higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
NQF LEVEL 4
(First paper)

(10501064)

9 November 2023 (X-paper)
09:00–12:00

Nonprogrammable scientific calculators may be used.

This question paper consists of 10 pages, 2 answer sheets and a formula sheet of 2 pages.

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)
MATHEMATICS
NQF LEVEL 4
PAPER 1
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Leave at least THREE lines after each question.
5. Diagrams are not drawn to scale.
6. Round off answers to TWO decimals where necessary unless stated otherwise.
7. Use only a black or blue pen.
8. Write neatly and legibly.
9. Answer QUESTION 2.5.3 on ANSWER SHEET 1 and QUESTION 6.2.3 on ANSWER SHEET 2.

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QUESTION 1

1.1 Various options are given as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5) in the ANSWER BOOK.

1.1.1 If $z_1 = 6 + 4i$ and $z_2 = 8 - 3i$, then $z_1 \times z_2$ in standard form is …

A $36 + 14i$.
B $60 + 14i$.
C $48 - 12i^2$.
D $36 - 50i$.

1.1.2 i^{15} simplified is equal to …

A $15i^{14}$.

B $\frac{i^{16}}{16}$.

C i.

D $-i$.

1.1.3 The factors for $x^2 + 25$ are …

A $x(x + 25)$.
B $(x - 5)(x + 5)$.
C $(x - 5i)(x + 5i)$.
D $(x + 5)(x + 5)$.

1.1.4 If $z = 5cis(60°)$, then $z^3 = \ldots$

A $5^3 cis(60°)^3$
B $5^3 cis(60° + 3)$
C $15cis(3 \times 60°)$
D $5^3 cis(3 \times 60°)$

1.1.5 Solve y if $5 - yi = 15 - 3i - 10$ where $i = \sqrt{-1}$.

A $y = -3i$

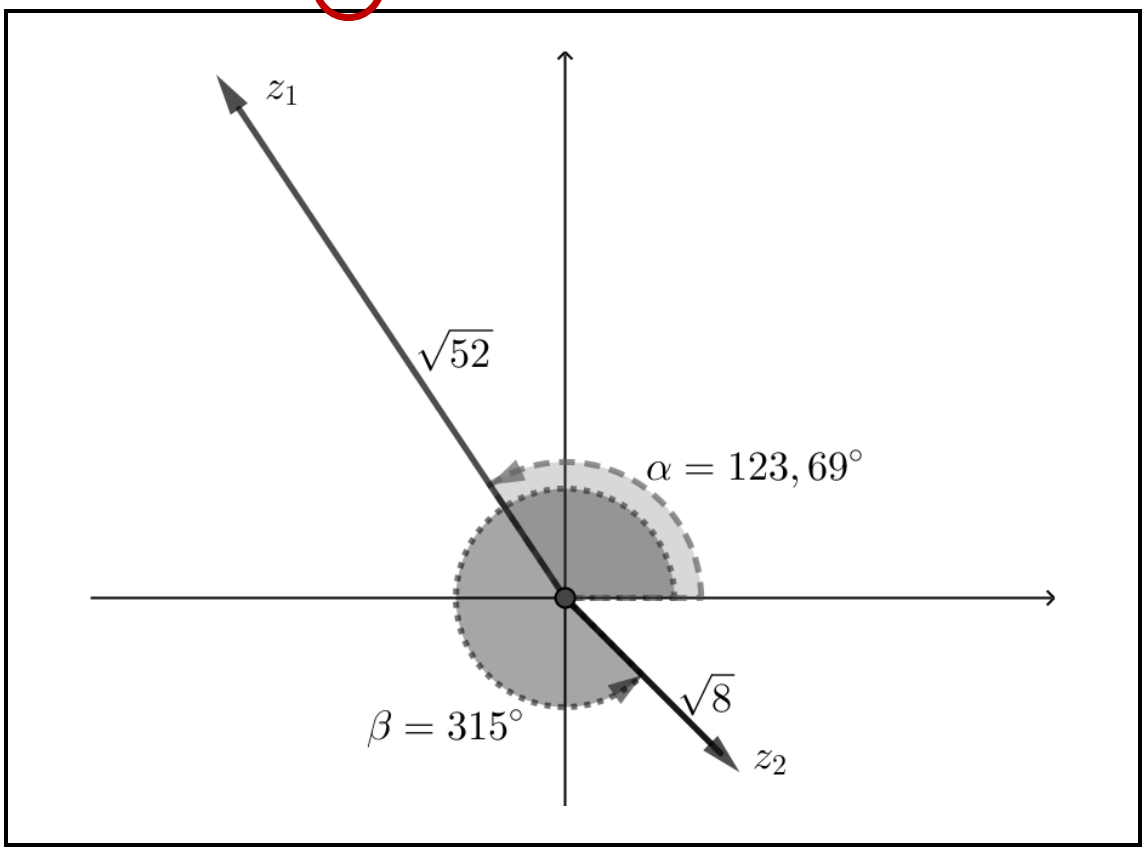
B $y = 3$

C $y = -3$

D $y = \frac{1}{3}$

(5 × 1) (5)

1.2 The Argand diagram below represents two complex numbers: $z_1 = r_1 \text{ cis } \alpha°$ and $z_2 = r_2 cis\ \beta°$.



1.2.1 Write z_1 in standard form. (1)

1.2.2 Determine $z_1 \times z_2$ and leave the answer in polar form with $0 \leq \theta \leq 360$. (2)

1.2.3 Determine $z_1 + z_2$ and leave the answer in the form $a \pm bi$. (2)

1.3 Simplify to the following complex expression and write the answer in the form $a \pm bi$. Use THREE decimals and leave the answer in polar form with $0 \leq \theta \leq 360$. Calculations in rectangular form are excluded.

$$\frac{(1-i)^5 \times 2cis30°}{(12-8i)}$$ (3)

1.4 Solve for x if: $x^3 - 6x^2 = -34x$. (3)

1.5 Solve for x and y in the following equation and express any nonreal solutions in the form $a \pm bi$ for all values where $y \in Z$:

$$3 - ix^2y = -(x^2 + y) + 4i$$ (4)

[20]

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QUESTION 2

2.1 Given $f(x) = 12x^3 - 28x^2 - 3x + 7$

Prove that $(2x - 1)$ is a factor of $f(x)$ by applying the factor theorem.
Long division is excluded. (2)

2.2 The expression $x^3 - ax^2 + ax + b$ is divisible by $(x + 1)$, $(x - 3)$ and $(x + m)$.

2.2.1 Determine the values of a and b. (3)

2.2.2 Hence determine the value of m. (2)

2.3 Given the following set of ordered pairs: $f = \{(-3; 5); (-2; 5); (0; 7); (2; 9)\}$

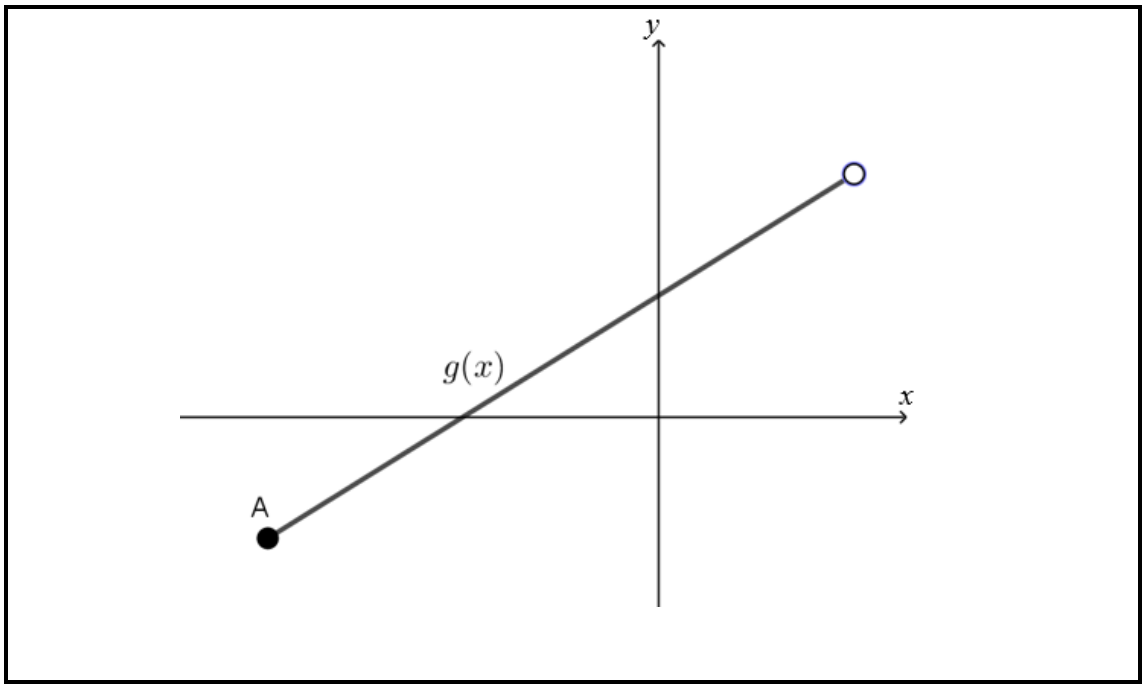
2.3.1 Write f^{-1} as a set of ordered pairs. (1)

2.3.2 State whether f^{-1} is a function or nonfunction. (1)

2.3.3 Write down the x-intercept for f^{-1}. (1)

2.3.4 Write down the domain of f^{-1}. (1)

2.4 The graph below represents the following function with the general equation given: $g(x): y = 2x + 4$ with the domain $\{x: -4 \leq x < 2; x \in R\}$.



2.4.1 Determine the equation of $g^{-1}(x)$ in the form $y = \ldots$ (1)

2.4.2 Analyse the graph and write down the domain of $g^{-1}(x)$. (2)

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2.5 Given $h(x) = -4x^2$ where $x \in (-2; 0]$

2.5.1 Determine the equation of $h^{-1}(x)$ in the form $y = \ldots$ (1)

2.5.2 Determine the point of intersection of $h(x)$ and $h^{-1}(x)$. (3)

2.5.3 Draw a neat graph of $h(x)$ and $h^{-1}(x)$ on the same system of axes on ANSWER SHEET 1 (attached). Clearly show the line about which h^{-1} reflects $h(x)$ and label it. Show at least one point on the line of symmetry. (3)

[21]

QUESTION 3

3.1 Given the following equation: $f(x) = 2x^3$

Determine the derivative of $f(x)$ by using first principles. Show all steps. (3)

3.2 Use differentiation rules to determine the derivatives of each of the following leaving the answer with positive exponents and in surd form where applicable:

3.2.1 $y = \frac{4}{5}x^5 + \frac{x}{2} + e^{4x} + \cos 30°$ (2)

3.2.2 $y = 3\ln 5x - 3\sin 5x - \frac{2}{x^5} + 3\sqrt{x}$ (4)

3.2.3 $y = \frac{1}{(2x^3 - 4)^4}$ (3)

[12]

QUESTION 4

4.1 Determine and simplify the following integrals, leaving answers with positive exponents and in surd form where applicable:

4.1.1 $\int \left(\frac{2}{x} - sec^2\, 5x + 4e^{2x} + 2x + 5\right) dx$ (3)

4.1.2 $\int \left(\frac{4}{x^2} - \frac{5}{2\sqrt{x}} + 6\, sin\, 3\, x\right) dx$ (3)

4.1.3 $\int \left(\frac{x^2 - 5x + 6}{x - 3}\right) dx$ (2)

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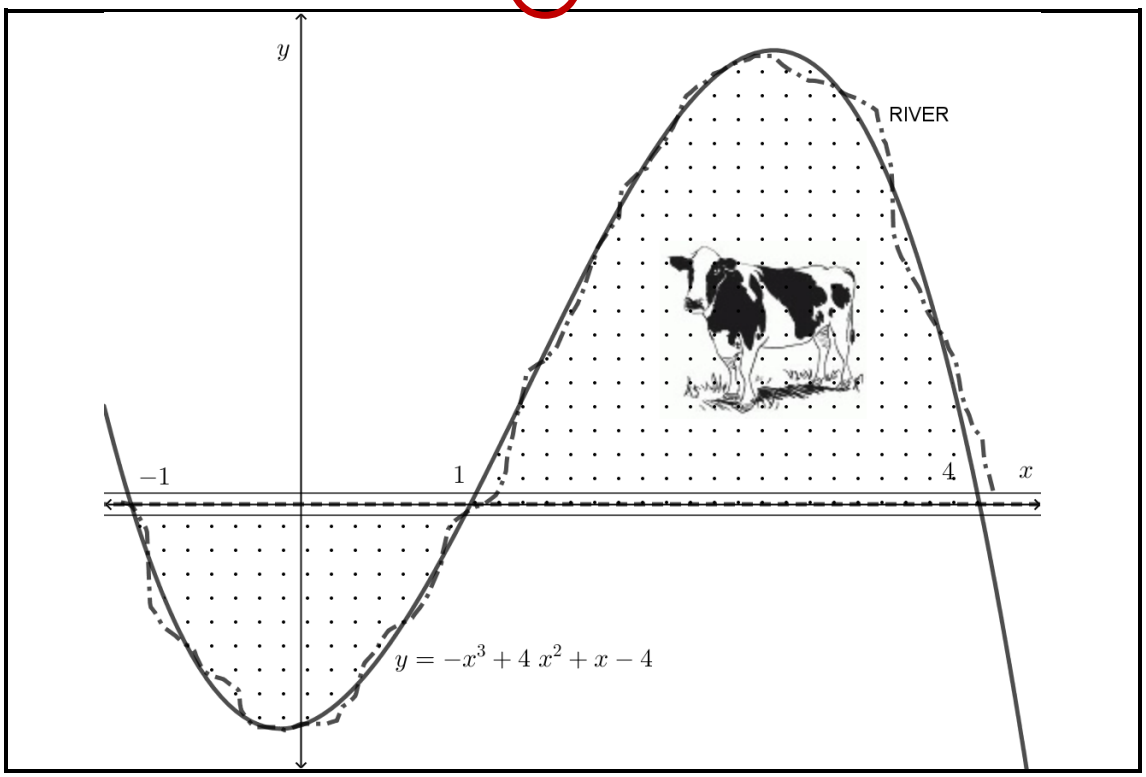
4.2 A cattle farm's borders are formed by a river. The equation that represents the borders is $y = -x^3 + 4x^2 + x - 4$.

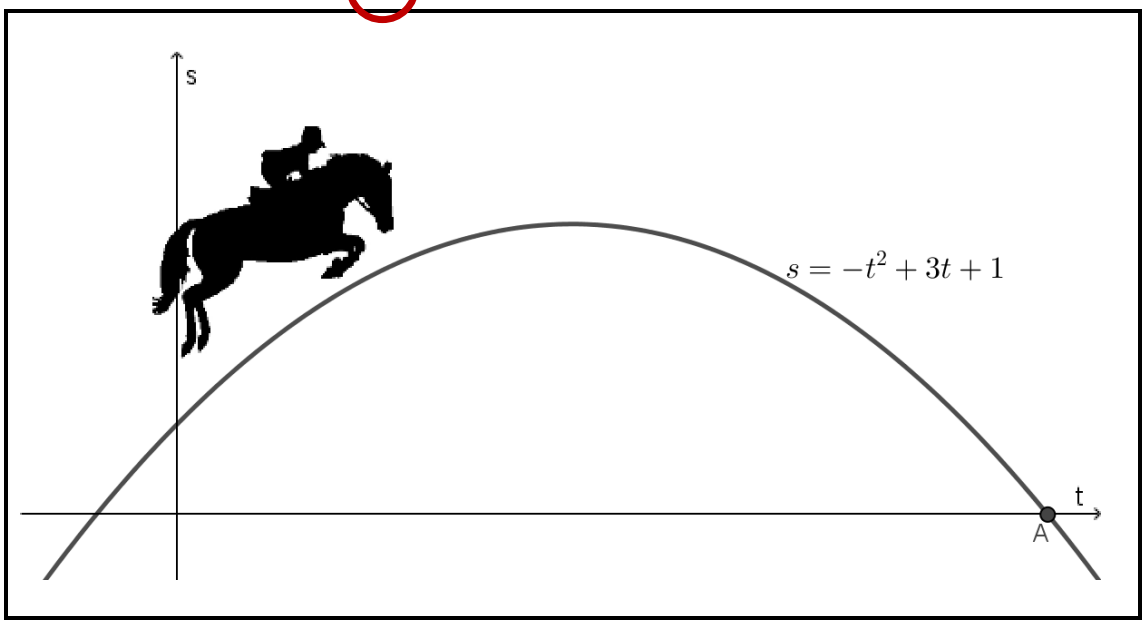


Calculate the magnitude of the farm area by using integration. The farm is bounded by the graph and the *x*-axis, the lines $x = -1$ and $x = 4$. (5)

[13]

QUESTION 5

5.1 A show jumper attempts to break a world record. His motion is expressed in terms of distance (s) in meters and time (t) in seconds. The equation of motion is $s(t) = -t^2 + 3t + 1$.



5.1.1 Determine the height of the show jumper at $t = 3$. (1)

5.1.2 (a) Determine the maximum height of the show jumper. (3)

(b) The current world record is 2,47 m.

By how many centimetres did he exceed the world record or was short of to reach the world record? (1)

5.1.3 Determine the time when the show jumper lands at point A. (2)

5.2 The third-degree polynomial $f(x) = 2x^3 + 3x^2 - px - 20$ is given.

5.2.1 Given that $f'(x) = 6x^2 + 6x + 12$, determine the value of p in $f(x)$. (2)

5.2.2 $(x + 2)$ is a factor of $f(x)$. Hence, determine the other factors of $f(x)$. (3)

5.3 Given $g(x) = 2x^3 - 7x^2 - 14x + 40$

5.3.1 Determine the gradient of $g(x)$ at $x = -2$ and $x = 1$ by applying differentiation. (3)

5.3.2 Hence, explain what happened with the curve between $x = -2$ and $x = 1$ and give a reason for the conclusion. (2)

5.3.3 Determine the point of inflection of $g(x)$. (2)

[19]

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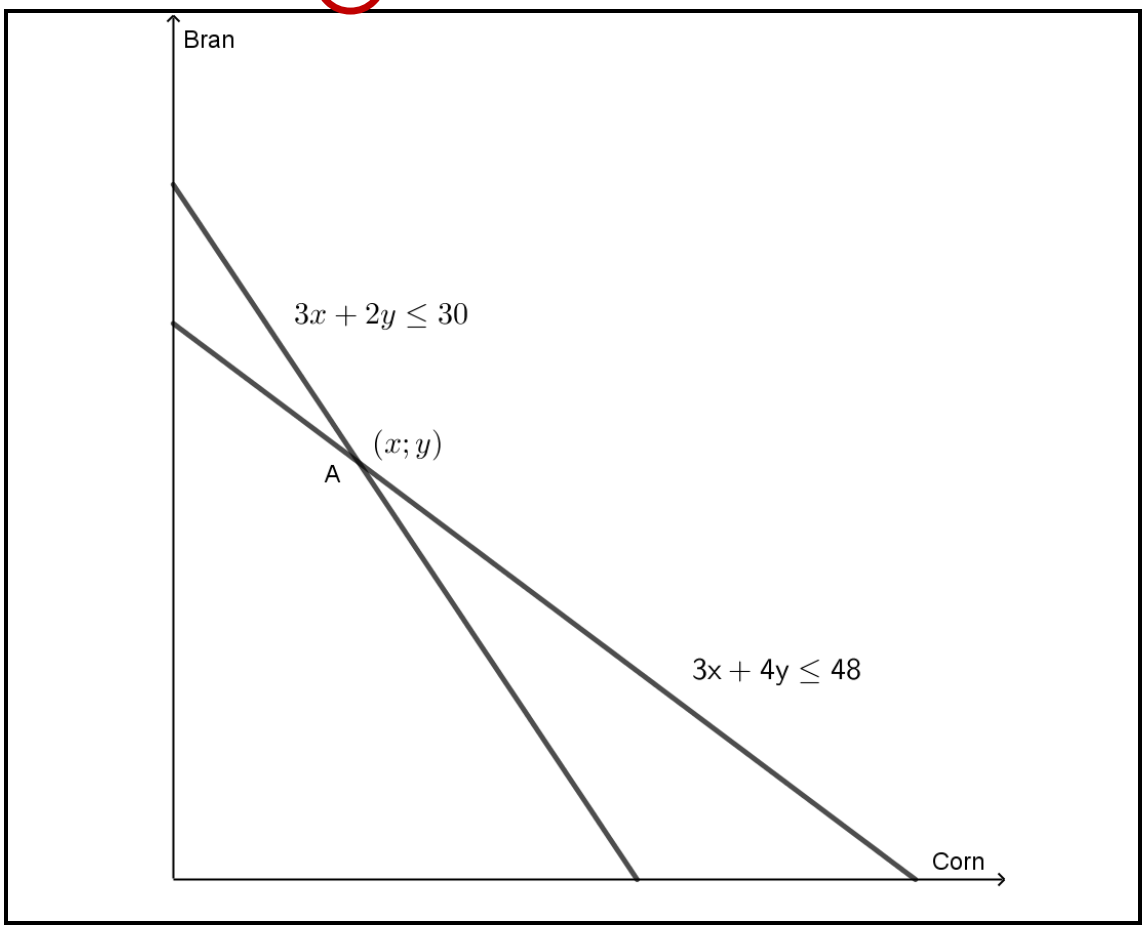
QUESTION 6

6.1 A baker bakes bran and corn muffins. A batch of corn muffins requires three cups of milk and three cups of flour. A batch of bran muffins needs four cups of flour and two cups of milk. In the bakery he has 48 cups of flour and 30 cups of milk. The baker makes T rand profit on each batch of bran muffins and Q rand on each batch of corn muffins. Let x be the number of batches of corn muffins and y be the number of batches of bran muffins.



6.1.1 Consider the points $(10; 2)$ and $(2; 10)$. (1)

(a) Which ONE of these points is not within the feasible region?

(b) Prove the answer to QUESTION 6.1.1 a) with a calculation. (1)

6.1.2 Determine the values of Q and T in the objective function $\boldsymbol{P = Qx + Ty}$ if it is known that the maximum profit at point $(\mathbf{4}; \mathbf{9})$ is R43 and the profit for five bran muffins and five corn muffins is R35. (3)

6.2 An aeroplane with executive and economy class seats can carry a maximum of 180 passengers. The profit made on an executive class ticket is R1 600 and on an economy class ticket R800. The airline reserves at most 150 seats for the economy class. The airline has a policy that the number of executive seats should not exceed one third of that of the economy class.

Let x be the number of executive class tickets and y the number of economy class tickets.

6.2.1 The profit made on an executive class ticket is R1 600 and on an economy class ticket R800.

Write the profit equation in terms of x and y. (1)

6.2.2 Two of the constraints are $x \geq 0$ and $y \geq 0$. What are the other constraints in respect of the given information in terms of x and y. (3)

6.2.3 Use ANSWER SHEET 2 (attached) to complete the following questions:

(a) Represent the inequalities graphically. (3)

(b) Indicate the feasible region on the graph. (1)

(c) Use the search line to determine the point that will inform you on how many tickets of each type would maximise the profit of the airline company. Indicate the search line and the maximum point on the graph. What is the maximum profit? (2)

[15]

TOTAL: 100

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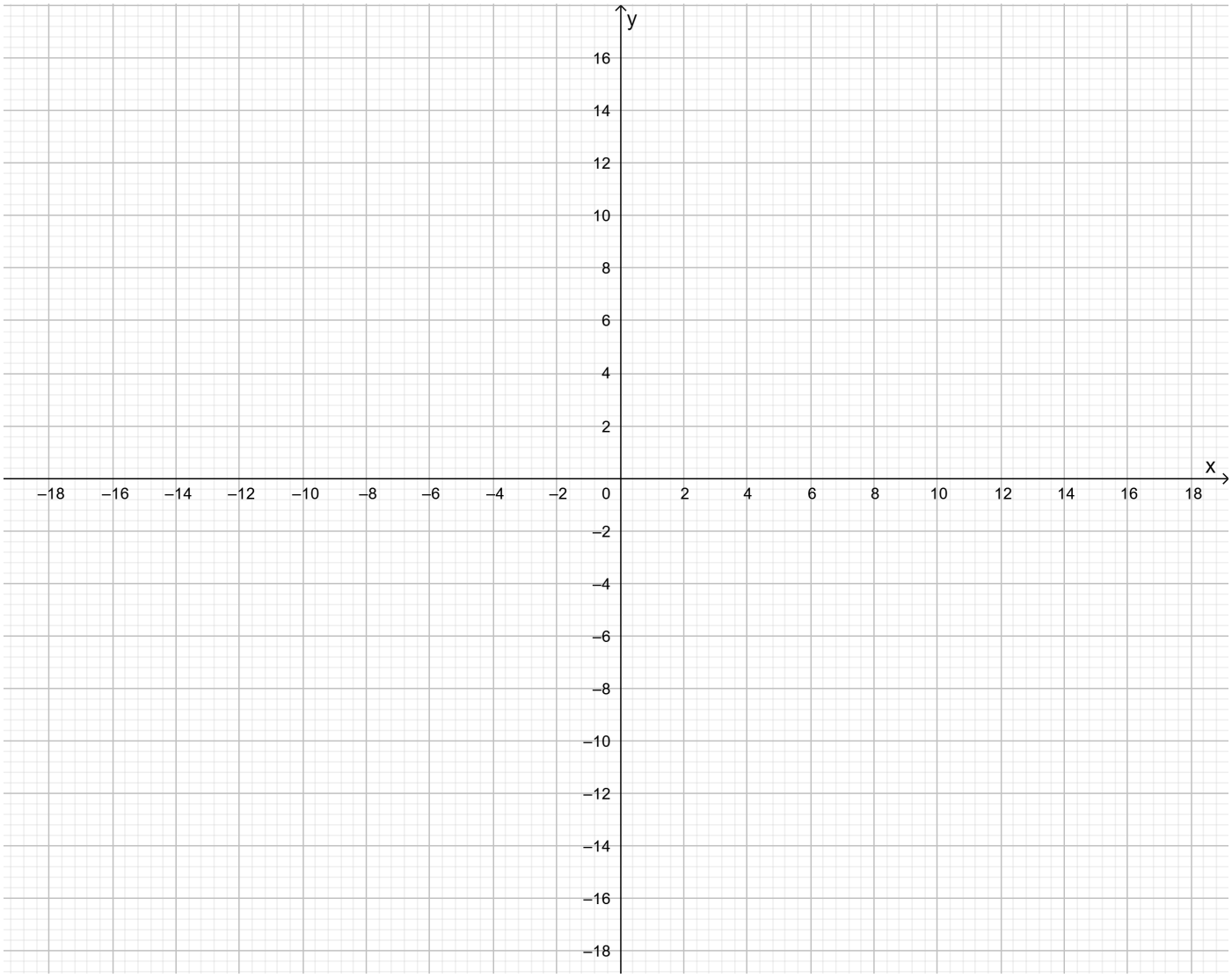
ANSWER SHEET 1 **EXAMINATION NUMBER:**

2.5.3



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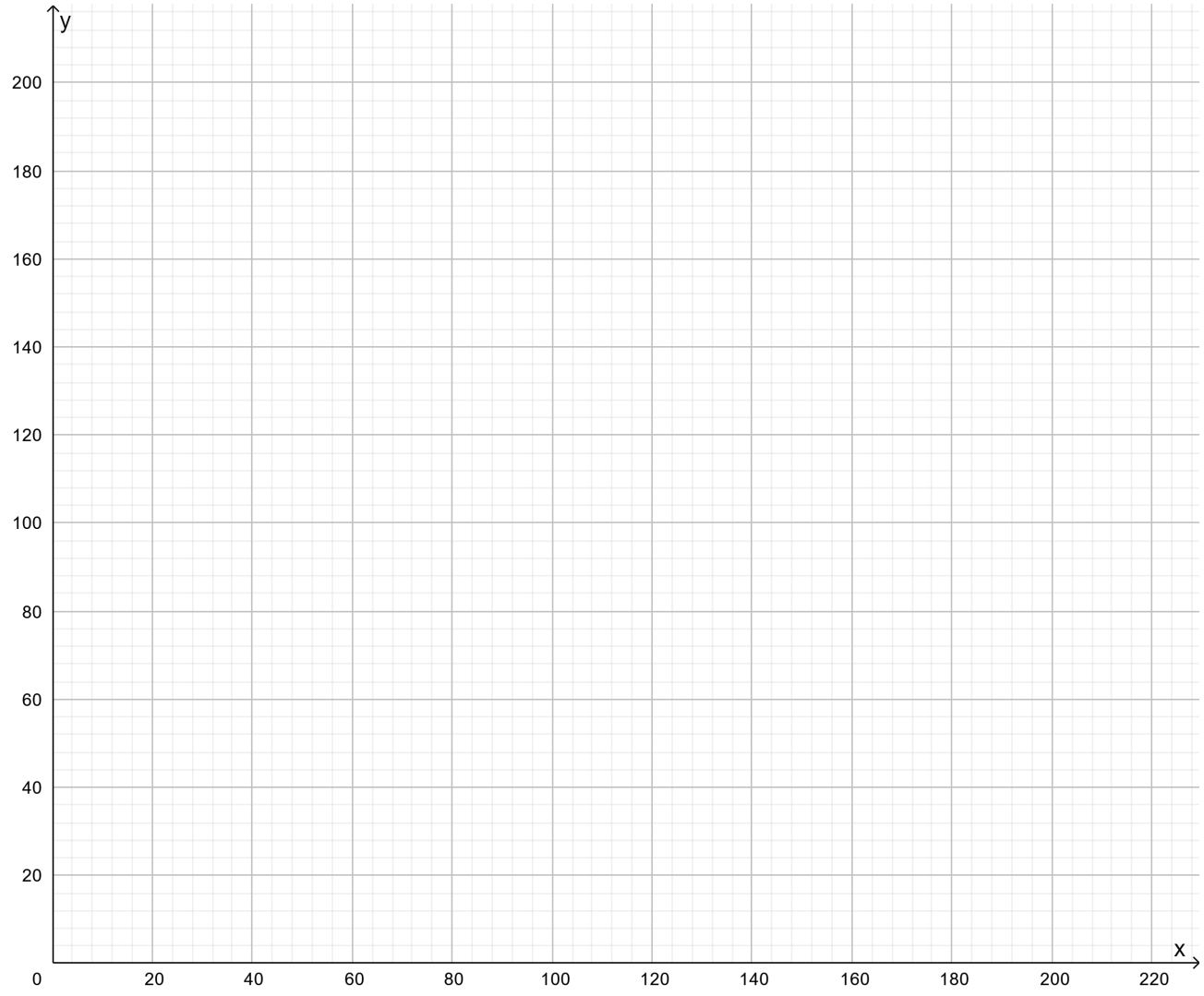
ANSWER SHEET 2 **EXAMINATION NUMBER:**

6.2.3



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FORMULA SHEET

1. $z = r\cos\theta \pm rj\sin\theta$ or $z = r\cos\theta \pm ri\sin\theta$

2. $z = a \pm bj$ or $z = a \pm bi$ where $i = j = \sqrt{-1}$

3. $r\angle\theta = rcis\theta$ where r represent modulus and θ represent argument

4. $m = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

5. $\dfrac{d}{dx}\,x^n = nx^{n-1}$

6. $\dfrac{d}{dx}\,k = 0$

7. $\dfrac{dy}{dx} = u.\dfrac{dv}{dx} + v.\dfrac{du}{dx}$ or $\dfrac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$

8. $\dfrac{dy}{dx} = \dfrac{v.\dfrac{du}{dx} - u.\dfrac{dv}{dx}}{v^2}$ or $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

9. $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

10. If $y = \big(f(x)\big)^n$ then $\dfrac{dy}{dx} = n\big(f(x)\big)^{n-1}f'(x)$

11. If $y = ln\,kx$ then $\dfrac{dy}{dx} = \dfrac{k}{kx}$ or If $f(x) = \ln kx$ then $f'(x) = \dfrac{k}{kx}$

12. If $y = e^x$ then $\dfrac{dy}{dx} = e^x$ or If $f(x) = e^x$ then $f'(x) = e^x$

13. If $y = e^{kx}$ then $\dfrac{dy}{dx} = ke^{kx}$ or If $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$

14. If $y = \sin x$ then $\dfrac{dy}{dx} = \cos x$ or If $f(x) = \sin x$ then $f'(x) = \cos x$

15. If $y = \cos x$ then $\dfrac{dy}{dx} = -\sin x$ or If $f(x) = \cos x$ then $f'(x) = -\sin x$

16. If $y = \tan x$ then $\dfrac{dy}{dx} = \sec^2 x$ or If $f(x) = \tan x$ then $f'(x) = \sec^2 x$

17. If $y = \cot x$ then $\dfrac{dy}{dx} = -\mathrm{cosec}^2 x$ or If $f(x) = \cot x$ then $f'(x) = -\mathrm{cosec}^2 x$

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18. If $y = \sec x$ then $\dfrac{dy}{dx} = \sec x \tan x$ or If $f(x) = \sec x$ then $f'(x) = \sec x \tan x$

19. If $y = \text{cosec}\, x \rightarrow \dfrac{dy}{dx} = -\text{cosec}\, x \cot x$ or If $f(x) = \text{cosec}\, x \rightarrow f'(x) = -\text{cosec}\, x \cot x$

20. $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c$

21. $\int kx^n dx = k \int x^n dx$ where k is a constant value

22. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

23. $\int \dfrac{k}{x} dx = k \ln x + c$

24. $\int e^{kx} dx = \dfrac{e^{kx}}{k} + c$

25. $\int \sin kx \, dx = -\dfrac{\cos kx}{k} + c = -\dfrac{1}{k}\cos kx + c$

26. $\int \cos kx \, dx = \dfrac{\sin kx}{k} + c = \dfrac{1}{k}\sin kx + c$

27. $\int \sec^2 kx \, dx = \dfrac{\tan kx}{k} + c = \dfrac{1}{k}\tan kx + c$

28. $A = \int_a^b y \, dx$

29. $[f(x)]_a^b = f(b) - f(a)$

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