higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N3

22 November 2024

This marking guideline consists of 12 pages.

MARKING LEGEND	
SYMBOL	**EXPLANATION OF SYMBOL**
A	Accuracy
M	Method
CA	Consistent accuracy (follow up)
B/D	Breakdown
S	Statement
R	Reason
S/R	Statement and reason
F	Choosing correct formula
SF	Substitution into correct formula

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QUESTION 1

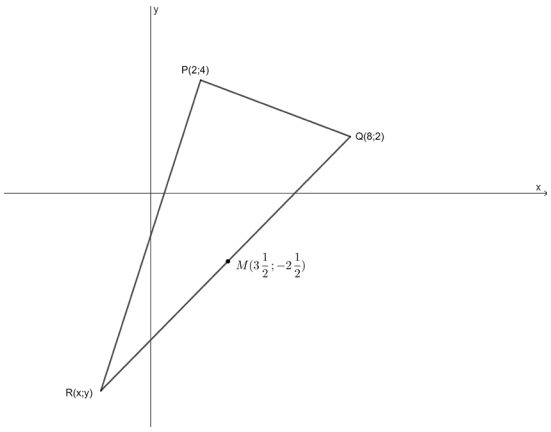
			MARKING INSTRUCTIONS	
1.1	$(3x^2+7x)^2-4(3x^2+7x)-12$ $=(3x^2+7x-6)(3x^2+7x+2)$ ✓ ✓ $=(3x-2)(x+3)(3x+1)(x+2)$ ✓ ✓ ✓ ✓	**ALTERNATIVE SOLUTION** $(3x^2+7x)^2-4(3x^2+7x)-12$ Let $a=3x^2+7x$ Then $a^2-4a-12$ $=(a-6)(a+2)$ $=(3x^2+7x-6)(3x^2+7x+2)$ ✓ ✓ $=(3x-2)(x+3)(3x+1)(x+2)$ ✓ ✓ ✓ ✓	1M for each correct factor 1 M for each correct factor when fully factorising the product.	(6)
1.2	$f(x)=x^3-2x^2-5x+6$ $\therefore f(1)=(1)^3-2(1)^2-5(1)+6$ $=1-2-5+6$ $=0$ ✓ Therefore $x-1$ is a factor of $f(x)$ ✓ x^2-x-6 ✓ $x-1\overline{)\,x^3-2x^2-5x+6}$ x^3-x^2 $-x^2-5x$ $-x^2+x$ ✓ $-6x+6$ $-6x+6$ $f(x)=(x-1)(x^2-x-6)$ ✓ ✓ $=(x-1)(x-3)(x+2)$ Synthetic division can also be used in the place of long division $1 \mid 1 \quad -2 \quad -5 \quad 6$ $\quad \mid 0 \quad 1 \quad -1 \quad -6$ ✓ ✓ ✓ $1 \quad -1 \quad -6 \quad 0$	**ALTERNATIVE SOLUTION** $f(x)=x^3-2x^2-5x+6$ $\therefore f(3)=(1)^3-2(1)^2-5(1)+6$ $=1-2-5+6$ $=0$ ✓ Therefore $x-3$ is a factor of $f(x)$ ✓ x^2+x-2 ✓ $x-3\overline{)\,x^3-2x^2-5x+6}$ x^3-3x^2 x^2-5x x^2-3x ✓ $-2x+6$ $-2x+6$ $f(x)=(x-3)(x^2+x-2)$ ✓ ✓ ✓ $=(x-3)(x+2)(x-1)$	1 mark for using the Remainder Theorem to find the first factor. 1 mark for first factor 2 marks for correctly using long division, synthetic division, or trial and error -1 M for dividend and 1 M for method. 2 marks for accurately factorising the dividend	(6)

			MARKING INSTRUCTIONS	
1.3	$2x^2 - 3x = 5$ $\therefore x^2 - \frac{3}{2}x = \frac{5}{2}$ ✓ $\therefore x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{9}{16} + \frac{5}{2}$ ✓ $\therefore \left(x - \frac{3}{4}\right)^2 = \frac{9+40}{16}$ ✓ $\therefore \left(x - \frac{3}{4}\right)^2 = \frac{49}{16}$ $\therefore x - \frac{3}{4} = \pm\frac{7}{4}$ ✓ $\therefore x = +\frac{3}{4} + \frac{7}{4}$ or $x = +\frac{3}{4} - \frac{7}{4}$ $\therefore x = \frac{10}{4}$ or $x = -\frac{4}{4}$ $\therefore x = \frac{5}{2}$ ✓ or $x = -1$ ✓		1M divide both sides by coefficient of x^2 1M for adding $(\frac{1}{2} \times \text{coefficient of } x)^2$ to both sides of the equation 1 M for writing the LHS as a perfect square trinomial 1 M for determining the square root of both sides of the equation 1M for each correct x value	(6)
				[18]
QUESTION 2				
2.1	$\frac{x^{\frac{1}{2}} - 64x^{-\frac{1}{2}}}{x^{\frac{1}{2}} - 16 + 64x^{-\frac{1}{2}}}$ $= \frac{x^{\frac{1}{2}} - \frac{64}{x^{\frac{1}{2}}}}{x^{\frac{1}{2}} - 16 + \frac{64}{x^{\frac{1}{2}}}}$ ✓ $= \frac{x - 64}{x - 16x^{\frac{1}{2}} + 64}$ ✓ $= \frac{(x^{\frac{1}{2}} - 8)(x^{\frac{1}{2}} + 8)}{(x^{\frac{1}{2}} - 8)(x^{\frac{1}{2}} - 8)}$ ✓ ✓ $= \frac{x^{\frac{1}{2}} + 8}{x^{\frac{1}{2}} - 8}$ ✓	**ALTERNATIVE SOLUTION** $\frac{x^{\frac{1}{2}} - 64x^{-\frac{1}{2}}}{x^{\frac{1}{2}} - 16 + 64x^{-\frac{1}{2}}}$ $= \frac{\left(x^{\frac{1}{4}}\right)^2 - \left(8x^{-\frac{1}{4}}\right)^2}{\left(x^{\frac{1}{4}}\right)^2 - 16 + \left(8x^{-\frac{1}{4}}\right)^2}$ ✓ ✓ $= \frac{\left(x^{\frac{1}{4}} - 8x^{-\frac{1}{4}}\right)\left(x^{\frac{1}{4}} + 8x^{-\frac{1}{4}}\right)}{\left(x^{\frac{1}{4}} - 8x^{-\frac{1}{4}}\right)\left(x^{\frac{1}{4}} - 8x^{-\frac{1}{4}}\right)}$ ✓ ✓ $= \frac{x^{\frac{1}{4}} + 8x^{-\frac{1}{4}}}{x^{\frac{1}{4}} - 8x^{-\frac{1}{4}}}$ ✓	2 M for simplifying 2 M for factorising all numerators and denominators 1M for answer	(5)

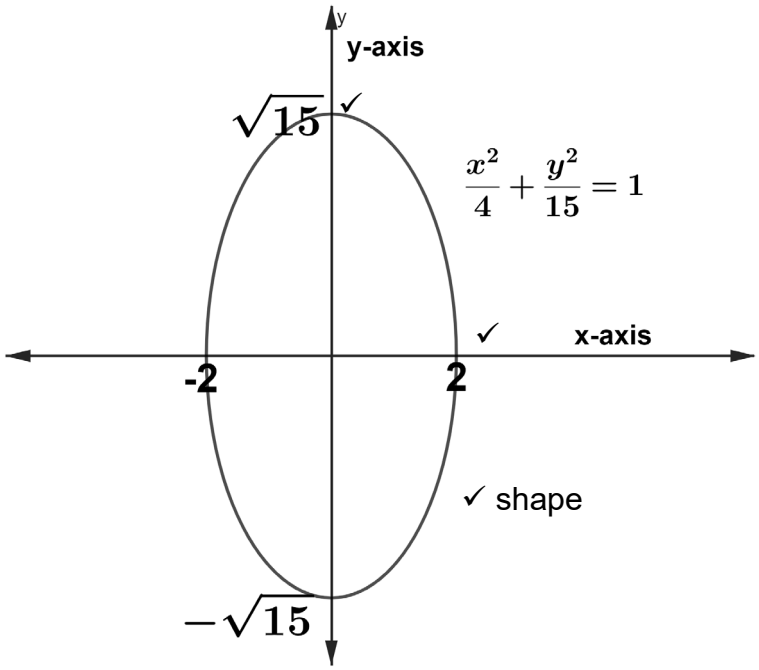
			MARKING INSTRUCTIONS	
2.2	$log_{\frac{1}{2}}(3-x) = log_2\left(\frac{1}{x^2}\right) + 2$	**ALTERNATIVE SOLUTION** $log_{\frac{1}{2}}(3-x) = log_2\left(\frac{1}{x^2}\right) + 2$		
	$log_{\frac{1}{2}}(3-x) = \frac{log_{\frac{1}{2}}\left(\frac{1}{x^2}\right)}{\log_{\frac{1}{2}} 2} + 2$ ✓	$\frac{log_2(3-x)}{\log_2\left(\frac{1}{2}\right)} = log_2\left(\frac{1}{x^2}\right) + 2$ ✓	1M -writing the logs to the base ½ or 2 by using the log law [$\log_b a = \frac{\log_c a}{\log_c b}$]	
	$log_{\frac{1}{2}}(3-x) = \frac{log_{\frac{1}{2}}\left(\frac{1}{x^2}\right)}{-1} + 2$	$\frac{log_2(3-x)}{\log_2 2^{-1}} = log_2\left(\frac{1}{x^2}\right) + 2$		
	$log_{\frac{1}{2}}(3-x) = -log_{\frac{1}{2}}\left(\frac{1}{x^2}\right) + 2$ ✓	$-log_2(3-x) = log_2\left(\frac{1}{x^2}\right) + 2$ ✓	1M simplifying	
	$log_{\frac{1}{2}}(3-x) + log_{\frac{1}{2}}\left(\frac{1}{x^2}\right) = 2$ ✓		1M -transpose log-term to LHS	
		$-log_2(3-x) - log_2(x^{-2}) = 2$	1M-write the LHS as single logarithm	
	$log_{\frac{1}{2}}\frac{(3-x)}{x^2} = +2$ ✓	$-log_2(3-x) + log_2 x^2 = 2$ ✓	1M -rewrite in exponential form	
	$\frac{(3-x)}{x^2} = \left(\frac{1}{2}\right)^2$ ✓	$log_2\frac{x^2}{(3-x)} = +2$ ✓		
	$12 - 4x = x^2$ $x^2 + 4x - 12 = 0$ ✓ $(x+6)(x-2) = 0$	$2^2 = \frac{x^2}{(3-x)}$	1 M -write in general form	
	$x \neq -6$ ✓or $x = 2$ ✓	$4 = \frac{x^2}{(3-x)}$ ✓	1M-valid root 1M-invalid root	
		$12 - 4x = x^2$ $x^2 + 4x - 12 = 0$ ✓ $(x+6)(x-2) = 0$ $x \neq -6$ ✓or $x = 2$ ✓		(8)
				[13]

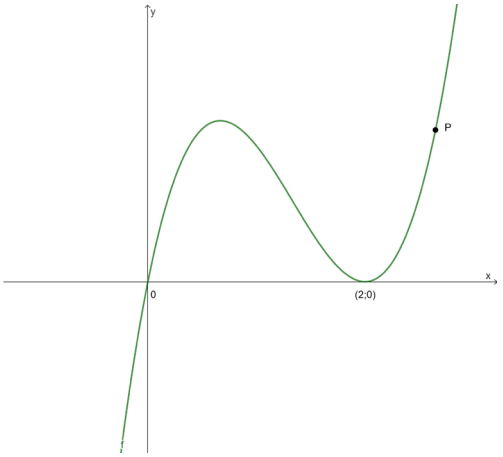
QUESTION 3

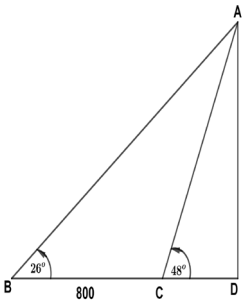
			MARKING INSTRUCTIONS	
3.1	$A = De^{rt}$ $\frac{A}{D} = e^{r\,t}$ ✓ $ln\frac{A}{D} = ln\,e^{r\,t}$ ✓ $ln\frac{A}{D} = rt\,ln\,e$ $ln\frac{A}{D} = rt$ ✓ $\frac{ln\frac{A}{D}}{r} = t$ ✓ $or \quad \frac{ln\,A - ln\,D}{r} = t$	**ALTERNATIVE SOLUTION** $A = De^{r\,t}$ $\frac{A}{D} = e^{r\,t}$ ✓ $log\frac{A}{D} = log\,e^{r\,t}$ ✓ $log\frac{A}{D} = rt\,log\,e$ ✓ $\frac{log\frac{A}{D}}{r\,log\,e} = t$ ✓ $or \quad \frac{log\,A - log\,D}{r\,log\,e} = t$	1M -divide both sides by D 1M write the logarithm ln on both sides 1M simplification 1M final answer	(4)
3.2	Let the number be x . $\therefore 3x^2 + 7x = 20$ ✓ $\therefore 3x^2 + 7x - 20 = 0$ $\therefore (3x - 5)(x + 4) = 0$ ✓ $\therefore 3x = 5 \quad or \quad x = -4$ $\therefore x = \frac{5}{3}$ ✓ or $x = -4$ ✓		1M -set up an equation 1M -Solve the equation 1M for each correct root	(4)
3.3	$\frac{3}{x-1} - \frac{1}{x^2-4x+3} - \frac{1}{1-x} = 1$ $\therefore \frac{3}{x-1} - \frac{1}{(x-1)(x-3)}$ ✓ $+ \frac{1}{x-1}$ ✓ $= 1$ $\therefore 3(x-3)$ √ $- 1$ √ $+ (x-3)$ √ $= (x-3)(x-1)$ √ $\therefore 3x - 9 - 1 + x - 3 = x^2 - 4x + 3$ $\therefore x^2 - 8x + 16 = 0$ ✓ $\therefore (x-4)(x-4) = 0$ $\therefore x = 4$ √ $or\ x = 4$ √		2M -factorising 2nd and 3rd fraction ½ M for multiplying each term by the LCD 1M write it in general form 1M for writing solution	(6)
				[14]

QUESTION 4			
		MARKING INSTRUCTIONS	
			
4.1	$M_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4-2}{2-8}$ ✓ $= \frac{2}{-6}$ ✓ $= -\frac{1}{3}$ $\theta = tan^{-1}\left(-\frac{1}{3}\right) = 180° - 18{,}43°$ ✓ $\theta = 161{,}6°$ ✓	1M SF (substituting in the correct formula) 1M answer 1M CA(follow up from gradient) 1M answer	(4)
4.2	$x = \frac{x_1+x_2}{2}$ $y = \frac{y_1+y_2}{2}$ $\frac{7}{2} = \frac{8+x}{2}$ ✓ $-\frac{5}{2} = \frac{2+y}{2}$ ✓ $x = -1$ ✓ $y = -7$ ✓ $\therefore$ The coordinates of $R = (-1; -7)$	1M for x and one marks for y, substituting in the correct formula 1 M each for answers	(4)
4.3	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $PQ = \sqrt{(8-2)^2 + (2-4)^2}$ ✓ $= \sqrt{36+4}$ $= \sqrt{40}$ =6,325 or $2\sqrt{5}$ ✓	1M SF 1 M for answer	(2)

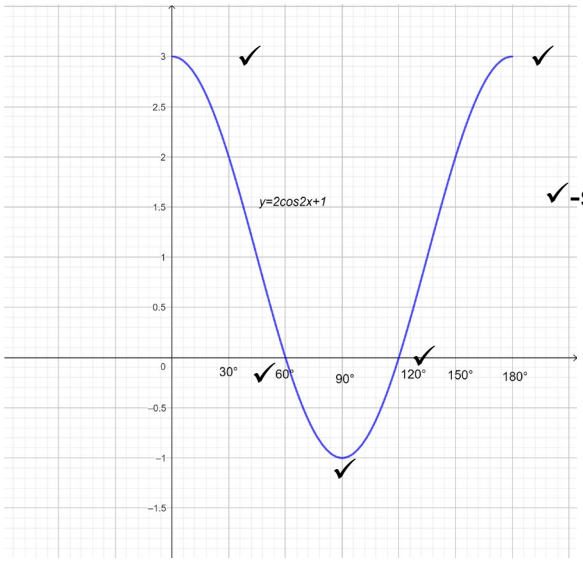
			MARKING INSTRUCTIONS	
4.4	$y - y_1 = m(x - x_1)$ But $M_{PQ} \times M = -1$ (lines $\perp$) $\therefore -\frac{1}{3} \times M = -1$ $\therefore M = 3$ ✓ Midpoint $of\ PQ$: $(x; y)$ $= \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $= \left(\frac{8+2}{2}; \frac{2+4}{2}\right)$ ✓ ✓ $= (5; 3)$ $\therefore y - 3 = 3(x - 5)$ $y - 3 = 3x - 15$ ✓ $0 = 3x - y - 12$ or $y = 3x - 12$ ✓	**ALTERNATIVE SOLUTION** $y = mx + c$ But $M_{PQ} \times M = -1$ (lines $\perp$) $\therefore -\frac{1}{3} \times M = -1$ $\therefore M = 3$ ✓ Midpoint $of\ PQ$: $(x; y)$ $= \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $= \left(\frac{8+2}{2}; \frac{2+4}{2}\right)$ ✓ ✓ $= (5; 3)$ $\therefore y = 3x + c$ $\therefore 3 = 3(5) + c$ $\therefore 3 = 15 + c$ or ✓ $\therefore c = -12$ $\therefore y = 3x - 12$ ✓	1M CA(follow up from gradient in 4.1) 2M for midpoint(x;y) 1M SF 1M solution	(5)
4.5	$MN \parallel PQ$ $\therefore M_{MN} = M_{PQ}$ $\therefore M_{MN} = -\frac{1}{3}$ ✓ $\therefore y - y_1 = m(x - x_1)$ $\therefore y + \frac{5}{2} = -\frac{1}{3}\left(x - \frac{7}{2}\right)$ ✓ $\therefore y + \frac{5}{2} = -\frac{x}{3} + \frac{7}{6}$ $\therefore 6y + 15 = -2x + 7$ ✓ $\therefore 2x + 6y + 8 = 0$ *or* $\therefore x + 3y + 4 = 0$✓	**ALTERNATIVE SOLUTION** $MN \parallel PQ$ $\therefore M_{MN} = M_{PQ}$ $\therefore M_{MN} = -\frac{1}{3}$ ✓ $\therefore y = mx + c$ $\therefore y = -\frac{1}{3}x + c$ $\therefore -\frac{5}{2} = -\frac{1}{3} \times \frac{7}{2} + c$ ✓ $\therefore -15 = -7 + 6c$ $\therefore -8 = 6c$ $\therefore c = -\frac{8}{6} = -\frac{4}{3}$ $\therefore y = -\frac{1}{3}x - \frac{4}{3}$ ✓ $\therefore 3y = -x - 4$ $\therefore x + 3y + 4 = 0$ ✓	1M CA from 4.1 1M SF 1M simplify 1M writing it in general form	(4)
				[19]

QUESTION 5			
		MARKING INSTRUCTIONS	
5.1	$\frac{x^2}{4}+\frac{y^2}{15}=1$  	1M : x-intercepts at $(2;0),(-2;0)$ 1M: y-intercept at $(0;\sqrt{15})$; $(0;-\sqrt{15})$ 1M for shape	(3)
5.2	$y=\frac{\sqrt{x}}{2}+\frac{2}{\sqrt[4]{x}}$ $y=\frac{x^{\frac{1}{2}}}{2}+\frac{2}{x^{\frac{1}{4}}}$ $=\frac{1}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{4}}$ $\frac{dy}{dx}=\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}+2\times\left(-\frac{1}{4}\right)x^{-\frac{5}{4}}$ $=\frac{1}{4x^{\frac{1}{2}}}-\frac{1}{2x^{\frac{5}{4}}}$ $=\frac{1}{4\sqrt{x}}-\frac{1}{2\sqrt[4]{x^5}}$	½ marks for each term , converting the radical to fractional form(the preparation step) 1 Mark for differentiating each term, using the power rule Deduct 1 mark if the derivative notation is placed incorrectly or is missing ½ mark for each correct derivative in radical form	(4)

			MARKING INSTRUCTIONS	
5.3				
	5.3.1	$f(x) = x(x-2)(x-2)$ ✓ $f(x) = x(x^2 - 4x + 4)$ $f(x) = x^3 - 4x^2 + 4x$ ✓ From $f(x) = x^3 + bx^2 + cx$ follows that $b = -4$ and $c = 4$ ✓	1M SF 1M simplify 1 mark for correctly reading off the values of **b** and **c**.	(3)
	5.3.2	$f(x) = x^3 - 4x^2 + 4x$ $\therefore f'(x) = 3x^2 - 8x + 4$ $\therefore 0 = 3x^2 - 8x + 4$ ✓ ✓ $\therefore 3x^2 - 8x - 16 = 0$ $\therefore (3x+4)(x-4) = 0$ ✓ $\therefore 3x = -4$ or $x = 4$ $\therefore x \neq -\frac{4}{3}$ $x = 4$ ✓ *If* $x = 4$ then $f(4) = (4)^3 - 4(4)^2 + 4(4)$ $= 64 - 64 + 16$ $= 16$ ✓ The coordinates of P are $(4; 16)$	1M for derivative and 1 mark for gradient 1M factorisation 1M for valid x 1M for y	(5)
				[15]

QUESTION 6					
				MARKING INSTRUCTIONS	
6.1	$sin^4x - cos^4x = 1 - 2cos^2x$ LHS = $sin^4x - cos^4x$ $= (sin^2x + cos^2x)(sin^2x - cos^2x)$ ✓ $=1 \times$ ✓$(sin^2x - cos^2x)$ $= 1 - cos^2x - cos^2x$ ✓ $= 1 - 2cos^2x$ ✓ $=$ RHS			1M factorisation 1M for applying the square identities 2M simplification	(4)
6.2	$5\sin\theta = 8\cos\theta$ $\therefore \frac{\sin\theta}{\cos\theta} = \frac{8}{5}$ ✓ $\therefore \tan\theta = 1{,}6$ ✓ $\theta = 57{,}995°$ or $\theta = 180° + 57{,}995°$ $\theta = 57{,}995°$ ✓ or $\theta = 237{,}995°$ ✓			1M cross multiply 1M convert to one ratio 2M for correct angles	(4)
6.3	6.3.1	In △ ABC : $B\hat{A}C = 48^o - 26^o = 22^o$ ✓ $\therefore \frac{AC}{\sin 26^o} = \frac{800}{\sin 22^o}$ ✓ $\therefore AC = \frac{800 \sin 26^o}{\sin 22^o}$ $\therefore AC = 936{,}174$ ✓ In ΔACD: $\frac{AD}{AC} = sin\ 48^o$ ✓ $AD = 936{,}174 \times sin\ 48^o$ $= 695{,}713m$ ✓		1M SF 1 mark for calculating angle ∠BAC either in the formula or in the initial steps. 1M value of A 1M SF 1M value of AD	(5)

				MARKING INSTRUCTIONS	
	6.3.2	In Δ ACD: $\frac{\text{CD}}{\text{AD}} = \cot 48^{\text{o}}$ $\text{CD} = 695{,}713 \times \cot 48^{\text{o}}$ ✓ $= 626{,}423m$ ✓ **ALTERNATIVE SOLUTION 2** In Δ ABD $\tan 26^{\text{o}} = \frac{\text{AD}}{\text{BD}}$ $\therefore tan\, 26^{o} = \frac{695{,}713}{BD}$ $\therefore BD = \frac{695{,}713}{tan\, 26^{o}}$ $\therefore BD = 1426{,}423$ ✓ $\therefore CD = 1426{,}423 - 800$ $= 626{,}423m$ ✓	**ALTERNATIVE SOLUTION 1** $\frac{\text{CD}}{\text{AC}} = \cos 48^{\text{o}}$ $\therefore \text{CD} = 936{,}174 \times \cos 48^{\text{o}}$ ✓ $= 626{,}423m$ ✓	1 M SF 1M answer	(2)

6.4	$y = 2cos2x + 1$  	2 Marks for x-intercepts (60°;0) and (120°; 0) 1 Mark for y-intercept (0°;3) 2 Marks for turning points (90°; -1) and (180°;3) The other turning point is already described under the y-intercept (do not count it again) 1M shape- The graph is drawn over the interval 0° and 180° The graph have a minimum TP at (90°;-1) and maximum TPs at (0°;3) and (180°;3) and x-intercepts at (60°;0) and (120°; 0)	(6)
			[21]
		TOTAL:	**100**

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