2) = 0$

$\therefore x = 6;\ x = -2$

$\therefore y = 0;\ y = 8$

$\therefore$ (6; 0); (−2; 8)



$$A = \int_{-2}^{6} \left[-\tfrac{1}{4}x^2 + 9 - (6 - x)\right] dx$$

$$= \int_{-2}^{6} \left[-\tfrac{1}{4}x^2 + 3 + x\right] dx$$

$$= \left[-\tfrac{1}{4}.\tfrac{x^3}{3} + 3x + \tfrac{x^2}{2}\right]_{-2}^{6}$$

$$= \left(-\tfrac{1}{12}(6)^3 + 3(6) + \tfrac{(6)^2}{2}\right) - \left(-\tfrac{1}{12}(-2)^3 + 3(-2) + \tfrac{(-2)^2}{2}\right)$$

$$= (-18 + 18 + 18) - (0{,}667 - 6 + 2)$$

$$= 21{,}333 \text{ units}^2$$

Summative assessment: Module 6 — SB page 424

1. 1.1 $\int \left[(x^4 - 2 + x)3x^{-1}\right] dx$

$= \int \left[3x^3 - 6.x^{-1} + 3\right] dx$

$= 3\int x^3\, dx - 6\int \frac{1}{x}\, dx + \int 3\, dx$

$= 3.\frac{x^{3+1}}{3+1} - 6 \ln x + 3x + c$

$= \frac{3}{4}x^4 - 6 \ln x + 3x + c$ (5)

1.2 $\int\left(\frac{1}{x^3} + \frac{3}{2}\sin 3x + 3\sqrt{x} - 10.10^x + \frac{1}{\sec 2x} - 2x^0\right) dx$

$= \frac{-1}{2x^2} - \frac{1}{2}\cos 3x + 2x^{\frac{3}{2}} - \frac{10.10^x}{\ln 10} + \frac{1}{2}\sin 2x - 2x + c$ (7)

1.3 $\int\left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right) dx$

$= \int \frac{1}{\cos^2 x}\, dx$

$= \int \sec^2 x\, dx$

$= \tan x + c$ (3)

1.4 $\int(3 \cos 4x - 3e^{2x} + 4.3^{-x})\, dx$

$= 3\int \cos 4x\, dx - 3\int e^{2x}\, dx + 4\int 3^{-x}\, dx$

$= \frac{3}{4}\sin 4x - \frac{3e^{2x}}{2} - 4.\frac{3^{-x}}{\ln 3} + c$

$= \frac{3}{4}\sin 4x - \frac{3}{2}e^{2x} - \frac{4}{(\ln 3)3^x} + c$ (5)

2. 2.1 $\int_1^4 \left[\left(\frac{1}{x} + 3x - 4\right)\right] dx$

$= \int_1^4 \left[(1 - \frac{4}{x} + 3)(x - 12)\right] dx$

$= \int_1^4 \left[3x - \frac{4}{x} - 11\right] dx$

$= \left[3.\frac{x^{1+1}}{1+1} - 4 \ln x - 11x\right]_1^4$

$= \left[\frac{3x^2}{2} - 4 \ln x - 11x\right]_1^4$

$= \left(\frac{3(4)^2}{2} - 4 \ln(4) - 11(4)\right) - \left(\frac{3(1)^2}{2} - 4 \ln|(1)| - 11(1)\right)$

$= -16{,}045$ (5)

2.2 $\int_1^3 \left(\frac{5}{x} + 2\right) dx$

$= [5 \ln x + 2x]_1^3$

$= (5 \ln(3) + 2(3)) - (5 \ln(1) + 2(1))$

$= 9{,}493$ (3)

2.3 $\int_0^{\frac{\pi}{2}} 2 \sin \theta \cos \theta \, d\theta$

$= \int_0^{\frac{\pi}{2}} \sin 2\theta \, d\theta$

$= \left[-\frac{\cos 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= -\frac{1}{2}\left[\cos 2\left(\frac{\pi}{2}\right) - \cos 2(0)\right]$

$= -\frac{1}{2}[\cos \pi - \cos 0]$

$= -\frac{1}{2}[-1 - 1]$

$= -\frac{1}{2}\left[-2\right]$

$= 1$ (4)

2.4 $\int_{-\frac{\pi}{3}}^{\frac{\pi}{9}} \left(2 \sin 3x - 2.8^{5x} + \frac{2}{x^3}\right) dx$

$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{9}} (2 \sin 3x - 2.8^{5x} + 2x^{-3}) \, dx$

$= \left[2.\frac{-\cos 3x}{3} - 2.\frac{8^{5x}}{5 \ln 8} + 2.\frac{x^{-3+1}}{-3+1}\right]_{-\frac{\pi}{3}}^{\frac{\pi}{9}}$

$= \left[-\frac{2}{3} \cos 3x - \frac{2}{5 \ln 8}.8^{5x} - \frac{1}{x^2}\right]_{-\frac{\pi}{3}}^{\frac{\pi}{9}}$

$= \left[-\frac{2}{3}\cos\left(3.\frac{\pi}{9}\right) - \frac{2}{5 \ln 8}.8^{5\left(\frac{\pi}{9}\right)} - \frac{1}{\left(\frac{\pi}{9}\right)^2}\right] - \left[-\frac{2}{3} \cos \left(3.-\frac{\pi}{3}\right) - \frac{2}{5 \ln 8}.8^{5\left(-\frac{\pi}{3}\right)} - \frac{1}{\left(-\frac{\pi}{3}\right)^2}\right]$

$= -15{,}545$ (5)

3. $S = \int_1^4 (\sqrt{t} - 5)^2 \, dt$

$= \int_1^4 (\sqrt{t} - 5)(\sqrt{t} - 5) \, dt$

$= \int_1^4 (t - 5\sqrt{t} - 5\sqrt{t} + 25) \, dt$

$= \int_1^4 (t - 10\sqrt{t} + 25) \, dt$

$= \left[\frac{t^2}{2} - \frac{20t^{\frac{3}{2}}}{3} + 25t\right]_1^4$

$= \left(\frac{4^2}{2} - \frac{20(4)^{\frac{3}{2}}}{3} + 25(4)\right) - \left(\frac{(1)^2}{2} - \frac{20(1)^{\frac{3}{2}}}{3} + 25(1)\right)$

$= 8 - 53{,}333 + 100 - \frac{1}{2} + 6{,}667 - 25$

$= 35{,}834$ (3)

4. 4.1 $A = \int_1^2 y \, dx$

$= \int_1^2 (-x + 2) \, dx$

$= \left[-\frac{x^2}{2} + 2x\right]_1^2$

$= \left(-\frac{(2)^2}{2} + 2(2)\right) - \left(-\frac{(1)^2}{2} + 2(1)\right)$

$= 0{,}5 \text{ units}^2$ (4)

4.2 $A_1 = \int_0^{-4} (x^3 + x^2 - 12x) \, dx$

$= \left[\frac{x^4}{4} + \frac{x^3}{3} - 6x^2\right]_{-4}^0$

$= 0 - (-53{,}33)$

$= 53{,}33 \text{ units}^2$

$A_2 = \int_0^3 (x^3 + x^2 - 12x) \, dx$

$= \left[\frac{x^4}{4} + \frac{x^3}{3} - 6x^2\right]_0^3$

$= |-24{,}75 - 0|$

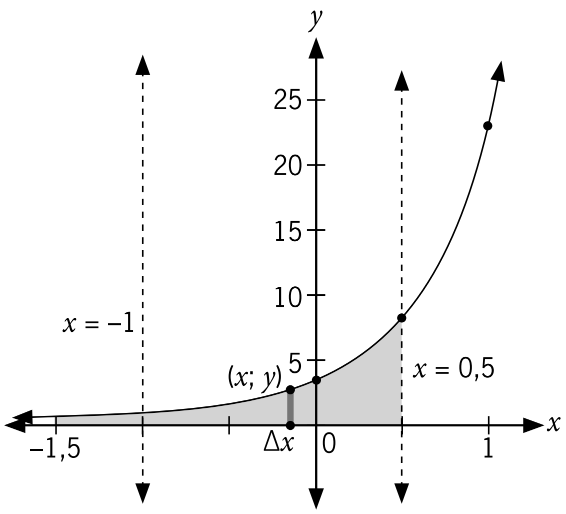
$= |-24{,}75|$

$= 24{,}75 \text{ units}^2$

$A_T = 78{,}08 \text{ units}^2$ (6)

5. 5.1 $y = 3.e^{2x}$

x	y
−1,500	0,149
−1,000	0,406
−0,500	1,104
0,000	3,000
0,500	8,155
1,000	22,167



$\Delta A = y\Delta x$

$A = \int_{-1}^{0,5} y\,dx$ • Area above the x-axis

$= \int_{-1}^{0,5} 3.e^{2x}\,dx$

$= \left[\frac{3e^{2x}}{2}\right]_{-1}^{0,5}$

$= \left[\frac{3e^{2(0,5)}}{2}\right] - \left[\frac{3e^{2(-1)}}{2}\right]$

$A = 3{,}874 \text{ units}^2$ (6)

5.2 $y = -(x-2)^2 - 1$

x-intercepts, $y = 0$:

$y = -(x-2)^2 - 1$

$0 = -(x-2)^2 - 1$

$(x-2)^2 = -1$

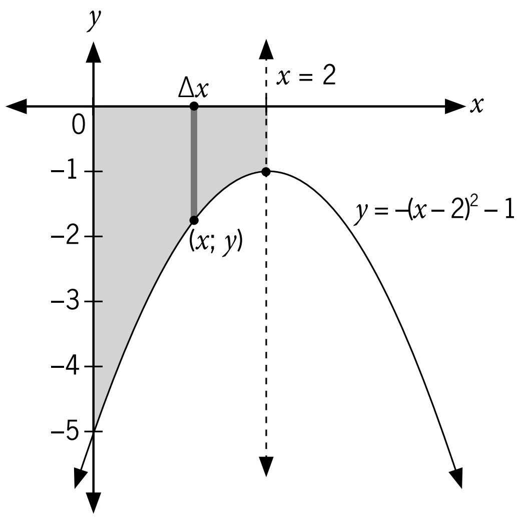
$\sqrt{(x-2)^2} = \sqrt{-1}$

$\therefore$ There are no real x-intercepts.

$x - 2 = \pm\sqrt{-1}$

$x - 2 = \pm i$

$x = 2 \pm i$



y-intercept, $x = 0$:

$y = -(x-2)^2 - 1$

$= -((0) - 2)^2 - 1$

$y = -5$

$\therefore (0; -5)$

Axis of symmetry:

$y = -(x-2)^2 - 1$

$\therefore x - 2 = 0$

$x = 2$

Turning point(s):

$y = -(x-2)^2 - 1$

Since the coefficient of the bracket is negative, therefore $y = -1$ is a maximum value.

$\therefore (2; -1)$

$\Delta A = |y\Delta x|$

$A = \left|\int_0^2 y\,dx\right|$ • Area below the x-axis

$= \left|\int_0^2 [-(x-2)^2 - 1]\,dx\right|$

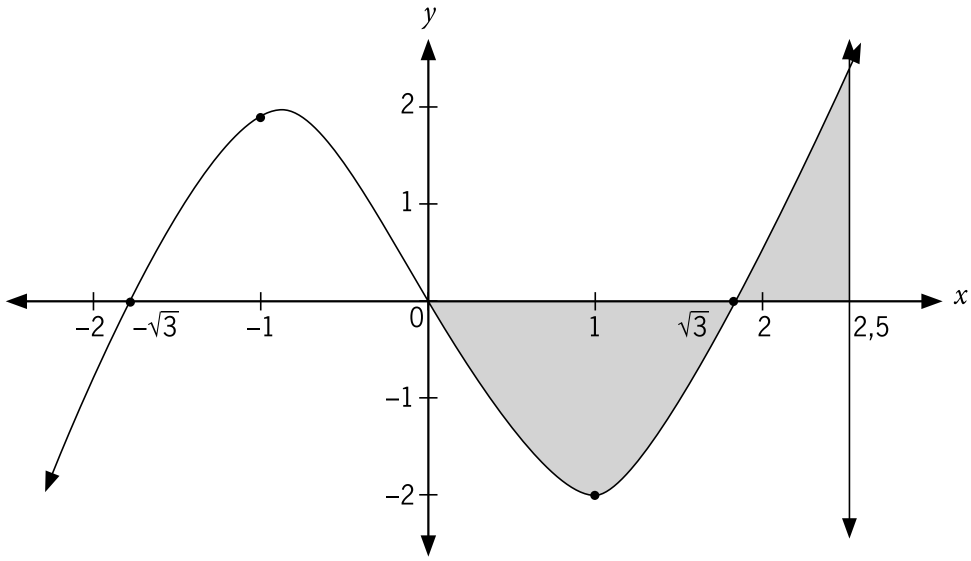
$= \left|\int_0^2 [-x^2 + 4x - 5]\,dx\right|$

$= \left|\left[-\frac{x^3}{3} + 2x^2 - 5x\right]_0^2\right|$

$= \left|\left[-\frac{(2)^3}{3} + 2(2)^2 - 5(2)\right] - \left[-\frac{(0)^3}{3} + 2(0)^2 - 5(0)\right]\right|$

$A = 4\frac{2}{3}$ units^2 (6)

5.3 $y = x(x^2 - 3)$



x-intercepts, $y = 0$:

$y = x(x^2 - 3)$

$0 = x(x^2 - 3)$

$0 = x(x + \sqrt{3})(x - \sqrt{3})$

$0 = x$ or $0 = x + \sqrt{3}$ or $0 = x - \sqrt{3}$

$-\sqrt{3} = x$ $\sqrt{3} = x$

$\therefore (0; 0)$ $\therefore (-\sqrt{3}; 0)$ $\therefore (\sqrt{3}; 0)$

y-intercept, $x = 0$:

$y = x(x^2 - 3)$

$y = (0)((0)^2 - 3)$

$y = 0$

$\therefore (0; 0)$

Turning point(s):

$y = x(x^2 - 3)$

$y = x^3 - 3x$

$\frac{dy}{dx} = 3x^2 - 3 = 0$

$x^2 - 1 = 0$

$(x + 1)(x - 1) = 0$

$x + 1 = 0 \quad \text{or} \quad x - 1 = 0$

$x = -1 \qquad\qquad x = 1$

Substitute $x = -1$ and $x = 1$ into $y = x(x^2 - 3)$.

$y = (-1)((-1)^2 - 3) = 2 \qquad \therefore (-1; 2)$

$y = (1)((1)^2 - 3) = -2 \qquad \therefore (1; -2)$

Area below the x-axis,

$\Delta A = |y\Delta x|$

$$
\begin{aligned}
A &= \left|\int_0^{\sqrt{3}} y\,dx\right| \\
&= \left|\int_0^{\sqrt{3}} [x(x^2 - 3)]\,dx\right| \\
&= \left|\int_0^{\sqrt{3}} [x^3 - 3x]\,dx\right| \\
&= \left|\left[\frac{x^4}{4} - \frac{3x^2}{2}\right]_0^{\sqrt{3}}\right| \\
&= \left|\left[\frac{(\sqrt{3})^4}{4} - \frac{3(\sqrt{3})^2}{2}\right] - \left[\frac{(0)^4}{4} - \frac{3(0)^2}{2}\right]\right|
\end{aligned}
$$

$A = 2\frac{1}{4}$ units^2

Area above the x-axis,

$\Delta A = y\Delta x$

$$
\begin{aligned}
A &= \int_{\sqrt{3}}^{2,5} y\,dx \\
&= \int_{\sqrt{3}}^{2,5} [x(x^2 - 3)]\,dx \\
&= \int_{\sqrt{3}}^{2,5} [x^3 - 3x]\,dx \\
&= \left[\frac{x^4}{4} - \frac{3x^2}{2}\right]_{\sqrt{3}}^{2,5} \\
&= \left[\frac{(2,5)^4}{4} - \frac{3(2,5)^2}{2}\right] - \left[\frac{(\sqrt{3})^4}{4} - \frac{3(\sqrt{3})^2}{2}\right]
\end{aligned}
$$

$A = 2\frac{41}{64}$ units^2

Therefore,

$A_{NET} = 2\frac{1}{4} + 2\frac{41}{64}$

$A_{NET} = 4\frac{57}{64}$ units^2 (8)

6. $x^2 = x + 2$

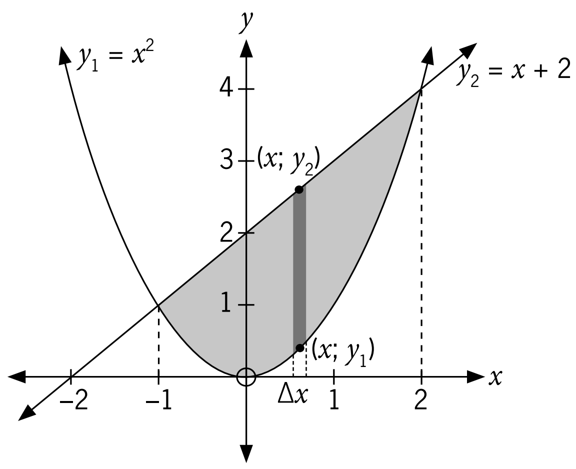
$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$\therefore x = 2;\ x = -1$

$\therefore y = 4;\ y = 1$

$\therefore (2; 4); (-1; 1)$



$A = \int_{-1}^{2} [(x + 2) - x^2]\ dx$

$= \int_{-1}^{2} (x + 2 - x^2)\ dx$

$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$

$= \left(\frac{(2)^2}{2} + 2(2) - \frac{(2)^3}{3}\right) - \left(\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3}\right)$

$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$

$= 4{,}5$ units^2 (10)

TOTAL: [80]

Exemplar examination paper

Time: 3 hours **Marks: 100**

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions in full. Show ALL the calculations and intermediary steps and simplify where possible.
2. For graph work, the values of the intercept(s) with the system of axes and turning point(s) MUST be shown on the graph.
3. ALL final answers must be rounded to THREE decimal places.
4. Questions may be answered in any order but subsections must NOT be separated.
5. A formula sheet is attached to this question paper. You are NOT compelled to use the formulae and the list is NOT necessarily complete.

QUESTION 1

1.1 Given: $x + 3y - 2z = -13$

$2x - 6y + 3z = 32$

$3x - 4y - z = 12$

Solve for y by using determinants. (8)

1.2 Simplify the following and leave your answer in $a + bj$ form:
$\sqrt{-144} + \sqrt{169} - \sqrt{-1}$. (2)

1.3 Given: $z = -1 - 4j$

1.3.1 Calculate the modulus and the argument of z. The argument must be positive. Show ALL steps. (3)

1.3.2 Express z in polar form. (1)

1.3.3 Represent z on an Argand diagram. (1)

1.4 Solve for x and y:

$x + yj = \frac{3 - 2j}{1 - j} - \frac{1 - 3j}{1 + 3j}$ (5)

[20]

QUESTION 2

2.1 Prove that: $\frac{2 \sin x + 1}{\sin 2x + \cos x} = \sec x$ (3)

2.2 If $\tan x = \frac{5}{12}$ and $\tan y = \frac{3}{4}$, with both x and y acute angles, determine without using a calculator, $\sin(x - y)$. (3)

2.3 Given: $2 \cos^2\theta + 5 \sin \theta = -1; 0° \leq \theta \leq 360°$

Solve for θ. (5)

2.4 Simplify $(\sec \theta + \tan \theta)^2$ (3)

2.5 Derive a formula for $\tan 2\theta$. (3)

2.6 Determine $\cot 105°$ WITHOUT using a calculator. (3)

[20]

QUESTION 3

3.1	3.1.1 Sketch the graph of $y = -x^2 - 2x + 3$	(3)
	3.1.2 Is the graph of $y = -x^2 - 2x + 3$ continuous?	(1)
	3.1.3 What is the domain of the graph $y = -x^2 - 2x + 3$?	(1)
3.2	Sketch the graph of $x = \ln y$	(3)
3.3	Sketch the graph of $3x^2 = 9y^2 + 27$	(2)
		[10]

QUESTION 4

4.1	Differentiate from first principles: $y = -x^2 - x + 1$	(5)
4.2	Given: $y = 4\cos(7x + 2)$	
	Differentiate by the use of the chain rule or function of a function.	(4)
4.3	Given: $(2y^2 - 3)^4$	
	Use the binomial theorem to expand to FOUR terms.	(5)
4.4	Differentiate with respect to x:	
	$y = 2\sec 3x - 4\operatorname{cosec} 2x + 6\ln x + p$	(4)
4.5	Given: $y = 2x^3 - 8x$	
	Calculate, using differentiation, the coordinates of the maximum and minimum turning points. Distinguish between the maximum and minimum turning points by the use of the second derivative.	(7)
		[25]

QUESTION 5

5.1	Determine:	
	$\int_0^{\frac{\pi}{2}} (\sin\theta + \cos\theta)\, d\theta$	(4)
5.2	Integrate:	
	$\int\left(2\cos 2x + 6k - \frac{1}{e^{-x}} + 10.10^x - \frac{1}{x^2}\right) dx$	(6)
5.3	5.3.1 Sketch, and clearly indicate the area bounded by the graph of $y = 2.3^{2x}$; the x-axis, the y-axis and the line $x = 2$. Also indicate the representative strip to be used to calculate the area.	(3)
	5.3.2 Calculate, using integration, the value of the area in QUESTION 5.3.1.	(4)
5.4	5.4.1 Sketch the graphs of $y = -x + 3$ and $xy = 2$ on the same system of axes. Clearly indicate the area bounded by the two graphs. Show the representative strip used to calculate the area.	(4)
	5.4.2 Calculate, using integration, the value of the area shown in QUESTION 5.4.1.	(4)
		[25]

TOTAL: 100

Possible formula sheet

$a^x = b \Leftrightarrow \log a^x = \log b$

$\ln x = \log_e x$

$(r\angle\theta)^n = r^n\angle n\theta$

$a + bj = c + dj \Leftrightarrow a = c$ and $b = d$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \mathrm{cosec}^2 x$

$1 + \tan^2 x = \sec^2 x$

$\sin 2A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$\cos 2A = \frac{1}{2} + \frac{1}{2}\cos 2A$

y	$\frac{dy}{dx}$
ax^n	nax^{n-1}
ka^x	$ka^x \ln a$
$k \ln x$	$\frac{k}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

$y = u(x).v(x)$

$\therefore \frac{dy}{dx} = u(x)v'(x) + u'(x)v(x)$

$y = \frac{u(x)}{v(x)}$

$\therefore \frac{dy}{dx} = \frac{v(x)u'(x) - u(x)v'(x)}{[v(x)]^2}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\int ax^n\, dx = \frac{ax^{n+1}}{n+1} + c$

$\int \frac{a}{x}\, dx = a \ln x + c$

$\int ka^x\, dx = \frac{ka^x}{\ln a} + c$

$\int \sin x\, dx = -\cos x + c$

$\int \cos x\, dx = \sin x + c$

$A_{OX} = \int_a^b y\, dx$

$A = \int_a^b (y_2 - y_1)\, dx$

Exemplar examination paper Memorandum

QUESTION 1

1.1 $|D| = \begin{vmatrix} 1 & 3 & -2 \\ 2 & -6 & 3 \\ 3 & -4 & -1 \end{vmatrix}$ ✓

- Expand using any row or column

$= +(1)\begin{vmatrix} -6 & 3 \\ -4 & -1 \end{vmatrix} - (3)\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} + (-2)\begin{vmatrix} 2 & -6 \\ 3 & -4 \end{vmatrix}$ ✓

- Positional sign (element) | minor of element for the entire row or column

$= 1(6 + 12) - 3(-2 - 9) - 2(-8 + 18)$

$= 18 + 33 - 20$

$= 31$ ✓

$|D_y| = \begin{vmatrix} 1 & -13 & -2 \\ 2 & 32 & 3 \\ 3 & 12 & -1 \end{vmatrix}$ ✓

- $|D_y|$: replace the 'y' column by inserting the constant column in its place

$= (+1)\begin{vmatrix} 32 & 3 \\ 12 & -1 \end{vmatrix} - (-13)\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 32 \\ 3 & 12 \end{vmatrix}$ ✓

$= 1(-32 - 36) + 13(-2 - 9) - 2(24 - 96)$

$= -68 - 143 + 144$

$= -67$ ✓

$\therefore y = \frac{D_y}{D}$ ✓

$= \frac{-67}{31}$

$= -2{,}161$ ✓ (8)

1.2 $\sqrt{-144} + \sqrt{169} - \sqrt{-1}$

- $\sqrt{-1} = j$

$= 12\sqrt{-1} + 13 - \sqrt{-1}$ ✓

$= 12j + 13 - j$

$= 13 + 11j$ ✓ (2)

1.3 1.3.1 $z = -1 - 4j$

- Modulus = radius; argument = angle

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(-1)^2 + (-4)^2}$

$= \sqrt{17}$

$= 4{,}123$ ✓

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

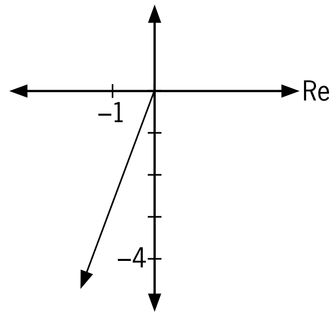
$= \tan^{-1}\left(\frac{4}{1}\right)$

$= 75{,}964°$ ✓

Note

To know in which quadrant, make a sketch of z on an Argand diagram.

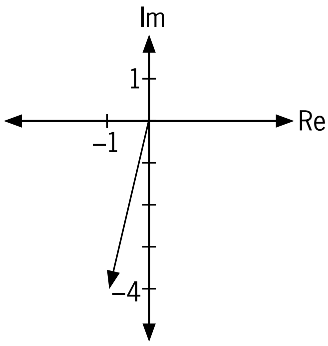
Now you see that it is in the 3rd quadrant.
Therefore, 180 + 75,964 = 255,964°.



$z = 4{,}123\underline{|255{,}964°}$
Modulus = 4,123
Argument = 255,964° ✓ (3)

1.3.2 $z = 4{,}123\underline{|255{,}964°}$ (1)

1.3.3



(1)

1.4 $x + yj = \frac{3-2j}{1-j} - \frac{(1-3j)}{(1+3j)}$

$= \frac{(3-2j)(1+j)}{(1-j)(1+j)} - \frac{(1-3j)(1-3j)}{(1+3j)(1-3j)}$ ✓

- Rationalise each fraction by multiplying by the conjugate of the denominator

$= \frac{3+j-2j^2}{1-j^2} - \frac{1-6j+9j^2}{1-9j^2}$ ✓

- Multiply out and add like terms

$= \frac{3+j-2(-1)}{1-(-1)} - \frac{1-6j+9(-1)}{1-9(-1)}$

- $j^2 = -1$

$= \frac{5+j}{2} - \frac{-8-6j}{10}$

$= \frac{5(5+j)-1(-8-6j)}{10}$

$= \frac{25+5j+8+6j}{10}$

$= \frac{33+11j}{10}$

$= \frac{33}{10} + \frac{11}{10}j$ ✓

$\therefore x = \frac{33}{10}$ and $y = \frac{11}{10}$

$= 3{,}3$ ✓ $= 1{,}1$ ✓

- Equate real terms and imaginary terms. (5)

[20]

QUESTION 2

2.1 LHS $= \frac{2\sin x + 1}{\sin 2x + \cos x}$

$= \frac{2\sin x + 1}{2\sin x\cos x + \cos x}$ ✓ • Double angle identity $\sin 2x = 2\sin x\cos x$

$= \frac{2\sin x + 1}{\cos x(2\sin x + 1)}$ ✓ • Factorise

$= \frac{1}{\cos x}$ ✓

$= \sec x =$ RHS (3)

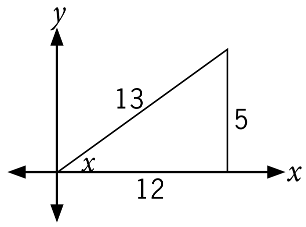
2.2 $\sin(x - y)$

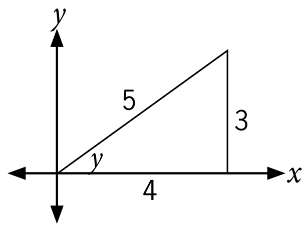
$= \sin x\cos y - \cos x\sin y$ ✓

$= \frac{5}{13}\cdot\frac{4}{5} - \frac{12}{13}\cdot\frac{3}{5}$ ✓

$= \frac{20}{65} - \frac{36}{65}$

$= -\frac{16}{65}$ ✓ (3)





2.3 $2\cos^2\theta + 5\sin\theta = -1$

$2(1 - \sin^2\theta) + 5\sin\theta + 1 = 0$ ✓ • Change to same ratio $\cos^2\theta = 1 - \sin^2\theta$

$2 - 2\sin^2\theta + 5\sin\theta + 1 = 0$

$-2\sin^2\theta + 5\sin\theta + 3 = 0$ • (× –1): LHS and RHS

$2\sin^2\theta - 5\sin\theta - 3 = 0$ ✓½ • Trinomial

$(2\sin\theta + 1)(\sin\theta - 3) = 0$ ✓½

$\therefore 2\sin\theta = -1$ ✓½ • $\sin\theta = 3$ ✓½

$\sin\theta = -\frac{1}{2}$ • Not possible: maximum of $\sin\theta$ is 1

$\therefore$ in 3rd and 4th quadrants

$\theta = 180° + \sin^{-1}0{,}5$ and $\theta = 360° - \sin^{-1}0{,}5$

$= 210°$ ✓ $= 330°$ ✓ (5)

2.4 $(\sec\theta + \tan\theta)^2$

$= (\sec\theta + \tan\theta)(\sec\theta + \tan\theta)$

$= \sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta$ ✓

$= \frac{1}{\cos^2\theta} + \left(\frac{2}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right) + \frac{\sin^2\theta}{\cos^2\theta}$ • Change to $\sin\theta$ and $\cos\theta$

$= \frac{1 + 2\sin\theta + \sin^2\theta}{\cos^2\theta}$ ✓ • Common denominator

$= \frac{1 + 2\sin\theta + \sin^2\theta}{1 - \sin^2\theta}$ • $\cos^2\theta = 1 - \sin^2\theta$

$= \frac{(1 + \sin\theta)(1 + \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}$ • Factorise top and bottom

$= \frac{1 + \sin\theta}{1 - \sin\theta}$ ✓ (3)

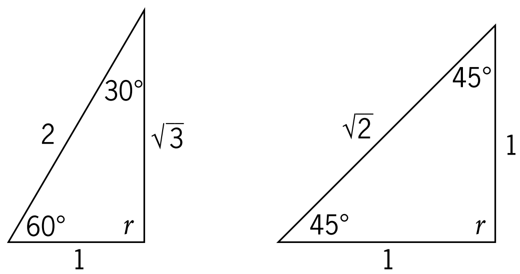
2.5 $\tan 2\theta = \tan(\theta + \theta)$ ✓

$= \frac{\tan\theta + \tan\theta}{1 - \tan^2\theta}$ ✓ • Compound angles

$= \frac{2\tan\theta}{1 - \tan^2\theta}$ ✓ (3)

2.6 $\cot 105°$

$= \cot(60° + 45°)$ ✓



$= \frac{\cos(60° + 45°)}{\sin(60° + 45°)}$

$= \frac{\cos 60° \cos 45° - \sin 60° \sin 45°}{\sin 60° \cos 45° + \cos 60° \sin 45°}$ ✓½

$= \frac{\frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}}$ ✓½

$= \frac{\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}}{\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}}$

$= \frac{\frac{1 - \sqrt{3}}{2\sqrt{2}}}{\frac{\sqrt{3} + 1}{2\sqrt{2}}}$ • Everything over LCM

$= \frac{1 - \sqrt{3}}{2\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{3} + 1}$ • Invert

$= \frac{1 - \sqrt{3}}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$ • Rationalise

$= \frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{3 - 1}$

$= \frac{2\sqrt{3} - 4}{2}$

$= \frac{2(\sqrt{3} - 2)}{2}$ • Factorise

$= \sqrt{3} - 2$ ✓

Alternatively

$\cot 105° = \frac{1}{\tan 105°}$

$= \frac{1}{\tan(45° + 60°)}$

$= \frac{1}{\frac{\tan 45° + \tan 60°}{1 - \tan 45° \tan 60°}}$

$= \frac{1 - \tan 45° \tan 60°}{\tan 45° + \tan 60°}$

$= \frac{1 - (1)\sqrt{3}}{1 + \sqrt{3}}$

$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$ • Rationalise denominator

$= \frac{1 - 2\sqrt{3} + 3}{1 - 3}$

$= \frac{4 - 2\sqrt{3}}{-2}$

$= -2 + \sqrt{3}$

$= \sqrt{3} - 2$

(3)

[20]

QUESTION 3

3.1 3.1.1 $y = -x^2 - 2x + 3$

- Exponent of $y = 1$; exponent of $x = 2$ $\therefore$ parabola

***x*-intercepts** ($y = 0$)

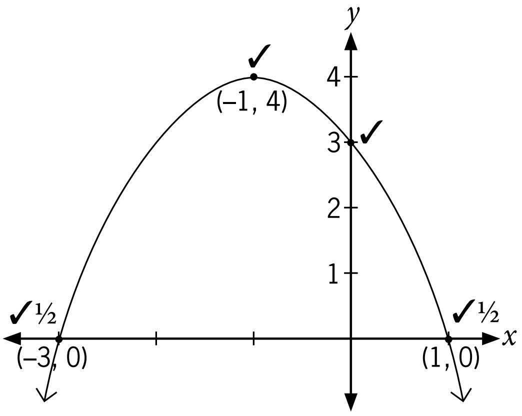
$$-x^2 - 2x + 3 = 0$$
$$x^2 + 2x - 3 = 0$$
$$(x + 3)(x - 1) = 0$$
$$\therefore x = -3 \text{ or } x = 1$$

***y*-intercept** ($x = 0$)

$$y = 3$$

Turning point: $x = \frac{-b}{2a} = \frac{-(-2)}{-2} = -1$

$$y = f(-1) = -(-1)^2 - 2(-1) + 3 = 4$$



(3)

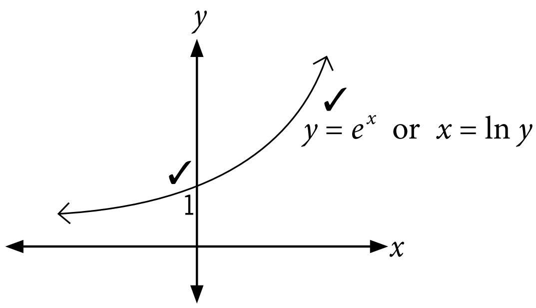
3.1.2 Continuous ✓ (1)

3.1.3 $-\infty \leq x \leq \infty$ or $x \in \mathbb{R}$ ✓ (1)

3.2 $x = \ln y$

$x = \log_e y$

$e^x = y$ ✓

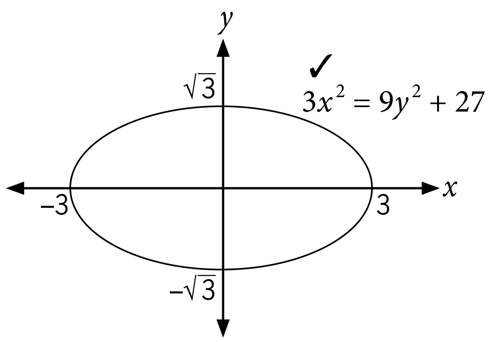


(3)

3.3 $3x^2 = 9y^2 + 27$

$\frac{3x^2}{27} = \frac{9}{27}y^2 + \frac{27}{27}$

$\frac{x^2}{9} - \frac{y^2}{3} = 1$ ✓



(2)

[10]

QUESTION 4

4.1

$$y = -x^2 - x + 1$$

$$f(x) = -x^2 - x + 1$$

$$f(x+h) = -(x+h)^2 - (x+h) + 1 \checkmark$$

$$= -(x^2 + 2xh + h^2) - x - h + 1$$

$$= -x^2 - 2xh - h^2 - x - h + 1 \checkmark$$

$$f(x+h) - f(x) = -x^2 - 2xh - h^2 - x - h + 1 - (-x^2 - x + 1) \checkmark$$

$$= -x^2 - 2xh - h^2 - x - h + 1 + x^2 + x - 1$$

$$= -2xh - h^2 - h \checkmark$$

$$\frac{f(x+h) - f(x)}{h} = -2x - h - 1$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = -2x - 1 \checkmark$$

- Substitute $h = 0$

(5)

4.2

$y = 4\cos(7x + 2)$ Let $u = 7x + 2$

$y = 4\cos u$ then $\frac{du}{dx} = 7 \checkmark$

$$\frac{dy}{du} = -4\sin u \checkmark$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -4\sin u.7 \checkmark$$

$$= -4\sin(7x + 2).7$$

$$= -28\sin(7x + 2) \checkmark$$

(4)

4.3 $(2y^2 - 3)^4$

$$= \overset{\checkmark}{(2y^2)^4} + \overset{\checkmark}{4(2y^2)^3(-3)} + \overset{\checkmark}{\frac{4(3)(2y^2)^2(-3)^2}{2!}} + \overset{\checkmark}{\frac{4(3)(2)(2y^2)(-3)^3}{3!}} + \ldots$$

$$= 16y^8 + (-12)(8y^6) + \frac{12(4y^4)(9)}{2} + \frac{24(2y^2)(-27)}{3 \times 2} + \ldots$$

$$= 16y^8 - 96y^6 + 216y^4 - 216y^2 + \ldots \checkmark$$

(5)

4.4 $y = 2\sec 3x - 4\operatorname{cosec} 2x + 6\ln x + p$

$$\frac{dy}{dx} = 2.3\sec 3x \overset{\checkmark}{\tan} 3x - 4.2(-\operatorname{cosec} 2\overset{\checkmark}{x} \cot 2x) + 6\frac{1}{x}\overset{\checkmark}{} + 0$$

$$= 6\sec 3x \tan 3x + 8\operatorname{cosec} 2x \cot 2x + \frac{6}{x} \checkmark$$

(4)

4.5 $y = 2x^3 - 8x$

For turning points $\frac{dy}{dx} = 6x^2 - 8 = 0$ ✓

$\therefore$ Critical values of x are $6x^2 = 8$

$$x^2 = \frac{8}{6} = \frac{4}{3}$$

$$x = \pm\sqrt{\frac{4}{3}}$$

$$\therefore x = \pm 1{,}155 \text{ ✓}$$

For the nature of the turning points, examine $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = 12x$ ✓

- $\frac{dy}{dx} = 6x^2 - 8$ ✓

At $x = 1{,}155$ $\frac{d^2y}{dx^2} = 12(1{,}155)$ ✓½

positive $\therefore$ it will have a minimum turning point

Min T.P $(1{,}155; f(1{,}155))$

$= (1{,}155; -6{,}158)$ ✓

- substitute $y = 2(1{,}155)^3 - 8(1{,}155)$
  $= -6{,}158$

At $x = -1{,}155$ $\frac{d^2y}{dx^2} = 12(-1{,}155)$ ✓½

negative $\therefore$ it will have a maximum turning point

Max T.P $(-1{,}155; f(-1{,}155))$

$= (-1{,}155; 6{,}158)$ ✓

- substitute $y = 2(-1{,}155)^3 - 8(-1{,}155)$
  $= 6{,}158$ (7)

[25]

QUESTION 5

5.1 $\int_0^{\frac{\pi}{2}} (\sin\theta + \cos\theta)\, d\theta$

$= [-\cos\theta + \sin\theta]_0^{90°}$ ✓

$= [-\cos 90° + \sin 90°] - [-\cos 0° + \sin 0°]$ ✓

$= [0 + 1] - [-1 + 0]$ ✓

$= 1 + 1$

$= 2$ ✓ (4)

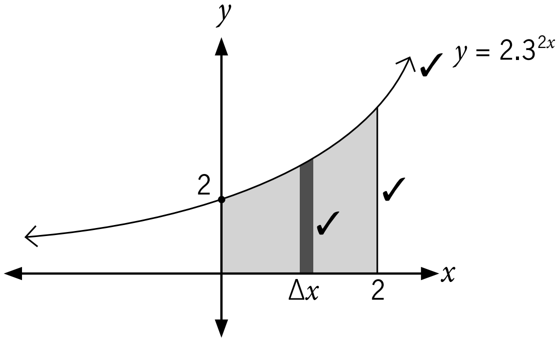
5.2 $\int \left(2\cos 2x + 6k - \frac{1}{e^{-x}} + 10.10^x - \frac{1}{x^2}\right) dx$

$= \int (2\cos 2x + 6k - e^x + 10.10^x - x^{-2})\, dx$

$= \frac{2\sin 2x}{2} + 6kx - e^x + \frac{10.10^x}{\ln 10} - \frac{x^{-1}}{-1} + c$ ✓ ✓ ✓ ✓ ✓

$= \sin 2x + 6kx - e^x + \frac{10.10^x}{\ln 10} + \frac{1}{x} + c$ ✓ (6)

5.3 5.3.1



(3)

5.3.2 Area of strip: $\Delta A = y\Delta x$

$$A = \int_0^2 y\,dx$$

$$= \int_0^2 2.3^{2x}\,dx \checkmark \checkmark$$

$$= \left[\frac{2.3^{2x}}{2\ln 3}\right]_0^2 \checkmark$$

$$= \left(\frac{3^{2(2)}}{\ln 3}\right) - \left(\frac{3^{2(0)}}{\ln 3}\right)$$

$$= 72{,}819 \text{ units}^2 \checkmark$$

(4)

5.4 5.4.1 $y = -x + 3$ $y = \frac{2}{x}$

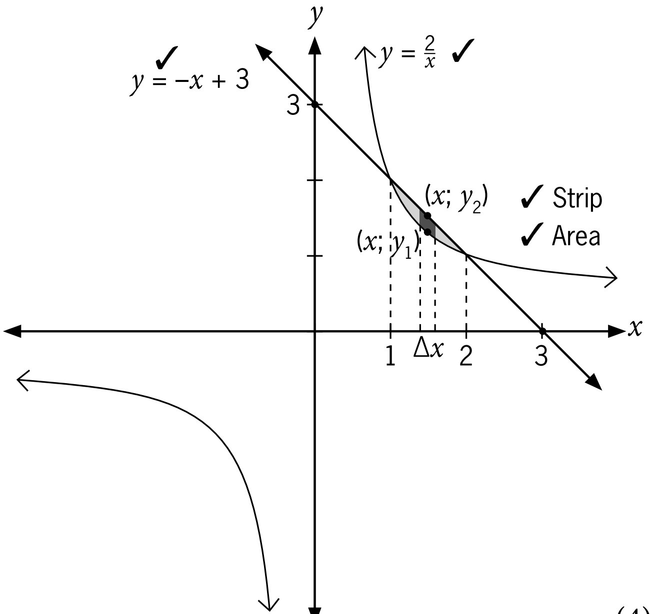
$$\therefore -x + 3 = \frac{2}{x}$$

$$-x^2 + 3x - 2 = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x - 2)(x - 1) = 0$$

$$\therefore x = 1 \text{ or } x = 2$$



(4)

5.4.2 Area $= \int_1^2 (y_2 - y_1)\,dx$

$$= \int_1^2 \left(-x + 3 - \frac{2}{x}\right) dx \checkmark$$

$$= \left[-\frac{x^2}{2} + 3x - 2\ln x\right]_1^2 \checkmark$$

$$= (-2 + 6 - 2\ln 2) - \left(-\frac{1}{2} + 3 - 0\right)$$

$$= 0{,}114 \text{ units}^2 \checkmark$$

(4)

[25]

TOTAL: 100

Glossary

A

Acute angle – an angle that lies between 0 and $\frac{\pi}{4}$ radians, or between 0° and 90°

Angle of inclination – the angle between the line and the positive x-axis

Area – the amount of space inside the boundary of a flat (two-dimensional) object

Argand diagram – a graphical representation of complex numbers

Axis – a horizontal line and a vertical line that intersect at the origin are used to describe the position of points in a coordinate system

Axis of symmetry – a line that divides the graph into two symmetrical halves

C

Cartesian coordinate system – a system in which the location of a point is given by coordinates that represent its distance from the axes

Centre point – the point inside the circle that is the same distance from all points on the circle

Coefficient – a number that is multiplied by a variable

Complex numbers ($\mathbb{C}$) – numbers that can be expressed in the form $a + bi$, where a and b are real numbers; the imaginary part is i; in the expression $a + bi$, a is the real part and b is the imaginary part of the complex number

Congruent – has exactly the same shape and size

Coordinates – a set of values that shows the exact position of a point in relation to the axes

Co-terminal angle – an angle in standard position (angle with the initial side on the positive x-axis) that have a common terminal side

D

Delta – the symbol used to indicate a change in value or the differences in values

Dependent variable – the output value of a function that is dependent on the input value

Derivative – the gradient of a curve at any point, or the gradient of the tangent to the curve at that point; the instantaneous rate of change of a function

Determinant – a function of which the input is a square matrix and the output is a number

Diameter – the distance from one side of the circumference of a circle, through the centre of the circle, to the other side

Differentiate – the process of finding the derivative of a function

Differentiating by first principles – finding the derivative of a function using the definition

E

Equation – a mathematical statement that two equations have the same value

Exponent – shows how many times something is multiplied by itself

F

Frequency – the number of cycles completed by a graph over a given interval

Function – a rule or relationship for which any input value results in one unique output value

G

Gradient – the slope or steepness and is the ratio of vertical change to horizontal change

I

Imaginary number – a complex number that can be written as a real number multiplied by an imaginary unit i, defined by $i^2 = -1$

Imaginary numbers ($\mathbb{I}$) – the square root of negative numbers

Inclination – the slope or gradient of a line

Independent variable – the input value of a function that does not depend on anything

Inflection point – a point where the concavity of a curve changes; can be a stationary point, but not always

Integers ($\mathbb{Z}$) – positive and negative numbers

Irrational numbers ($\mathbb{Q}'$) – any number that cannot be written as a fraction in the form $\frac{a}{b}$, $b \neq 0$ with $a, b \in \mathbb{Z}$

L

Limit – the value that a function or sequence approaches as the input approaches some value

Logarithm – a number that is the exponent by which another fixed value, the base, must be raised to produce that number, so a logarithm is an exponent

M

Midpoint – the point exactly halfway along a line segment, which divides the line segment into two equal pieces

N

Natural numbers ($\mathbb{N}$) – numbers used for counting; starts at 1

P

Phase shift – indicates how far the function is shifted horizontally from the usual position

Q

Quadratic equation – any equation in the form $ax^2 + bx + c = 0$, where x represents an unknown (variable) and a, b and c are constants, with $a \neq 0$

R

Radian – the angle at the centre of an angle subtended by an arc of the same length as the radius

Radius – the distance from the centre to any point on a circle

Rational numbers ($\mathbb{Q}$) – any number that can be written as a fraction in the form $\frac{a}{b}$, $b \neq 0$ with $a, b \in \mathbb{Z}$

Real numbers ($\mathbb{R}$) – rational and irrational numbers

S

Secant – a line that intersects a circle at two points

Second derivative – the derivative of the derivative; it tells us about the concavity of the graph

Simultaneous equations – two or more equations with the same variables; when there are at least as many equations as variables they may be solvable

Stationary point – a point where the derivative of the function is zero; i.e. the gradient at a stationary point is zero

T

Tangent – a line that intersects a circle at only one point

Turning point – the point at which the graph has a maximum or minimum value

V

Variable – a letter that represents or stands for a number

W

Whole numbers ($\mathbb{N}_0$) – natural numbers plus the number 0