higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

# NASIENRIGLYN

## NASIONALE SERTIFIKAAT

## WISKUNDE N5

## 21 November 2024

**Hierdie nasienriglyn bestaan uit 12 bladsye.**

Kopiereg voorbehou

**VRAAG 1**

1.1 Definieer die volgende limiete:

1.1.1 $\lim\limits_{x\to\infty} \dfrac{\ln(1+e^x)}{5x}$ $\dfrac{\infty}{\infty}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{e^x}{1+e^x}}{5}$ ✓

$= \lim\limits_{x\to\infty} \dfrac{e^x}{5(1+e^x)}$ $\dfrac{\infty}{\infty}$

$= \lim\limits_{x\to\infty} \dfrac{e^x}{5e^x}$ ✓

$= \dfrac{1}{5}$ ✓ (3)

1.1.2 $\lim\limits_{x\to 0} \dfrac{3^x - 2^x}{x^2 - x}$

$= \lim\limits_{x\to 0} \dfrac{3^x \ln 3 - 2^x \ln 2}{2x-1}$ ✓

$= \ln 2 - \ln 3 \ \ or - 0{,}405$ ✓ (2)

1.2 $f(x) = \dfrac{3x + \sin x}{\sqrt{2-x}}$

$x < 2 \ \ or(-\infty; 2)$ ✓ (1)
**[6]**

**VRAAG 2**

2.1 2.1.1 $y = \cot^4 \sqrt{\tan 3x^2}$

$$\frac{dy}{dx} = 4\cot^3 \sqrt{\tan 3x^2}\checkmark \times -\operatorname{cosec}^2 \sqrt{\tan 3x^2}\checkmark \times \frac{1}{2\sqrt{\tan 3x^2}}\checkmark \times \sec^2 3x^{2\checkmark} \times 6x\checkmark$$ (5)

2.1.2 $y = \tan\left(\sqrt{\frac{1-e^{2x}}{1+e^{2x}}}\right)$

$$\frac{dy}{dx} = \sec^2\left(\sqrt{\frac{1-e^{2x}}{1+e^{2x}}}\right)\checkmark \times \frac{1}{2\sqrt{\frac{1-e^{2x}}{1+e^{2x}}}}\checkmark \times \frac{2e^{2x}(1+e^{2x})\checkmark - 2e^{2x}(1-e^{2x})\checkmark}{(1+e^{2x})^{2\checkmark}}$$ (5)

2.2 $y = \left(\frac{1}{x}\right)^{3x^2+2}$

$y = x^{-(3x^2+2)}$

$\ln y = -(3x^2+2)\ln x\checkmark$

$\frac{1}{y}\frac{dy}{dx}\checkmark = -6x\ln x\checkmark - \frac{3x^2+2}{x}\checkmark$

$\frac{dy}{dx} = y\left(-6x\ln x - \frac{3x^2+2}{x}\right)\checkmark$

$\frac{dy}{dx} = \left(\frac{1}{x}\right)^{3x^2+2}\left(-6x\ln x - \frac{3x^2+2}{x}\right)$ (5)

2.3 2.3.1 $x = 3 + \sqrt{x^2+y^2}$

$1 = 0 + \frac{1}{2\sqrt{x^2+y^2}} \times \left(2x + 2y\frac{dy}{dx}\right)\checkmark$

$2x + 2y\frac{dy}{dx} = 2\sqrt{x^2+y^2}\checkmark$

$2y\frac{dy}{dx} = 2\sqrt{x^2+y^2} - 2x\checkmark$

$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}-x}{y}\checkmark$ (4)

2.3.2 $\frac{dy}{dx} = \frac{\sqrt{3^2 + 4^2} - 3}{4} = \frac{1}{2}$ ✓

$y = \frac{1}{2}x + c$

$4 = \frac{1}{2}(3) + c$

$c = \frac{5}{2}$ ✓

$y = \frac{1}{2}x + \frac{5}{2}$ ✓ (3)

**[22]**

**VRAAG 3**

3.1 3.1.1 $f(x) = 2x^3 - 3x - 2$
$f'(x) = 6x^2 - 3 = 0$
$x = \pm\frac{1}{\sqrt{2}}$

$x = -0{,}707 \quad of \quad x = 0{,}707$

$y = -0{,}586$ ✓ $y = -3{,}414$

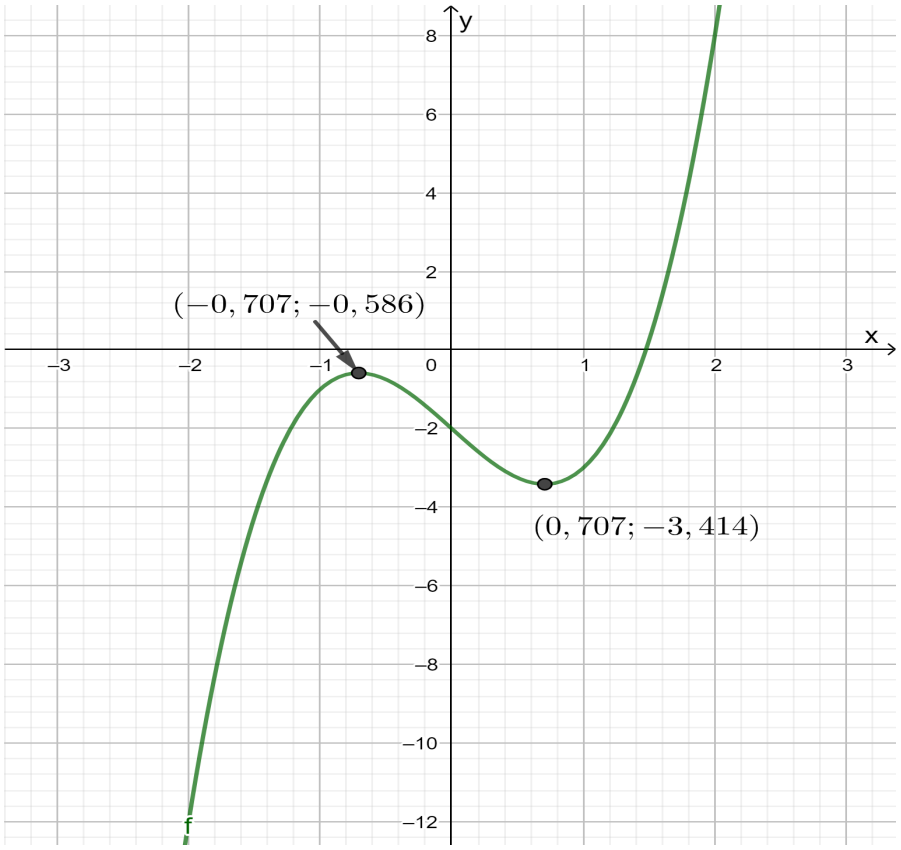
Dus, die koördinate van die draaipunte is $(-0{,}707\,;\,-0{,}586)$✓ en $(0{,}707;-3{,}414\,)$.✓ (3)

3.1.2

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | -12 | -1 | -2 | -3 | 8 |

✓= 1 punt vir enige 2 korrekte antwoorde. (2)

3.1.3




✓ = 1 punt vir die vorm.
✓ = 1 punt vir die aanduiding van die draaipunte op die grafiek. (2)

3.1.4 Laat $x_0 = 1{,}4$

$f(1{,}4) = -0{,}712$

$f'(1{,}4) = 8{,}760$✓

$$x_1 = 1{,}4 - \frac{-0{,}712\ ✓}{8{,}760}$$

$$= 1{,}481✓$$

Laat $x_1 = 1{,}481$

$f(1{,}481) = 0{,}057$

$f'(1{,}481) = 10{,}165$

$$x_2 = 1{,}481 - \frac{0{,}057}{10{,}165}$$

$$= 1{,}476✓$$ (4)

3.2
$$P = 2l + 2w$$
$$300 = 2l + 2w$$
$$150 = l + w\checkmark$$
$$w = 150 - l$$
$$A = lw$$
$$= l(150 - l)$$
$$= 150l - l^2\checkmark$$
$$A' = 150 - 2l = 0\checkmark$$
$$2l = 150$$
$$l = 75m\checkmark$$
$$w = 75m\checkmark$$ (5)

3.3 3.3.1
$$s\,(t) = e^{-t}\ln 3t$$
$$s'(t) = \frac{1}{t}e^{-t} - e^{-t}\ln 3t\checkmark$$ (1)

3.3.2
$$s''(t) = e^{-t}\ln 3t - \frac{2}{t}e^{-t} - \frac{1}{t^2}e^{-t}\checkmark$$

$$s''(2) = e^{-2}\ln 3(2) - \frac{2}{2}e^{-2} - \frac{1}{2^2}e^{-2}\checkmark$$

$$= 0{,}021m/s^2\checkmark$$ (3)

**[20]**

**VRAAG 4**

4.1 4.1.1
$$\int \left[2\sec^2\left(\frac{\pi}{3}x\right)\tan\left(\frac{\pi}{3}x\right)\right]dx$$

$$let\ u = \tan\left(\frac{\pi}{3}x\right)$$
$$du = \sec^2\left(\frac{\pi}{3}x\right) \times \frac{\pi}{3}dx\checkmark$$
$$\frac{3}{\pi}du = \sec^2\left(\frac{\pi}{3}x\right)dx$$

$$= 2 \times \frac{3}{\pi}\int u\,du\checkmark$$

$$= \frac{6}{\pi}\left[\frac{1}{2}u^2 + C\right]$$

$$= \frac{3}{\pi}\tan^2\left(\frac{\pi}{3}x\right) + C\checkmark$$ (3)

4.1.2 $\int [\sin(-6x)\sin(3x)]\,dx$

$= \frac{1}{2}\int (\cos(-3x) - \cos(-6)x)\,dx$✓

$= \frac{1}{2}\left(\frac{1}{3}\sin(3x)\checkmark - \frac{1}{6}\sin(6x)\checkmark\right) + C$

$= \frac{1}{6}\sin(3x) - \frac{1}{12}\sin(6x) + C$ (3)

4.1.3 $\int \sin^2\left(\frac{3}{4}x\right)dx$

$= \frac{1}{2}\int \left[1 - \cos\frac{3}{2}x\right]dx$✓

$= \frac{1}{2}x\checkmark - \frac{1}{3}\sin\frac{3}{2}x\checkmark + C$ (3)

4.1.4 $\int \frac{4x^3 - 6x^2 + 10x + 9}{2x - 1}\,dx$

$= \int \left(2x^2 - 2x + 4 + \frac{13}{2x-1}\right)dx$ ✓

$= \frac{2}{3}x^3\checkmark - x^2\checkmark + 4x\checkmark + \frac{13}{2}\ln(2x-1)\checkmark + C$ (5)

4.1.5 $$\int \cos(-x)\, e^{-4x}\, dx$$

| | |
|---|---|
| $let\ u = \cos(-x)$ | $dv = e^{-4x}\, dx$ |
| $du = -\sin(-x)\, dx$ | $v = -\frac{1}{4}e^{-4x}$ ✓ |

$$\int \cos(-x)\, e^{-4x}\, dx = -\frac{1}{4}e^{-4x}.\cos(-x) - \int -\sin(-x).-\frac{1}{4}e^{-4x}\, dx$$ ✓

$$= -\frac{1}{4}e^{-4x}.\cos(-x) - \frac{1}{4}\int \sin(-x).e^{-4x}\, dx$$

| | |
|---|---|
| $let\ u = \sin(-x)$ | $dv = e^{-4x}\, dx$ |
| $du = -\cos(-x)\, dx$ | $v = -\frac{1}{4}e^{-4x}$ ✓ |

$$= -\frac{1}{4}e^{-4x}.\cos(-x) - \frac{1}{4}\left[-\frac{1}{4}e^{-4x}.\sin(-x) - \int -\cos(-x).-\frac{1}{4}e^{-4x}\, dx\right]$$ ✓

$$= -\frac{1}{4}e^{-4x}.\cos(-x) + \frac{1}{16}e^{-4x}.\sin(-x) + \frac{1}{16}\int \cos(-x).e^{-4x}\, dx$$

$$\int \cos(-x)\, e^{-4x}\, dx + \frac{1}{16}\int \cos(-x).e^{-4x}\, dx = -\frac{1}{4}e^{-4x}.\cos(-x) + \frac{1}{16}e^{-4x}.\sin(-x)$$

$$\frac{17}{16}\int \cos(-x).e^{-4x}\, dx = -\frac{1}{4}e^{-4x}.\cos(-x) + \frac{1}{16}e^{-4x}.\sin(-x)$$

$$\int \cos(-x).e^{-4x}\, dx = -\frac{4}{17}e^{-4x}.\cos(-x) + \frac{1}{17}e^{-4x}.\sin(-x) + C$$ ✓ (5)

4.2 $$\int \frac{x-4}{2x^2-5x-3}dx$$

$$\frac{x-4}{(x-3)(2x+1)} = \frac{A}{x-3} + \frac{B}{2x+1}$$

$$x-4 = A(2x+1) + B(x-3)\checkmark$$

| |
|---|
| $laat\ x = 3; \qquad B = -\frac{7}{2}\checkmark$ <br> $of$ <br> $x = -\frac{1}{2}; \qquad A = \frac{9}{7}\checkmark$ |

$$\int \left(\frac{-\frac{7}{2}}{x-3} + \frac{\frac{9}{7}}{2x+3}\right)dx$$

$$= -\frac{7}{2}\ln(x-3)\checkmark + \frac{9}{14}\ln(2x+3)\checkmark + C$$ (5)

**[15]**

**VRAAG 5**

5.1 $$\int_0^8 xf'(x)\,dx$$

| |
|---|
| $let\ u = x \qquad dv = f'(x)\,dx$ <br> $du = dx \qquad v = f(x)\checkmark$ |

$$\int_0^5 xf'(x)\,dx = [xf(x)]_0^8 - \int_0^8 f(x)\,dx\checkmark$$

$$= 8 \times f(8) - 0 \times f(0) - (-12)\checkmark$$

$$= 12\ \checkmark$$ (4)

5.2 $\int_0^{\infty} 4^{-x}\,dx$

$= \lim\limits_{b\to\infty} \int_0^{b} 4^{-x}\,dx$ ✓

$= \lim\limits_{b\to\infty} \left[-\frac{4^{-x}}{\ln 4}\right]_0^{b}$ ✓

$= -\frac{1}{\ln 4} \lim\limits_{b\to 0}(4^{-b} - 4^{0})$

$= \frac{1}{\ln 4}$ ✓ (3)

5.3 5.3.1 $A = \int_a^{b} (y_1 - y_2)\,dx$

$A = \int_2^{4} (-x^2 + 7x - 8 - x)\,dx$

$= \int_2^{4} (-x^2 + 6x - 8)\,dx$ ✓

$= \left[-\frac{1}{3}x^3 + 3x^2 - 8x\right]_2^{4}$ ✓

$= \frac{4}{3} eenhede^2$ ✓ (3)

5.3.2 $V = \pi \int_a^{b} ({y_1}^2 - {y_2}^2)\,dx$

$= \pi \int_2^{4} [(-x^2 + 7x - 8)^2 - x^2]\,dx$

$= \pi \int_2^{4} (x^4 - 14x^3 + 15x^2 - 112x + 64)\,dx$ ✓✓

$= \pi \left[\frac{1}{5}x^5 - \frac{7}{2}x^4 + \frac{57}{3}x^3 - 56x^2 + 64\right]_2^{4}$ ✓

$= \frac{136\pi}{15} eenhede^3 \; of \; 28{,}484\, eenhede^3$ ✓ (4)

5.4 $d)_z = r^2 dA$
$dA = 2\pi x\, dx$✓

$x^2 . 2\pi x\, dx$✓

$2\pi \int_0^{15} x^3\, dx$✓

$= 2\pi \left[\frac{x^4}{4}\right]_0^{15}$ ✓

$= 2\pi \left[\frac{50625}{4} - 0\right]$ ✓

$= 79521.564\ cm^4$✓ (6)
**[20]**

**VRAAG 6**

6.1 $\frac{dy}{dx} = \frac{2x + 2xy^2}{y} \quad y(0) = -2$

$y\, dy = 2x(1 + y^2)\, dx$

$\frac{y\, dy}{1 + y^2} = 2x\, dx$ ✓

$\frac{1}{2}\ln(1 + y^2)$✓ $= x^2 + C$ ✓

$\frac{1}{2}\ln(1 + (-2)^2) = 0^2 + C$

$C = \ln\sqrt{5}$ ✓

$\frac{1}{2}\ln(1 + y^2) = x^2 + \ln\sqrt{5}$ ✓ (5)

6.2 $\frac{d^2y}{dx^2} = (x+2)^2 - 3$

$\frac{d^2y}{dx^2} = x^2 + 4x + 1$✓

$\frac{dy}{dx} = \frac{1}{3}x^3 + 2x^2 + x + A$✓

$y = \frac{1}{12}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2 + Ax + B$✓ (3)

**[8]**

**TOTAAL: 100**

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