# N4
# *Engineering Science*
## *Lecturer Guide*

**Sparrow Consulting**
*in collaboration with Johan Els*

Additional resource material for this title includes:
- PowerPoint presentation
- Interactive tests
- Past exam papers.

Scan the QR code below or visit this link: futman.pub/N4EngSciLG

SCAN ME

ISBN 978-0-6391-0711-0



**Acknowledgements**
Sparrow Consulting would like to thank author Johan Els for their contributions to this book.



FutureManagers
SIYAFUNDA • SIYAKHULA

**Published by**
**Future Managers (Pty) Ltd**
**PO Box 13194, Mowbray, 7705**
**Tel (021) 462 3572**
**Fax (021) 462 3681**
**E-mail: info@futuremanagers.com**
**Website: www.futuremanagers.com**

CONTENTS

# Lecturer guidance

## 1. General aims

This subject builds onto the basic knowledge attained in Engineering Science N1, N2 and N3. This subject involves knowledge of various systems and components, hence when presenting modules for the subject, attention should be taken that the students understand each basic scientific principle in such a way that they will be able to integrate this knowledge in their applied subjects.

## 2. Specific aims

On completion of all the modules in Engineering Science N4, the students should be able to apply the scientific principles mastered to his specific trade theory. Students should be able to apply SI units and derived units correctly. Students should be able to demonstrate understanding of subject content through the application of acquired knowledge. Students should also be able to solve problems by using subject content. Students should be able to acquire in-depth knowledge of the following content:

1.2.1 Kinematics
1.2.2 Angular motion
1.2.3 Dynamics
1.2.4 Statics
1.2.5 Hydraulics
1.2.6 Stress, strain and Young's modulus
1.2.7 Heat

## 3. Prerequisites

Students must have passed N3 Engineering Science.

## 4. Duration

The duration of the subject is one trimester on full-time, part-time or distance learning mode.

## 5. Evaluation

Candidates must be evaluated continually as follows:

### 5.1 ICASS Trimester Mark

5.1.1 **Two** formal class tests for full-time and part-time students (or **two** assignments for distance learning students only)
5.1.2 Obtain a minimum of 40% in order to qualify to write the final examination.

5.1.3 Assessment marks are valid for a period of one year and are referred to as ICASS trimester marks.

5.1.4 Calculation of trimester mark
Weight of test or assignment 1 = 30% of the syllabus
Weight of test or assignment 2 = 70% of the syllabus

## 5.2 Examination

5.2.1 The examination shall consist of 100 % of the syllabus
5.2.2 Duration shall be 3 hours
5.2.3 Minimum pass percentage shall be 40%
5.2.4 Closed book examination
5.2.5 Knowledge, understanding, application and evaluation are important aspects of the subject and should be weighted as follows:

| Knowledge | Understanding | Application | Evaluation |
|---|---|---|---|
| 60% | 20% | 15% | 5% |

## 5.3 Promotion Mark

The promotion mark consisting of the combination of the trimester and examination marks, shall be a minimum of 40%.

# 6. Mark allocation and weighted value of modules

| MODULES | WEIGHTING |
|---|---|
| 1. Kinematics | 15 |
| 2. Angular motion | 12 |
| 3. Dynamics | 14 |
| 4. Statics | 15 |
| 5. Hydraulics | 15 |
| 6. Stress, strain and Young's modulus | 14 |
| 7. Heat | 15 |
| **TOTAL** | **100%** |

# 7. Work schedule

| Week | Topic | Content | Hours |
|---|---|---|---|
| 1–2 | **Module 1**<br>Kinematics | 1.1 Relative velocity<br>1.2 Resulting velocity<br>1.3 Projectiles | 15 hours |
| 3–4 | **Module 2**<br>Angular motion | 2.1 Angular displacement<br>2.2 Angular velocity<br>2.3 Angular acceleration<br>2.4 Relationship between linear and angular quantities<br>2.5 Torque, work done and power | 12 hours |
| 5–6 | **Module 3**<br>Dynamics | 3.1 Newton's three laws of motion<br>3.2 Kinetic and potential energy<br>3.3 Conservation of energy | 14 hours |
| 6–7 | **Module 4**<br>Statics | 4.1 Supported beams and cantilevers<br>4.2 Centroids and centres of gravity | 15 hours |
| 8–9 | **Module 5**<br>Hydraulics | 5.1 Hydraulic presses<br>5.2 Hydraulic pumps<br>5.3 Hydraulic accumulators | 15 hours |
| 9–10 | **Module 6**<br>Stress, strain and Young's modulus | 6.1 Stress types<br>6.2 Young's modulus | 14 hours |
| 11–12 | **Module 7**<br>Heat | 7.1 Volumetric change in solids<br>7.2 Volumetric change in liquids<br>7.3 Volumetric change in gases<br>7.4 Gas processes | 15 hours |
| **TOTAL** | | | **100 hours** |

# 8. Lesson plan template

| | |
|---|---|
| **CAMPUS** | |
| **LECTURER** | |
| **SUBJECT AND LEVEL** | N4 Engineering Science |
| **PRESCRIBED TEXTBOOK: TITLE AND AUTHOR** | *N4 Engineering Science* by Sparrow Consulting |

| LESSON | CONTENT/OUTCOMES TO BE COVERED THIS WEEK | LIST OF EXAMPLES TO BE DONE IN CLASS BY THE LECTURER TO EXPLAIN THE OUTCOME/CONCEPT | FACILITATION METHOD (PLEASE TICK) | | TEACHING RESOURCES/AIDS (PLEASE TICK) | | STUDENT ACTIVITY (EXERCISE IN TEXTBOOK/ADDITIONAL SUPPORTING TASK) TO BE DONE THIS WEEK |
|---|---|---|---|---|---|---|---|
| WEEK 1 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | INTRODUCTION TO LESSONS | | | | |
| | | | | | | | |
| | | | RECAPPING/REINFORCEMENT | | | | |
| | | | | | | | |

| LESSON | CONTENT/OUTCOMES TO BE COVERED THIS WEEK | LIST OF EXAMPLES TO BE DONE IN CLASS BY THE LECTURER TO EXPLAIN THE OUTCOME/CONCEPT | FACILITATION METHOD (PLEASE TICK) | | TEACHING RESOURCES/AIDS (PLEASE TICK) | | STUDENT ACTIVITY (EXERCISE IN TEXTBOOK/ADDITIONAL SUPPORTING TASK) TO BE DONE THIS WEEK |
|---|---|---|---|---|---|---|---|
| WEEK 2 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | **INTRODUCTION TO LESSONS** | | | | |
| | | | | | | | |
| | | | **RECAPPING/REINFORCEMENT** | | | | |
| | | | | | | | |

This page may be photocopied.

| LESSON | CONTENT/OUTCOMES TO BE COVERED THIS WEEK | LIST OF EXAMPLES TO BE DONE IN CLASS BY THE LECTURER TO EXPLAIN THE OUTCOME/CONCEPT | FACILITATION METHOD (PLEASE TICK) | | TEACHING RESOURCES/AIDS (PLEASE TICK) | | STUDENT ACTIVITY (EXERCISE IN TEXTBOOK/ADDITIONAL SUPPORTING TASK) TO BE DONE THIS WEEK |
|---|---|---|---|---|---|---|---|
| WEEK 3 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | **INTRODUCTION TO LESSONS** | | | | |
| | | | | | | | |
| | | | **RECAPPING/REINFORCEMENT** | | | | |
| | | | | | | | |

| LESSON | CONTENT/OUTCOMES TO BE COVERED THIS WEEK | LIST OF EXAMPLES TO BE DONE IN CLASS BY THE LECTURER TO EXPLAIN THE OUTCOME/CONCEPT | FACILITATION METHOD (PLEASE TICK) | | TEACHING RESOURCES/AIDS (PLEASE TICK) | | STUDENT ACTIVITY (EXERCISE IN TEXTBOOK/ADDITIONAL SUPPORTING TASK) TO BE DONE THIS WEEK |
|---|---|---|---|---|---|---|---|
| WEEK 4 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | **INTRODUCTION TO LESSONS** | | | | |
| | | | | | | | |
| | | | **RECAPPING/REINFORCEMENT** | | | | |
| | | | | | | | |

This page may be photocopied.

| LESSON | CONTENT/OUTCOMES TO BE COVERED THIS WEEK | LIST OF EXAMPLES TO BE DONE IN CLASS BY THE LECTURER TO EXPLAIN THE OUTCOME/CONCEPT | FACILITATION METHOD (PLEASE TICK) | | TEACHING RESOURCES/AIDS (PLEASE TICK) | | STUDENT ACTIVITY (EXERCISE IN TEXTBOOK/ADDITIONAL SUPPORTING TASK) TO BE DONE THIS WEEK |
|---|---|---|---|---|---|---|---|
| WEEK 5 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | INTRODUCTION TO LESSONS | | | | |
| | | | | | | | |
| | | | RECAPPING/REINFORCEMENT | | | | |
| | | | | | | | |

<table>
<tr><th>LESSON</th><th>CONTENT/OUTCOMES TO BE COVERED THIS WEEK</th><th>LIST OF EXAMPLES TO BE DONE IN CLASS BY THE LECTURER TO EXPLAIN THE OUTCOME/CONCEPT</th><th colspan="2">FACILITATION METHOD (PLEASE TICK)</th><th colspan="2">TEACHING RESOURCES/AIDS (PLEASE TICK)</th><th>STUDENT ACTIVITY (EXERCISE IN TEXTBOOK/ADDITIONAL SUPPORTING TASK) TO BE DONE THIS WEEK</th></tr>
<tr><td rowspan="8">WEEK 6</td><td rowspan="8"></td><td rowspan="8"></td><td>Lecture</td><td></td><td>White board/ OHP</td><td></td><td rowspan="8"></td></tr>
<tr><td>Group work</td><td></td><td>Models</td><td></td></tr>
<tr><td>Demonstration</td><td></td><td>Handouts</td><td></td></tr>
<tr><td>Simulation</td><td></td><td>Multimedia</td><td></td></tr>
<tr><td colspan="4">INTRODUCTION TO LESSONS</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="4">RECAPPING/REINFORCEMENT</td></tr>
<tr><td colspan="4"></td></tr>
</table>

<table>
<tr><th>LESSON</th><th>CONTENT/OUTCOMES TO BE COVERED THIS WEEK</th><th>LIST OF EXAMPLES TO BE DONE IN CLASS BY THE LECTURER TO EXPLAIN THE OUTCOME/CONCEPT</th><th colspan="2">FACILITATION METHOD (PLEASE TICK)</th><th colspan="2">TEACHING RESOURCES/AIDS (PLEASE TICK)</th><th>STUDENT ACTIVITY (EXERCISE IN TEXTBOOK/ADDITIONAL SUPPORTING TASK) TO BE DONE THIS WEEK</th></tr>
<tr><td rowspan="8">WEEK 7</td><td rowspan="8"></td><td rowspan="8"></td><td>Lecture</td><td></td><td>White board/ OHP</td><td></td><td rowspan="8"></td></tr>
<tr><td>Group work</td><td></td><td>Models</td><td></td></tr>
<tr><td>Demonstration</td><td></td><td>Handouts</td><td></td></tr>
<tr><td>Simulation</td><td></td><td>Multimedia</td><td></td></tr>
<tr><td colspan="4">INTRODUCTION TO LESSONS</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="4">RECAPPING/REINFORCEMENT</td></tr>
<tr><td colspan="4"></td></tr>
</table>

This page may be photocopied.

| LESSON | CONTENT/OUTCOMES TO BE COVERED THIS WEEK | LIST OF EXAMPLES TO BE DONE IN CLASS BY THE LECTURER TO EXPLAIN THE OUTCOME/CONCEPT | FACILITATION METHOD (PLEASE TICK) | | TEACHING RESOURCES/AIDS (PLEASE TICK) | | STUDENT ACTIVITY (EXERCISE IN TEXTBOOK/ADDITIONAL SUPPORTING TASK) TO BE DONE THIS WEEK |
|---|---|---|---|---|---|---|---|
| WEEK 8 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | INTRODUCTION TO LESSONS | | | | |
| | | | | | | | |
| | | | RECAPPING/REINFORCEMENT | | | | |
| | | | | | | | |

| LESSON | CONTENT/OUTCOMES TO BE COVERED THIS WEEK | LIST OF EXAMPLES TO BE DONE IN CLASS BY THE LECTURER TO EXPLAIN THE OUTCOME/CONCEPT | FACILITATION METHOD (PLEASE TICK) | | TEACHING RESOURCES/AIDS (PLEASE TICK) | | STUDENT ACTIVITY (EXERCISE IN TEXTBOOK/ADDITIONAL SUPPORTING TASK) TO BE DONE THIS WEEK |
|---|---|---|---|---|---|---|---|
| WEEK 9 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | INTRODUCTION TO LESSONS | | | | |
| | | | | | | | |
| | | | RECAPPING/REINFORCEMENT | | | | |
| | | | | | | | |

This page may be photocopied.

| LESSON | CONTENT/OUTCOMES TO BE COVERED THIS WEEK | LIST OF EXAMPLES TO BE DONE IN CLASS BY THE LECTURER TO EXPLAIN THE OUTCOME/CONCEPT | FACILITATION METHOD (PLEASE TICK) | | TEACHING RESOURCES/AIDS (PLEASE TICK) | | STUDENT ACTIVITY (EXERCISE IN TEXTBOOK/ADDITIONAL SUPPORTING TASK) TO BE DONE THIS WEEK |
|---|---|---|---|---|---|---|---|
| WEEK 10 | | | Lecture | | White board/ OHP | | |
| | | | Group work | | Models | | |
| | | | Demonstration | | Handouts | | |
| | | | Simulation | | Multimedia | | |
| | | | **INTRODUCTION TO LESSONS** | | | | |
| | | | | | | | |
| | | | **RECAPPING/REINFORCEMENT** | | | | |
| | | | | | | | |

**This page may be photocopied.**

MODULE

# 1 Kinematics

**By the end of this module, students should be able to:**

- solve problems dealing with linear motion analytically
- determine the relative velocity, shortest distance, time intersections, overtaking and actual velocity
- calculate resultant velocity and direction
- do calculations dealing with projectiles that are launched horizontally from a certain vertical height or launched at an angle from the horizontal landing on the same horizontal plane
- calculate the maximum height reached by an object as well as time of flight and range, as well as the velocity of projection, the angle of projection and the height and velocity at any part of the projectile path

Motion refers to the action or process of moving or of changing place or position. Motion can be seen in many different situations everywhere around you in sport, nature and industry to name but a few areas.

In this module, you will learn about the study of motion called kinematics. This describes the way objects move without considering the forces responsible for that motion.

## Exercise 1.1 — SB page 12

1. $v_{TI} = v_T - v_I$
   $= 54 \text{ km/h} - 0 \text{ km/h} = 54 \text{ km/h} = 54 \times \frac{1\,000}{3\,600}$
   $= 15 \text{ m/s}$ (2)
2. $v_{GB} = v_G - v_B = -2 \text{ m/s} - 1{,}4 \text{ m/s}$
   $= -3{,}4 \text{ m/s}$ (2)
3. 3.1 $v_{AP} = v_A - v_P$
   $= 126 \text{ km/h} - 108 \text{ km/h}$
   $= 18 \text{ km/h}$ (2)

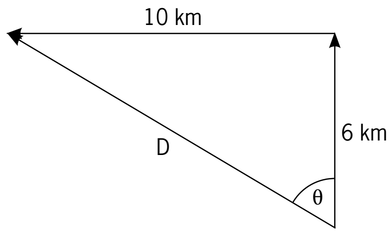
   3.2 $v_{AP} = v_A - v_P$
   $= -126 \text{ km/h} - 108 \text{ km/h}$
   $= -234 \text{ km/h}$ (2)

4. 4.1 E
   4.2 G
   4.3 B
   4.4 A
   4.5 F
   4.6 C
   4.7 D (7)

**Total: 15 marks**

## Exercise 1.2 SB page 16

1. 1.1 16 km (1)
   1.2 Her movements can be represented by the following right-angled triangle:




By measurement D = 11,7 km and θ = 59°.

By calculation:

$D = \sqrt{6^2 + 10^2} = 11{,}66$ km

$\theta = a\tan\frac{10}{6} = 59{,}0°$

Her displacement is 11,66 km at an angle of 59,0° towards the northwest. (3)

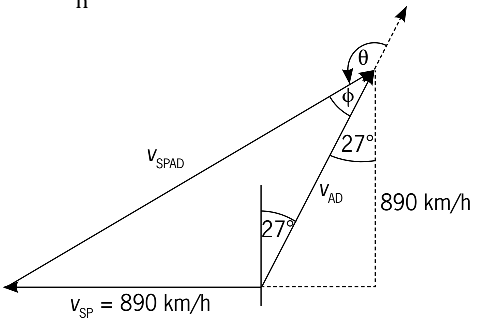
   1.3 $t = \frac{6}{6} + \frac{10}{5} = 3$ hours

$\bar{v} = \frac{11{,}7}{3} = 3{,}9$ km/h (1)

   1.4 $v = \frac{16}{3} = 5{,}33$ km/h (1)

2. 2.1 Since the bus is moving and the hop-off point is motionless, the relative velocity of the bus with reference to the hop-off point will remain as 45 km/h. (1)
   2.2 Soso's velocity with respect to his hop-off point is equal to the sum of the velocity of the bus plus his own velocity, which is: 45 + 2 = 47 km/h. (2)
   2.3 Soso's relative velocity with reference to the cyclist is equal to his own velocity with respect to the hop-off point, $V_{MH}$, plus the velocity of the cyclist, $V_C$, which is: 47 + 10 = 57 km/h. (2)

3. $v_{SP} = 950\frac{km}{h}, v_{AD} = 890\frac{km}{h}$




$$v_{SPADx} = v_{SPx} - v_{ADx} = -950 - 890\cos(90 - 27) = -1\ 354 \text{ km/h}$$
$$v_{SPADy} = v_{SPy} - v_{ADy} = 0 - 890\sin(90 - 27) = -793 \text{ km/h}$$
$$v_{SPAD} = \sqrt{1\ 354^2 + 793^2} = 1\ 569 \text{ km/h}$$
$$\phi + 27° = a\tan\frac{1\ 354}{793} = 59{,}64°$$
$$\therefore \phi = 32{,}64°$$
$$\theta = 180° - \phi = 180 - 32{,}64 = 147{,}36°$$

The relative velocity of SP to AD is 1 569 km/h in a 147,36° direction of AD. (10)

4. The graphical or drawing method and the analytical or calculation method. (2)

5. 5.1 The graphical method involves drawing an accurate diagram of the given **vectors** and measuring the resulting vector size and angle on the diagram. (1)

   5.2 The analytical method involves using **trigonometric ratios** to calculate the resulting vector. (1)

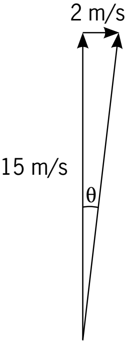
**Total: 25 marks**

## Exercise 1.3 SB page 31

1. 1.1 The velocity of the boat:

$$v = \sqrt{15^2 + 2^2}$$
$$= 15{,}13 \text{ km/h}$$
$$\tan\theta = \frac{2}{15}$$
$$\therefore \theta = 7{,}6° \text{ east of north}$$

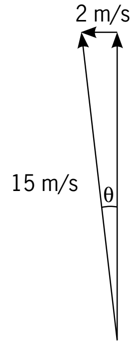
The boat will move at a resultant velocity of 15,13 km/h in a direction 7,6° east of north. (5)



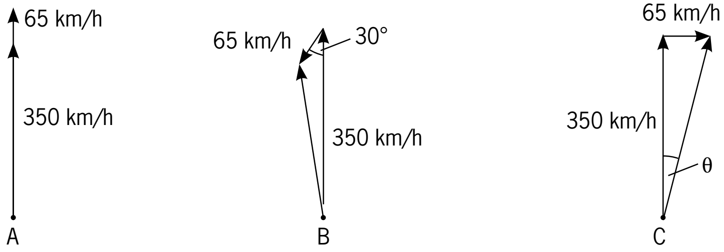
1.2 The direction in which the sailor must steer to get to point B:

$\sin\theta = \frac{2}{15}$

$\therefore \theta = 7{,}7°$ west of north (2)




2.




2.1 $v_{Ry} = v_{py} + v_{wy} = 350 + 65 = 415$ km/h

Since there are no components in the $x$ direction, the resultant velocity is 415 km/h directly north. (2)

2.2 $v_{Ry} = v_{py} + v_{wy} = 350 - 65\cos 30° = 293{,}71$ km/h north

$v_{Rx} = v_{px} + v_{wx} = 0 + 65\sin 30° = 32{,}5$ km/h west

$v_R = \sqrt{293{,}71^2 + 32{,}5^2} = 295{,}5$ km/h

$\tan\theta = \frac{32{,}5}{293{,}71}$

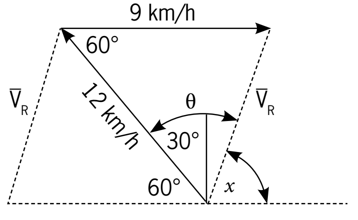
$\therefore \theta = 6{,}3°$ west of north (4)

2.3 $v_R = \sqrt{350^2 + 65^2} = 356$ km/h

$\tan\theta = \frac{65}{350}$

$\therefore \theta = 10{,}5°$ east of north (3)

3. 3.1




$\overline{V}_R^2 = 12^2 + 9^2 - 2(12)(9)\cos 60$

$= \sqrt{117}$

$= 10{,}8$ km/h

Use the sine rule:

$$\frac{\sin\theta}{9\text{ km/h}} = \frac{\sin 60}{V_R}$$

$$\frac{\sin\theta}{9} = \frac{\sin 60}{10{,}8}$$

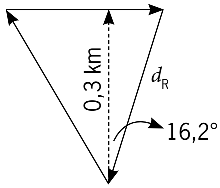
$$\theta = 46{,}2°$$

We can now find the direction of the resultant velocity, α, is:

$$\alpha = 180° - 46{,}2° - 60°$$

$$= 73{,}8°$$ (5)

3.2




The width of the river has to be converted to make the units similar:

$$\text{Width} = \frac{300}{1\,000}$$

$$= 0{,}3\text{ km}$$

We first find the displacement:

$$\cos 16{,}2 = \frac{0{,}3\text{ km}}{d_R}$$

$$d_R = 0{,}312\text{ km}$$ (3)

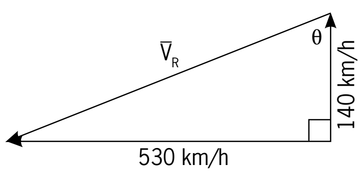
3.3 Now we can find the time:

$$d_R = V_R \times t$$

$$0{,}312\text{ km} = 10{,}8\,\frac{\text{km}}{\text{h}} \times t$$

$$t = 0{,}0289\text{ s}$$ (2)

4.




This is a right-angled diagram, so we can use the Pythagoras theorem:

$$\overline{V}_R^2 = 140^2 + 530^2$$

$$\overline{V}_R = \sqrt{300\,500}$$

$$= 548{,}2\text{ km/h}$$

To find the direction we can use a simple trigonometric function:

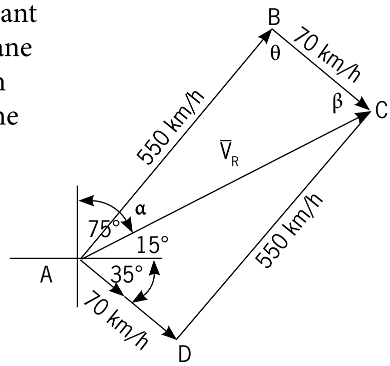
$$\tan\theta = \frac{530}{140}$$

$$\theta = \tan^{-1} 3{,}79$$

$$= 75{,}2° \text{ west of south}$$

Therefore, the wind will result in the aeroplane travelling at 548,2 km/h at 75,2° west of south. (5)

5. 5.1 In the diagram, AC represents the resultant velocity. AB is the velocity of the aeroplane at the direction that will allow it to reach its destination C. AD is the velocity of the wind.




We can use the sine rule:

$$\frac{\sin\alpha}{BC} = \frac{\sin\beta}{AB}$$

We can find the angle, β:

$$\beta = 15° + 35°$$
$$= 50°$$

$$\frac{\sin\alpha}{70 \text{ km/h}} = \frac{\sin 50}{550 \text{ km/h}}$$

$$\alpha = \sin^{-1} 0{,}0975$$
$$= 5{,}6°$$

Therefore, the direction of the aeroplane must be:

75° − 5,6° = 69,4° east of north. (5)

5.2 Use the cosine rule: $\overline{V}_R^2 = 550^2 + 70^2 - 2(550)(70)\cos\theta$

We can find θ:

$$\theta = 180° - \beta - \alpha$$
$$= 180° - 50° - 5{,}6°$$
$$= 124{,}4°$$

Therefore, the equation becomes:

$$\overline{V}_R^2 = 550^2 + 70^2 - 2(550)(70)\cos 124{,}4$$
$$\overline{V}_R = \sqrt{350\ 902{,}5}$$
$$\overline{V} = 592{,}4 \text{ km/h}$$ (7)

5.3 Use the velocity equation:

We know that $d_R = 480$ km.

Then, $d_R = \overline{V}_R \times t$

$$480 \text{ km} = (592{,}4 \text{ km/h}) \times t$$
$$t = 0{,}81 \text{ hours} = 48{,}6 \text{ min}$$

It would take 48,6 minutes for the plane to reach its destination. (2)

**Total: 45 marks**

## Exercise 1.4 SB page 42

1. 1.1 Vertical values:

$$y = ?$$
$$v_{ty} = 12 \text{ m/s}$$
$$= v_{fy} = -12 \text{ m/s}$$
$$a_y = -9{,}8 \text{ m/s}^2$$
$$t = ?$$
$$v_{fy} = v_{ty} + a_y \cdot t$$
$$-12 \text{ m/s} = 12 \text{ m/s} + (-9{,}8 \text{ m/s}^2) \times t$$
$$-24 \text{ m/s} = -9{,}8 \text{ m/s}^2 \times t$$
$$t = 2{,}45\ s$$

This means that the stone will be in the air for 2,45 seconds before it hits the ground. (2)

1.2 First calculate the upward time:

$$t = \frac{2{,}45}{2}$$
$$= 1{,}22 \text{ s}$$

Then:

$$y = v_{ty} \times \frac{1}{2} a_y \times t^2$$
$$= (12 \text{ m/s})(1{,}22 \text{ s}) + (0{,}5)(-9{,}8 \text{ m/s}^2)(1{,}22)^2$$
$$= 7{,}35 \text{ m}$$

This means that the stone will reach maximum height of 7,35 m above the ground. (3)

2. 2.1

| Horizontal values | Vertical values |
|---|---|
| $x = ?$ | $y = ?$ |
| $v_{ix} = 460 \cos 28 = 406{,}2$ m/s | $v_{ty} = 460 \sin 28 = 216$ m/s |
| $v_{fx} = 406{,}2$ m/s | $v_{fy} = -216$ m/s |
| $a_x = 0$ m/s$^2$ | $a_y = -9{,}8$ m/s$^2$ |
| $t = ?$ | $t = ?$ |

$$v_{fy} = v_{ty} + a_y \times t$$
$$-216 \text{ m/s} = 216 \text{ m/s} + (-9{,}8 \text{ m/s}^2) \times t$$
$$-432 \text{ m/s} = -9{,}8 \text{ m/s} \times t$$
$$t = 44{,}08 \text{ s}$$

This means that the bullet will be in the air for 44,08 seconds. (2)

2.2 We first calculate the upward time:

$t = \frac{44{,}08}{2}$

$= 22{,}04 \text{ s}$

Then:

$y = v_{ty} \times t + \frac{1}{2}a_y \times t^2$

$= (216 \text{ m/s})(22{,}04 \text{ s}) + (0{,}5)(-9{,}8 \text{ m/s})(22{,}04)$

$= 2\ 380{,}4 \text{ m}$

This means that the bullet reaches a maximum height of 2 380,4 m above the ground. (3)

2.3 $x = v_{ix} \times t + \frac{1}{2}a_x \times t$

$= \left(406{,}2 \frac{m}{s}\right)(44{,}08 \text{ s}) + (0{,}5)(0)(44{,}08)^2$

$= 17\ 905{,}3 \text{ m}$

This means that the bullet travels a horizontal distance of 17 905,3 m. (3)

3. 3.1 Horizontal value: (2)

$v_{ix} = 8{,}5\cos 30° = 7{,}36 \text{ m/s}$

$v_{fx} = 7{,}36 \text{ m/s}$

$a_x = 0 \text{ m/s}^2$

Vertical values:

$v_{iy} = 8{,}5\sin 30° = 4{,}25 \text{ m/s}$

$v_{fy} = -4{,}25 \text{ m/s}$

$a_y = -9{,}8 \text{ m/s}^2$

$v_{fy} = v_{iy} + a_y t$

$\therefore -4{,}25 = 4{,}25 - 9{,}8t$

$\therefore t = 0{,}867 \text{ s}$

This means that the long-jumper will be in the air for 0,867 seconds. (2)

3.2 $x = \text{v}_{ix} t + \frac{1}{2}a_x t^2$

$= 7{,}36 \times 0{,}867 + 0$

$= 6{,}38 \text{ m}$

This means that the horizontal distance of the long-jumper is 6,38 m. (3)

3.3 The upward time is half of the total time:

$t_u = \frac{0{,}867}{2} = 0{,}434 \text{ s}$

Then the height after half the time:

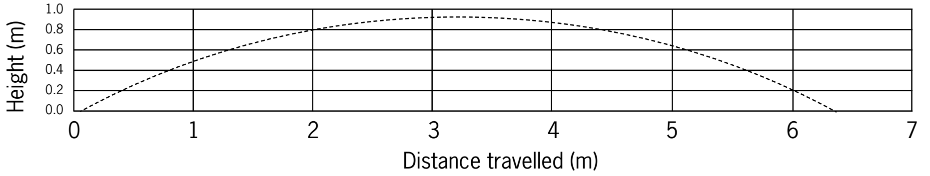
$y = v_{iy} t + \frac{1}{2}a_y t_u^2$

$= 4{,}25 \times 0{,}434 - \frac{1}{2} \times 9{,}8 \times 0{,}43\,4^2$

$= 0{,}92 \text{ m}$

This means that the long-jumper reaches a peak height of 0,92 m above the ground. (3)

3.4




(3)

4. False. For projectiles, you must always use the point from where the projectile was launched as the reference point (1)

**Total: 25 marks**

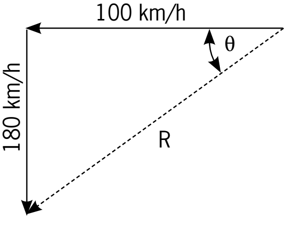
## Summative assessment — SB page 45

1. We draw a velocity vector diagram for the motion of the train relative to the aeroplane.

$R^2 = (100 \text{ km/h})^2 + (180 \text{ km/h})^2$

$= \sqrt{42\ 400}$

$= 205{,}9 \text{ km/h}$




For direction:

$\tan\theta = \frac{180}{100}$

$= \tan^{-1}(1{,}8)$

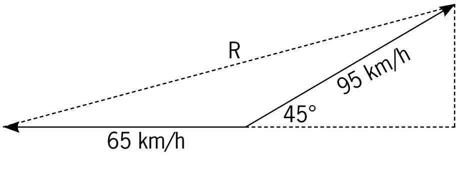
$= 60{,}9°$

Therefore, the actual velocity of the aeroplane is 205,9 km/h at approximately 60,9° south of west. (4)

2. $C_v = 95\sin45$

$= 67{,}18 \text{ km/h}$

Since these are moving away from each other, we add the two horizontal components of the velocities.




$C_H = 95\cos45 + 65 \text{ km/h}$

$= 132{,}18 \text{ km/h}$

$R^2 = (132{,}18 \text{ km/h})^2 + (67{,}18 \text{ km/h})^2$

$R = \sqrt{21\ 984{,}7}$

148,27 km/h

For direction:

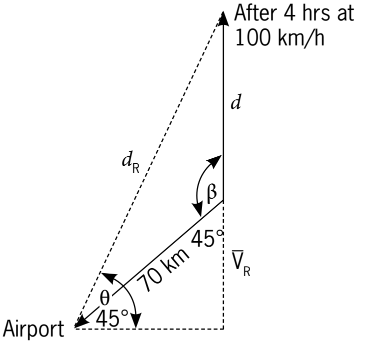
$\tan\theta = \frac{67{,}18}{132{,}18}$

$= \tan^{-1}(0{,}508)$

$= 26{,}9°$

Therefore, the velocity of car P with reference to car Q is 148,27 km/h at 26,9° north of east. (5)

3. The figure below represents the displacement vector diagram of the aircraft in relation to the airport. In the diagram, $d$ is the distance travelled by the aircraft after 4 hours; $q$ is the extended distance of the aircraft and $d_R^2$ is the displacement of the aircraft with reference to the airport. Theta (θ) represents the direction of the aircraft with reference to the airport, while β is the angle between the distance, $d$, and the initial distance of the aircraft from the airport.




To get the displacement of the aircraft with reference to the airport, we have to use the cosine rule. But first, we need to calculate the value of $d$.

$d = v \times t$

$= (100 \text{ km/h})(4\text{h})$

$= 400 \text{ km}$

Then,

$d_R^2 = a^2 + b^2 - 2ab\cos\beta$

$= (70 \text{ km})^2 + (400 \text{ km})^2 - 2(70)(400)\cos 135$

$d_R = \sqrt{204\ 497{,}98}$

$= 452{,}2 \text{ km}$

For direction, we need to know the size of $q$ and the total vertical displacement vector.

$\sin 45 = \frac{q}{70 \text{ km}}$

$q = 49{,}5 \text{ km}$

Then:

$\sin\theta = \frac{(49{,}5 \text{ km} + 400 \text{ km})}{452{,}2 \text{ km}}$

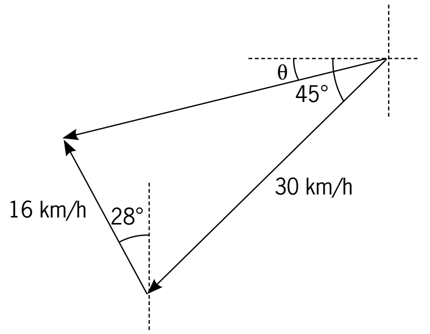
$= \frac{449{,}5}{452{,}2}$

$\theta = \sin^{-1}(0{,}994)$

$= 83{,}7°$

Therefore, the aircraft will be about 452,2 km away from the airport at 83,7° north of east. (5)

4. 4.1 Calculate the $x$ direction velocity:




$$v_{Sx} = 30\sin45° = 21{,}21 \text{ km/h}$$
$$v_{Cx} = 16\sin28° = 7{,}51 \text{ km/h}$$
$$v_{Rx} = 21{,}21 + 7{,}51 = 28{,}72 \text{ km/h}$$

Calculate the $y$ direction velocity:

$$v_{Sy} = -30\cos45° = -21{,}21 \text{ km/h}$$
$$v_{Cy} = 16\cos28° = 14{,}13 \text{ km/h}$$
$$v_{Ry} = -21{,}21 + 14{,}13 = -7{,}09 \text{ km/h}$$

Calculate the resultant velocity:

$$v_R = \sqrt{28{,}72^2 + 7{,}09^2} = 29{,}59 \text{ km/h}$$

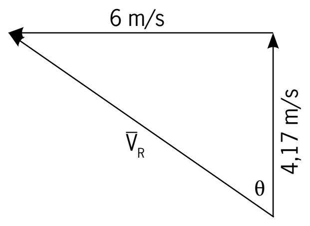
Calculate the direction:

$$\theta = a\tan\frac{7{,}09}{28{,}72} = 13{,}9°$$

Therefore, the velocity is 29,59 km/h and the direction is 13,9° south of west. (5)

4.2 $d = v_R t = 29{,}59 \times 3$
$= 88{,}76$ km (2)

5. 5.1 First, convert units to be the same:




$$\text{Velocity of boat} = 15 \times \frac{1\,000}{3\,600} = 4{,}17 \text{ m/s}$$
$$\overline{V}_R^2 = (6 \text{ m/s})^2 + (4{,}17 \text{ m/s})^2$$
$$\overline{V} = \sqrt{53{,}29} = 7{,}3 \text{ m/s}$$

For direction:

$$\tan\theta = \frac{6 \text{ m/s}}{4{,}17 \text{ m/s}}$$
$$\theta = \tan^{-1}(1{,}44) = 55{,}2° \text{ west of north}$$ (4)

5.2 $d_R = \overline{V}_R \times t$

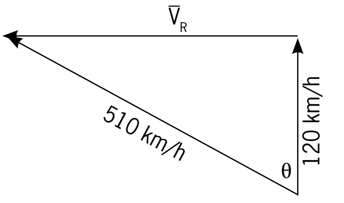
$(350 \text{ m}) = (7{,}3 \text{ m/s}) \times t$

$t = 48$ s rounded off from 47,95 s (2)

6. 6.1 $\overline{V}^2_R = (510\text{ km/h})^2 - (120\text{ km/h})^2$

$\overline{V} = \sqrt{245\,700}$

$= 495{,}7\text{ m/s}$ (2)

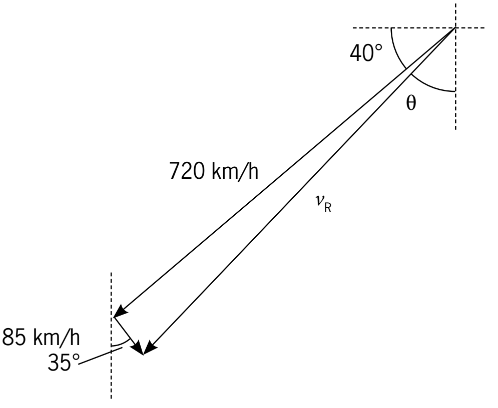



6.2 $\cos\theta = \frac{120\text{ km/h}}{510\text{ km/h}}$

$\theta = \cos^{-1}(0{,}235)$

$= 76{,}4°$ west of north (3)

7. 7.1 Calculate the $x$ direction velocity:




$v_{Ax} = -720\cos40°$

$= -551{,}55\text{ km/h}$

$v_{Wx} = 85\sin35°$

$= 48{,}75\text{ km/h}$

$v_{Rx} = -551{,}55 + 48{,}75$

$= -502{,}8\text{ km/h}$

Calculate the $y$ direction velocity:

$v_{Ay} = -720\sin40°$

$= -462{,}81\text{ km/h}$

$v_{Wy} = 85\cos35°$

$= -69{,}63\text{ km/h}$

$v_{Ry} = -462{,}81 - 69{,}63$

$= -532{,}44\text{ km/h}$

Calculate the resultant velocity:

$v_R = \sqrt{502{,}8^2 + 532{,}44^2}$

$= 732{,}32\text{ km/h}$

Calculate the direction:

$\theta = a\tan\frac{502{,}8}{532{,}44} = 43{,}36°$ (6)

Therefore, the velocity is 732,32 km/h and the direction is 43,36° west of south.

7.2 $d = v_R t = 732{,}32 \times 5$

$= 3\,662\text{ km}$ (2)

8. 8.1 **Vertical values**

$y = ?$

$v_{ty} = 12\text{ m/s}$

$v_{fy} = -12\text{ m/s}$

$a_y = -9{,}8\text{ m/s}$

$t = ?$

$v_{fy} = v_{ty} + a_y \times t$

$$-12\text{ m/s} = 12\text{ m/s} + (-9{,}8\text{ m/s}^2) \times t$$
$$t = 2{,}45\text{ s}$$ (3)

8.2 Get the time for upward movement:

$$t = \frac{2{,}45\text{ s}}{2}$$
$$= 1{,}225\text{ s}$$

Then:

$$y = v_{ty} \times t + \frac{1}{2}a_y \times t^2$$
$$= (12)(1{,}225) + (0{,}5)(-9{,}8)(1{,}225)^2$$
$$= 7{,}35\text{ m}$$ (4)

9. 9.1

| **Horizontal values** | **Vertical values** |
|---|---|
| $x = ?$ | $y = ?$ |
| $v_{ix} = 21\cos28 = 18{,}54$ m/s | $v_{ty} = 21\sin28 = 9{,}89$ m/s |
| $v_{fx} = 18{,}54$ m/s | $v_{fy} = -9{,}89$ m/s |
| $a_x = 0$ m/s | $a_y = -9{,}8$ m/s$^2$ |
| $t = ?$ | $t = ?$ |

$$v_{fy} = v_{ty} + a_y \times t$$
$$-9{,}8\text{ m/s} = 9{,}89\text{ m/s} + (9{,}8\text{ m/s}^2) \times t$$
$$t = 2{,}02\text{ s}$$ (3)

9.2 First get the time for the upward movement:

$$t = \frac{2{,}02\text{ s}}{2}$$
$$= 1{,}009\text{ s}$$

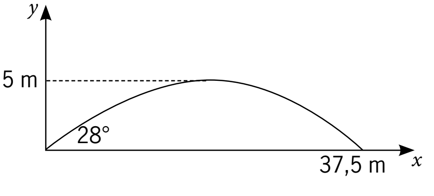
Then:

$$y = v_{ty} \times t + \frac{1}{2}a_y \times t$$
$$= (9{,}89)(1{,}009) + (0{,}5)(-9{,}8)(1{,}009)^2$$
$$= 5\text{ m}$$ (4)

9.3 $x = v_{ix} \times t + \frac{1}{2}a_x \times t^2$

$$= (18{,}54)(2{,}02) + (0{,}5)(0)(1{,}009)^2$$
$$=37{,}5\text{ m}$$ (3)

9.4




(3)

10. 10.1 Horizontal value

$v_x = 40$ m/s

Vertical values

$d_{yi} = 1{,}2$ m

$d_{yf} = 0$ m

$v_{iy} = 0$ m/s

$a_y = -9{,}8$ m/s$^2$

Calculate the time:

$d_{yf} = d_{yi} + v_{yi}t + \frac{1}{2}a_y t^2$

$\therefore 0 = 1{,}2 + 0 - \frac{1}{2}9{,}8\,t^2$

$\therefore t = 0{,}495$ s

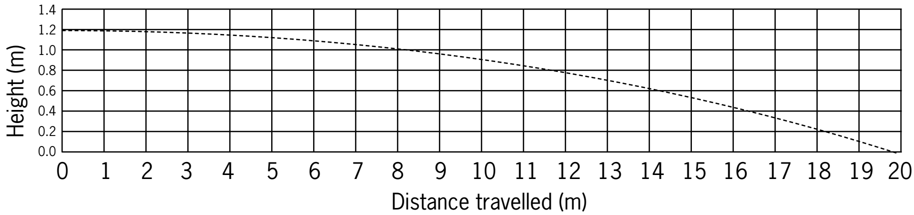
This means that the pebbles will be in the air for 0,495 seconds. (3)

10.2 $x = v_{ix}t$

$= 40 \times 0{,}495$

$= 19{,}8$ m

The horizontal distance the pebble travels is 19,8 m.




(2)

**Total: 65 marks**

## Additional examples with solutions for lecturers and tutors:

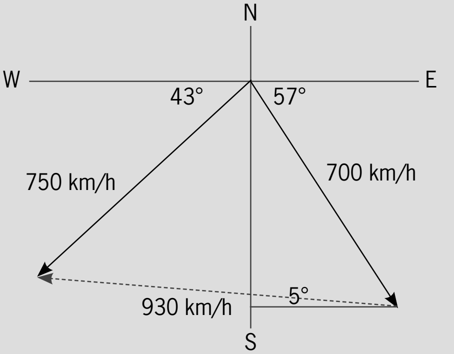
Give your learners these problems for extra practice and work through the solutions with them in class.

### Additional example 1

1. Two planes take off from OR Tambo Airport shortly after each other. The one flies to Durban and the other flies to Cape Town. The one flying to Durban maintains an average speed of 700 km/h for the 550-km route, while the Cape Town flight maintains 750 km/h for its 1 450-km flight. The direction from OR Tambo to Durban is 57° south of east, while the direction from OR Tambo to Cape Town is 43° south of west.
   1.1 Use the graphical method to determine the velocity of the Cape Town flight relative to the Durban flight.
   1.2 Calculate the flight time for each of the flights.

**Solution**

1. 1.1 Draw a vector diagram to scale and measure the length and direction of the relative velocity vector.




***Vector diagram***

$v_{DCT}$ = 930 km/h direction 5° north of west.

   1.2 Use the time formula to calculate each flight time.

   Durban:

   $t = \frac{d_D}{v_D} = \frac{550}{700}$ = 0,786 hours = 47 minutes

   Cape Town:

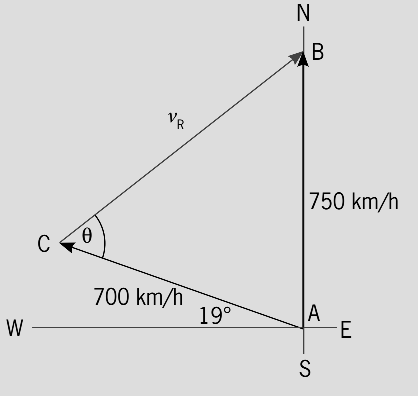
   $t = \frac{d_{CT}}{v_{CT}} = \frac{1\,450}{750}$ = 1,933 hours = 1 hour 56 minutes

## Additional example 2

2. Two planes take off from OR Tambo Airport shortly after each other. The one flies to Lusaka in Zambia and the other flies to Windhoek in Namibia. The one flying to Lusaka maintains an average speed of 750 km/h, while the Windhoek flight maintains 700 km/h for its flight. The direction from OR Tambo to Lusaka is due north, while the direction from OR Tambo to Windhoek is 19° north of west.
   2.1 Use an analytical method to determine the velocity of the Lusaka flight relative to the Windhoek flight.

### Solution

Draw a vector diagram of the different flights.




***Vector diagram of the different flights***

The $y$-component for Windhoek is:

$v_{Wy} = 700\sin 19° = 227{,}9$ km/h

The $y$-component for Lusaka is:

$v_{Ly} = 750$ km/h

The resultant $y$-velocity is then:

$$\begin{aligned} v_{WLy} &= v_{Wy} - v_{Ly} \\ &= 227{,}9 - 750 \text{ km/h} \\ &= -522{,}1 \text{ km/h} \end{aligned}$$

The $x$-component for Windhoek is:

$v_{Wx} = 700\cos 19° = 661{,}9$ km/h

The $x$-component for Lusaka is:

$v_{Lx} = 0$ km/h

The resultant $x$-velocity is then:

$$v_{WLx} = v_{Wx} - v_{Lx}$$
$$= 661{,}9 - 0 \text{ km/h}$$
$$= 661{,}9 \text{ km/h}$$

The resultant velocity is then:

$$v_R^2 = (522{,}1)^2 + (661{,}9)^2$$
$$= 710\ 650$$
$$v_R = 843 \text{ km/h}$$

There are different ways to work out the direction, but the cosine rule works well here:

$$c^2 = a^2 + b^2 - 2ab \times \cos\theta$$
$$\therefore \cos\theta = \frac{c^2 - a^2 - b^2}{-2ab} = \frac{750^2 - 843^2 - 700^2}{-2 \times 843 \times 700} = 0{,}541$$
$$\therefore \theta = \cos^{-1} 0{,}541 = 57{,}27°$$

The relative velocity of the Windhoek flight relative to the Lusaka flight is therefore:

$v_{WL}$ = 843 km/h in a direction 57,27°.

## Additional example 3

3. As winter approaches, a flock of swallows departs from South Africa for the north. Using their internal magnetic orientation system, they set their course for 21° west of north. They fly at a constant speed of 20 km/h but a 6-km/h wind from the southwest steers them off course.

   3.1 Calculate their resultant velocity.

   3.2 Calculate the distance travelled after 12 hours of flight.

**Solution**

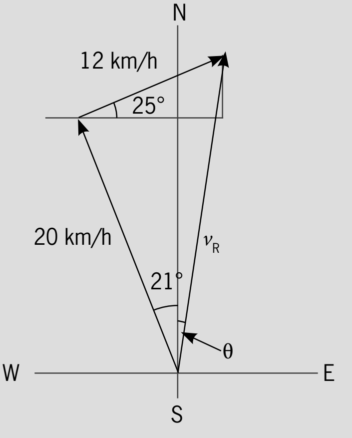
Draw the vector diagram

Calculate the $y$-component of the swallows' velocity:

$$v_{sy} = 20\cos21° = 18{,}672 \text{ km/h}$$

Calculate the $y$-component of the wind velocity:

$$v_{wy} = 12\sin25° = 5{,}071 \text{ km/h}$$



*Vector diagram*

The resultant $y$ velocity is then:

$$v_{Ry} = v_{sy} + v_{wy}$$
$$= 18{,}672 + 5{,}071 \text{ km/h}$$
$$= 23{,}743 \text{ km/h}$$

Calculate the $x$-component of the swallows' velocity:

$$v_{sx} = -20\sin 21° = -7{,}167 \text{ km/h}$$

Calculate the $x$-component of the wind velocity:

$$v_{wy} = 12\cos 25° = 10{,}876 \text{ km/h}$$

The resultant $x$ velocity is then:

$$v_{Rx} = v_{sx} + v_{wx}$$
$$= -7{,}167 + 10{,}876 \text{ km/h}$$
$$= 3{,}709 \text{ km/h}$$

The overall resultant velocity is then:

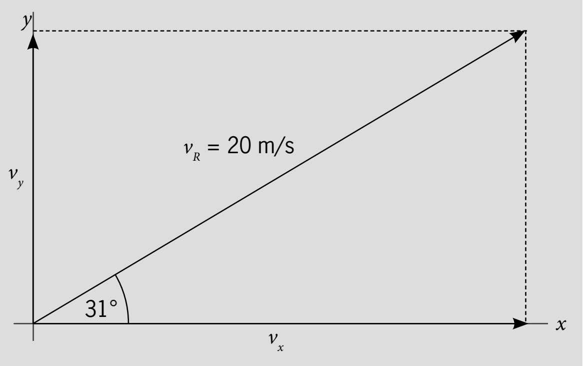
$$v_R^2 = (23{,}743)^2 + (3{,}709)^2$$
$$= 577{,}49$$
$$v_R = 24{,}031 \text{ km/h}$$

Calculate the angle:

$$\tan\theta = \frac{3{,}709}{23{,}743} = 0{,}156$$
$$\therefore \theta = 8{,}9°$$

## Additional example 4

4. A goalkeeper kicks the ball from his goal area at a velocity of 20 m/s and at an angle of 31° to the horizontal. Calculate:
   - 4.1 The time of flight
   - 4.2 The maximum height that the stone reaches
   - 4.3 The horizontal displacement of the stone.




*Vector diagram of the kicked ball*

**Solution**

4. $v_x = v_{xi} = 20 \cos 31° = 17,143$ m/s

$v_{yi} = 20 \sin 31° = 10,301$ m/s

$v_{yf} = -10,301$ m/s

4.1 Calculate the total flight time by using the vertical velocity equation.

$$v_y = v_{yi} + at$$

$$\therefore t = \frac{v_{yf} - v_{yi}}{a}$$

$$= \frac{-10,301 - 10,301}{-9,8} = 2,1 \text{ s}$$

This means that the ball is in the air for a total time of 2,1 s.

4.2 Calculate the maximum height.

$$d_y = v_y t + \frac{1}{2}at^2$$

$$d_{ymax} = v_{yi}\frac{t}{2} + \frac{1}{2}a\left(\frac{t}{2}\right)^2$$

$$= 10,301 \times \frac{2,1}{2} - \frac{1}{2} \times 9,8 \times \left(\frac{2,1}{2}\right)^2 = 5,4 \text{ m.}$$

This means that the ball's highest point is 5,4 m.

4.3 Calculate the horizontal displacement.

= 17,143 × 2,1 = 36,0 m.

This means that the ball travels 36 m horizontally before it hits the ground.

**Additional example 5**

5. The police investigates a shooting incident where a bullet was shot through the door of a house and was found in the opposite wall. The angle measured was 0,07° and the type of bullet has a muzzle velocity of 948 m/s.

5.1 Determine the distance from where it was shot assuming a flat area.

**Solution**

First calculate its horizontal velocity:

$v_x = v_R \times \cos\theta = 948\cos 0{,}07 = 948$ m/s

Calculate its vertical velocity:

$v_{yi} = v_R \times \sin\theta = 948\sin 0{,}07 = 1{,}158$ m/s

$\therefore v_{yf} = -1{,}158$ m/s

Calculate the time:

$v_{yf} = v_{yi} + at$

$\therefore t = \frac{v_{yf} - v_{yi}}{a} = \frac{-1{,}158 - 1{,}158}{-9{,}8} = 0{,}236$ s

Now calculate the horizontal distance:

$d_x = v_x t = 948 \times 0{,}236 = 224$ m

The bullet was therefore fired from 224 m from the point of impact.

MODULE

# 2 *Angular motion*

**By the end of this module, students should be able to:**

- calculate angular displacement
- calculate angular velocity
- calculate angular acceleration
- calculate linear acceleration and distance moved by an object
- understand the relationship between linear and angular quantities
- calculate accelerating and decelerating torque if the mass of inertia of an object is given
- calculate work done

Angular motion can be defined as the motion of an object around a fixed axis or pivot point. This movement is described as angular rather than linear (in a straight line). Examples of objects with rotational motion are pendulums, wheels, motors and even the planets that rotate around the sun.

## Exercise 2.1 SB page 55

1. 1.1 Angular velocity

$v = \frac{40 \times 1\,000}{3\,600} = 11{,}11$ m/s (3)

1.2 Rotational frequency

$\omega = \frac{v}{r} = \frac{11{,}11}{0{,}62} = 17{,}92$ rad/s (2)

1.3 Angular displacement

$n = \frac{\omega}{2\pi} = \frac{17{,}92}{2\pi} = 2{,}85$ revs/s (= 171 rpm)

$\theta = \omega t = 17{,}92 \times 80 = 1\,434$ rad (2)

1.4 Linear displacement

$s = \theta r = 1\,434 \times 0{,}62 = 888{,}9$ m (2)

2. 2.1 Rotational frequency

$n = \frac{\text{rpm}}{60} = \frac{240}{60} = 4$ revs/s (2)

2.2 Rotational period

$T = \frac{1}{n} = \frac{1}{4} = 0{,}25$ s (2)

2.3 Angular velocity

$\omega_i = 2\pi n = 2\pi \times 4 = 25{,}13$ rad/s (2)

2.4 Angular displacement

$\omega_f = 0$ rad/s

$\theta = \omega_{avg} t = \frac{25{,}13 + 0}{2} \times 24 = 301{,}6$ rad (3)

2.5 Angular deceleration

$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 - 25.13}{24} = -1{,}047$ rad/s$^2$ (2)

**Total: 20 marks**

## Exercise 2.2 SB page 62

1. 1.1 Angular acceleration

$\omega = \omega_i + \alpha t$

$\therefore \alpha = \frac{\omega - \omega_i}{t}$

$= \frac{400 - 600}{30} = -6{,}67$ rad/s$^2$ (3)

1.2 Angular displacement

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$= 600 \times 30 - \frac{1}{2} \times 6{,}67 \times 30^2$

$= 15\ 000$ rad

or

$\theta = \omega_{avg} t = \frac{\omega_i + \omega_f}{2} t$

$= \frac{600 + 400}{2} \times 30$

$= 15\ 000$ rad (2)

1.3 Number of revolutions

$n = \frac{\omega}{2\pi}$

$\therefore$ revolutions $= \frac{\omega_{avg} t}{2\pi} = \frac{\theta}{2\pi}$

$= \frac{15\ 000}{2\pi} = 2\ 387$ revolutions (3)

2. 2.1 Angular velocity

$\omega = \frac{\Delta\theta}{\Delta t}$

$= \frac{2\pi}{8} = 0{,}785$ rad/s (2)

2.2 Linear displacement

$s = \theta r = 2\pi r$

$= 2\pi \times 20 = 125,66$ m (2)

2.3 Linear velocity

$v = \frac{\Delta s}{\Delta t}$

$= \frac{125,66}{8} = 15,71$ m/s

or:

$v = \omega r$

$= 0,785 \times 20 = 15,71$ m/s (2)

3. 3.1 C
   3.2 E
   3.3 D
   3.4 F
   3.5 A
   3.6 G
   3.7 B (7 × 1)

**Total: 21 marks**

## Exercise 2.3 SB page 68

1. 1.1 Average angular velocity

$\omega_i = 0$

$\omega_f = \frac{2\pi n}{60}$

$= \frac{2\pi \times 500}{60} = 52,36$ rad/s

$\omega_{avg} = \frac{\omega_i + \omega_f}{2}$

$= \frac{0 + 52,36}{2} = 26,18$ rad/s (3)

1.2 Angular acceleration

$\alpha = \frac{\omega_f - \omega_i}{t}$

$= \frac{52,36 - 0}{8} = 6,545$ rad/s$^2$ (2)

1.3 Torque required

$\tau = mr^2\alpha$

$= 250 \times 0,03^2 \times 6,545 = 1,47$ Nm (5)

1.4 Work done

$\theta = \omega_{avg} t$

$= 26{,}18 \times 8 = 209{,}44$ rad

$W = \tau\theta$

$= 1{,}47 \times 209{,}44 = 308{,}43$ J (4)

1.5 Power required

$P = \frac{W}{t}$

$= \frac{308{,}43}{8} = 38{,}55$ W (2)

2. 2.1 Angular acceleration

$\tau = mr^2\alpha$

$\therefore \alpha = \frac{\tau}{mr^2}$

$= \frac{400}{650 \times 2^2} = 0{,}154$ rad/s² (2)

2.2 Angular velocity

$\alpha = \frac{\omega_f - \omega_i}{t}$

$\therefore \omega_f = \omega_i + \alpha t$

$= 0 + 0.154 \times 45 = 6{,}92$ rad/s (2)

2.3 Work done

$\theta = \omega_{avg} t$

$= \frac{6{,}92 + 0}{2} \times 45 = 155{,}77$ rad

$W = \tau\theta$

$= 400 \times 155{,}77 = 62\ 308$ J (2)

2.4 Power required

$P = \frac{W}{t}$

$= \frac{62\ 308}{45} = 1\ 385$ W (2)

3. 3.1 Angular acceleration

$\omega_f = \frac{2\pi n}{60}$

$= \frac{2\pi \times 30}{60} = 3{,}142$ rad/s

$\alpha = \frac{\omega_f - \omega_i}{t}$

$= \frac{3{,}142 - 0}{4} = 0{,}785$ rad/s² (2)

3.2 Average angular velocity

$\omega_{avg} = \frac{\omega_i + \omega_f}{2}$

$= \frac{0 + 3{,}142}{2} = 1{,}571$ rad/s (2)

3.3 Angular displacement

$\theta = \omega_{avg} t$

$= 1{,}571 \times 4 = 6{,}283$ rad (2)

3.4 Number of revolutions

$\therefore$ revolutions $= \frac{\theta}{2\pi}$

$= \frac{6{,}283}{2\pi} = 1$ revolutions (2)

3.5 Linear acceleration

$a = \alpha r$

$= 0{,}785 \times 0{,}035 = 0{,}0275$ m/s (2)

3.6 Linear displacement

$s = \theta r$

$= 6{,}283 \times 0{,}8 = 5{,}0265$ m (2)

3.7 Torque required

$\tau = mk^2\alpha$

For a flat, solid metal disc:

$k = \frac{r}{\sqrt{2}}$

$\therefore k^2 = \frac{r^2}{2}$

$\tau = mk^2\alpha = \frac{mr^2\alpha}{2}$

$= \frac{100 \times 0{,}8^2 \times 0{,}785}{2} = 25{,}13$ Nm (2)

3.8 Work done

$W = \tau\theta$

$= 25{,}13 \times 6{,}283 = 157{,}9$ J (2)

3.9 Power requirement

$P = \frac{W}{t}$

$= \frac{157{,}9}{4} = 39{,}48$ W (2)

4. 4.1 C
4.2 G
4.3 F
4.4 E
4.5 H
4.6 A
4.7 D
4.8 B (8 × 1)

**Total: 50 marks**

## Summative assessment SB page 73

1. 1.1 Angular displacement is the angle in degrees or radians through which a point or a line travels about an axis.

   1.2 Angular velocity is the rate at which angular displacement changes.

   1.3 Angular acceleration is the rate at which angular velocity changes with time.

   1.4 Angular or rotational frequency is referred to as the number of revolutions per unit time.

   1.5 Torque is the force needed to affect the angular acceleration of an object in a circular motion. (5 × 2)

2. 2.1 Angular velocity

   $$\omega = \frac{\Delta\theta}{\Delta t}$$

   $$= \frac{2\pi}{12} = 0{,}524 \text{ rad/s}$$ (2)

   2.2 Linear velocity

   $$v = \omega r$$

   $$= 0{,}524 \times 10 = 5{,}24 \text{ m/s}$$ (2)

3. Given information:

   $r = 6$ m

   $t = 3$ s

   $v_i = 0$ m/s

   $v_f = 6$ m/s

   3.1 Average angular velocity

   $$\omega = \frac{v}{r}$$

   $$= \frac{6}{6} = 1 \text{ rad/s}$$ (2)

   3.2 Angular acceleration

   $$\alpha = \frac{\omega_f - \omega_i}{t}$$

   $$= \frac{1 - 0}{3} = 0{,}333 \text{ rad/s}^2$$ (2)

   3.3 Linear acceleration

   $$a = \alpha r$$

   $$= 0{,}333 \times 6 = 2 \text{ m/s}^2$$ (2)

3.4 Angular displacement

$$\theta = \omega_{avg}$$

$$t = \frac{\omega_f + \omega_i}{2} t$$

$$= \frac{1 + 0}{2} \times 3 = 1{,}5 \text{ rad}$$ (2)

3.5 Linear displacement

$$s = \theta r$$

$$= 1{,}5 \times 6 = 9 \text{ m}$$ (2)

4. Given information:

$t = 5$ s

$n_i = 12$ rpm

$n_f = 35$ rpm

4.1 Angular acceleration

$$\omega_i = \frac{2\pi n}{60}$$

$$= \frac{2\pi \times 12}{60} = 1{,}257 \text{ rad/s}$$

$$\omega_f = \frac{2\pi \times 35}{60}$$

$$= 3{,}665 \text{ rad/s}$$

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

$$= \frac{3{,}665 - 1{,}257}{5} = 0{,}482 \text{ rad/s}^2$$ (4)

4.2 Average angular velocity

$$\omega_{avg} = \frac{\omega_f + \omega_i}{2}$$

$$= \frac{1{,}257 + 3{,}665}{2} = 2{,}461 \text{ rad/s}$$ (2)

4.3 Angular displacement

$$\theta = \omega_{avg} t$$

$$= 2{,}461 \times 5 = 12{,}305 \text{ rad}$$ (2)

4.4 Number of revolutions completed

$$\text{revolutions} = \frac{\theta}{2\pi}$$

$$= \frac{12{,}305}{2\pi} = 1{,}958 \text{ revolutions}$$ (3)

5. 5.1 Torque developed

$$\omega = \frac{2\pi n}{60}$$
$$= \frac{2\pi \times 1\ 200}{60} = 125{,}66 \text{ rad/s}$$
$$P = \tau\omega$$
$$\therefore \tau = \frac{P}{\omega}$$
$$= \frac{72\ 000}{125{,}66} = 572{,}96 \text{ Nm} \qquad (4)$$

5.2 Work done in 3 s

$$P = \frac{W}{t}$$
$$\therefore W = Pt$$
$$= 72\ 000 \times 3 = 216\ 000 \text{ J} \qquad (2)$$

5.3 The force on the road

$$\tau = Fr$$
$$\therefore F = \frac{\tau}{r} = \frac{572{,}96}{0{,}31} = 1\ 848 \text{ N} \qquad (2)$$

**Total: 45 marks**

## Additional examples with solutions for lecturers and tutors:

Give your learners these problems for extra practice and work through the solutions with them in class.

### Additional example 1

The 800-mm diameter, 100-kg flywheel of a stamping machine accelerates from 0 to its full operating speed of 30 rpm in 4 seconds.

Calculate:

1.1 The angular acceleration of the flywheel.

1.2 The average angular velocity during start-up.

1.3 The angular displacement during start-up.

1.4 The number of revolutions completed during start-up.

1.5 The linear acceleration of lever connected 35 cm from the centre.

1.6 The total linear displacement of the lever during start-up.

1.7 The torque required for start-up if $k = 0{,}71r$.

1.8 The work done during start-up.

1.9 The power requirement during start-up.

**Solution**

Given: $r = 0{,}4$ m, $m = 100$ kg, $n_i = 0$, $n_f = 30$, $t = 4$ s.

1.1 Calculate the angular acceleration of the flywheel.

$\omega_f = 2\pi n = 2\pi \times 30 = 188{,}5$ rad/s

$\alpha = \frac{188{,}5 - 0}{4} = 47{,}124 \text{ rad/s}^2$

1.2 Calculate the average angular velocity during start-up.

$\omega_{avg} = \frac{0 + 188{,}5}{2} = 94{,}25$ rad/s

1.3 Calculate the angular displacement during start-up.

$\theta = \omega_{avg} t = 94{,}25 \times 4 = 377$ rad

1.4 Calculate the number of revolutions completed during start-up.

$n = \frac{377}{2\pi} = 60$ revolutions

1.5 Calculate the linear acceleration of lever connected 35 centimetres from the centre.

$a = \alpha r = 47{,}124 \times 0{,}35 = 16{,}49 \text{ m/s}^2$

1.6 Calculate the total linear displacement of the lever during start-up.

$s = \theta r = 377 \times 0{,}35 = 131{,}95$ m

1.7 Calculate the torque required for start-up if $k = 0{,}71r$.

$\tau = mk^2\alpha = 100 \times (0{,}71 \times 0{,}4)^2 \times 47{,}124 = 380$ Nm

1.8 Calculate the work done during start-up.

$W = \tau\theta = 380 \times 377 = 143{,}3$ kJ

1.9 Calculate the power requirement during start-up.

$P = \frac{W}{t} = \frac{143{,}3}{4} = 35{,}8$ kW

**Additional example 2**

A flywheel with a mass of 320 kg and a radius of gyration of 26 cm accelerates from rest to 750 rpm in 10 seconds.

Calculate:

2.1 The average angular velocity.

2.2 The angular acceleration.

2.3 The torque required to accelerate the flywheel.

2.4 The work done.

2.5 The power required.

**Solution**

Given: $m$ = 320 kg, $k$ = 0,26 m, $n_i$ = 0, $n_f$ = 750 rpm, $t$ = 10 s.

2.1 Calculate the average angular velocity.

$$\omega_f = \frac{2\pi \times 750}{60} = 78{,}54 \text{ rad/s}$$

$$\omega_{avg} = \frac{78{,}54 + 0}{2} = 39{,}27 \text{ rad/s}$$

2.2 Calculate the angular acceleration.

$$\alpha = \frac{\Delta\omega}{t} = \frac{78{,}54 - 0}{10} = 7{,}854 \text{ rad/s}^2$$

2.3 Calculate the torque required to accelerate the flywheel.

$$\tau = mr^2\alpha = 320 \times 0{,}26^2 \times 7{,}854 = 169{,}9 \text{ Nm}$$

2.4 Calculate the work done.

$$\theta = \omega_{avg} t = 39{,}27 \times 10 = 392{,}7 \text{ rad}$$

$$W = \tau\theta = 169{,}9 \times 392{,}7 = 66\,719 \text{ j}$$

2.5 Calculate the power required.

$$P = \frac{W}{t} = \frac{66\,719}{10} = 6\,672 \text{ W}$$

MODULE

# 3 Dynamics

**By the end of this module, students should be able to:**

- state Newton's three Laws of Motion
- apply Newton's second law of motion
- sketch free body diagrams of vehicles travelling on horizontal or inclined planes while moving at constant speed, accelerating, or decelerating
- calculate the tractive effort and braking effort required for motion on horizontal and inclined planes with regard to practical problems, including gravitational force, rolling resistance force and inertia force
- calculate work done over a given distance or during a given time.
- calculate power at a given instant or at a given velocity
- define the concept of kinetic and potential energy
- calculate kinetic and potential energies and energy loss
- apply the conservation of energy equation for vehicles travelling on horizontal and inclined planes due to its own weight only (no tractive effort or braking effort applied)

In this module students will learn more about the three Laws of Motion and how to do different motion calculations. You will also get to know and use the principles of energy to help you calculate an object's motion.

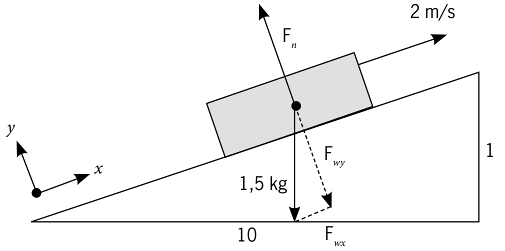
Since an object needs energy to move, they must also understand the different types of energy and how they affect the motion of an object.

**Important concepts and pre-knowledge in this module include:**

- Vector quantity concepts: including *x* and *y* quantities.
- Units of displacement, velocity, time and acceleration.
- Unit conversions.
- Trigonometric ratios: including sine, cosine and tangent.

## Exercise 3.1 SB page 87

1. Given: $\tau = 600$ Nm, $r = 0{,}3$ m, $f = -200 \times 0{,}9 = -180$ N, $m = 900$ kg
   1.1 Calculate the acceleration of the car.
   $F = \frac{\tau}{r} = \frac{600}{0{,}3} = 2\,000$ N
   $a = \frac{F}{m} = \frac{2\,000 - 180}{900} = 2{,}022$ m/s$^2$ (4)
   1.2 Calculate the final velocity of the car.
   $v = at = 2{,}022 \times 10$
   $= 20{,}22$ m/s (= 72,8 km/h) (2)
2. Given: $m = 1{,}5$ kg, $\theta = 1{:}10$, $v_i = 2$ m/s
   Step 1: Draw a vector diagram:

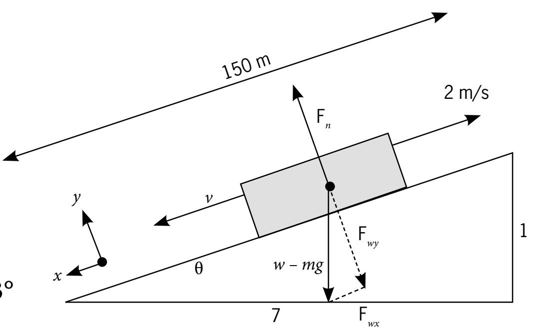



   Step 2: Calculate the distance that the car will move up the incline before coming to rest.
   $\theta = \tan^{-1}\frac{1}{10} = 5{,}71$
   $F_{wx} = mg\sin\theta$
   $= 1{,}5 \times 9{,}8 \times \sin 5{,}71 = 1{,}46$ N
   $a_x = \frac{F_{wx}}{m} = \frac{-1{,}46}{1{,}5} = -0{,}975$ m/s$^2$
   $v_f^2 = v_i^2 + 2as$
   $\therefore s = \frac{v_f^2 - v_i^2}{2a} = \frac{0 - 4}{2 \times -0{,}975} = 2{,}051$ m (9)

**Total: 15 marks**

## Exercise 3.2 SB page 95

1. Draw a general vector diagram:
   Given: $s = 150$ m, $\theta = 1{:}7$,
   $v_i = 12$ km/h, $m$ (boxcar A) = 65,
   $f_A = 30$, $m$ (boxcar B) = 55,
   $f_B = 20$




   1.1 The acceleration of each car.
   $\theta = \tan^{-1}\left(\frac{1}{7}\right) = 8{,}13°$
   $v_i = 12 \times \frac{1\,000}{3\,600} = 3{,}33$ m/s
   $\Sigma F_{Ax} = F_{wx} - f_x = ma$
   $\therefore m_A g\sin\theta - f_x = 65 \times 9{,}8 \times \sin 8{,}13° - 30 = 60{,}1$ N
   $a_A = \frac{F_{Ax}}{m_A} = \frac{60{,}085}{65} = 0{,}924$ m/s$^2$

$$\Sigma F_{wBx} = m_B g\sin\theta - f_B = 55 \times 9{,}8 \times \sin 8{,}13° - 20 = 56{,}2 \text{ N}$$

$$a_B = \frac{F_{Bx}}{m_B} = \frac{56{,}226}{55} = 1{,}022 \text{ m/s}^2 \quad (9)$$

1.2 The top speed of both cars.

$$v_f^2 = v_i^2 + 2ad$$

$$v_{fA}^2 = 3{,}33^2 + 2 \times 0{,}924 \times 150 = 288{,}43$$

$$\therefore v_{fA} = 17{,}0 \text{ m/s } (= 61 \text{ km/h})$$

$$v_{fB}^2 = 3{,}33^2 + 2 \times 1{,}022 \times 150 = 317{,}8$$

$$\therefore v_{fB} = 17{,}8 \text{ m/s } (= 64 \text{ km/h}) \quad (5)$$

1.3 The time it takes to complete the distance for each car.

$$v_f = v_i + at$$

$$t_A = \frac{v_{fA} - v_i}{a_A} = \frac{17{,}0 - 3{,}33}{0{,}924} = 14{,}77 \text{ s}$$

$$t_B = \frac{17{,}8 - 3{,}33}{1{,}022} = 14{,}18 \text{ s} \quad (3)$$

1.4 The losses of both cars.

$$h = 150 \times \sin 8{,}13° = 21{,}2 \text{ m}$$

$$\% \text{ losses}_A = \frac{\Sigma E_{losses}}{\Sigma E_{in}} \times 100\,\% = \frac{E_{losses} \times 100}{E_{pi} + E_{ki}}\%$$

$$= \frac{f_A d \times 100}{mgh + \frac{1}{2} m v_i^2} = \frac{30 \times 150 \times 100}{65 \times 9{,}8 \times 21{,}2 + \frac{1}{2} \times 65 \times 3{,}33^2} = 32{,}4\%$$

$$\% \text{ losses}_B = \frac{f_B d \times 100}{mgh + \frac{1}{2} m v_i^2} = \frac{20 \times 150 \times 100}{55 \times 9{,}8 \times 21{,}2 + \frac{1}{2} \times 55 \times 3{,}33^2} = 25{,}6\% \quad (4)$$

1.5 Calculate the efficiency:

$$\eta = \frac{E_{kf}}{E_{pi} + E_{ki}} \times 100\%$$

$$\eta_A = \frac{\frac{1}{2} \times 65 \times 17{,}0^2 \times 100}{65 \times 9{,}8 \times 21{,}2 + \frac{1}{2} \times 65 \times 3{,}33^2} = 67{,}6\%$$

$$\eta_B = \frac{\frac{1}{2} \times 55 \times 17{,}8^2 \times 100}{55 \times 9{,}8 \times 21{,}2 + \frac{1}{2} \times 55 \times 3{,}33^2} = 74{,}4\% \quad (4)$$

2. Given: $m = 900$ kg, $\theta = 1{:}30$, $v_i = 0$ km/h, $v_f = 65$ km/h, $t = 3$ min.

2.1 The acceleration of the car.

$$v_f = 65 \times \frac{1\,000}{3\,600} = 18{,}06 \text{ m/s}$$

$$a = \frac{v_f - v_i}{t} = \frac{18{,}06 - 0}{3 \times 60} = 0{,}10 \text{ m/s}^2 \quad (3)$$

2.2 The distance travelled by the car up the slope.

$$\therefore s = \frac{v_f^2 - v_i^2}{2a} = \frac{18{,}06^2 - 0}{2 \times 0{,}10} = 1\,625 \text{ m}$$

Or, from the following equation:

$s = v_{avg}t = \frac{18,06 + 0}{2} \times 180 = 1\,625$ m (2)

2.3 The potential energy that the car gains after 3 minutes.

$\theta = \tan^{-1}\left(\frac{1}{30}\right) = 1,91°$

$h = 1\,625 \sin 1,91° = 54,1$ m

$E_p = mgh = 900 \times 9,8 \times 54,1 = 477\,485$ J (4)

3. Given: $m = 3\,000$ kg, $\theta = 5\%$, $v_i = 0$ km/h, $s = 200$ m, $t = 3$ min.

3.1 The potential energy lost by the truck in the 3 minutes

$\theta = \tan^{-1}\left(\frac{5}{100}\right) = 2,86°$

$h = 200 \sin 2,86° = 10$ m

$\therefore E_{pi} - E_{pf} = mgh - 0 = 3\,000 \times 9,8 \times 10 = 293\,633$ J (4)

3.2 The final velocity of the truck.

$E_{pi} + E_{ki} = E_{pf} + E_{kf}$

$E_{pf} = 0$

$E_{ki} = \frac{1}{2}mv_i^2 = \frac{1}{2} \times m \times 0^2 = 0$

$\therefore E_{kf} = E_{pi}$

$\therefore \frac{1}{2}mv_f^2 = 293\,633$

$\frac{1}{2} \times 3\,000 \times v_f^2 = 293\,633$

$v_f^2 = 195,8$

$\therefore v_f = 14,0$ m/s (4)

4. Given: $m = 150$ kg, $v_i = 2$ m/s, $\mu = 0,4$

$f = \mu F_n = \mu mg = 0,4 \times 150 \times 9,8 = 588$ N

$P = Fv = 588 \times 2 = 1\,176$ W (3)

**Total: 45 marks**

**Summative assessment** **SB page 98**

1. 1.1 Newton's First Law – an object will indefinitely continue in its state of motion unless an unbalanced force acts on it.
   1.2 Newton's Second Law – the rate at which the momentum of a body changes, is directly proportional to the force applied and this change is in the direction of the applied force.
   OR when an external force is applied on an object in its state of movement, the force will cause the object to change its movement and accelerate in the direction of the resultant.
   1.3 Newton's Third Law – when one body exerts a force on a second body, the second body simultaneously exerts the same amount of force on the first body but in the opposite direction.
   1.4 Law of conservation of energy – the total energy of an object during its movement in an isolated system remains constant.
   1.5 Coefficient of friction – is a scalar value that describes the ratio of the frictional force between two bodies and the force pressing them together.
   1.6 Normal force – is a contact force exerted by a surface on an object to prevent it from falling.

(6 × 2)

2. Given information:

   Mass = 3 tonnes × 1 000 = 3 000 kg

   $v = 20$ m/s

   $f = 300$ N

   $f_k = \mu.FN$

   $f_k = \mu(mg)$

   $300 = \mu(3\,000 \times 9{,}8)$

   $\mu = 0{,}01$ (2)

3. 3.1 Given information:

   Mass = 90 kg

   $\theta = 5°$

   $t = 10$ s

   $v_i = 0$ m/s

   $v_f = 1{,}5$ m/s

   First find the acceleration:

   $$a = \frac{v_f - v_i}{t}$$

$= \frac{1,5 - 0}{10}$

$= 0,15 \text{ m/s}^2$

Then:

$v_f^2 = v_i^2 + 2ax$

$(1,5)^2 = (0)^2 + 2(0,15) \times x$

$x = 7,5 \text{ m}$ (3)

3.2 We use the sum of all forces acting on the box along the surface as the $x$-axis:

$$\Sigma F_x = ma$$

$$F - w_x - f = ma$$

Note that:

$$w_x = w \times \sin 5 = (90)(9,8) \sin 5$$

$$F - (90)(9,8) \sin 5 - 50 = (90)(0,15)$$

$$= 140,4 \text{ N}$$ (5)

4. Given information:

Mass = 150 kg

$\theta = 12°$

$v = 3$ m/s

$f_k = 220$ N

Get the force required:

$\Sigma F = ma$

$F - w_x - f_k = ma$

$F - mg \sin 12 - f_k = ma$

$F - (150)(9,8) \sin 12 - 220 = (150)(0)$

$F = 525,63$ N

Then, $P = F.v$

$= (525,63)(3) = 1\ 576,9$ W (4)

5. Given information: $m = 22$ tonnes = 22 000 kg

$F = 60$ kN = 60 000 N

$\theta = a\tan\frac{1}{30} = 1,91°$

$F_f = 70 \times 22 = 1\ 540$ N

5.1 Use the sum of forces acting along the identified $x$-axis.

$$ma = \Sigma F = F_e - F_f - mg\sin 1{,}91$$
$$= 60\,000 - 1\,540 - 22\,000 \times 9{,}81 \times \sin 1{,}91$$
$$= 51\,270$$
$$\therefore a = \frac{51\,270}{22\,000} = 2{,}33 \text{ m/s}^2$$ (4)

5.2 Gain in potential energy

$$E_P = mgh$$
$$= 22\,000 \times 9{,}8 \times 300$$
$$= 64{,}68 \times 10^6 \text{ J } (= 64{,}68 \text{ MJ})$$ (4)

6. Given information:

$v = 2$ m/s

Mass = 65 kg

$$E_k = \frac{1}{2} \text{m} \times v^2$$
$$\frac{1}{2}(65)\,(2)^2$$
$$= 130 \text{ J}$$ (3)

**Total: 37 marks**

## Additional examples with solutions for lecturers and tutors:

Give your learners these problems for extra practice and work through the solutions with them in class.

### Additional example 1

1. A student is driving home after a sporting event when the car's engine suddenly stops running. At that moment the car is doing 108 km/h and he knows it is exactly 2,75 km from home over a stretch of road that is slightly downhill with a gradient of 3,5%. He guesses the car weighs about 900 kg plus his own weight of 75 kg. He wants to calculate whether he will be able to free the car all the way home since there are no stops in between. The rolling resistance of the car is 300 N and the average air drag is 200 N.

   Calculate the following:

   1.1 The deceleration of the car.

   1.2 The distance the car will travel before it stops.

1.3 The time it will take to come to a standstill.

1.4 The initial potential energy.

1.5 The initial kinetic energy.

1.6 The distance to walk home.

**Solution**

Given: $v_i = 108$ km/h, $m = 975$ kg, $\theta = 1{:}30$, $f = 500$ N.

1. 1.1 Calculate the deceleration of the car.

$\theta = \tan^{-1}\left(\frac{3{,}5}{100}\right) = 2°$

$v_i = 108 \times \frac{1\,000}{3\,600} = 30$ m/s

$\Sigma F_x = F_{wx} - f_x = ma$

$\therefore ma = mg\sin\theta - f_x = 975 \times 9{,}8 \times \sin^2 - 500 = -166{,}544$ N

$\therefore a = \frac{-166{,}54}{975} = -0{,}171$ m/s$^2$

1.2 Calculate the distance the car will travel before it stops.

$v_f^2 = v_i^2 + 2ad$

$\therefore d = \frac{v_f^2 - v_i^2}{2a} = \frac{0^2 - 30^2}{2 \times (-0{,}171)} = 2\,635$ m

1.3 Calculate the time it will take to come to a standstill.

$v_f = v_i + at$

$t_A = \frac{v_f - v_i}{a} = \frac{0 - 30}{-0{,}17} = 176$ s $\left( = 2 \text{ minutes } 56 \text{ s}\right)$

1.4 Calculate the initial potential energy.

$h = 2\,635 \times \sin^2 = 91{,}96$ m

$E_{pi} = mgh = 975 \times 9{,}8 \times 91{,}96 = 878\,680$ J

1.5 Calculate the initial kinetic energy.

$E_{ki} = \frac{1}{2}mv^2 = \frac{1}{2} \times 975 \times 30^2 = 438\,750$ J

1.6 Calculate the distance to walk home.

$d_{walk} = 2\,750 - 2\,635 = 115$ m

## Additional example 2

2. A girl with a mass of 50 kg slides down a super tube water slide with a 1:10 angle and a 0,10 kinetic friction factor. Assume the drop is 12 m over its full distance of 80 m.

Determine:

2.1 What is her velocity at the bottom?

2.2 What is her kinetic energy at the bottom?

2.3 How long does it take her to get to the bottom?

2.4 What energy does she need to exert in the afternoon to get back home?

2.5 What energy losses did she incur?

2.6 What percentage losses did she incur?

2.7 What was her efficiency?

**Solution**

Given information: $m = 50$ kg, $d = 80$ m, $h = 12$ m, μ = 0,10.

2\. 2.1 $E_p = E_k + f_k d$

but $f_k = \mu_k F_n$

and $F_n = mg\cos\theta$

$$\theta = \tan^{-1}\left(\frac{12}{80}\right) = 8{,}53°$$

$$\therefore F_n = 50 \times 9{,}8\cos 8{,}53° = 484{,}6 \text{ N}$$

$$\therefore mgh = \frac{1}{2}mv^2 + \mu_k d F_n$$

$$50 \times 9{,}8 \times 12 = \frac{1}{2} \times 50 \times v_f^2 + 0{,}1 \times 80 \times 484{,}6$$

$$\therefore v = 8{,}95 \text{ m/s } (= 32 \text{ km/h})$$

2.2 $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 50 \times 8{,}95^2 = 2\ 003$ J

2.3 $v_{avg} = \frac{v_f + v_i}{2} = \frac{8{,}95 + 0}{2} = 4{,}475$ m/s

$t = \frac{d}{v} = \frac{80}{4{,}475} = 17{,}9$ s

2.4 $E_p = mgh = 50 \times 9{,}8 \times 12 = 5\ 880$ J

2.5 $E_{losses} = fd = \mu_k F_n d = \mu_k d \times mg\cos\theta$

$= 0{,}1 \times 80 \times 50 \times 9{,}8 \times \cos 8{,}53° = 3\ 877$ J

2.6 % losses $= \frac{3\ 877}{5\ 880} \times 100 = 65{,}9\%$

2.7 $\eta = \frac{\Sigma E_f}{\Sigma E_i} \times 100\%$

$$= \frac{\frac{1}{2} \times 50 \times 8{,}95^2}{5\ 880} \times 100\,\% = 34{,}1\%$$

## Additional example 3

3. A motocross bike develops 30 Nm torque at 35 km/h as it approaches a 10-m long 30° incline. It weighs 106 kg and the rider weighs another 60 kg, it has a 480-mm back tyre and loses 10 N per 100 kg of weight due to friction.
   3.1 Calculate the deceleration of the bike.
   3.2 Calculate the velocity of the bike at the top of the incline.
   3.3 Calculate the time it will take to reach the top

**Solution**

Given: $\tau = 30$ Nm, $r = 0{,}24$ m, $v_i = 35$ km/h, $d = 10$ m, $\theta = 30°$,
friction loss = –10 N per 100 kg, $m_B = 106$ kg, $m_R = 60$ kg

3. 3.1 Calculate the deceleration of the bike.

$$v_i = \frac{35 \times 1\,000}{3\,600} = 9{,}72 \text{ m/s}$$

$$m = 106 + 60 = 166 \text{ kg}$$

$$f = \frac{-10}{100} \times 166 = -16{,}6 \text{ N}$$

$$F = \frac{\tau}{r} = \frac{30}{0.24} = 125 \text{ N}$$

$$F_w = mg\sin\theta = -166 \times 9{,}8 \times \sin 30 = -813{,}4 \text{ N}$$

$$a = \frac{\Sigma F}{m} = \frac{125 - 16{,}6 - 813{,}4}{166} = -4{,}247 \text{ m/s}^2$$

3.2 Calculate the velocity of the bike at the top of the incline.

$$v_f^2 = v_i^2 + 2ad = 9{,}72^2 - 2 \times 4{,}247 \times 10$$

$$\therefore v_f = 3{,}10 \text{ m/s } (= 11{,}1 \text{ km/h})$$

3.3 Calculate the time it will take to reach the top

$$v_{avg} = \frac{3{,}10 + 9{,}72}{2} = 6{,}41 \text{ m/s}$$

$$t = \frac{s}{v_{avg}} = \frac{10}{6{,}41} = 1{,}56 \text{ s}$$

# 4 Statics

**By the end of this module, students should be able to:**

- calculate reactions at each support;
- draw a shear force diagram;
- calculate and draw a bending moment diagram;
- calculate the point of inflection;
- draw a loaded beam from given shear-force diagram; and
- calculate the centre of gravity of different shapes like cylinders, cones, spheres and semi-spheres, including compound shapes made up from such shapes.

Statics is the engineering field that focuses on analysing the loads acting on physical shapes. For this course, these loads and shapes are at rest and do not accelerate or move. Therefore, statics has to do with the effect of forces on a stationary object where no motion is involved. In this module students will focus on forces exerted on beams, which in real life, may be part of a structure such as a bridge or a roof.

Apart from the effect of forces working directly to try and displace an object, the effect of forces working at a distance on an object is to try and rotate it or bend it. The effect of such forces is known as the moment of force or the bending moment. This bending moment is equal to the product of the force and the perpendicular distance of force to the pivot point.

## Exercise 4.1 SB page 121

1. 1.1

$$\Sigma M_A = 0$$
$$-(30)(4)(2) - (20)(5) - (40)(7) + R_E(8) = 0$$
$$-240 - 100 - 280 + 8R_E = 0$$
$$R_E = 77{,}5 \text{ kN}$$
$$\Sigma Fy = 0$$
$$R_A - (30)(4) - 20 - 40 + 77{,}5 = 0$$
$$R_A = 102{,}5 \text{ kN}$$ (3)

1.2 $BM_A = 0$ kNm

$BM_B = (102{,}5)(4) - (30)(4)(2)$

$= 170$ kNm

$BM_C = (102{,}5)(5) - (30)(4)(3)$
$= 152{,}5$ kNm
$BM_D = (102{,}5)(7) - (30)(4)(5) - (20)(2)$
$= 77{,}5$ kNm
$BM_E = (102{,}5)(8) - (30)(4)(6) - (20)(3) - (40)(1)$
$= 0$ kNm (3)

1.3 Before drawing the diagrams, we first calculate the shear forces at each point.

$F_A = 102{,}5$ kN
$F_B = 102{,}5 - (30)(4)$
$= -17{,}5$ kN
$F_C = 102{,}5 - (30)(4) - 20$
$= -37{,}5$ kN
$F_D = 102{,}5 - (30)(4)$
$= -77{,}5$ kN
$F_E = -77{,}5 + 77{,}5$
$= 0$ kN

Then:

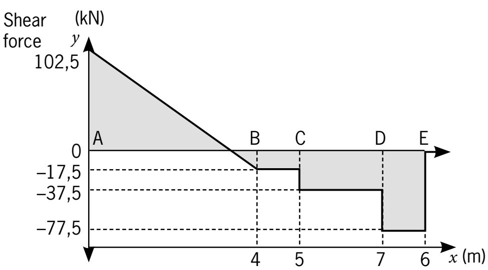



*Figure 1.3a*

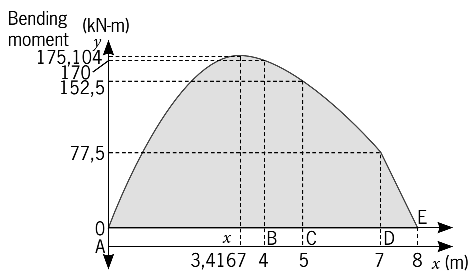



*Figure 1.3b*

(5)

1.4 $\Sigma Mx = 0$

$R_A(4 - x) - 17{,}5 \times x = 0$
$102{,}5(4 - x) - 17{,}5 \times x = 0$
$x = 3{,}42$ m
$M_x = (102{,}5)(3{,}42) - (30)(3{,}42)\left(\frac{3{,}42}{2}\right)$
$= 175{,}1$ kNm (3)

2. 2.1 $\Sigma M_A = 0$

$-(60)(3)(1{,}5) - (20)(3)(4{,}5) + RC(6) - (100)(7{,}5) = 0$
$R_C = 215$ kN
$\Sigma Fy = 0$

$$-(60)(3) - (20)(3) + 215 + R_A - 100 = 0$$

$$R_A = 125 \text{ kN} \qquad (3)$$

2.2 $BM_B = (125)(3) - (60)(3)(1{,}5)$

$= 105 \text{ kNm}$

$BM_C = (125)(6) - (60)(3)(4{,}5) - (20)(3)(1{,}5)$

$= -150 \text{ kNm}$

$BM_D = (125)(7{,}5) - (60)(3)(6) - (20)(3)(3) + (215)(1{,}5)$

$= 0 \text{ kNm}$ (3)

2.3 $F_B = 125 - (60)(3)$

$= -55 \text{ kN}$

$F_C = 125 - (60)(3) - (20)(3)$

$= -115 \text{ kN}$

$F_D = -115 + 215 - 100$

$= 0 \text{ kN}$ (3)

2.4

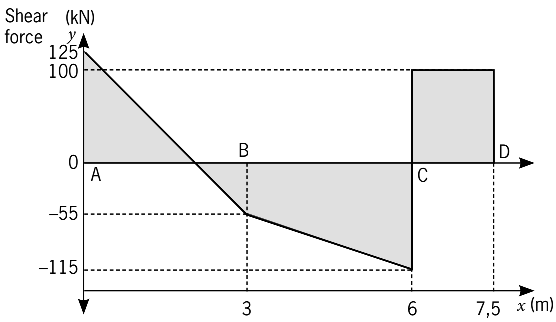



*Figure 2.4a*

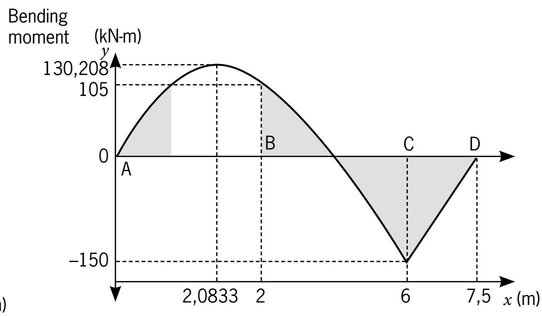



*Figure 2.4b*

(5)

**Total: 28 marks**

## Exercise 4.2 SB page 127

1. $V_1 = \pi r^2 h$

$= \pi(25)^2(80)$

$= 157\ 079{,}63 \text{ m}^3$

$x_1 = \frac{d}{2}$

$= \frac{80}{2}$

$s = 40 \text{ m}$

$V_2 = \frac{4}{6}\pi r^3$

$= \frac{4}{6}\pi(25)^3$

$= 32\ 724{,}92\ \text{m}^3$

$x_2 = 80 + \frac{3}{8}r$

$= 80 + \frac{3}{8}(25)$

$= 89{,}375\ \text{m}$

$V_T = 157\ 079{,}63 + 32\ 724{,}92$

$= 189\ 804{,}55\ \text{m}^3$

So,

$\overline{x} = \frac{(v_1 \cdot x_1 + v_2 \cdot x_2)}{v_{\text{total}}}$

$= \frac{(157\ 079{,}63)(40) + (32\ 724{,}92)(89{,}375)}{189\ 804{,}55}$

$= 48{,}51\ \text{m}$ (5)

2. $A_1 = l \times b$

$= 90 \times 45$

$= 4\ 050\ \text{cm}^2$

$= 45\ \text{cm}$

$x_1 = \frac{b}{2} = \frac{90}{2}$

$A_2 = \pi r^2 = \pi(20)^2$

$= 1\ 256{,}63\ \text{cm}^2$

$x_2 = 20\ \text{cm}$

$A_T = 4\ 050 + 1\ 256{,}63$

$\overline{x} = \frac{(A_1 \cdot x_1 + A_2 \cdot x_2)}{A_{\text{total}}}$

$= \frac{(4\ 0505)(45) + (1\ 256{,}63)(20)}{5\ 306{,}637}$

$= 5\ 306{,}63\ \text{cm}^2$

$= 29{,}61\ \text{cm}$ (5)

**Total: 10 marks**

## Exercise 4.3 SB page 130

1. Assume the density of all the objects are 1 kg/m³.

For the bottom cone:

$m_1 = \rho V_1 = \rho \times \frac{1}{3}\pi r^2 h_1$

$= 1 \times \frac{\pi}{3} \times 1{,}5^2 \times 2 = 4{,}712\ \text{kg}$

Centre of gravity of a cone $= \frac{1}{4}h$

$= \frac{1}{4} \times 2 = 0{,}5\ \text{m}$

Since this is from the base of a cone:

$y_1 = 2 - 0{,}5 = 1{,}5$ m from A

For the cylinder:

$m_2 = \rho V_2 = \rho \times \pi r^2 h_2$

$= 1 \times \pi \times 1{,}5^2 \times 1 = 7{,}069$ kg

$y_2 = 2 + \frac{1}{2} \times 1 = 2{,}5$ m from A

For the top cone:

$m_3 = \rho V_3 = \rho \times \frac{1}{3}\pi r^2 h_3$

$= 1 \times \frac{\pi}{3} \times 1{,}5^2 \times 3 = 7{,}069$ kg

$y_3 = 3 + \frac{1}{4} \times 3 = 3{,}75$ m from A

The centre of gravity is:

$$y_T = \frac{\Sigma my}{\Sigma m} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3}$$

$$= \frac{1{,}5 \times 4{,}712 + 2{,}5 \times 7{,}069 + 3{,}75 \times 7{,}069}{4{,}712 + 7{,}069 + 7{,}069} = 2{,}719 \text{ m from A} \quad (10)$$

2. Assume the density of all the parts are 1 g/cm$^3$.

For the left half sphere:

$m_1 = \rho V_1 = \rho \times \frac{2}{3}\pi r_1^3$

$= 1 \times \frac{2}{3}\pi \times 2^3 = 16{,}755$ g

For the centre of gravity:

$x_1 = 2 - \frac{3}{8}r = 2 - \frac{3}{8} \times 2 = 1{,}25$ cm

For the first cylinder:

$m_2 = \rho V_2 = \rho \times \pi r^2 l$

$= 1 \times \pi \times 1^2 \times 4 = 12{,}57$ g

For the centre of gravity:

$x_2 = 2 + \frac{4}{2} = 4$ cm

For the second cylinder:

$m_3 = \rho V_3 = \rho \times \pi r^2 l$

$= 1 \times \pi \times 2^2 \times 2 = 25{,}13$ g

For the centre of gravity:

$x_3 = 6 + \frac{2}{2} = 7$ cm

For the right half sphere:

$m_4 = \rho V_4 = \rho \times \frac{2}{3}\pi r_4^3$

$= 1 \times \frac{2}{3}\pi \times 1{,}5^3 = 7{,}069$ g

For the centre of gravity:

$x_4 = 8 + \frac{3}{8}r = 8 + \frac{3}{8} \times 1{,}5 = 8{,}563$ cm

The centre of gravity is:

$$x_T = \frac{\Sigma mx}{\Sigma m} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + x_4 m_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{1{,}25 \times 16{,}755 + 4 \times 12{,}57 + 7 \times 25{,}13 + 8{,}563 \times 7{,}069}{16{,}755 + 12{,}57 + 25{,}13 + 7{,}069}$$

= 5 cm from the left (15)

**Total: 25 marks**

## Summative assessment SB page 134

1. 1.1 Centre of gravity is the point where the weights of all the particles forming the object are centred. (2)

   1.2 Centre of mass is a point where the resultant mass of an object or system of particles is located. (2)

   1.3 Centroid is a point where the geometric centre of an object lies. (2)

   1.4 Shear force is a type of force that pushes one part of a body to one direction and the other part to the opposite direction. (2)

   1.5 Bending moment is a bending reaction that is caused by an applied force. (2)

2. 2.1

$$\Sigma M_A = 0$$
$$-(10)(5)(2{,}5) - (5)(8) + R_D(12) - (5)(14) = 0$$
$$R_D = 19{,}58 \text{ kN}$$
$$\Sigma Fy = 0$$
$$R_A - (10)(5) - 5 + 19{,}58 - 5 = 0$$
$$R_A = 40{,}42 \text{ kN}$$

(3)

   2.2 $M_B = (40{,}42)(5) - (10)(5)(2{,}5)$

$= 77{,}1$ kNm

$M_C = (40{,}42)(8) - (10)(5)(5{,}5)$

$= 48{,}36$ kNm

$M_D = (40{,}42)(12) - (10)(5)(9{,}5) - (5)(4)$

$= -9{,}96$ kNm (3)

2.3 $F_B = 40{,}42 - (10)(5)$

$= -9{,}58$ kN

$F_C = 40{,}42 - (10)(5) - 5$

$= -14{,}58$ kN

$F_D = 40{,}42 - (10)(5) - 5 + 19{,}58$

$= 5$ kN (3)

2.4

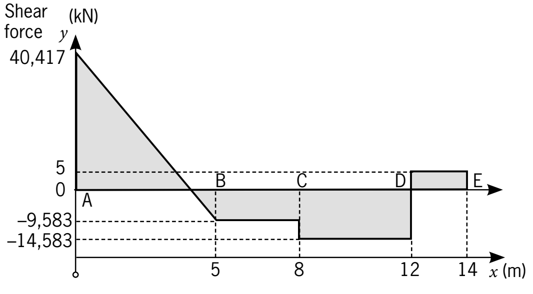



*Figure 2a*

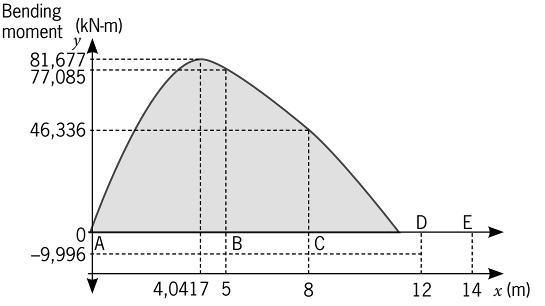



*Figure 2b*

(6)

2.5 $\Sigma M x = 0 \; R_A(5 - x) - F_B \times x = 0$

$40{,}42(5 - x) - 9{,}58 \times x = 0$

$x = 4{,}04$ m

$M_x = (40{,}42)(4{,}04) - (10)(4{,}04)\left(\frac{4{,}04}{2}\right)$

$= 81{,}69$ kNm (3)

3\. 3.1 $\Sigma M_A = 0$

$-(15)(2) - (15)(5) - (12)(3)(6{,}5) + R_D(8) = 0$

$R_D = 42{,}38$ kN

$\Sigma F y = 0$

$R_A - 15 - 15 - (12)(3) + 42{,}38 = 0$

$R_A = 23{,}62$ kN (3)

3.2 $M_B = (23{,}62)(2)$

$= 47{,}24$ kNm

$M_C = (23{,}62)(5) - (15)(3)$

$= 73{,}1$ kNm (3)

3.3 $F_B = 23{,}62 - 15$

$= 8{,}62$ kN

$F_C = 8{,}62 - 15 = -6{,}38$ kN

$F_D = -6{,}38 - (12)(3)$

$= -42{,}38$ kN (3)

3.4

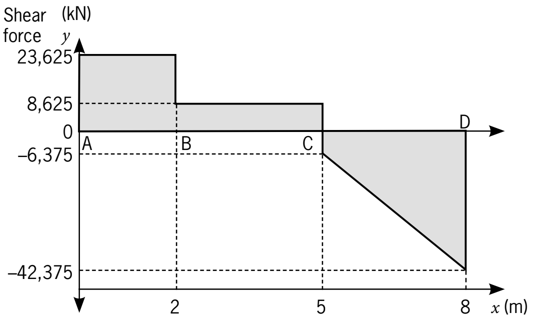



*Figure 3a*

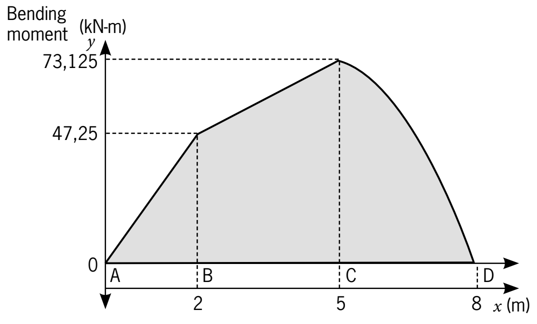



*Figure 3b*

(6)

4. $A_1 = 50 \times 10$ $\quad x_1 = \frac{10}{2}$

$= 500\ m^2$ $\quad = 5$ m

$A_2 = 20 \times 30$ $\quad x_2 = \frac{10 + 30}{2}$

$= 600\ m^2$ $\quad = 25$ m

$A_3 = 35 \times 25$ $\quad x_3 = \frac{10 + 30 + 25}{2}$

$= 875\ m^2$ $\quad = 32{,}5$ m

So:

$$\bar{x} = \frac{A_1 \times x_1 \times A_2 \times x_2 \times A_3 \times x_3}{A_{total}}$$

$$= \frac{((500)(5) + (600)(25) + (875)(32{,}5))}{(500 + 500 + 875)}$$

$= 24{,}5$ m (6)

5. Assume the density of all the parts are 1 g/cm³ and use the left end as reference.

For the half sphere:

$$m_1 = \rho V_1 = \rho \times \frac{2}{3}\pi r_1^3$$

$$= 1 \times \frac{2}{3}\pi \times 3{,}5^3 = 89{,}80\ g$$

For the centre of gravity:

$$x_1 = 3{,}5 - \frac{3}{8}r = 3{,}5 - \frac{3}{8} \times 3{,}5 = 2{,}19\ cm$$

For the cylinder:

$m_2 = \rho V_2 = \rho \times \pi r^2 l$

$= 1 \times \pi \times 3{,}5^2 \times 10 = 384{,}9$ g

For the centre of gravity:

$x_2 = 3{,}5 + \frac{10}{2} = 8{,}5$ cm

For the cone:

$m_3 = \rho V_3 = \rho \times \frac{1}{3}\pi r^2 h_3$

$= 1 \times \frac{\pi}{3} \times 3{,}5^2 \times 12 = 153{,}9$ g

$y_3 = 13{,}5 + \frac{1}{4} \times 12 = 16{,}5$ cm

The centre of gravity is:

$$x_T = \frac{\Sigma mx}{\Sigma m} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3}$$

$$= \frac{2{,}19 \times 89{,}80 + 8{,}5 \times 384{,}9 + 16{,}5 \times 153{,}9}{89{,}80 + 384{,}9 + 153{,}9}$$

= 9,557 cm from the left (11)

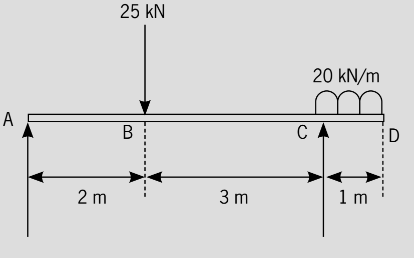
**Total: 60 marks**

## Additional examples with solutions for lecturers and tutors:

Give your learners these problems for extra practice and work through the solutions with them in class.

### Additional example 1

A 6-m beam is supported at point A on it left end and point C 1 m from the right as shown in the figure below. It has a uniformly distributed load of 20 kNm for 1 m at the right between points C and D, and a point load of 25 kN at point B, 2 m from the left.




Using this information:

- Calculate the reactions at A and C;
- Calculate the bending moments at each point from A to D;

- Draw a shear force diagram and indicate all the major point values; and
- Draw a bending moment diagram and indicate all the major point values;
- Determine the position and magnitude of the maximum bending moment.

**Solution**

**Step 1:** Find the reactions by using the bending moment (around A) and sum of forces equations:

$$\Sigma M_{CW} = \Sigma M_{ACW}$$

$$F_{CD}\frac{d_C + d_D}{2} + F_B d_B = F_C d_C$$

$$\therefore 20 \times 1 \times \frac{5+6}{2} + 25 \times 2 = F_C \times 5$$

$$\therefore F_C = 32 \text{ kN}$$

$$\uparrow\Sigma F = \downarrow\Sigma F$$

$$F_A + F_C = F_B + F_{CD}$$

$$\therefore F_A = F_B + F_{CD} - F_C$$

$$= 25 + 20 - 32 = 13 \text{ kN}$$

**Step 2:** Take each segment from A to D and calculate the bending moments for each:

$$M_A = 0$$

$$M_B = F_A d_{AB} = 13 \times 2 = 26 \text{ kNm}$$

$$M_C = F_A d_{AC} - F_B d_{BC} = 13 \times 5 - 25 \times 3 = -10 \text{ kNm}$$

$$M_D = 0$$

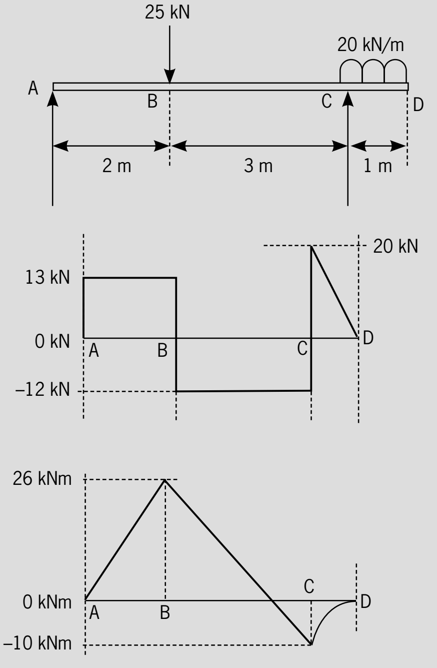
**Step 3:** List all the vertical forces to help draw the force diagram.

$F_A = 13$ kN, $F_B = -25$ kN, $F_C = 32$ kN, $F_D = 0$ kN

**Step 4:** List all the bending moments to help draw the bending moment diagram.

$M_A = 0$ kNm, $M_B = 26$ kNm, $M_C = -10$ kNm, $M_D = 0$ kNm

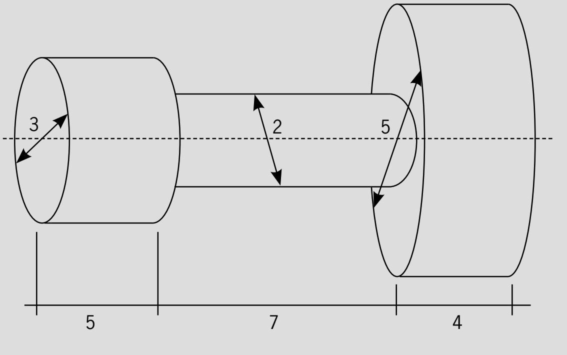
**Step 5:** Draw the two diagrams below each other with the shear diagram first and then the bending moment diagram.




On the diagram it can be seen that the maximum bending moment is at point B. The bending moment at B is 26 kNm.

## Additional example 2

The following metal article is a combination of three cylinders, with the measurements in cm. Determine the centre of gravity of the object.



**Solution**

Since you do not know the density of the material, you must work with the volumes rather than the weights in the equation. Also, every weight term includes other constants which will divide out above and below the line. Therefore rewrite the equation in a suitable form. Start on the left-hand side for $x = 0$.

$$x = \frac{x_1 w_1 + x_2 w_2 + x_3 w_3}{w_1 + w_2 + w_3}$$

$$= \frac{\frac{\pi\rho g}{4}\left(x_1 l_1 d_1^2 + x_2 l_2 d_2^2 + x_3 l_3 d_3^2\right)}{\frac{\pi\rho g}{4}\left(l_1 d_1^2 + l_2 d_2^2 + l_3 d_3^2\right)}$$

$$= \frac{x_1 l_1 d_1^2 + x_2 l_2 d_2^2 + x_3 l_3 d_3^2}{l_1 d_1^2 + l_2 d_2^2 + l_3 d_3^2}$$

$$= \frac{2{,}5 \times 5 \times 3^2 + 8{,}5 \times 7 \times 2^2 + 14 \times 4 \times 5^2}{5 \times 3^2 + 7 \times 2^2 + 4 \times 5^2}$$

$$= 10{,}12 \text{ cm}$$

MODULE

# 5 *Hydraulics*

**By the end of this module, students should be able to:**

- calculate the volume of liquid required per stroke by the press (slip to be taken into account);
- calculate the diameter of the ram of the process or force exerted by the ram;
- calculate pressure in the liquid to overcome a given load;
- calculate the work done by the press (efficiency to be taken into account);
- calculate the volume of liquid delivered per stroke for a reciprocating pump (slip to be taken into account;
- calculate the theoretical and actual flow rate of a reciprocating pump;
- calculate the pressure in the liquid and force on the plunger;
- calculate rotational frequency of a pump to deliver a given flow rate of water;
- calculate the time taken to fill or empty a reservoir for a given flow rate;
- calculate power required and efficiency with input power given in the case of single, double and three-cylinder single acting pumps;
- name the types of accumulators found in industry;
- state the functions of an accumulator in a hydraulic system;
- determine the volume of liquid delivered by the accumulator per working stroke of the machine it serves or the volume delivered in a given time;
- calculate the diameter of the ram and the load required on the accumulator to keep the pressure required constant during the working stroke;
- calculate the distance travelled by the accumulator ram during the working stroke;
- calculate the transfer of pressure in the liquid between the accumulator and the machine;
- calculate the work done and power with efficiency (slip to be taken into account); and
- draw sketches, which illustrate the working of an accumulator.

With your understanding of mechanics, you can now learn about hydraulics**,** which will require some of the skills you have gained so far. This module will give you an understanding of the science behind pumps, pressure and fluid behaviour. It will also provide you with the knowledge to work out the force needed in a hydraulic system so that you can select suitable pumps, presses and accumulators in the workplace.

**Important concepts and pre-knowledge in the module:**

- Hydraulic presses and hydraulic pumps (from N3).
- Definition of pressure and its units.
- Understanding of forces from previous modules.
- Basic distance, area, volume, flow rate, pressure, force, work done and power calculations.
- Conversions for distance, area, volume, flow rate, pressure, force, work done and power.

## Exercise 5.1 SB page 151

1. The area of the master cylinder is:

$$A_m = \frac{\pi d^2}{4}$$
$$= \frac{\pi \times 0{,}08^2}{4} = 5{,}027 \times 10^{-3}\ \text{m}^2$$

The pressure on the master cylinder is:

$$F_m = \text{MA} \times F$$
$$= 12 \times 800\ \text{N} = 9\ 600\ \text{N}$$
$$P_m = \frac{F_m}{A}$$
$$= \frac{9\ 600}{5{,}027 \times 10^{-3}} = 1{,}91 \times 10^6\ \text{Pa}$$

The area of the ram is:

$$A_R = \frac{\pi d^2}{4}$$
$$= \frac{\pi \times 0{,}2^2}{4} = 0{,}0314\ \text{m}^2$$

The force on the ram is:

$$F_R = \eta_V P_m A_R$$
$$= 0{,}8 \times 1{,}91 \times 10^6 \times 0{,}0314 = 48\ 000\ \text{N}$$

The mass that can be lifted is:

$$m = \frac{F}{g}$$
$$= \frac{48\ 000}{9{,}8} = 4\ 898\ \text{kg}$$ (10)

2. 2.1 Given information:

$$d_2 = \frac{20\ \text{cm}}{100} = 0{,}2\ \text{m}$$
$$d_1 = \frac{10\ \text{cm}}{100} = 0{,}1\ \text{m}$$
$$\text{Stroke} = \frac{60\ \text{cm}}{100} = 0{,}6\ \text{m}$$
$$\text{MA} = 14\ \text{N}$$

Additional information:

$w = 3\ 100\ \text{kg} \times 9{,}8 = 30\ 380\ \text{N}$ $\eta = 70\%$

$$\frac{F_1}{d^2 1} = \frac{w}{d_2^2}$$

$$\frac{F_1}{(0{,}1)^2} = \frac{30\ 380}{(0{,}2)^2}$$

$$F_1 = 7\ 595 \text{ N}$$

But, $\eta = 70\%$

$$F = 7\ 595 \text{ N} \times \frac{70}{100} = 5\ 316{,}50 \text{ N}$$ (6)

2.2 The volume has to be multiplied by the number of strokes (which is 4) and the percentage difference of the slip (100 – 5 = 95%).

$$V = A \times \text{stroke} \left(4 \times \frac{95}{100}\right)$$

$$= \pi \left(\frac{0{,}1}{2}\right)^2 \times 4 \times 3{,}8$$

$$= 0{,}12 \text{ m}$$

This volume is that at the plunger and is equal to the volume of liquid delivered at the ram. (2)

2.3 Since the volume of liquid delivered to the ram is equal to the volume at the plunger.

$$V_{\text{plunger}} = V_{\text{ram}}$$

$$0{,}12 \text{ m} = \pi r \text{ ram} \times h$$

$$0{,}12 \text{ m}^3 = \pi \left(\frac{0{,}2}{2}\right)^2 \times h$$

$$h = 3{,}8 \text{ m}$$

Then:

$$h = 3{,}8 \text{ m} \times 100$$

$$= 380 \text{ cm}$$

The distance that the ram moves is 380 cm after 4 strokes. (2)

3. The force on the master cylinder is:

$$w = mg$$

$$= 60 \times 9{,}8 = 588 \text{ N}$$

$$F_m = \text{MA} \times w$$

$$= 8 \times 588 = 4\ 704 \text{ N}$$

The pressure of the press:

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0{,}06^2}{4}$$

$$= 2{,}83 \times 10^{-3} \text{ m}^2$$

$$P = \frac{F}{A}$$

$$= \frac{4\ 704}{2{,}83 \times 10^{-3}} = 1{,}66 \times 10^6 \text{ Pa}$$ (2)

Master cylinder volume:

$$V = Al$$

$$= 2{,}83 \times 10^{-3} \times 0{,}25 = 0{,}707 \times 10^{-3} \text{ m}^3$$

The slave volume would be the same as the master cylinder volume. (2)

The ram area is:

$$A = \frac{\pi d^2}{4}$$

$$= \frac{\pi \times 0{,}1\,5^2}{4} = 0{,}0177 \text{ m}^2$$

The lift is:

$$l = \frac{V}{A} = \frac{0{,}707 \times 10^{-3}}{0{,}0177}$$

$$= 0{,}04 \text{ m } (= 4 \text{ cm})$$ (2)

The ram force is:

$$F = PA$$

$$= 1{,}66 \times 10^6 \times 0{,}0177 = 29\,400 \text{ N}$$ (2)

The fruit table pressure is:

$$P = \frac{F}{A} = \frac{29\,400}{0{,}8 \times 0{,}8} = 45\,938 \text{ Pa}$$ (2)

**Total: 30 marks**

## Exercise 5.2 SB page 160

1. Given information:

$$= \frac{160 \text{ cm}}{1\,000} = 0{,}16 \text{ m}$$

$$\text{Stroke} = \frac{190 \text{ cm}}{1\,000} = 0{,}19 \text{ m}$$

$$\text{Pressure} = 280 \text{ kPa} = 280\,000 \text{ Pa}$$

$$\text{Work done per stroke} = \text{pressure} \times \text{area} \times \text{stroke}$$

$$W = \text{pressure} \times \pi r \times \text{stroke}$$

$$280\,000 \times \pi \left(\frac{0{,}16}{2}\right)^2 \times 0{,}19 = 1\,069{,}65 \text{ J}$$ (3)

2. 2.1 Given information

$$\text{Work} = 8{,}3 \text{ kJ} = 8\,300 \text{ J}$$

$$= \frac{300 \text{ cm}}{1000} = 0{,}3 \text{ m}$$

$$F = 17 \text{ kN} = 17\,000 \text{ N}$$

$$\text{Pressure} = \frac{F}{A}$$

$$= \frac{17\,000}{\pi \left(\frac{0{,}3}{2}\right)^2}$$

$$= 240\,500{,}8 \text{ Pa}$$

$$= 240{,}5 \text{ kPa}$$ (2)

2.2 Work = force × $sl$

$8\,300 = 17\,000 \times sl$

$sl = 0{,}488 \text{ m} = 488 \text{ mm}$ (2)

2.3 Work = pressure × volume

$8\ 300 = 240\ 500{,}80 \times V$

$V = 0{,}0345\ m^3 = 34{,}5\ l$ (3)

2.4 $\rho = \frac{m}{V}$

$1\ 000 = \frac{m}{0{,}0345}$

$m = 34{,}5$ kg (2)

3. 3.1 Given information:

Cylinders = 3

$d = \frac{140\text{ mm}}{1\ 000} = 0{,}14$ m

Stroke ($sl$) $= \frac{500\text{ mm}}{1\ 000} = 0{,}5$ m

$= 2\ 100\text{ rpm} = \frac{2\ 100}{60} = 35$ rad/s

Slip = 5%

Static head = 28 m

Volume = area × $sl \times N \times n \times \eta_{slip}$

$= \pi\left(\frac{0{,}14}{2}\right)^2 \times 0{,}5 \times 3 \times 35 \times \frac{100}{95}$

$= 0{,}8507$ m /s  1 $m^3$/s ~3,6 × $10^6$ $l/h$

$= 3{,}06 \times 10^6\ l/h$ (3)

3.2 Power = $\rho . g . h \times (A \times \text{stroke length} \times n \times N \times \eta)$

$= \rho . g . h \times$ volume

$= (1\ 000)(9{,}8)(28) \times 0{,}8507$

$= 233\ 432{,}08$ W

But, $\eta = 90\%$

Power $= 233\ 432{,}08 \times \frac{90}{100}$

$= 210\ 088{,}87 = 210{,}09$ kW (4)

**Total: 19 marks**

## Exercise 5.3 SB page 167

1. Given information:

Mass = 1,5 tonnes = 1 500 kg

$d = \frac{80\text{ cm}}{100} = 0{,}8$ m

Pressure $= \frac{\text{weight}}{\text{area}}$

$= \frac{(1\ 500)(9{,}8)}{\pi\left(\frac{0{,}8}{2}\right)^2}$

$= 29\ 244{,}72$ Pa (3)

2. Given information:

$$d_{\text{ram}} = \frac{25 \text{ cm}}{100} = 0{,}25 \text{ m}$$

$$m_{\text{ram}} = 460 \text{ kg}$$

$$\text{Pressure} = 0{,}8 \text{ MPa} = 800\ 000 \text{ Pa}$$

$$\text{Pressure} = \frac{\text{weight}_{\text{total}}}{\text{area}}$$

$$= \frac{(m_{\text{total}} \times g)}{\text{area}}$$

$$m_{\text{total}} = \frac{\text{pressure} \times \text{area}}{g}$$

$$= \frac{\left(800\ 000 \times \pi\left(\frac{0{,}25}{2}\right)^2\right)}{9{,}8}$$

$$= 4\ 007{,}13 \text{ kg}$$

So:

$$m_{\text{total}} = m_{\text{ram}} + m_{\text{additional}}$$

$$4\ 007{,}13 = 460 + m_{\text{additional}}$$

$$m_{\text{additional}} = 3\ 437{,}13 \text{ kg}$$ (4)

3. 3.1 Given information:

$$\text{Volume}_{\text{pump}} = 130\ l/\text{min}$$

$$d_{\text{ram}} = \frac{300 \text{ mm}}{1\ 000} = 0{,}3 \text{ m}$$

$$\text{Volume}_{\text{lift}} = 300\ l$$

$$\text{Height} = 2 \text{ m}$$

$$\text{Volume}_{\text{pump}} = \frac{(130 \times 15)}{60}$$

$$= 32{,}5\ l$$

So,

$$\text{Volume}_{\text{total}} = \text{volume}_{\text{pump}} + \text{volume}_{\text{acc}}$$

$$300 = 32{,}5 + \text{volume}_{\text{acc}}$$

$$\text{Volume}_{\text{acc}} = 267{,}5\ l \ (2)$$

3.2 Volume = area × height

$$\frac{267{,}5}{1\ 000} = \pi\left(\frac{0{,}3}{2}\right)^2 \times \text{height}$$

$$\text{Height} = 3{,}78 \text{ m}$$ (2)

4. 4.1 The pressure in the liquid

The ram area is:

$$A = \frac{\pi d^2}{4}$$

$$= \frac{\pi \times 0{,}12^2}{4} = 0{,}0113 \text{ m}^2$$

The pressure at the ram is:

$$P = \frac{F}{A}$$

$$= \frac{40\ 000}{0{,}0113} = 3{,}537 \times 10^6\ \text{Pa} \qquad (4)$$

4.2 The volume of liquid delivered in litres

Ram volume:

$$V_R = Al$$

$$= 0{,}0113 \times 0{,}6 = 6{,}786 \times 10^{-3}\ \text{m}^3$$

Delivered volume:

$$V_P = \frac{V_R}{\eta_V}$$

$$= \frac{6{,}786 \times 10^{-3}}{0{,}92} = 7{,}376 \times 10^{-3}\ \text{m}^3 \qquad (4)$$

4.3 The speed of the pump crank

Volume for a single plunger crank rotation:

$$V = 2 \times \frac{\pi d^2}{4} \times l$$

$$= 2 \times \frac{\pi \times 0{,}03^2}{4} \times 0{,}1 = 1{,}414 \times 10^{-4}\ \text{m}^3$$

Number of rotations required every minute:

$$n = \frac{60}{20} \times \frac{7{,}376 \times 10^{-3}}{1{,}414 \times 10^{-4}} = 156{,}5\ \text{rpm} \qquad (5)$$

4.4 The height of the accumulator ram

Accumulator area:

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0{,}08^2}{4} = 5{,}027 \times 10^{-3}\ \text{m}^2$$

Accumulator height:

$$h = \frac{V_P}{A} = \frac{7{,}376 \times 10^{-3}}{5{,}027 \times 10^{-3}} = 1{,}47\ \text{m} \qquad (2)$$

4.5 The mass

Mass on accumulator:

$$F = PA$$

$$= 3{,}537 \times 10^6 \times 5{,}027 \times 10^{-3} = 17\ 778\ \text{N}$$

$$m = \frac{F}{g} = \frac{17\ 778}{9{,}8} = 1\ 814\ \text{kg} \qquad (4)$$

4.6 The power of the pump

$$P_{out} = \frac{W}{t} = \frac{PV}{t}$$

$$= \frac{3{,}537 \times 10^6 \times 7{,}376 \times 10^{-3}}{20} = 1\ 304\ \text{W}$$

$$\text{but } \eta = \frac{P_{out}}{P_{in}}$$

$$\therefore P_{in} = \frac{P_{out}}{\eta}$$

$$= \frac{1\ 304}{0{,}85} = 1\ 535\ \text{W} \qquad (5)$$

**Total: 35 marks**

## Summative assessment SB page 171

1. 1.1 Pascal's law states that the pressure of a liquid in a closed container spreads throughout the liquid with the same intensity in all directions. (2)

   1.2 Hydraulic accumulators are used to store hydraulic energy until it is needed.

   Hydraulic accumulators can be used as shock and pulsation absorbers in industries to prevent damage and wear in pipes. (4)

   1.3 A liquid can take the shape of its container, has a definite volume which is important for specific hydraulic systems and can be used repeatedly. (3)

2. 2.1 Given information:

$$d_{plunger} = \frac{35\text{ mm}}{1\,000} = 0{,}035\text{ m}$$

$$sl = \frac{20\text{ cm}}{100} = 0{,}2\text{ m}$$

$$\text{Force} = 180\text{ N}$$

$$d = \text{ram} = \frac{650\text{ mm}}{1\,000} = 0{,}65\text{ m}$$

$$\text{Volume} = \text{area} \times sl \times \text{strokes}$$

$$= \pi\left(\frac{0{,}035}{2}\right)^2 \times 0{,}2 \times 20$$

$$= {,}00385\text{ m}$$ (2)

   2.2

$$\text{Volume}_{ram} = \text{volume}_{plunger}$$

$$\pi\left(\frac{0{,}65}{2}\right)^2 \times \text{height} = 0{,}00385$$

$$\text{Height} = 0{,}0116\text{ m}$$ (2)

   2.2

$$\frac{F_{ram}}{d^2_{ram}} = \frac{F_{plunger}}{d^2_{plunger}}$$

$$\therefore F_{ram} = \frac{F_{plunger} \times d^2_{ram}}{d^2_{plunger}}$$

$$= \frac{180 \times 0{,}65^2}{0{,}035^2}$$

$$= 62\,081{,}63\text{ N}$$

So, work = force$_{ram}$ × height

$$= 62\,081{,}63 \times 0{,}0116$$

$$= 720{,}15\text{ J}$$ (3)

3. 3.1 Given information

$MA = 60 \text{ N}$

$d^2 = \frac{150 \text{ mm}}{1\,000} = 0{,}15 \text{ m}$

$d^1 = \frac{34 \text{ mm}}{1\,000} = 0{,}034 \text{ m}$

$\text{Stroke} = \frac{48 \text{ mm}}{1\,000} = 0{,}048 \text{ m}$

Additional information: η = 90%; mass = 2 000 kg

$$\frac{F_1}{d_1^2} = \frac{w_{ram}}{d_2^2}$$

$$\frac{F_1}{(0{,}034)^2} = \frac{2\,000 \times 9{,}8}{(0{,}15)^2}$$

$$F_1 = 1\,007 \text{ N}$$

$$MA = \frac{\text{load force (F)}}{\text{handle effort}}$$

$$\text{Handle effort} = \frac{F_1}{MA} = \frac{1\,007}{60} = 16{,}78 \text{ N}$$

Given 90% efficiency of lifting effort $= 16{,}78 \text{ N} \times \frac{90}{100}$

$=15{,}10 \text{ N}$ (4)

3.2 The volume of liquid per each stroke at the plunger would be the same as the volume that would lift the ram or load to the height. This means that:

$\pi r \times \text{stroke} \times \text{number of strokes} = \pi r_2^2 \times \text{height}$

$$\text{Number of strokes} = \frac{r_2^2 \times \text{height lifted}}{r_1^2 \times \text{stroke}}$$

$$= \frac{\left(\left(\frac{0{,}15}{2}\right)^2 \times \left(\frac{75}{1\,000}\right)\right)}{\left(\frac{0{,}034}{2}\right)^2 \times 0{,}048} = 30{,}4$$ (4)

4. 4.1 Given information:

$\text{Height}_{static} = 26 \text{ m}$

$d_{plunger} = \frac{75 \text{ mm}}{1\,000} = 0{,}075 \text{ m}$

$\text{Stroke} = \frac{28 \text{ cm}}{100} = 0{,}28 \text{ m}$

$n_{rot.fr} = 120 \text{ rpm}$

$$\text{Volume}_{per\ sec} = \text{volume}_{per\ stroke} \times n$$

$$= \text{area} \times \text{stroke} \times n$$

$$= \pi \left(\frac{0{,}075}{2}\right)^2 \times 0{,}28 \times \frac{120}{60}$$

$$= 0{,}00247 \text{ m}^3/\text{s}$$

$$= 2{,}47 \times 10^{-3} \text{ m}^3/\text{s}$$ (2)

4.2 The formula, weight = ρ.V.$g$, can be written as:

$$\text{Force}_{\text{per sec}} = \rho.V.g$$
$$= 1\,000 \times 0{,}00247 \times 9{,}8$$
$$= 24{,}21 \text{ N/s}$$

4.3 Power = $(\rho . g . h \times A) \times$ stroke length

$$= \text{force}_{\text{per sec}} \times sl$$
$$= (24{,}21 \text{ N/s}) \times (26)$$
$$= 629{,}36 \text{ W}$$ (2)

5. 5.1 Given information:

$$\text{Force} = 12 \text{ kN} = 12\,000 \text{ N}$$
$$d_{\text{plunger}} = \frac{85 \text{ mm}}{1\,000} = 0{,}085 \text{ m}$$
$$\text{Work} = 2{,}5 \text{ kJ} = 2\,500 \text{ J}$$
$$\text{Pressure} = \frac{\text{force}}{\text{area}}$$
$$= \frac{2\,500}{\pi\left(\frac{0{,}085}{2}\right)^2}$$
$$= 440\,567{,}32 \text{ W}$$ (3)

5.2 We know that: work done per stroke = pressure × (area × stroke)

So, work done per stroke = pressure × (volume)

2 500 = 440 567,32 × volume

Volume = 0,00567 $m^3$ = 5,67 × $10^{-3}$ $m^3$ (2)

5.3 Volume = area × stroke

$$0{,}00567 = \pi\left(\frac{0{,}085}{2}\right)^2 \times \text{stroke}$$

Stroke = 0,999 m = 1 m (2)

6. 6.1 Given information:

$$d_{\text{pump}} = \frac{50 \text{ mm}}{1\,000} = 0{,}05$$
$$d_{\text{acc}} = \frac{100 \text{ mm}}{1\,000} = 0{,}1 \text{ m}$$
$$\text{Stroke}_{\text{pump}} = 65 \text{ mm}$$
$$\text{Stroke}_{\text{press}} = \frac{800 \text{ mm}}{1\,000} = 0{,}8 \text{ m}$$
$$\text{Time} = 10 \text{ s}$$
$$d_{\text{press}} = \frac{150 \text{ mm}}{1\,000} = 0{,}15 \text{ m}$$
$$\text{Force}_{\text{press}} = 28 \text{ kN} = 28\,000 \text{ N}$$
$$\text{Volume}_{\text{press}} = \text{area} \times \text{stroke}$$
$$= \pi\left(\frac{15}{2}\right)^2 \times 80 \text{ cm}$$
$$= 14\,137{,}17 \text{ cm}^3$$

$= \frac{14\ 137,17\ \text{cm}^3}{1\ 000}$

$= 14,14\ \ell$

But slip 5%, so

$\text{Volume}_{\text{press}} = 14,14 \times \frac{100}{95} = 14,88\ \ell$ (4)

6.2 Additional information: N = 3

Volume = area × stroke × N × $n$

$$14,88 = \frac{\pi\left(\frac{5\ \text{cm}}{2}\right)^2 \times 6,5 \times 3 \times n}{1\ 000}$$

$n = 38,86$ revs/10 s

$= 38,86 \times \frac{1}{10}$

$= 3,89$ r/s (4)

6.3 Pressure $= \frac{\text{force}}{\text{area}}$

$= \frac{28\ 000\ \text{N}}{\pi\left(\frac{0,15}{2}\right)^2}$

$= 1\ 584\ 475,88$ Pa $= 1,585$ MPa (2)

6.4 Additional information: $\eta = 90\%$

Work = force × stroke

= 28 000 N × 0,8 m

= 22 400 K

$= \frac{22\ 400\ \text{J}}{10} = 2\ 240$ J/s

So, power = work × $\frac{100}{90}$

$= 2\ 240 \times \frac{100}{90}$

$= 2\ 488,89$ W (4)

6.5 Volume = area × height

$$14,88\ l = \frac{\left(\pi\left(\frac{10\ \text{cm}}{2}\right)^2 \times \text{height}\right)}{1\ 000}$$

Height $= \frac{189,46\ \text{cm}}{100}$

$= 1,9$ m (2)

6.6 Pressure $= \frac{\text{weight}}{\text{area}}$

$= \frac{mg}{\text{area}}$

$$1\ 584\ 475,88\ \text{Pa} = \frac{m \times 9,8}{\pi\left(\frac{0,1}{2}\right)^2}$$

$m = 1\ 269,84$ kg (2)

**Total: 55 marks**

## Additional examples with solutions for lecturers and tutors:

Give your learners these problems for extra practice and work through the solutions with them in class.

### Additional example 1

A hydraulic jack is used to lift a platform with baggage to the 4-m high storage door of a Boeing 737. The pump has a 3,5-cm diameter master cylinder with a stroke length of 12 cm and produces a pressure of 10 MPa. It drives a 6-cm diameter ram to lift the pallet.

- What is the lift of the ram for a single stroke of the master cylinder assuming a 5% slip?
- How many strokes are required to lift the platform to a 4 m height?
- What is the mass that the jack can lift?

**Solution**

**Step 1:** Calculate the volume of the master cylinder:

$$V_m = \frac{\pi \times d^2 \times l}{4}$$
$$= \frac{\pi \times 0{,}035^2 \times 0{,}12}{4}$$
$$= 1{,}15 \times 10^{-4}\ \text{m}^3$$

**Step 2:** Calculate the volume transferred to the ram:

$$V_s = \eta_V V_m$$
$$= 95\,\% \times 1{,}15 \times 10^{-4}$$
$$= 1{,}097 \times 10^{-4}\ \text{m}^3$$

**Step 3:** Calculate the lift of the ram:

$$V_s = A_s l_s$$
$$\therefore l_s = \frac{4 \times V_s}{\pi d^2}$$
$$= \frac{4 \times 1{,}097 \times 10^{-4}}{\pi \times 0{,}06^2} = 3\ 879 \times 10^{-2}\ \text{m}$$

**Step 4:** Calculate the strokes:

$$n = \frac{4}{3{,}879 \times 10^{-2}} = 128{,}9 \text{ strokes}$$

**Step 5:** Calculate the force for a 6 cm ram:

$$P = \frac{F}{A}$$

$$\therefore F = PA$$

$$= 10 \times 10^{6} \times \frac{\pi \times 0{,}06^{2}}{4} = 28\ 274 \text{ N}$$

**Step 6:** Calculate the mass:

$$m = \frac{F}{g} = \frac{28\ 274}{9{,}8} = 2\ 885 \text{ kg}$$

## Additional example 2

An engineer designs the hydraulic system to tip the bucket of a dump truck. At present, he plans the following setup:

When the truck wants to tip its cargo, it engages a gearbox that drives a single action hydraulic pump from its diesel engine. The pump has a single piston with a 4,5-cm diameter and 11 cm stroke which provides a spring-loaded accumulator with a 15-cm diameter piston with hydraulic fluid. The spring delivers a constant force of 28 kN to the accumulator piston. The ram of the bucket cylinder has a 20-cm diameter ram and takes 25 s to lift its bucket 1,8 m high.

Calculate:

- The pressure of the hydraulic system.
- The volume of liquid required for the lift if the slip is 11%.
- The speed that the pump must run at.
- The distance that the accumulator piston will travel.
- The number of accumulators needed if an accumulator is 900 mm long.
- The force of the ram and the maximum mass it can lift.
- The power of the pump if the efficiency is 92%.

### Solution

Given information for the press:

- $D = 0{,}2$ m, $l = 1{,}8$ m, $t = 25$ s.
- For the accumulator:

- $d$ = 0,15 m, F = 28 000 N
- For the pump:
- $d$ = 0,045 m, $l$ = 0,11 m, slip = 11%, η = 92%.

**Step 1:** The pressure provided by the accumulator is:

$$P = \frac{F}{A}$$

But

$$A = \frac{\pi d^2}{4}$$

$$= \frac{\pi \times 0{,}15^2}{4} = 0{,}0177 \text{ m}^2$$

$$\therefore P = \frac{28\ 000}{0{,}0177} = 1{,}58 \times 10^6 \text{ Pa } (= 1{,}58 \text{ MPa})$$

**Step 2:** The volume for the lift is:

$$V = A \times l$$

$$= \frac{\pi \times 0{,}2^2}{4} \times 1{,}8 = 0{,}0565 \text{ m}^3 (= 56{,}5\ l)$$

**Step 3:** Add the 11% slip in:

$$\eta = 100 - 11 = 89\%$$

But

$$\eta_h = \frac{V_{ram}}{V_{total}}$$

$$\therefore V_{total} = \frac{V_{ram}}{\eta} = \frac{0{,}0565}{89\%} = 0{,}0635 \text{ m}^3 (= 63{,}5\ l)$$

**Step 4:** Determine the pump output volume per stroke:

$$V = \frac{\pi d^2 l}{4}$$

$$= \frac{\pi \times 0{,}045^2 \times 0{,}11}{4} = 1{,}750 \times 10^{-4} \text{ m}^3$$

**Step 5:** To determine the number of cycles required, divide the volume used by the bucket cylinder by the stroke volume:

$$n = \frac{V_{ram}}{V_{pump}} = \frac{6{,}35 \times 10^{-3}}{1{,}750 \times 10^{-4}} = 363 \text{ cycles for 25 s}$$

**Step 6:** The speed of the pump is therefore:

$$n = \frac{363}{25}$$

$$= 14{,}5 \text{ rev/s } (= 872 \text{ rpm}).$$

**Step 7:** To get the accumulator piston distance, the volume can be divided by the piston area:

$$V = Al$$

$$\therefore l = \frac{V}{A} = \frac{0{,}0565}{0{,}0177} = 3{,}19 \text{ m}$$

**Step 8:** The number of accumulators needed are therefore:

$$n = \frac{L}{l}$$

$$= \frac{3{,}19}{0{,}9} = 3{,}5$$

$\therefore$ 4 accumulators

**Step 9:** The mass can be determined from the system pressure and the accumulator ram diameter:

$$F = PA \text{ and } F = mg$$

$$\therefore F = 1{,}58 \times 10^6 \times \frac{\pi \times 0{,}2^2}{4} = 49\,778 \text{ N } (= 49{,}78 \text{ kN})$$

and

$$m = \frac{F}{g} = \frac{49\,778}{9{,}8} = 5\,079 \text{ kg}$$

**Step 10:** The work done by the press for one stroke is:

$$W = Fl$$

$$= 28\,000 \times 3{,}19 = 89\,600 \text{ J } (= 89{,}6 \text{ kJ})$$

**Step 11:** So the power required to do this work is:

$$P_{out} = \frac{W}{t}$$

$$= \frac{89\,600}{25} = 3\,584 \text{ W}$$

**Step 12:** Taking the inefficiency into consideration:

$$\eta = \frac{P_{out}}{P_{in}}$$

$$\therefore P_{in} = \frac{P_{out}}{\eta} = \frac{3\,584}{92\%} = 3\,896 \text{ W } (= 3{,}896 \text{ kW})$$

# 6 Stress, strain and Young's modulus

**By the end of this module, students should be able to:**

- name the three main types of stresses (direct, shear and bending);
- calculate direct stresses including determination of cross-sectional areas as well as load or dimensions of a member (the nature of the stresses should be mentioned);
- calculate shear stress including determination of cross-sectional areas as well as load or dimensions of a member;
- calculations on single and double shear should be included;
- calculations on stress, strain and Young's modulus for the material;
- calculate stress and strain;
- state Hooke's Law and define Young's modulus;
- draw stress and strain graph or force-distance graph. (limited to elastic limit) with clear reference to the direct proportionality between stress and strain; and
- calculate change in length or total change in length for compound rods.

As you know from personal experience, when you try to break, bend, or stretch an object, its internal binding forces resist your efforts. The amount of stress you can apply to an object depends on your strength, the size of the object it is applied to and the material it is made of. The direction of the applied forces also affects the result that will be achieved.

Important concepts and pre-knowledge for this module includes:

- a good understanding of forces from previous modules
- standard units and conversions of *length* or *distance* and *area*

## Exercise 6.1 SB page 184

1. Given information:

$$\frac{36\text{ mm}}{1\,000} = 0{,}036\text{ m}$$

$$\frac{17\text{ mm}}{1\,000} = 0{,}017\text{ m}$$

Force = 65,5 kN = 65 500 N

$$\text{Stress} = \frac{\text{force}_{\text{load}}\ (\text{N})}{\text{area}\ (\text{m}^2)}$$

$$\sigma_C = \frac{\text{force}_{\text{load}}\ (\text{N})}{\text{area}\ (\text{m}^2)}$$

$$= \frac{65\ 500}{0{,}036 \times 0{,}017}$$

$$= 107\ 026\ 143{,}8 \text{ Pa} = 107{,}03 \text{ MPa}$$ (3)

2. Given information:

$$\sigma_C = 8 \text{ MPa} = 8\ 000\ 000 \text{ Pa}$$

$$\text{Force} = 5 \text{ kN} = 5\ 000 \text{ N}$$

$$\text{Stress} = \frac{\text{force}_{\text{load}}\ (\text{N})}{\text{area}\ (\text{m}^2)}$$

$$\sigma_C = \frac{\text{force}}{\text{area}}$$

$$8\ 000\ 000 = \frac{5\ 000}{\text{area}}$$

$$\text{Area} = 0{,}000625 \text{ m}^2$$

$$= 6{,}25 \times 10^{-4} \text{ m}^2$$ (2)

3. $\varepsilon = \frac{\Delta l}{l}$

$$= \frac{0{,}7}{3\ 000} = 2{,}33 \times 10^{-4}$$ (2)

4. Given information:

$$x = \frac{5 \text{ mm}}{1\ 000} = 0{,}005 \text{ m}$$

$$\varepsilon = 0{,}00186$$

$$\text{Strain} = \frac{\text{length}_{\text{change}}\ (\text{m})}{\text{length}_{\text{original}}\ (\text{m})}$$

$$0{,}00186 = \frac{0{,}005}{l}$$

$$l = 2{,}7 \text{ m}$$ (2)

5. Given information:

$$\varepsilon = 0{,}00025 = 2{,}5 \times 10^{-4}$$

$$l = 4{,}1 \text{ m}$$

$$\text{Strain} = \frac{\text{length}_{\text{change}}\ (\text{m})}{\text{length}_{\text{original}}\ (\text{m})}$$

$$0{,}00025 = \frac{x}{4{,}1}$$

$$x = 1{,}025 \times 10^{-3} \text{ m}$$

$$= 1{,}025 \text{ mm}$$ (2)

6. 6.1 $\sigma_T = \frac{\text{F}}{\text{A}} = \frac{4 \times mg}{\pi d^2}$

$$= \frac{4 \times 10 \times 9{,}8}{\pi \times 0{,}00135^2}$$

$$= 68{,}47 \times 10^6 \text{ Pa } (= 68{,}47 \text{ MPa})$$ (5)

6.2 $\varepsilon = \frac{\Delta l}{l}$

$$= \frac{10{,}5}{1\ 000} = 0{,}0105$$ (4)

7. 7.1 $\sigma = \frac{F}{A}$

$= \frac{15\,000}{0{,}02 \times 0{,}005} = 150 \times 10^6 \text{ Pa} \equiv 150 \text{ MPa}$ (2)

7.2 $\varepsilon = \frac{\Delta l}{l} = \frac{1{,}3}{1\,700} = 7{,}65 \times 10^{-4}$ (1)

7.3 $E = \frac{\sigma}{\varepsilon}$

$= \frac{150 \times 10^6}{7{,}65 \times 10^{-4}} = 196 \times 10^9 \text{ Pa} \equiv 196 \text{ GPa}$ (2)

8. $\varepsilon = \frac{\Delta l}{l}$

$= \frac{1}{300} = 0{,}00333$

$E = \frac{\sigma}{\varepsilon}$

$\therefore \sigma = E\varepsilon$

$= 110 \times 10^9 \times 0{,}00333 = 366{,}7 \times 10^6 \text{ Pa}$

$\sigma = \frac{F}{A}$

$\therefore F = \sigma A = \sigma \times \frac{\pi d^2}{4}$

$= 366{,}7 \times 10^6 \times \frac{\pi \times 0{,}002^2}{4}$

$= 1\,152 \text{ N}$ (5)

9. 9.1 $\sigma = \frac{F}{A}$

$= \frac{400\,000}{0{,}04 \times 0{,}005} = 2{,}0 \times 10^9 \text{ Pa}$ (2)

9.2 $\varepsilon = \frac{\Delta l}{l}$

$= \frac{1{,}4}{150} = 0{,}00933$ (1)

9.3 $E = \frac{\sigma}{\varepsilon}$

$= \frac{2{,}0 \times 10^9}{0{,}00933}$

$= 214{,}3 \times 10^9 \text{ Pa } (= 214{,}3 \text{ GPa})$ (2)

**Total: 35 marks**

## Exercise 6.2 SB page 189

1. $\tau = \frac{F}{2 \times A_R} = \frac{4F}{2\pi d^2}$

$= \frac{4 \times 150\,000}{2 \times \pi \times 0{,}015^2}$

$= 424{,}4 \times 10^6 \text{ Pa } (= 424{,}4 \text{ MPa})$ (2)

2. $\tau = \frac{F}{2 \times A_R} = \frac{4F}{2\pi d^2}$

$= \frac{4 \times 23\,000}{2 \times \pi \times 0{,}005^2}$

$= 585{,}7 \times 10^6 \text{ Pa}$

$$\gamma = \frac{\Delta l}{l}$$
$$= \frac{0,9}{9} = 0,1$$ (5)

**Total: 7 marks**

## Exercise 6.3 SB page 193

1. Corresponding stresses and strains:

   Stress

   $$\sigma = \frac{F}{A}$$
   $$= \frac{3\,000}{0,01 \times 0,01} = 30 \times 10^6 \text{ Pa}$$
   $$\varepsilon = \frac{\Delta l}{l}$$
   $$= \frac{0,25}{4\,500} = 5,56 \times 10^{-5}$$

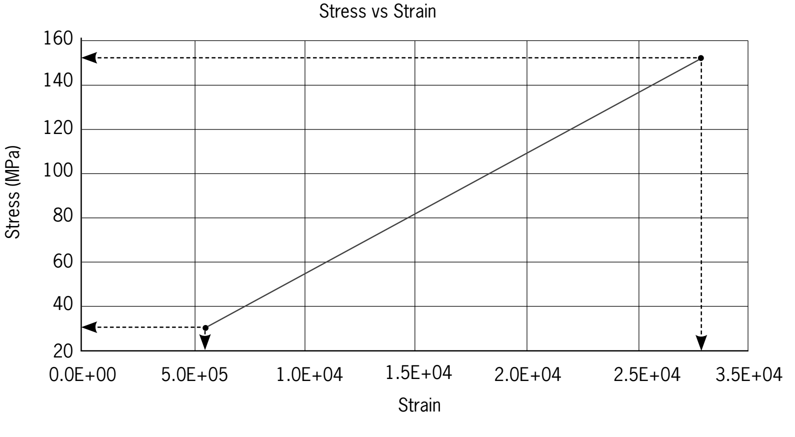
   Strain

   $$E = \frac{\sigma}{\varepsilon}$$
   $$= \frac{30 \times 10^6}{5,56 \times 10^{-5}}$$
   $$= 540 \times 10^9 \text{ Pa } (= 540 \text{ GPa})$$

| **Load (kN)** | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|
| **Deformation (mm)** | 0,25 | 0,50 | 0,75 | 1,00 | 1,25 |
| **Stress (MPa)** | 30 | 60 | 90 | 120 | 150 |
| **Strain** | $5,56 \times 10^{-5}$ | $1,11 \times 10^{-4}$ | $1,67 \times 10^{-4}$ | $2,22 \times 10^{-4}$ | $2,78 \times 10^{-4}$ |

*Table of corresponding stresses and strains* (5)

2.




*Stress-strain graph*

Young's modulus

$E = \frac{\Delta\sigma}{\Delta\varepsilon}$

$= \frac{(150 - 30) \times 10^6}{(27{,}8 - 5{,}56) \times 10^{-5}}$

$= 540 \times 10^9$ Pa (= 540 GPa) (5)

**Total: 10 marks**

## Summative assessment SB page 197

1. 1.1 Tensile, compressive and shear stress. (3)

   1.2 Elasticity is the property of an object which allows it to regain its original size and shape after a force that resulted to a strain is removed. (2)

   1.3 Young's Modulus of elasticity is a constant where the strain is proportional to the stress which produces it, limited to the elastic limits of the material. (2)

   1.4 Hooke's law states that for an elastic object the stress is proportional to the strain producing it. (2)

   1.5 Tensile stress is the type of stress on an object caused by an extensive force that acts to increase the original length of the object, while compressive stress is the stress on an object caused by a compressive force that act to decrease the original length of the object. (2)

   1.6 Tensile stress and compressive stress are called 'direct stresses' since their forces work in direct opposition to each other. Tensile stress tends to make the object longer and thins it out in the middle while compressive stress shortens the object and makes it bulge in the middle. (2)

2. Given information:

   Force = 30 kN = 30 000 N

   $\sigma = 12$ MPa $= 12 \times 10^6$ Pa

   Stress $= \frac{\text{force}_{\text{load}}(\text{N})}{\text{area }(\text{m}^2)}$

   $\sigma = \frac{F}{A}$

   $12 \times 10^6 \text{ Pa} = \frac{30\,000 \text{ N}}{L^2}$

   $L = \sqrt{2{,}5 \times 10^{-3}}$

   $= 0{,}05$ mm = 50 mm (3)

3. Given information:

   0,062 m × 0,045 m

   Force = 50 kN = *50 000 N*

   $\text{Stress} = \frac{\text{force}_{\text{load}}\ (\text{N})}{\text{area}\ (\text{m}^2)}$

   $\tau = \frac{F}{A}$

   $= \frac{50\ 000\ \text{N}}{0{,}062\ \text{m} \times 0{,}045\ \text{m}}$

   = 17 921 146,95 MPa = 17,92 Pa (3)

4. Given information:

   $d = \frac{36\ \text{mm}}{1\ 000} = 0{,}036\ \text{m}$

   Force = 65,5 kN = *65 500 N*

   $\text{Stress} = \frac{\text{force}_{\text{load}}\ (\text{N})}{\text{area}\ (\text{m}^2)}$

   $\sigma_C = \frac{F}{A}$

   $\frac{65\ 500\ \text{N}}{\pi\left(\frac{0{,}036}{2}\right)^2}$

   = 64 349 683,78 Pa

   = 64,35 MPa (3)

5. Given information:

   Force = 20 kN = *20 000 N*

   $\sigma_T = 10\ \text{MPa} = 10 \times 10^6\ \text{Pa}$

   $\text{Stress} = \frac{\text{force}_{\text{load}}\ (\text{N})}{\text{area}\ (\text{m}^2)}$

   $\sigma_T = \frac{F}{A}$

   $10 \times 10^6\ \text{Pa} = \frac{20\ 000\ \text{N}}{\pi\left(\frac{d}{2}\right)^2}$

   $d = \sqrt{2{,}55 \times 10^{-3}}$

   = 0,0505 m = 50,5 mm (3)

6. Given information:

   $l = 6\ \text{m}$

   $x = \frac{28\ \text{mm}}{1\ 000} = 0{,}028\ \text{m}$

   Force = 8 kN = 8 000 N

$$\text{Strain} = \frac{\text{length}_{\text{change}}\ (\text{m})}{\text{length}_{\text{original}}\ (\text{m})}$$

$$\varepsilon = \frac{x}{l}$$

$$= \frac{0{,}028\ \text{m}}{6\ \text{m}}$$

$$= 4{,}67 \times 10^{-3}$$ (2)

7. 7.1 Given information:

$l = 12$ m

Area = 1,5 $\text{cm}^2$

$$x = \frac{0{,}1\ \text{mm}}{1\ 000\ \text{mm}} = 1 \times 10^{-4}\ \text{m}$$

Mass = 800 kg

$$\text{Stress} = \frac{\text{force}_{\text{load}}\ (\text{N})}{\text{area}\ (\text{m}^2)}$$

$$\sigma = \frac{800\ \text{kg} \times 9{,}8}{1 \times 10^{-4}\ \text{m}}$$

$$= 78\ 400\ 000\ \text{MPa}$$

$$= 78{,}4\ \text{MPa}$$ (2)

7.2 $$\text{Strain} = \frac{\text{length}_{\text{change}}\ (\text{m})}{\text{length}_{\text{original}}\ (\text{m})}$$

$$\varepsilon = \frac{x}{l}$$

$$= \frac{1 \times 10^{-4}\ \text{m}}{12\ \text{m}}$$

$$= 8{,}3 \times 10^{-6}$$ (2)

7.3 $$E = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{\sigma}{\varepsilon}$$

$$= \frac{78\ 400\ 000\ \text{Pa}}{8{,}3 \times 10^{-6}}$$

$$= 9{,}445 \times 10^{12}\ \text{Pa}$$ (2)

8. 8.1 Given information:

$$d_r = \frac{50\ \text{mm}}{1\ 000} = 0{,}05$$

$l_r = 2{,}5$ m

$$l_{sq} = \frac{20\ \text{mm}}{1\ 000} = 0{,}02\ \text{m}$$

$l_{sq} = 1{,}4$ m

$\sigma_{max} = 300$ MPa

$$\text{Stress} = \frac{\text{force}_{\text{load}}\ (\text{N})}{\text{area}\ (\text{m}^2)}$$

$$\sigma_{\max} = \frac{F_{sq}}{\text{area}_{sq}}$$

$$300 \times 10^6\ \text{Pa} = \frac{F_{sq}}{(0{,}02)^2}$$

$$F_{sq} = 120\ 000\ \text{N}$$

But:

$$F_r = F_{sq}$$

$$\sigma_r = \frac{F_{sq}}{\text{area}_r}$$

$$= \frac{120\ 000\ \text{N}}{\pi\left(\frac{0{,}05}{2}\right)^2}$$

$$= 61\ 115\ 498{,}15\ \text{Pa}$$

$$= 61{,}115\ \text{MPa}$$ (4)

8.2 From the Modulus formula one substitutes the strain formula to find the extension on each section of the bar:

$$E = \frac{\sigma}{\varepsilon}$$

$$x_r = \frac{F_r \times l_r}{\text{area}_r \times E}$$

$$= \frac{120\ 000 \times 2{,}5}{\pi(0{,}05)^2(100 \times 10^9)}$$

and

$$x_{sq} = \frac{F_{sq} \times l_{sq}}{\text{area}_{sq} \times E}$$

$$= \frac{120\ 000 \times 1{,}4}{(0{,}02)^2(100 \times 10^9)}$$

$$= 4{,}2 \times 10^{-3}\ \text{m}$$

Then:

$$x_T = x_r + x_{sq}$$

$$= 1{,}528 \times 10^{-3}\ \text{m} + 4{,}2 \times 10^{-3}\ \text{m}$$

$$= 5{,}73 \times 10^{-3}\ \text{m} = 5{,}73\ \text{mm}$$ (5)

9. 9.1 Given information:

$E = 50\ \text{GPa} = 50 \times 10^9\ \text{Pa}$

Force = 4 kN

$d = \frac{30 \text{ mm}}{1\ 000} = 0{,}03 \text{ m}$

$l = 1{,}5 \text{ m}$

$\text{Stress} = \frac{\text{force}_{\text{load}} \text{ (N)}}{\text{area (m}^2\text{)}}$

$\sigma = \frac{F}{A}$

$\frac{4\ 000 \text{ N}}{\pi \left(\frac{0{,}03}{2}\right)^2}$

$= 5\ 658\ 842{,}42 \text{ Pa}$

$= 5{,}66 \text{ MPa}$ (2)

9.2 $E = \frac{\text{stress}}{\text{strain}}$

$= \frac{\sigma}{\varepsilon}$

But: $\varepsilon = \frac{x}{l}$.

So:

$E = \sigma \times \frac{l}{x}$

$50 \times 10^9 \text{ Pa} = \frac{5\ 658\ 842{,}42 \text{ Pa} \times 1{,}5 \text{ m}}{x}$

$x = 1{,}7 \times 10^{-4} \text{ mm}$

$= 0{,}17 \text{ m}$ (4)

10. 10.1 Given information:

$l = 5 \text{ m}$

Dimensions = 2 cm × 2 cm

$\text{Stress} = \frac{\text{force}_{\text{load}} \text{ (N)}}{\text{area (m}^2\text{)}}$

$\sigma = \frac{15\ 000 \text{ N}}{(0{,}02)^2}$

$= 37\ 500\ 000 \text{ Pa} = 37{,}5 \text{ MPa}$

and

$\text{Strain} = \frac{\text{length}_{\text{change}} \text{ (m)}}{\text{length}_{\text{original}} \text{ (m)}}$

$\varepsilon = \frac{x}{l}$

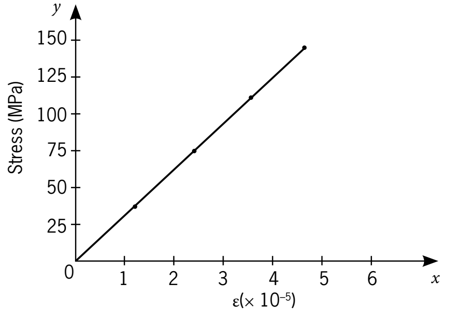
$= \frac{0{,}055 \text{ mm}}{5\ 000 \text{ mm}}$

$= 1{,}1 \times 10^{-5}$

| **Stress (MPa)** | 37,5 | 75 | 112,5 | 150 |
|---|---|---|---|---|
| **Strain ($\times 10^{-5}$)** | 1.1 | 2.2 | 3.3 | 4.4 |

(2)

10.2




*Stress-strain graph* (3)

10.3 $E = \frac{\text{stress}}{\text{strain}}$

$= \frac{\sigma}{\varepsilon}$

$\frac{37\ 500\ 000 \text{ Pa}}{1{,}1 \times 10^{-5}}$

$= 3{,}41 \times 10^{12}$ Pa (2)

**Total: 55 marks**

## Additional examples with solutions for lecturers and tutors:

Give your learners these problems for extra practice and work through the solutions with them in class.

### Additional example 1

A 540-kg ladle is suspended by a steel cable from a hoist in a factory. The cable is 3,2 m long and has a diameter of 5 mm. The cable stretches 3 mm due to the weight.

Calculate:

- The stress
- The strain
- Young's modulus

#### Solution

**Step 1:** Write down the given information:

- $m = 540$ kg
- $d = 0{,}005$ m
- $l = 3{,}2$ m
- $\Delta l = 0{,}003$ m

**Step 2:** Calculate the stress:

$$\sigma_T = \frac{F}{A} = \frac{4mg}{\pi d^2}$$
$$= \frac{4 \times 540 \times 9{,}8}{\pi \times 0{,}005^2}$$
$$= 269{,}5 \times 10^6 \text{ Pa } (= 269{,}5 \text{ MPa})$$

**Step 3:** Calculate the strain:

$$\varepsilon = \frac{\Delta l}{l} = \frac{0{,}003}{3{,}2} = 9{,}38 \times 10^{-4}$$

**Step 4:** Calculate Young's modulus:

$$E = \frac{\sigma}{\varepsilon}$$
$$= \frac{269{,}5 \times 10^6}{9{,}38 \times 10^{-4}}$$
$$= 287{,}5 \times 10^9 \text{ Pa } (= 287{,}5 \text{ GPa})$$

## Additional example 2

A 73-kg signboard for a shop hangs on two 1,5-m aluminium sections with a surface area of $2 \times 10^{-6}$ m² each. Young's modulus for aluminium is 70 GPa. Calculate the elongation of the aluminium under this weight.

### Solution

**Step 1:** Write down the given information:

- $m = 73$ kg
- $A = 2 \times 10^{-6}$ m²
- $l = 1{,}5$ m
- $n = 2$
- $E = 70 \times 10^9$ Pa

**Step 2:** Calculate the elongation of the aluminium under this weight

$$\sigma_T = \frac{F}{A} = \frac{mg}{A}$$
$$= \frac{73 \times 9{,}8}{2 \times 2 \times 10^{-6}} = 178{,}9 \times 10^6 \text{ Pa}$$
$$E = \frac{\sigma}{\varepsilon} = \frac{\sigma \times l}{\Delta l}$$
$$\therefore \Delta l = \frac{\sigma \times l}{E}$$
$$= \frac{178{,}9 \times 10^6 \times 1{,}5}{70 \times 10^9}$$
$$= 0{,}0038 \text{ m } (= 3{,}8 \text{ mm})$$

## Additional example 3

A tennis ball weighs 60 g and is accelerated from standstill to 180 km/h during the 5 milliseconds of the serve. During the serve, it touches 8 strings with a 1,3-mm diameter and an average length of 35 cm. Young's modulus for the strings is 7 GPa.

Calculate the following:

3.1 The force of the serve;

3.2 The stress of the strings;

3.3 The strain of the strings; and

3.4 The elongation of the strings.

### Solution

**Step 1:** Write down the given information:

- $m = 0{,}060$ kg
- $t = 0{,}005$ s
- $v_i = 0$ m/s
- $v_f = 180$ km/h
- $n = 8$
- $d = 0{,}0013$ m
- $l = 0{,}35$ m
- $\mathrm{E} = 7 \times 10^9$ Pa

**Step 2:** Convert 180 km/h to m/s:

$$v_f = \frac{180 \times 1\,000}{3\,600} = 50 \text{ m/s}$$

**Step 3:** Determine the force of the serve:

$$\mathrm{F} = ma = m\frac{\Delta v}{t}$$

$$= 0{,}060 \times \frac{50 - 0}{0{,}005} = 600 \text{ N}$$

**Step 4:** To determine the stress, first calculate the combined string area:

$$\mathrm{A} = n \times \frac{\pi \times d^2}{4}$$

$$= 8 \times \frac{\pi \times 0{,}0013^2}{4} = 10{,}62 \times 10^{-6} \text{ m}^2$$

**Step 5:** And now:

$$\sigma_\mathrm{T} = \frac{\mathrm{F}}{\mathrm{A}}$$

$$= \frac{600}{10{,}62 \times 10^{-6}}$$

$$= 56{,}51 \times 10^6 \text{ Pa } (= 56{,}51 \text{ MPa})$$

**Step 6:** Now calculate the strain:

$$\varepsilon = \frac{\sigma}{E} = \frac{56{,}51 \times 10^{6}}{7 \times 10^{9}} = 8{,}07 \times 10^{-3}$$

**Step 7:** From this you can calculate the elongation:

$$\varepsilon = \frac{\Delta l}{l}$$

$$\therefore \Delta l = \varepsilon \times l$$

$$= 8{,}07 \times 10^{-3} \times 0{,}35$$

$$= 0{,}0028 \text{ m } (= 2{,}8 \text{ mm})$$

## Additional example 4

A steel wire with a diameter of 4 mm is used to lower an object with a mass of 100 kg down a shaft. It is found that the 60-m wire stretched 26 mm.

Calculate:

- The stress
- The strain
- Young's modulus

### Solution

**Step 1:** Write down the given information:

- $d = 0{,}004$ m
- $m = 100$ kg
- $l = 60$ m
- $\Delta l = 0{,}026$ m

**Step 2:** Calculate the stress.

$$\sigma = \frac{F}{A} = \frac{4mg}{\pi d^2} = \frac{4 \times 100 \times 9{,}8}{\pi \times 0{,}004^2} = 77{,}99 \times 10^{6} \text{ Pa}$$

**Step 3:** Calculate the strain.

$$\varepsilon = \frac{\Delta l}{l} = \frac{0{,}026}{60} = 4{,}33 \times 10^{-4}$$

**Step 4:** Calculate Young's modulus.

$$E = \frac{\sigma}{\varepsilon} = \frac{77{,}99 \times 10^{6}}{4{,}33 \times 10^{-4}} = 180{,}0 \times 10^{9}$$

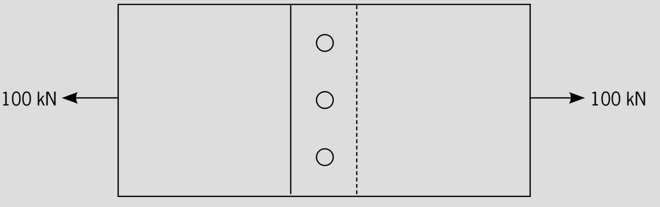
## Additional example 5

Two 12-mm plates riveted together with three 10-mm rivets are used as a structural part for a mine crusher. In extreme conditions, this part may experience a force of 100 kN.

The shear modulus for the rivets is 80 GPa.

Calculate the following:

- The shear stress on each rivet
- The strain on each rivet
- The expected distortion of the rivets




*Plates riveted together*

### Solution

**Step 1:** Write down the given information:

- F = 100 000 N
- $n = 3$
- $d = 0{,}01$
- $l = 0{,}024$ m
- $G = 80 \times 10^9$ P

**Step 2:** Calculate the single shear stress on the rivets:

$$\tau = \frac{F}{A} = \frac{4F}{3\pi d^2}$$
$$= \frac{4 \times 100\ 000}{3 \times \pi \times 0{,}01^2}$$
$$= 424{,}4 \times 10^6 \text{ Pa } (= 424{,}4 \text{ MPa})$$

**Step 3:** Calculate the strain:

$$G = \frac{\tau}{\gamma}$$
$$\therefore \gamma = \frac{\tau}{G}$$
$$= \frac{424{,}4 \times 10^6}{80 \times 10^9} = 5{,}305 \times 10^{-3}$$

**Step 4:** Calculate the deviation:

$$\gamma = \frac{\Delta l}{l}$$
$$\therefore \Delta l = \gamma \times l$$
$$= 5{,}305 \times 10^{-3} \times 0{,}024 = 1{,}27 \times 10^{-4} \text{ m } (= 0{,}127 \text{ mm})$$

MODULE

# 7 *Heat*

**By the end of this module, students should be able to:**

- calculate the volumetric coefficient of expansion given the linear coefficient of expansion;
- calculate the change in volume and final volume due to temperature change;
- calculate the percentage change in volume;
- explain the anomaly in the expansion of water;
- calculate the change in volume and final volume;
- calculate the change in the level of the liquid in a container due to temperature change;
- calculate overflow in a container filled with liquid due to temperature change;
- state the characteristic gas law with relevant formula;
- state Boyle's Law with relevant formula and PV graph;
- state Charles's Law with relevant formula and VT graph;
- state Gay-Lussac's Law with relevant formula and PT graph;
- do calculations on pressure, volume, temperature, mass and gas constant on all four different processes;
- define the three basic gas processes (isochoric, isobaric and isothermal);
- sketch the PV diagrams for isochoric, isobaric and isothermal processes;
- do calculations on work done, change in internal energy and heat flow for all three processes;
- sketch PV diagrams of a maximum of two successive gas processes; and
- do calculations on work done, change in internal energy and heat flow for the two successive processes.

Heat is a form of energy (measured in Joules) which depends on the rate of movement of the atoms or molecules in a substance. As the temperature of a substance increases, the atoms or molecules in the material gain kinetic energy and their rate of movement increases. This means that particles in a material move faster at higher temperatures. The opposite happens when the temperature decreases, the particles move slower as the temperature decreases.

This results in all substances, gases, liquids and solids, expanding when the temperature increases, and contracting when the temperature decreases. While the impact of pressure on the volume of liquids and solids is minimal, the volume of gases is affected directly by pressure.

The principle of conservation of energy determines that heat flowing into a system either adds to the internal energy of the system or is converted into work performed by the system on the environment. Real life gas processes such as internal combustion engines and fridges function successfully based on energy transfer taking place due to changes in pressure and temperature.

**Important concepts and pre-knowledge in the module**

Make sure that you understand the following concepts from previous science and mathematics studies before you start this module:

- The **Celsius** and **Kelvin** temperature measurement **scales** and conversions.
- Standard units and conversions for mass, volume, density, pressure, temperature, heat, work done and energy.
- Atmospheric pressure is 101 325 Pa and room temperature is 20 °C.
- Standard temperature and pressure (STP) is 0 °C and 101 325 Pa.
- 1 bar = 100 000 Pa, 1 atm = 101 325 Pa.
- The concepts of **heat capacity**, mole mass and molecular volume.
- The concepts of heat flow and -conservation.

## Exercise 7.1 SB page 209

1. Given information:
   $l_o = 3$ m
   $t_o = 300$ K – 273 = 27 °C
   $t_f = 540$ K – 273 = 267 °C
   $\alpha = 18 \times 10^{-6}$/K
   $\Delta l = l_o.\alpha.\Delta t$
   $l_f - l_o = l_o.\alpha.\Delta t$
   $l_f - 3 = (3)(18 \times 10^{-6})(267 - 27)$
   $l_f = 3{,}013$ m
   $l_f = 3\ 013$ mm (3)
2. $l_o = 3{,}2$ m
   $l_f = 3{,}3$ m
   $t_o = 28$ °C
   $t_f = 41$ °C
   $\Delta l = l_o.\alpha.\Delta t$

$$\alpha = \frac{\Delta l}{l_o \times \Delta t}$$
$$= \frac{3{,}3 \times 3{,}2}{(3{,}2) \times (41 - 28)}$$
$$= 2{,}4 \times 10^{-4}$$ (3)

3. The following information is given:

   $l_o = 2m$

   $d_o = \frac{600}{1\,000} = 0{,}6$ m

   $T_i = 25$ °C

   $\alpha = 17 \times 10^{-6}$/K

   Additional information: $l_f = 5{,}05$ m

   3.1 Metal temperature

   $$\Delta l = \alpha l_o \Delta T = \alpha l_o (T_f - T_i)$$
   $$\therefore T_f = T_i + \frac{\Delta l}{\alpha l_o}$$
   $$= 25 + \frac{5{,}05 - 5}{17 \times 10^{-6} \times 5} = 613{,}2 \text{ °C}$$ (3)

   3.2 Diameter of the hole

   The expansion of a hole in a metal is treated as if it was filled with the metal itself.

   $$\beta = 2\alpha$$
   $$= 2 \times 17 \times 10^{-6} = 34 \times 10^{-6}\text{/K}$$
   $$A_o = \frac{\pi d^2}{4} = \frac{\pi \times 0{,}6^2}{4} = 0{,}283 \text{ m}^2$$
   $$\Delta A = A_f - A_o = \beta A_o \Delta T$$
   $$= 34 \times 10^{-6} \times 0{,}283 \times (613{,}2 - 25)$$
   $$= 5{,}655 \times 10^{-3}$$
   $$\therefore A_f = A_o + \Delta A$$
   $$= 0{,}283 + 5{,}655 \times 10^{-3} = 0{,}288 \text{ m}^2$$
   $$d_f = \sqrt{\frac{4A_f}{\pi}} = \sqrt{\frac{4 \times 0{,}288}{\pi}} = 0{,}606 \text{ m}$$ (2)

4.1 Given information:

   $$d = \frac{65 \text{ cm}}{1\,000} = 0{,}065 \text{ m}$$
   $$l_o = 2 \text{ m}$$
   $$\Delta t = 95 \text{ °C}$$
   $$\Delta V = 15{,}05 \text{ mm}^3 = 15{,}05 \times 10^{-6} \text{ m}^3$$
   $$V_o = A_o \times l_o$$
   $$= \pi \left(\frac{0{,}065}{2}\right)^2 \times 2$$
   $$= 6{,}637 \times 10^{-3} \text{ m}^3$$

$\text{Percentage increase} = \frac{\Delta V}{V_o} \times 100$

$= \frac{15,05 \times 10^{-6}}{6,637 \times 10^{-3}} \times 100$

$= 0,227\%$ (2)

4.2 $\Delta V = V_o \cdot \tau \cdot \Delta t$

$15,05 \times 10^{-6} = 6,637 \times 10^{-3} \cdot \tau \cdot 95$

$\tau = 2,387 \times 10^{-5}$

$\tau = 3 \cdot a$

$2,387 \times 10^{-5} = 3a$

$a = 7,957 \times 10^{-6}$ (2)

**Total: 15 marks**

## Exercise 7.2 SB page 217

1. For aluminium:

$\Delta V_A = \beta V_o \Delta T$

$= 75 \times 10^{-6} \times 80 \times (30 - 5) = 0,15\ l$

For diesel:

$\Delta V_D = \beta V_o \Delta T$

$= 830 \times 10^{-6} \times 80 \times (30 - 5) = 1,66\ l$

Spillage:

$V_S = V_D - V_A$

$= 1,66 - 0,15 = 1,51\ l$ (6)

2. $\Delta V = \beta V_o \Delta T$

$= 190 \times 10^{-6} \times (8 \times 6 \times 2) \times (29 - 14)$

$= 0,273\ m^3 \equiv 273\ l$ (4)

3.1 Tank volume:

$V_T = \frac{\pi d^2}{4} h$

$= \frac{\pi \times 1,5^2}{4} \times 3 = 5,3\ m^3$ (2)

3.2 Fuel level:

$h = \frac{V_O}{A} = \frac{4V}{\pi d^2}$

$= \frac{4 \times 5,1}{\pi \times 1,5^2} = 2,89\ m$ (2)

3.3 Filled percentage:

$\text{Level} = \frac{2,89}{3} \times 100\,\% = 96,2\%$ (2)

3.4 Volume increase:

$\Delta V = \beta V_o \Delta T$

$= 745 \times 10^{-6} \times 5,1 \times (75 - 12)$

$= 0,239 \text{ m}^3$ (3)

3.5 Percentage increase:

$\%V = \frac{\Delta V}{V_o} \times 100\%$

$= \frac{0,239}{5,1} \times 100\,\% = 4,69\%$ (2)

3.6 Final volume:

$V_f = V_o + \Delta V$

$= 5,1 + 0,239 = 5,339 \text{ m}^3$ (2)

3.7 Spillage:

$V_S = V_f - V_T$

$= 5,339 - 5,3 = 0,039 \text{ m}^3 \equiv 39\ l$ (2)

**Total: 25 marks**

## Exercise 7.3 — SB page 242

1. $P_1V_1 = P_2V_2$

$\therefore V_2 = \frac{P_1V_1}{P_2}$

$= \frac{8 \times 10^6 \times 9,6}{101\,325} = 758\ l$ (2)

2. $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$\therefore T_2 = T_1\frac{V_2}{V_1}$

$= (20 + 273) \times \frac{1,8}{1,45}$

$= 363,7 \text{ K} \equiv 90,7\text{ °C}$ (3)

3. 3.1 The pressure required:

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\therefore P_2 = P_1\frac{T_2V_1}{T_1V_2}$

$= 100 \times \frac{(200 + 273) \times 45}{(30 + 273) \times 20} = 351,24 \text{ kPa}$ (3)

3.2 The maximum temperature:

$V_3 = V_2$

$\therefore T_3 = T_2\frac{P_3}{P_2}$

$= (200 + 273)\frac{350}{351,24}$

$= 471,3 \text{ K} \equiv 198,3\text{ °C}$ (2)

4. 4.1 The mole mass for oxygen gas is 32:

$$n = \frac{m}{M} = \frac{90}{32} = 2{,}81 \text{ mols}$$

$$PV = nRT$$

$$\therefore P = \frac{nRT}{V}$$

$$= \frac{2{,}81 \times 8{,}314 \times (70 + 273)}{10 \times 10^{-3}}$$

$$= 802\ 041 \text{ Pa}$$ (2)

4.2 The required temperature is:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\therefore T_2 = T_1 \frac{P_2}{P_1}$$

$$= (70 + 273) \times \frac{920\ 000}{802\ 041}$$

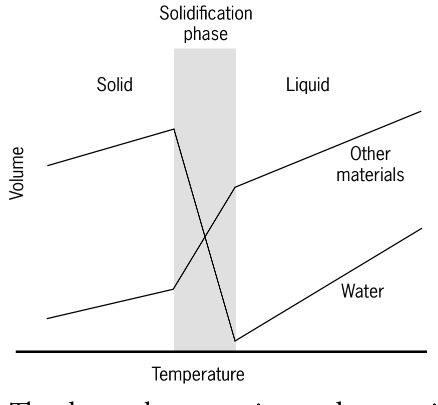
$$= 393{,}4 \text{ K} \equiv 120{,}4 \text{ °C}$$ (3)

**Total: 15 marks**

## Summative assessment SB page 246

1. Water exhibits the same expansion and contraction properties as other materials in its solid, liquid and gas phases except between 0 °C and 4 °C when it reacts differently from other materials. When it is cooled from room temperature, it continues to contract like other materials until it gets to 4 °C. It then starts expanding until it reaches 0 °C and turns into ice. As the temperature drops further, the volume of the ice then contracts like those of other materials. However, its density never drops below that of water. This behaviour of water is responsible for ice forming and floating on top of water.

   The reason for this strange behaviour is that at 4 °C, water molecules settle into a molecular structure that requires more space than when it is in a liquid state.




The thermal contraction and expansion of water (8)

2. The volume of the sphere:

$$V_o = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi \times 0{,}35^3 = 0{,}180\ \text{m}^3$$

Increase in volume:

$$\beta = 3\alpha$$
$$= 3 \times 19 \times 10^{-6} = 57 \times 10^{-6}$$
$$\Delta V = \beta V_o \Delta T$$
$$= 57 \times 10^{-6} \times 0{,}180 \times (520 - 298)$$
$$= 2{,}27 \times 10^{-3}\ \text{m}^3 \equiv 2\ 270\ \text{cm}^3$$ **(3)**

3. 3.1 Calculate the volume of the billet:

$$V_o = \frac{\pi d^2}{4} l$$
$$= \frac{\pi \times 0{,}5^2}{4} \times 3 = 0{,}589\ \text{m}^3$$

Calculate the coefficient of volumetric expansion:

$$\Delta V = \beta V_o \Delta T$$
$$\therefore \beta = \frac{\Delta V}{V_o \Delta T}$$
$$= \frac{3 \times 10^{-3}}{0{,}589 \times 100} = 50{,}9 \times 10^{-6}/\text{K}$$

Get the coefficient of linear expansion:

$$\beta = 3\alpha$$
$$\therefore \alpha = \frac{50{,}9 \times 10^{-6}}{3} = 17{,}0 \times 10^{-6}/\text{K}$$ (4)

3.2 Calculate the percentage volume increase:

$$\%V = \frac{\Delta V}{V_o} \times 100\%$$
$$= \frac{3 \times 10^{-3}}{0{,}589} \times 100\,\% = 0{,}51\%$$ (3)

4. 4.1 Temperature

$$\Delta V = \beta V_o \Delta T = 3\alpha V_o (T_f - T_i)$$
$$\therefore T_f = T_i + \frac{\Delta V}{3\alpha V_o}$$ (4)

4.2 Diameter

The expansion of a hole in a metal is treated as if it was filled with the metal itself.

$$A_o = \frac{\pi d^2}{4} = \frac{\pi \times 0{,}57^2}{4} = 0{,}255\ \text{m}^2$$

$\Delta A = A_f - A_o = \beta A_o \Delta T$

$= 43{,}5 \times 10^{-6} \times 0{,}255 \times (230 - 20)$

$= 2{,}33 \times 10^{-3}\ m^2$

$\therefore A_f = A_o + \Delta A$

$= 0{,}255 + 2{,}33 \times 10^{-3} = 0{,}258\ m^2$

$d_f = \sqrt{\frac{4\,A_f}{\pi}} = \sqrt{\frac{4 \times 0{,}258}{\pi}} = 0{,}573\ m$ (4)

5. 5.1 Excess liquid to be drained:

$\Delta V = \beta V_o \Delta T$

$= 214 \times 10^{-6} \times 30\,000 \times (35 - 5) = 192\ l$ (3)

5.2 Percent expansion:

$\%V = \frac{\Delta V}{V_o} \times 100\%$

$= \frac{192}{30\,000} \times 100\,\% = 0{,}64\%$ (2)

6. Glass expansion:

$\Delta V = \beta\, V_o \Delta T = 3\alpha\, V_o \left(T_f - T_i\right)$

$= 3 \times 7 \times 10^{-6} \times 1 \times (250 - 12) = 5 \times 10^{-3}\ l$

For the glycerine:

$\Delta\, V_G = V_{of} + \Delta\, V_B$

$= 112 + 5 = 117\ cm^3$

$\beta = \frac{\Delta\, V_G}{V_o \Delta T}$

$= \frac{117}{1\,000 \times (250 - 12)} = 492 \times 10^{-6}/K$ **(8)**

7. Pipe area:

$A = \frac{\pi d^2}{4}$

$= \frac{\pi \times 0{,}01\,9^2}{4} = 2{,}835 \times 10^{-4}\ m^2$

Original water volume:

$V_o = Al$

$= 2{,}835 \times 10^{-4} \times 24 = 6{,}805 \times 10^{-3}\ m^3$

Volume increase:

$\Delta V = \beta\, V_o \Delta T$

$= 210 \times 10^{-6} \times 6{,}805 \times 10^{-3} \times (60 - 5) = 78{,}6 \times 10^{-6}\ m^3$

$V_f = V_o + \Delta V$

$= 6{,}805 \times 10^{-3} + 78{,}6 \times 10^{-6} = 6{,}883 \times 10^{-3}\ \text{m}^3$

Pipe level:

$V = Al$

$\therefore l = \frac{V}{A} = \frac{6{,}883 \times 10^{-3}}{2{,}835 \times 10^{-4}} = 24{,}28\ \text{m}$ **(5)**

8. 8.1 The pressure of a gas is inversely proportional to its volume, provided the temperature remains the same. (3)

   8.2 The volume of an ideal gas is directly proportional to the absolute temperature, provided the pressure of the gas remains the same. (3)

   8.3 The pressure of a gas in a given volume goes up in direct relation to the absolute temperature going up. (3)

9. $P_1V_1 = P_2V_2$

   $\therefore V_1 = \frac{P_2V_2}{P_1}$

   $= \frac{300 \times 48}{50} = 288\ l$ **(3)**

10. $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

    $\therefore T_2 = T_1\frac{V_2}{V_1}$

    $= (22 + 273) \times \frac{15{,}6}{12{,}3}$

    $= 374\ \text{K} \equiv 101\ °\text{C}$ **(3)**

11. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

    $\therefore V_2 = V_1\frac{T_2P_1}{T_1P_2}$

    $= 1{,}1 \times \frac{(-10 + 273) \times 190}{(32 + 273) \times 80} = 2{,}25\ \text{m}^3$ **(3)**

12. Initial energy:

    $Q_1 = P_1V_1$

    $= 500\,000 \times 1{,}8 \times 10^{-3} = 900\ \text{J}$

    Mass of gas:

    $PV = nRT$

    $\therefore n = \frac{P_1V_1}{RT_1}$

    $= \frac{500\,000 \times 1{,}8 \times 10^{-3}}{286 \times 373} = 8{,}43 \times 10^{-3}\ \text{kg}$

Final energy:

$Q_2 = P_2V_2 = nRT_2$

$= 8{,}43 \times 10^{-3} \times 286 \times 293 = 706{,}97$ J

Energy change:

$Q = Q_2 - Q_1$

$= 706{,}97 - 900 = 193{,}03$ J **(5)**

13. 13.1 Mols in a big bag:

$PV = nRT$

$\therefore n = \frac{PV}{RT}$

$= \frac{270\,000 \times 100}{8{,}314 \times 310} = 10\,476$ mol

Compressed gas volume:

$V = \frac{nRT}{P}$

$= \frac{10\,476 \times 8{,}314 \times 310}{10\,000\,000} = 2{,}7\ m^3$ (3)

13.2 Cylinders filled:

$N = \frac{2{,}7}{0{,}2} = 13{,}5$ cylinders (2)

13.3 Moles per cylinder:

$n_c = \frac{10\,476}{13{,}5} = 776$ mols (3)

13.4 Combusted gas volume:

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$

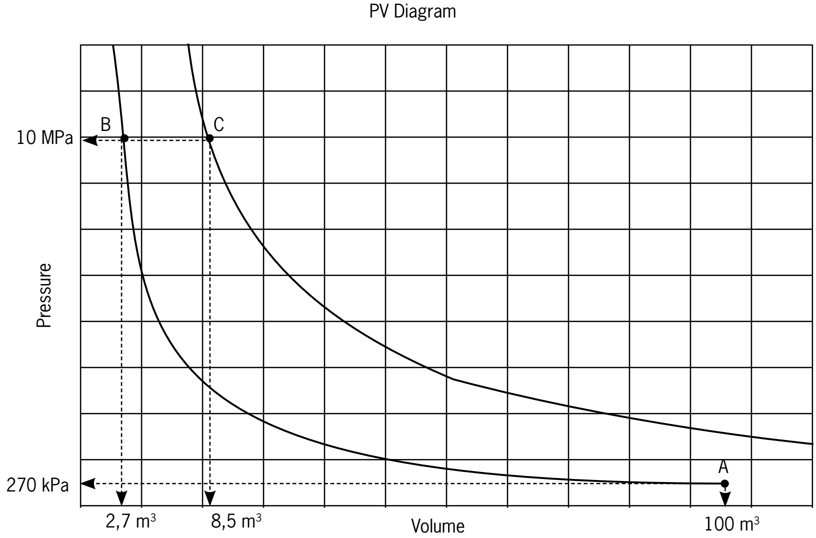
$\therefore V_2 = V_1\frac{T_2}{T_1}$

$= 2{,}7 \times \frac{700 + 273}{37 + 273} = 8{,}47\ m^3$ (3)

13.5 Volume of combusted gas from a single cylinder:

$V_c = \frac{8{,}47}{13{,}5} = 0{,}628\ m^3$ (3)

13.6 PV Diagram:




(9)

**Total: 90 marks**

## Additional examples with solutions for lecturers and tutors:

Give your learners these problems for extra practice and work through the solutions with them in class.

### Additional example 1

1. A new water geyser is 1 150 mm long and has a diameter of 550 mm. It is filled with cold water at 5°C and when switched on, heats up to 62 °C. Assume the volumetric coefficient of expansion for water is $210 \times 10^{-6}$/°C, and the linear coefficient of expansion for the geyser is $12 \times 10^{-6}$/°C.

   How much water will be released through the overflow?

#### Solution

**Step 1:** Write down the given information:

- $l = 1{,}15$ m
- $d = 0{,}55$ m
- $T_1 = 5$ °C
- $T_2 = 62$ °C
- $\alpha_g = 12 \times 10^{-6}$/°C
- $\beta_w = 210 \times 10^{-6}$/°C

**Step 2:** Calculate the expansion of the drum:

$\beta_g = 3\alpha_g$

$= 3 \times 12 \times 10^{-6}$

$= 36 \times 10^{-6}/°C$

Drum volume:

$V = \frac{\pi d^2}{4} \times l$

$= \frac{\pi \times 0{,}55^2}{4} \times 1{,}15$

$= 0{,}273 \text{ m}^3$

Change in volume:

$\Delta V_g = \beta_g V \Delta T$

$= 36 \times 10^{-6} \times 0{,}273 \times (62 - 5)$

$= 0{,}56 \times 10^{-3} \text{ m}^3$

**Step 3:** Calculate the expansion of the water:

$\Delta V_w = \beta_w V \Delta T$

$= 210 \times 10^{-6} \times 0{,}273 \times (62 - 5)$

$= 3{,}27 \times 10^{-3} \text{ m}^3$

**Step 4:** Since the geyser itself expanded, it will be able to contain some of the water that would have overflown. Therefore, the overflow is:

$V_{of} = \Delta V_w - \Delta V_g$

$= 3{,}27 \times 10^{-3} - 0{,}56 \times 10^{-3}$

$= 2{,}71 \times 10^{-3} \text{ m}^3$ (= 2,71 *l*)

## Additional example 2

2. A medical depot uses a high pressure, 150-litre oxygen feed cylinders at 14,1 MPa to refill a 10-litre patient cylinder at room temperature. The valve connecting the two cylinders regulates the patient cylinder pressure at 8 MPa. The patient cylinder is at 2 MPa when the process starts.

Calculate:

- The moles of oxygen in the patient cylinder before filling.
- The moles of oxygen in the patient cylinder after filling.
- The mass of oxygen in the patient cylinder after filling.
- The moles of oxygen in the feed cylinder before filling.

- The moles of oxygen in the feed cylinder after filling.
- The pressure in the feed cylinder after filling.
- The number of 10 litre cylinders that can be filled from the feed cylinder before its pressure is insufficient. Assume they all arrive with 2 MPa pressure left.

**Solution**

**Step 1:** Write down the given information:

- $T_1 = 298$ K
- $V_1 = 150$ litre
- $P_1 = 14{,}1$ MPa
- $V_2 = 10$ litre
- $P_2 = 2$ MPa
- $M = 32 \times 10^{-3}$ kg/mol

**Step 2:** Calculate the moles in the patient cylinder before filling:

$$n = \frac{PV}{RT}$$
$$= \frac{2 \times 10^6 \times 10 \times 10^{-3}}{8{,}314 \times 298}$$
$$= 8{,}07 \text{ mol}$$

**Step 3:** Calculate the moles in the patient cylinder after filling:

$$n = \frac{PV}{RT}$$
$$= \frac{8 \times 10^6 \times 10 \times 10^{-3}}{8{,}314 \times 298}$$
$$= 32{,}3 \text{ mol}$$

**Step 4:** Calculate the mass of oxygen in the patient cylinder after filling:

$$m = nM$$
$$= 32{,}3 \times 32 \times 10^{-3}$$
$$= 1{,}03 \text{ kg}$$

**Step 5:** Calculate the moles in the feed cylinder before filling:

$$n = \frac{PV}{RT}$$
$$= \frac{14{,}1 \times 10^6 \times 150 \times 10^{-3}}{8{,}314 \times 298}$$
$$= 853{,}7 \text{ mol}$$

**Step 6:** Calculate the moles in the feed cylinder after filling:

$$n_{Ff} = n_{Fi} - (n_{Pf} - n_{Pi})$$
$$= 853{,}7 - (32{,}3 - 8{,}07)$$
$$= 829{,}4 \text{ mol}$$

**Step 7:** Calculate the pressure in the feed cylinder after filling:

$$P = \frac{nRT}{V}$$

$$= \frac{829{,}4 \times 8{,}314 \times 298}{150 \times 10^{-3}}$$

$$= 13{,}7 \times 10^{6} \text{ Pa}$$

**Step 8:** Calculate the mols that will remain in the feed cylinder when exhausted:

$$n = \frac{PV}{RT}$$

$$= \frac{8 \times 10^{6} \times 150 \times 10^{-3}}{8{,}314 \times 298}$$

$$= 484{,}35 \text{ mol}$$

**Step 9:** Calculate the mols available in the feed cylinder for charging:

$$n = \frac{n_{Fi} - n_{Ff}}{n_{Pf} - n_{Pi}}$$

$$= \frac{853{,}7 - 484{,}35}{32{,}3 - 8{,}07}$$

$$= 15{,}25$$

$\therefore$ 15 cylinders

# Exemplar examination paper memorandum

Please note that the answers to theory questions have not been provided but can be accessed by students and lecturers alike in the Student Book.

## Calculation questions

1. **Direction east:**

$$v_{RE} = v_{PE} + v_{WE}$$
$$= 250\sin30° - 85\sin30°$$
$$= 82{,}5 \text{ km/h East}$$

**Direction north:**

$$v_{RN} = v_{PN} + v_{WN}$$
$$= 250\cos30° + 85\cos30°$$
$$= 290{,}1 \text{ km/h North}$$

**Resultant velocity:**

$$v_R = \sqrt{82{,}5^2 + 290{,}1^2}$$
$$= 301 \text{ km/h}$$

**Resultant direction:**

$$\theta = \tan^{-1}\frac{82{,}5}{290{,}1}$$
$$= 15{,}9° \text{ East of North}$$

**Distance after 45 minutes:**

$$d_R = \left(\frac{45}{60}\right)301 = 225{,}8 \text{ km}$$

2. **Initial vertical velocity:**

$$v_y^2 = v_{yi}^2 + 2a\,d_y$$
$$0^2 = v_{yi}^2 - 2(9{,}8)1{,}1$$
$$v_{yi} = \sqrt{21{,}56} = 4{,}643 \text{ m/s}$$

**Total flight time:**

$$v_{yf} = v_{yi} + at$$
$$-4{,}643 = 4{,}643 - 9{,}8t$$
$$t = 0{,}948 \text{ s}$$

**Horizontal velocity:**

$$d_x = v_x t$$

$\therefore v_x = \frac{d}{t}$

$= \frac{24}{0{,}948} = 25{,}33 \text{ m/s}$

**The angle is:**

$\tan\theta = \frac{v_{yi}}{v_{xi}} = \frac{4{,}643}{25{,}33} = 0{,}183$

$\theta = \sin^{-1}0{,}183 = 10{,}4°$

**Initial speed:**

$v_R^2 = v_{xi}^2 + v_{yi}^2$

$v_R = \sqrt{25{,}33^2 + 4{,}643^2} = 25{,}75 \text{ m/s}$

3. $v_f = 43{,}2 \times \frac{1\,000}{3\,600} = 12 \text{ m/s}$

$\omega_f = \frac{v_f}{r} = \frac{12}{0{,}7} = 17{,}14 \text{ rad/s}$

$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{17{,}14 - 0}{10} = 1{,}71 \text{ rad/s}^2$

$\theta = \frac{\omega_f + \omega_i}{2} \cdot t$

$= \frac{17{,}14}{2} \cdot 10 = 85{,}7 \text{ rad}$

$d = \theta r = 85{,}7 \times 0{,}7 = 60 \text{ m}$

4. $\omega_f = \frac{2\pi n_f}{60}$

$= \frac{2\pi \times 2\,500}{60} = 261{,}8 \text{ rad/s}$

$\alpha = \frac{\omega_f - \omega_i}{\Delta t}$

$= \frac{261{,}8 - 0}{20} = 13{,}09 \text{ rad/s}^2$

$\tau = mr^2\alpha$

$= 10 \times 0{,}45^2 \times 13{,}09 = 26{,}51 \text{ Nm}$

$P = \tau\omega$

$= 26{,}51 \times 261{,}8 = 6\,940 \text{ W}$

5. a. **Acceleration of the truck:**

$\theta = \tan^{-1}\left(\frac{1}{15}\right) = 3{,}81°$

$v_i = 66 \times \frac{1\,000}{3\,600} = 18{,}33 \text{ m/s}$

$\Sigma F = F_x - f$

$= mg\sin\theta - f$

$= 4\,900 \times 9{,}8 \times \sin 3{,}81° - 1\,600$

$= 1\,594 \text{ N}$

$a = \frac{F}{m}$

$= \frac{1\,594}{4\,900} = 0{,}325 \text{ m/s}^2$

b. **Top speed:**

$$v_f^2 = v_i^2 + 2ad$$

$$v_f^2 = 18{,}33^2 + 2 \times 0{,}325 \times 2\,000 = 1\,638$$

$$\therefore v_f = 40{,}47 \text{ m/s } \left( = 146 \text{ km/h} \right)$$

c. **The time taken:**

$$v_f = v_i + at$$

$$t = \frac{v_f - v_i}{a}$$

$$= \frac{40{,}47 - 18{,}33}{0{,}325} = 68{,}0 \text{ s}$$

d. **Friction losses:**

$$h = 2\,000 \times \sin 3{,}81° = 133 \text{ m}$$

$$\% \text{ losses} = \frac{\Sigma E_{\text{losses}}}{\Sigma E_{\text{in}}} \times 100\,\%$$

$$= \frac{fd}{mgh + \frac{1}{2}mv^2} \times 100\,\%$$

$$= \frac{1\,600 \times 2\,000 \times 100}{4\,900 \times 9{,}8 \times 133 + \frac{1}{2} \times 4\,900 \times 18{,}33^2} = 44{,}4\%$$

e. **Efficiency of truck:**

$$\eta = \frac{E_{k.\text{bot}}}{E_p + E_{k.\text{top}}} \times 100\,\%$$

$$= \frac{\frac{1}{2} \times 4\,900 \times 40{,}47^2 \times 100}{4\,900 \times 9{,}8 \times 133 + \frac{1}{2} \times 4\,900 \times 18{,}33^2}$$

$$= 55{,}6\%$$

6. $$\circlearrowleft \Sigma M = \circlearrowright \Sigma M$$

$$F_C d_C = F_B d_B + F_E d_E$$

$$F_C \times 2 = 40 \times 1 + 15 \times 4$$

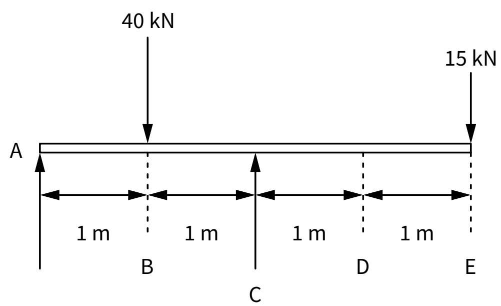
$$\therefore F_C = 50 \text{ kN}$$

$$\uparrow \Sigma F = \downarrow \Sigma F$$

$$F_A + F_C = F_B + F_E$$

$$\therefore F_A = 40 + 15 - 50$$

$$= 5 \text{ kN}$$




$$M_A = M_E = 0$$

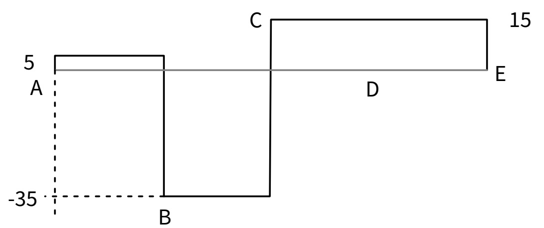
$$M_B = 5 \times 1 = 5 \text{ kNm}$$

$$M_C = 5 \times 2 - 40 \times 1 = -30 \text{ kNm}$$

$$M_D = 5 \times 3 - 40 \times 2 + 50 \times 1$$

$$= -15 \text{ kNm}$$

The maximum bending moment is at point C and $M_C = 30$ kNm (CCW).

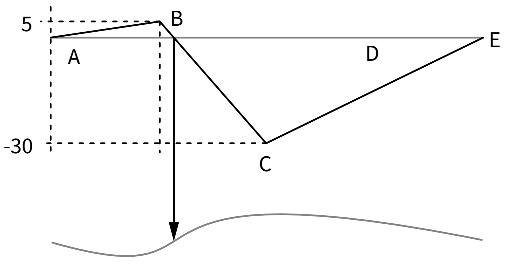


The point of inflection is between B and C.

$\frac{IB}{5} = \frac{1}{35} \therefore$

IB = 0,143 m

$\Rightarrow$ AI = 1,143 m




7. **Master cylinder volume:**

$V_m = \frac{\pi d^2}{4} l$

$= \frac{\pi \times 0{,}03^2}{4} \times 0{,}10$

$= 7{,}07 \times 10^{-5}\ \text{m}^3$

**Volume transferred to the ram:**

$V_s = \eta_V V_m$

$= 93\,\% \times 7{,}07 \times 10^{-5}$

$= 6{,}57 \times 10^{-5}\ \text{m}^3$

**Ram lift:**

$V_s = A_s l_s$

$\therefore l_s = \frac{4}{\pi d^2} \times V_s$

$= \frac{4}{\pi \times 0{,}06^2} \times 6{,}57 \times 10^{-5}$

$= 23{,}25 \times 10^{-3}\ \text{m}$

**Force on the plunger:**

$F_m = 4 \times mg$

$= 4 \times 60 \times 9{,}8 = 2\ 352\ \text{N}$

**System pressure:**

$P = \frac{F_m}{A_m} = \frac{4}{\pi \cdot d^2} \times F_m$

$= \frac{4}{\pi \cdot 0{,}03^2} \times 2\ 744 = 3{,}33 \times 10^6\ \text{Pa}$

**Force in the ram:**

$F_s = P \cdot A_s$

$= 3.33 \times 10^6 \times \frac{\pi \cdot 0{,}06^2}{4} = 9\ 408\ \text{N}$

**In terms of mass:**

$m = \frac{F}{g} = \frac{9\ 408}{9{,}8} = 960\ \text{kg}$

8. a. **Tensile stress:**

$\sigma_T = \frac{F}{A} = mg \times \frac{4}{\pi d^2}$

$= \frac{4 \times 1\ 200 \times 9{,}8}{\pi \cdot 0{,}012^2} = 1{,}04 \times 10^8\ \text{Pa}$

b. **Strain:**

$$\varepsilon = \frac{\Delta l}{l}$$
$$= \frac{0{,}8 \times 10^{-3}}{1{,}5} = 5{,}33 \times 10^{-4}$$

c. **Young's modulus:**

$$E = \frac{\sigma}{\varepsilon}$$
$$= \frac{1{,}04 \times 10^{8}}{5{,}33 \times 10^{-4}} = 195 \times 10^{9} \text{ Pa}$$

9. **Moles of nitrogen:**

$$n = \frac{PV}{RT}$$
$$= \frac{500\,000 \times 2}{8{,}314 \times (220 + 273{,}1)} = 243{,}9 \text{ mol}$$

**Calculate the new volume:**

$$V = \frac{nRT}{P}$$
$$= \frac{243{,}9 \times 8{,}314 \times (35 + 273{,}1)}{101\,325} = 6{,}17 \text{ m}^3$$

10. **Final temperature:**

$$\frac{T_1}{P_1} = \frac{T_2}{P_2}$$
$$\therefore T_2 = T_1\frac{P_2}{P_1}$$
$$= (22 + 273{,}1)\frac{120\,000}{300\,000} = 118 \text{ K}$$

**Work done:**

$W = P\Delta V = 0$

**Change in internal energy:**

$$\Delta U = nC_v\Delta T$$
$$C_v = \frac{5}{2}R$$
$$\therefore \Delta U = \frac{5}{2}nR\Delta T$$
$$= \frac{5}{2} \cdot 100 \cdot 8{,}314 \cdot (118 - 295{,}1)$$
$$= -368 \text{ kJ}$$

**Energy flow:**

$$Q = W + \Delta U$$
$$= 0 - 368 = -368 \text{ kJ}$$

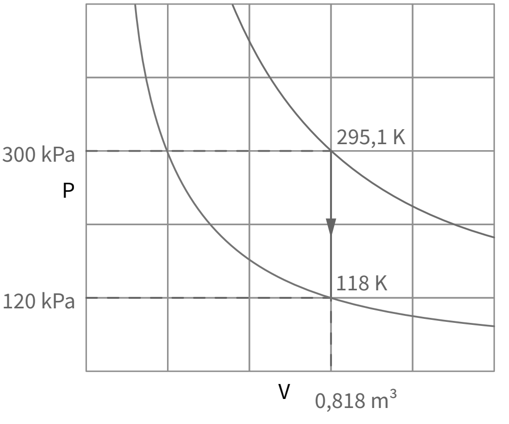
**The volume:**

$$V = \frac{nRT_i}{P_i}$$

$$= \frac{100 \times 8{,}314 \times 295{,}1}{300\ 000}$$

$$= 0{,}818\ m^3$$

Isochoric ammonia process



# Glossary

## A

**absolute temperature –** temperature on a scale where the absolute zero is taken as the initial point in Kelvins

**absolute zero –** the lowest theoretically possible temperature of a gas at –273,15 °C on the Celsius scale and zero on the Kelvin scale

**acceleration –** the rate at which velocity changes as a function of time

**angular acceleration –** the rate at which angular velocity changes

**angular displacement –** the angle in degree or radians that a point or line makes when it travels through an axis

**angular motion –** the motion of an object around a fixed axis or fixed point

**angular velocity** – the rate at which angular displacement changes

## B

**balanced force –** forces that cancel each other when they act on an object and result in no changes in its movement

**ballistics –** the scientific study of the propulsion, flight, and impact of projectiles.

**bending moment** – a bending reaction that is caused by an applied force or load

**brittle material** – materials that are hard but liable to break easily

## C

**cartesian plane –** a perpendicular axes system with positive and negative values spanning in opposite directions from the origin where the axes are all 0

**Celsius scale** – a metric scale, in which the 0 °C is the freezing point and 100 °C is the boiling point of water

**centre of gravity (CG)** – when gravitational force acts on the centre of mass of an object

**centre of mass (CM)** – the point 'right in the middle' of an object around which all its mass is distributed equally

**centroid** – the centre of mass of a flat object with a uniform density

**coefficient of friction** ($\mu$) **–** the ratio of the frictional force between two bodies and the force pressing them together

**coefficient of linear expansion (**$\alpha$**) –** the change in the length of a material by one unit per unit change in temperature

**compressible** – the ability to minimise the volume into a lesser space

**compressive stress** ($\sigma_C$) – stress on an object caused by a compressive force that acts to decrease the original length of the object

**concave** – a line having an inward curve like a half circle

**concentrated load** – a load that acts at a point such as a weight or a reaction

**crescent-shape** – a shape that is thicker in the middle and becomes thinner towards each end

## D

**density** (ρ) – the mass of the substance per unit volume

**displacement** – the shortest distance between the starting point and the end point of movement. It includes the *magnitude* and direction of movement

**distortion** – the alteration of the original shape (or other characteristic) of something

**distributed load** – a load that is spread across a beam. It can be evenly distributed or vary in some pattern over the length of the beam

**ductile material** – materials that can be drawn out into thin wires

## E

**elastic collision (of particles)** – one where there would be no change in the energy causing the particles to move

**elastic material** – materials that extend when being stretched and quickly return to their original state once the load is removed. *Elasticity* is a property of an object which allows it to regain its original form after the force that caused its deformation is removed

**electrostatic force** – the electrical field or force (associated with static electric charges) that exists between two differently charged objects that are close to each other. This field also forms around any single object that is electrically charged in relation to its environment

**equilibrium** – a state of no motion of an object due to the balance of forces that act on the object

## F

**frictional force** – the force due to contact between objects that restricts movement

## H

**heat** – a form of energy associated with the motion of atoms, molecules, or substances in a material

**heat capacity** – the amount of heat required by a substance to change its temperature by one degree

**Hooke's Law** – strain is directly proportional to the stress that causes it

**horizontal** – side-ways direction

**hydraulic accumulator** – a hydraulic device that stores fluids under pressure which is normally used to drive equipment

**hydraulic press** – a hydraulic machine usually used to crush or compress materials

**hydraulic pump** – a device that converts mechanical energy into hydraulic pressure to displace or compress fluids

**hydraulics** – a section in engineering that deals with the mechanical properties of fluids

## I

**intermolecular forces** – the forces that hold atoms together within a molecule

**inversely proportional** – inverse proportion occurs when one value increases and the other decreases

## K

**Kelvin scale** – a temperature scale in which 0 K corresponds to –273,15 °C and 273,15 K corresponds to 0 °C

**kinetic friction** $(f_k)$ – the force between two surfaces moving against each other. It is in the opposite direction of the motion of the object. For example, the force between the wheels of a braking vehicle and the ground

## L

**linear expansion** – the increase in length of a material due to the increase in temperature

**linear motion** – movement along a straight line

## M

**magnitude** – the size of a variable

**moment of force** – the force acting on an object that tries to rotate it around a pivot point

**momentum** – mass in motion

**motion** – the state of movement

## N

**Newton's Second Law** – an object will accelerate in the direction of the resultant force applied to it

**non-linear motion** – movement that is not along a straight line

**normal force (FN)** – the force that prevents an object from falling through a surface. For example, the force exerted by a table that prevents a book from falling to the ground

## O

**offset** – to cancel or reduce the effect of (something)

## P

**parabola** – a U-shaped curve where any point is at an equal distance from the focus point and a fixed line

**pendulum** – an object hung from a fixed point at the end of the hanging cord or rope so that it can swing freely

**pivot point** – the point of rotation of an object or system

**power** – the work done per unit of time in an angular motion

**pressure (P)** – the force that is exerted on a unit area of a surface

**PV diagram** – a pressure-volume diagram is used to describe corresponding changes in volume and pressure in a system

## R

**radian** – a unit of measure for angles that is based on the radius of a circle

**ram** – a heavy bar used in hydraulic devices to add pressure with its weight

**resultant force (FR)** – the sum of all the forces acting on an object in the same plane

## R

**shear force** – a force pair that pushes one part of a body into one direction and the other part in the opposite direction

**shear stress** (τ)– stress on an object caused by one or more forces that act to tear or separate the object into two or more pieces in a movement that is parallel to each other

**specific heat capacity** – the amount of heat required per unit mass of a given substance to raise its temperature by one degree

**static friction** $(f_s)$ – the force between stationary objects that prevents them from moving. For example, the static friction between a stationary bag of cement and the surface of a slightly inclined plane prevents it from sliding down

**STP** – standard temperature and pressure are standard sets of conditions for experimental measurements to be established to allow comparisons to be made between different sets of data

**strain** (ε) – the deformation of an object due to an internal state of stress

**stress (σ)** – the ability of an object to resist the effects of an external force and is given by the amount of load per unit area

**suction head** – the head or height to which water can be raised on the suction side of the pump by atmospheric pressure

## T

**tangential force** – a force that acts on the side of a body to either rotate the body or stop the rotation

**temperature** – the degree or intensity of heat present in a substance or object

**tensile stress** ($\sigma_T$) – stress on an object caused by an extensive force that acts to increase the original length of the object

**tension force** (T) – the pulling force exerted on objects at the opposite sides of a tight string, cable, or chain preventing them from moving away from each other

**torque** – the amount of force needed to change the angular acceleration of an object moving in a circle

**trajectory** – the path followed by an object moving in the air with only the force of gravity acting on it

## U

**unbalanced force** – a force that changes the motion of an object when acting on it

## V

**velocity** – the rate at which position changes as a function of time which includes both magnitude and direction

**vertical** – up and down direction

**volume (V)** – the amount of space in a container

**volume expansion** – the increase in the volume of a material due to the increase in temperature

**volumetric coefficient of expansion (γ)** – the change in the volume of a material by one unit per unit change in temperature

## W

**weight (w)** – the product of a body's mass and the gravitational acceleration

**weight (W)** – the product of an object's mass and gravitational acceleration

**work done** – the product of the force and the distance that the force is applied in order to accelerate or stop the object

## Y

**Young's modulus** – the property of a material that indicates how easily it can stretch and deform. It is defined as constant given by the ratio of tensile stress (σ) to tensile strain (ε)