higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

# MARKING GUIDELINE

## NATIONAL CERTIFICATE (VOCATIONAL)

## MATHEMATICS
**NQF LEVEL 4**
(First paper)

**9 November 2023**

**This marking guideline consists of 12 pages.**

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## INSTRUCTIONS AND INFORMATION

1. Examiners are reminded that the subject Mathematics lends itself to various methods and therefore several different alternative answers. Mark all mathematically correct answers.

2. The answers provided in this marking guideline are therefore not exhaustive. Marks are allocated for specific principles and markers must adhere to the mark allocation.

3. Please mark the questions and follow up, e.g.: An error in the first step of differentiation or a mistake in converting from surd to exponential, does not imply that the student cannot differentiate. Follow up.

4. Marks are allocated per step, but if a student omitted a step and there is evidence or a reason that he/she can derive at the next step without calculation, then the mark must still be given to the student.

5. Take note that marks have been allocated for simplification. This is to differentiate between an 80 percenter and 90 percenter.

6. Steps must be marked and not only the final answer. Ticks must correspond with the total. Put ticks and crosses neatly and legibly in red pen only.

7. If the answer does not look exactly the same as on the marking guideline, this could be due to an alternative method or a method that is not known to you as marker. Please take a moment before you mark to verify the students response.

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✓ Full mark √ Half mark A: Accuracy CA: Consistent accuracy or follow up

**QUESTION 1**

| | | | | |
|---|---|---|---|---|
| 1.1 | 1.1.1 | B | | |
| | 1.1.2 | D | | |
| | 1.1.3 | C | | |
| | 1.1.4 | D | | |
| | 1.1.5 | B | | |
| | | (5 × 1) | | (5) |

| | | | | |
|---|---|---|---|---|
| 1.2 | 1.2.1 | $z_1 = -4 + 6i$ | √ mark real value (A)<br>√ mark imaginary value (A) | (1) |
| | 1.2.2 | $\sqrt{52}cis\ 123{,}69° \times \sqrt{8}cis\ 315°$ √<br>$= 4\sqrt{26}cis\ 438{,}69°$√<br>$\cong 20{,}4\ cis\ 78\ cis\ 78{,}69°$ √<br>**OR**<br>$4\sqrt{26}\ cis\ 78{,}69°$ | √ mark correct readings (A)<br>√ mark argument not simplified<br>√ mark correct modulus<br>√ mark correct argument within given interval (CA) | (2) |
| | 1.2.3 | $(-4 + 6i) + (2 - 2i) = -2 + 4i$ ✓ | $(2 - 2i)$ 1 mark<br>1 mark correct answer CA 1.2.1 | (2) |

| | | | |
|---|---|---|---|
| 1.3 | $\frac{\{1{,}414cis(-45°)\}^5 \times 2\ cis\ 30°}{14{,}422\ cis\ (-33{,}690°)}$ √<br>$= \frac{5{,}657cis(-225°) \times 2\ cis\ 30°}{14{,}422\ cis\ (-33{,}690°)}$ √ √<br>$= \frac{11{,}314cis(-195°)}{14{,}422\ cis\ (-33{,}690°)}$<br>$= 0{,}784\ cis\ (-161{,}31°)$<br>$= 0{,}784\ cis\ (198{,}69°)$ √ √ | √ mark conversion numerator (A)<br>√ mark conversion denominator (A)<br>De Moivre:<br>√ mark modulus (A)<br>√ mark argument (A)<br>No marks for negative argument<br>Answer:<br>√ mark modulus (CA)<br>√ mark argument (CA)<br>Calculations in rectangular form are excluded. | (3) |

| | | | |
|---|---|---|---|
| 1.4 | $x(x^2 - 6x + 34) = 0$<br>$\therefore x = 0$ or $x^2 - 6x + 34 = 0$ ✓<br>$\therefore x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(34)}}{2}$<br>$\therefore x = \frac{6 \pm \sqrt{-100}}{2}$ ✓<br>$\therefore x = 3 \pm 5i$ ✓ | 1 mark arriving at quadratic equation and $x = 0$ (A)<br>1mark answer applying quadratic formula (A)<br>1 mark arriving at complex values of $x$ (A) | (3) |

| | | | |
|---|---|---|---|
| 1.5 | $3 - ix^2y = -(x^2 + y) + 4i$<br><br>$-(x^2 + y) = 3 \qquad eq\ 1\ \surd$<br><br>$-x^2y = 4 \ \rightarrow\ x^2 = -\frac{4}{y} \qquad eq2\ \surd$<br><br>$\therefore$ Substitute in $eq\ 1$<br><br>$\frac{4}{y} - y = 3 \ \ \therefore \ \ y^2 + 3y - 4 = 0$ ✓<br><br>$\therefore y = 1$ or $y = -4$<br><br>$y = 1$ in $eq\ 2 \therefore\ x^2 = -4 \ \rightarrow \ x = \pm 2i$<br><br>$y = -4$ in $eq\ 2 \ \rightarrow \ x^2 = -\frac{4}{-4} = 1 \ x = \pm 1$ | √ mark $eq1$ (A)<br><br>√ mark $eq2$ (A)<br><br>1 mark arriving at quadratic equation<br>√mark each $y$<br><br>√ mark complex roots<br><br>√ mark real | (4) |

**[20]**

**QUESTION 2**

| | | | |
|---|---|---|---|
| 2.1 | Let $(2x - 1) = 0 \therefore x = \frac{1}{2}$<br><br>$f\left(\frac{1}{2}\right) = 12\left(\frac{1}{2}\right)^3 - 28\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 7 = 0$✓<br><br>$\therefore$ Remainder $= 0$ ✓<br><br>$\therefore (2x - 1)$ is a factor. | 1 mark remainder zero<br><br>1 mark answer | (2) |

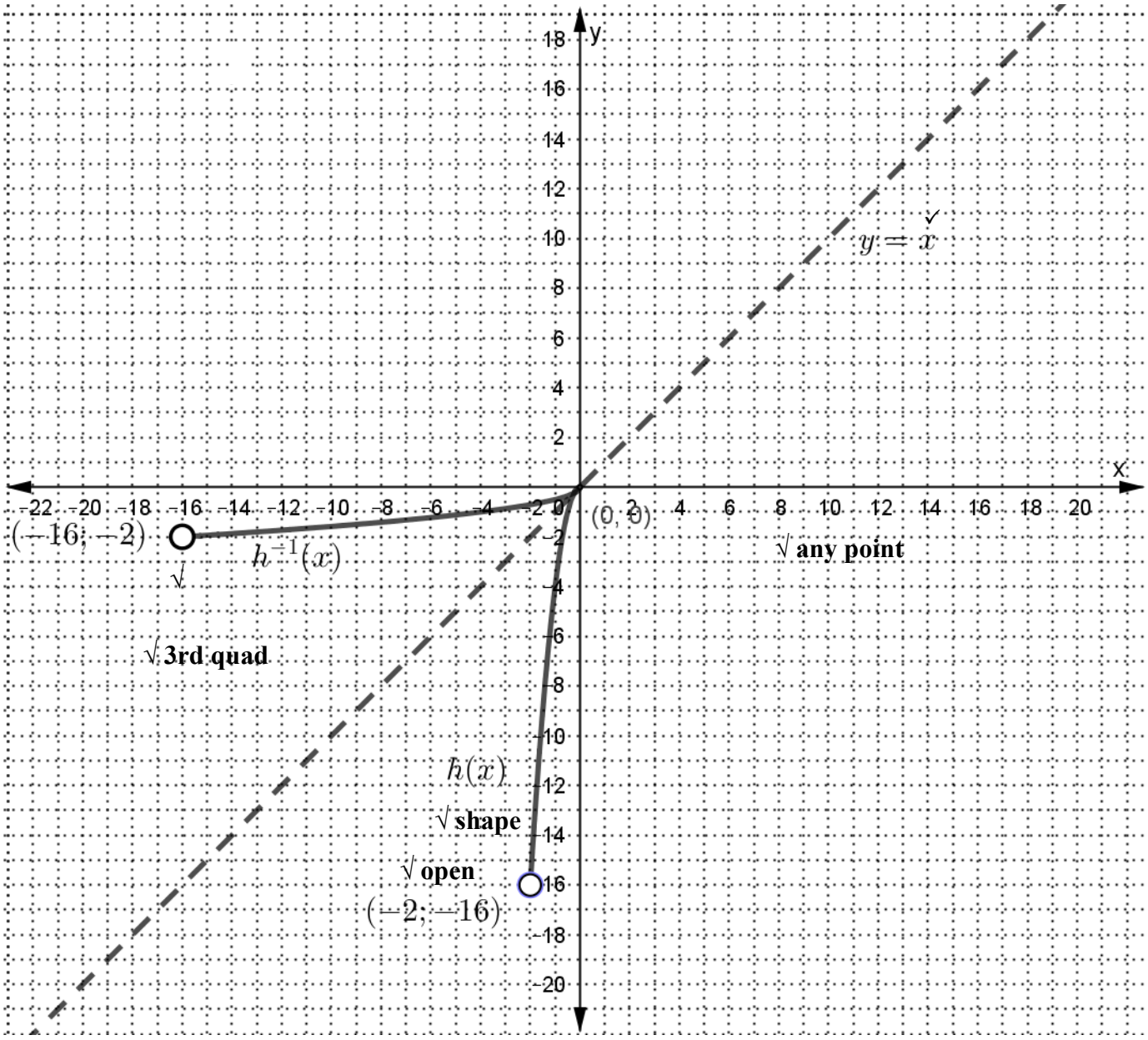
| | | | | |
|---|---|---|---|---|
| 2.2 | 2.2.1 | $x + 1 = 0$ given $\therefore x = -1$<br><br>$\therefore -2a + b = 1 \qquad eq\ 1$✓<br><br>$x - 3 = 0$ given $\therefore x = 3$<br><br>$\therefore -6a + b = -27 \ \ eq\ 2$✓<br><br>$eq1 - eq2$<br><br>$\therefore 4a = 28$✓ $\qquad \therefore a = 7$ ✓<br><br>$\therefore -14 + b = 1 \ \therefore b = 15$ | 1 mark $eq\ 1$<br><br>1 mark $eq\ 2$<br><br>1 mark answer $a$<br>1 mark answer $b$ | (3) |
| | 2.2.2 | Hence $x^3 - 7x^2 + 7x + 15 = (x + 1)(x - 3)(x + m)$✓<br>$\therefore 15 = -3m$ ✓<br>$m = -5$ | | (2) |

| | | | |
|---|---|---|---|
| 2.3 | 2.3.1 | $f^{-1} = \{(5;-3);(5;-2);(7;0);(9;2)\}$ | (1) |
| | 2.3.2 | Nonfunction | (1) |
| | 2.3.3 | $x = 7$ | (1) |
| | 2.3.4 | $5 \leq x \leq 9$ **OR** $x \in [5;9]$ | (1) |

| | | | | |
|---|---|---|---|---|
| 2.4 | 2.4.1 | $y = \frac{x-4}{2}$ ✓ or $y = \frac{1}{2}x - 2$ | Must be $y =$… | (1) |
| | 2.4.2 | $\{x;\ -4 \leq x < 8;\ x \in R\}$ (√ √ √ √)<br><br>**OR**<br><br>$x \in [-4;8)$ with $x \in R$ | √ mark min (A)<br>√ mark max (A)<br>√ mark each correct inequality or bracket (A) | (2) |

| | | | | |
|---|---|---|---|---|
| 2.5 | 2.5.1 | $h^{-1}(x)$: $y = \sqrt{\frac{x}{-4}}$ ✓ | Only marks for $y =$… | (1) |
| | 2.5.2 | $-4x^2 = \sqrt{\frac{x}{-4}}$<br><br>$\therefore 16x^4 = \frac{x}{-4}$ √<br><br>$\therefore 64x^4 - x = 0 \rightarrow x(64x^3 - 1) = 0$ √ √<br><br>$\therefore x = 0\ y = 0$ and $x^3 = \frac{1}{64}\ \therefore x = -\frac{1}{4}$ √<br><br>$\rightarrow y = -4\left(\frac{-1}{4}\right)^2 = -\frac{1}{4}$ √<br><br>$\left(-\frac{1}{4};-\frac{1}{4}\right)$<br><br>Positive answers are excluded because of the given domain. | √ mark equate $h$ and $h^{-1}$<br>√ mark factorising<br>√ mark each ordinate in $(0;0)$<br>√ mark each ordinate in $\left(-\frac{1}{4};-\frac{1}{4}\right)$ | (3) |

| | | | | |
|---|---|---|---|---|
| | 2.5.3 | | | |




| | |
|---|---|
| √ mark each open point<br>√ mark for using only 3rd quadrant<br>√ mark for curved shape of $h(x)$<br>√ mark $y = x$<br>√ mark point on $y = x$ | (3) |

**[21]**

## QUESTION 3

| | | | | |
|---|---|---|---|---|
| 3.1 | | $f(x) = 2x^3 \therefore f(x+h) = 2(x+h)^3$<br>$= 2x^3 + 6x^2h + 6xh^2 + 2h^3$ ✓<br>$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$<br>$= \lim_{h\to 0} \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x^3}{h}$ √<br>$= \lim_{h\to 0} \frac{6x^2h + 6xh^2 + 2h^3}{h}$ √<br>$= \lim_{h\to 0} \frac{h(6x^2 + 6xh + 2h^2)}{h}$ √<br>$= \lim_{h\to 0} 6x^2 + 6xh + 2h^2$<br>$= 6x^2$ √ | No marks for final answer if it cannot be derived from previous steps.<br>1 mark $f(x+h)$<br>√ mark correct substitution in first principles<br>√ mark addition simplified<br>√ mark taking $h$ as common factor<br>√ mark answer | (3) |
| 3.2 | 3.2.1 | $\frac{dy}{dx} = 4x^4 + \frac{1}{2} + 4e^{4x} + 0$<br>√ √ √ √ | √ mark each term differentiated | (2) |
| | 3.2.2 | $y = 3ln5x - 3\sin 5x - 2x^{-5} + 3x^{\frac{1}{2}}$<br>$\therefore \frac{dy}{dx} = \frac{3 \times 5}{5x}$ √ $- 15\cos 5x$ √ $+ 10x^{-6}$ √ $+ \frac{3}{2}x^{-\frac{1}{2}}$<br>$\therefore \frac{dy}{dx} = \frac{3}{x}$ √ $- 15\cos 5x$ √ $+ \frac{10}{x^6} + \frac{3}{2\sqrt{x}}$ √ | √ mark each term differentiated step 1<br>√ mark each term in surd and positive exponent where applicable<br>**NOTE:** Only √ mark for $-15\cos 5x$<br>√ mark to $\frac{dy}{dx}$ in final answer | (4) |

| | | | | |
|---|---|---|---|---|
| | 3.2.3 | $\frac{dy}{dx} = (2x^3 - 4)^{-4}$<br><br>$\frac{dy}{dx} = -4(2x^3 - 4)^{-5}(6x^2)$ ✓ ✓<br><br>$\frac{dy}{dx} = \frac{-24x^2}{(2x^3 - 4)^5}$ ✓<br><br>**ALTERNATIVE:**<br><br>Let $u = 2x^3 - 4 \ \therefore \frac{du}{dx} = 6x^2$ √<br><br>and $y = u^{-4} \ \therefore \frac{dy}{du} = -4\,u^{-5}$ √<br><br>$\therefore \frac{dy}{dx} = -4(2x^3 - 4)^{-5}(6x^2)$ ✓<br><br>$\therefore \frac{dy}{dx} = \frac{-24x^2}{(2x^3 - 4)^5}$ ✓ | 1 mark apply $y = f(x)^n$ rule (A)<br><br>1 mark $d/dx$ base (A)<br><br>1 mark simplified correctly with positive exponents (CA)<br><br>**OR**<br><br>√ mark $du/dx$ (A)<br><br>√ mark $dy/du$ base (A)<br><br>1 mark apply chain rule<br><br>1 mark simplified correctly with positive exponents | (3) |

**[12]**

**QUESTION 4**

| | | | | |
|---|---|---|---|---|
| 4.1 | 4.1.1 | $\int y dx = 2lnx - \frac{tan5x}{5} + 2e^{2x} + x^2 + 5x + c$ √ √ √ √ √ √ | √ mark each term integrated and half mark $c$ | (3) |
| | 4.1.2 | $\int y dx = \int \left(4x^{-2} - \frac{5}{2}x^{-\frac{1}{2}} + 6\sin 3x\right) dx$<br><br>$= \frac{4x^{-1}}{-1} - \frac{5x^{\frac{1}{2}}}{2 \times \frac{1}{2}} - \frac{6}{3}\cos 3x + c$ √ √ √<br><br>$= -\frac{4}{x} - 5\sqrt[2]{x} - 2\cos 3x + c$ √ √ √<br><br>No marks for $c$ in 4.1.2. | √ mark each term integrated<br><br>√ mark first two terms in surd and positive exponent,<br>√ mark negative sign for $2cos3x$ | (3) |
| | 4.1.3 | $\int \left(\frac{(x-3)(x-2)}{(x-3)}\right) dx$<br><br>$= \int x - 2\, dx$ √<br>$= \frac{x^2}{2} - 2x + c$ √ | √ mark simplified<br>√ mark each integrated<br>√ mark $c$ | (2) |

| | | | |
|---|---|---|---|
| 4.2 | $A_1 = \int_{-1}^{1} -x^3 + 4x^2 + x - 4 = \frac{-x^4}{4} + \frac{4x^3}{3} + \frac{x^2}{2} - 4x \Big\vert_{-1}^{1}$<br>$= \left[\frac{-1^4}{4} + \frac{4(1)^3}{3} + \frac{1^2}{2} - 4(1)\right] - \left[\frac{-(-1)^4}{4} + \frac{4(-1)^3}{3} + \frac{(-1)^2}{2} - 4(-1)\right]$<br>$= -\frac{16}{3} \overset{\surd}{=} +5\frac{1}{3}$<br>$A_2 = \int_{1}^{4} -x^3 + 4x^2 + x - 4 \overset{\surd}{=} \frac{-x^4}{4} + \frac{4x^3}{3} + \frac{x^2}{2} - 4x \Big\vert_{1}^{4}$<br>$= \left[\frac{-4^4}{4} + \frac{4(4)^3}{3} + \frac{4^2}{2} - 4(4)\right] - \left[\frac{-1^4}{4} + \frac{4(1)^3}{3} + \frac{1^2}{2} - 4(1)\right] = 15\frac{3}{4}$<br>Area total $= 21\frac{1}{12} = 21{,}083\ units^2$ | √ mark<br>$A_1 = \int_{-1}^{1} ydx$ (A)<br>correct boundaries<br>√ mark each integration in $A_1$<br>√ mark<br>$A_2 = \int_{1}^{4} ydx$<br>√ mark $-A_1$ (A)<br>√ mark$+A_1$ (CA)<br>√ mark$+A_2$ (A)<br>√ mark $A_t$ (CA) | |
| | | | (5) |

**[13]**

**QUESTION 5**

| | | | | |
|---|---|---|---|---|
| 5.1 | 5.1.1 | $\boldsymbol{s(2) = -(3)^2 + 3(3) + 1}$√<br>$\therefore\ \boldsymbol{s = 1\ m}$√ | √ mark substitute s' (A)<br>√ mark answer (A) | (1) |
| | 5.1.2 | (a) $s'(t) = -2t + 3 = 0$<br>$\rightarrow 2t = 3$ √<br>$t = \frac{5}{2}$ s $= 1{,}5$ s<br>$s = -(1{,}5)^2 + 3(1{,}5)$√<br>$s = 3\frac{1}{4}$ m $= 3{,}25$ m✓ | √ mark for s' (t) (A)<br>√ mark equate to zero (A)<br>√ mark solve $t$ (A)<br>√ mark substitute in $s$ (CA)<br>1 mark answer (A) | (3) |
| | | (b) 3,25 m − 2,47 m = 0,78 m = 78 cm ✓ | √ mark answer in meter (CA)<br>√ mark answer in cm (A) | (1) |
| | 5.1.3 | $s(t) = -(t)^2 + 3(t) + 1 = 0$ √<br>$\therefore t = \frac{-3 \pm \sqrt{13}}{-2}$ √<br>$\therefore t = -0{,}3$ s and $t = 3{,}302$ s<br>Time is not negative. ✓<br>$\therefore\ t = 3{,}302$ s✓ | √ mark equates to zero<br>√ mark applying quadratic formula<br>1 mark answer<br>Do not penalise if units are omitted. | (2) |

| | | | | |
|---|---|---|---|---|
| 5.2 | 5.2.1 | $f'(x) = 6x^2 + 6x - p = 6x^2 + 6x + 12$<br>$\therefore p = -12$ | 1 mark $f'(x)$<br>1 mark solution | (2) |
| | 5.2.2 | $-2 \left\lfloor \overrightarrow{2 + 3 - 12 - 20}$<br>$\downarrow \quad -4 - \ 2 + 20$<br>$2 - 1 - 10 \ \lfloor 0$<br>√ √ √<br>$\therefore 2x^2 - x - 10 = (x + 2)(2x - 5)(x + 2)$<br>√ √ √<br>**ALTERNATIVE:**<br>$x + 2 \left\lfloor \begin{array}{r} 2x^2 \quad -x \quad -10 \\ 2x^3 \quad 3x^2 \quad -12x \quad -20 \\ \underline{2x^3 \quad 4x^2} \\ 0 \quad -1x^2 \quad -12x \\ \underline{-1x^2 \quad -2x} \\ 0 \quad -10x \quad -20 \\ \underline{-10x \quad -20} \\ 0 \quad + \quad 0 \end{array} \right.$<br>√ √ √<br>$\therefore\therefore 2x^2 - x - 10 = (x + 2)(2x - 5)(x + 2)$<br>**ALTERNATIVE:**<br>$2x^3 + 3x^2 - 12x - 20 = (x + 2)(ax^2 + bx + c)$<br>$\therefore x^3\text{:} \ \ 2x^3 = ax^3 \ \therefore a = 2$ √<br>$\text{Constant} - 20 = 2 \times c \ \ \therefore c = -10$√<br>$x^2\text{:} + 2ax^2 + bx^2 = 3x^2$<br>√<br>$\therefore 4x^2 + bx^2 = 3x^2$<br>$\therefore \quad b = -1$<br>$\therefore 2x^2 - x - 10 = (2x - 5)(x + 2)$<br>√ √ √<br>$2x^2 - x - 10 = (x + 2)(2x - 5)(x + 2)$ | In $ax^2 + bx + c$<br>√ mark $a$<br>√ mark $b$<br>√ mark $c$<br>√ mark each factor | (3) |

| | | | | |
|---|---|---|---|---|
| 5.3 | 5.3.1 | $g(x) = 2x^3 - 7x^2 - 14x + 40$<br>$\therefore g'(x) = 6x^2 - 14x - 14$ ✓ ✓ ✓<br>$\therefore g'(-2) = 6(-2)^2 - 14(-2) - 14 = 38$ ✓<br>$g'(1) = 6(1)^2 - 14(1) - 14 = -22$ | √ mark each derivative in $g'$<br>√ mark correct use of $(-2)$<br>√ mark each gradient | (3) |
| | 5.3.2 | Gradient changed from positive ✓ to negative ✓.<br>∴ There is a maximum ✓ turning ✓ point. | √ mark each answer that indicate change in direction of curve<br>√ mark maximum<br>√ mark turning point | (2) |

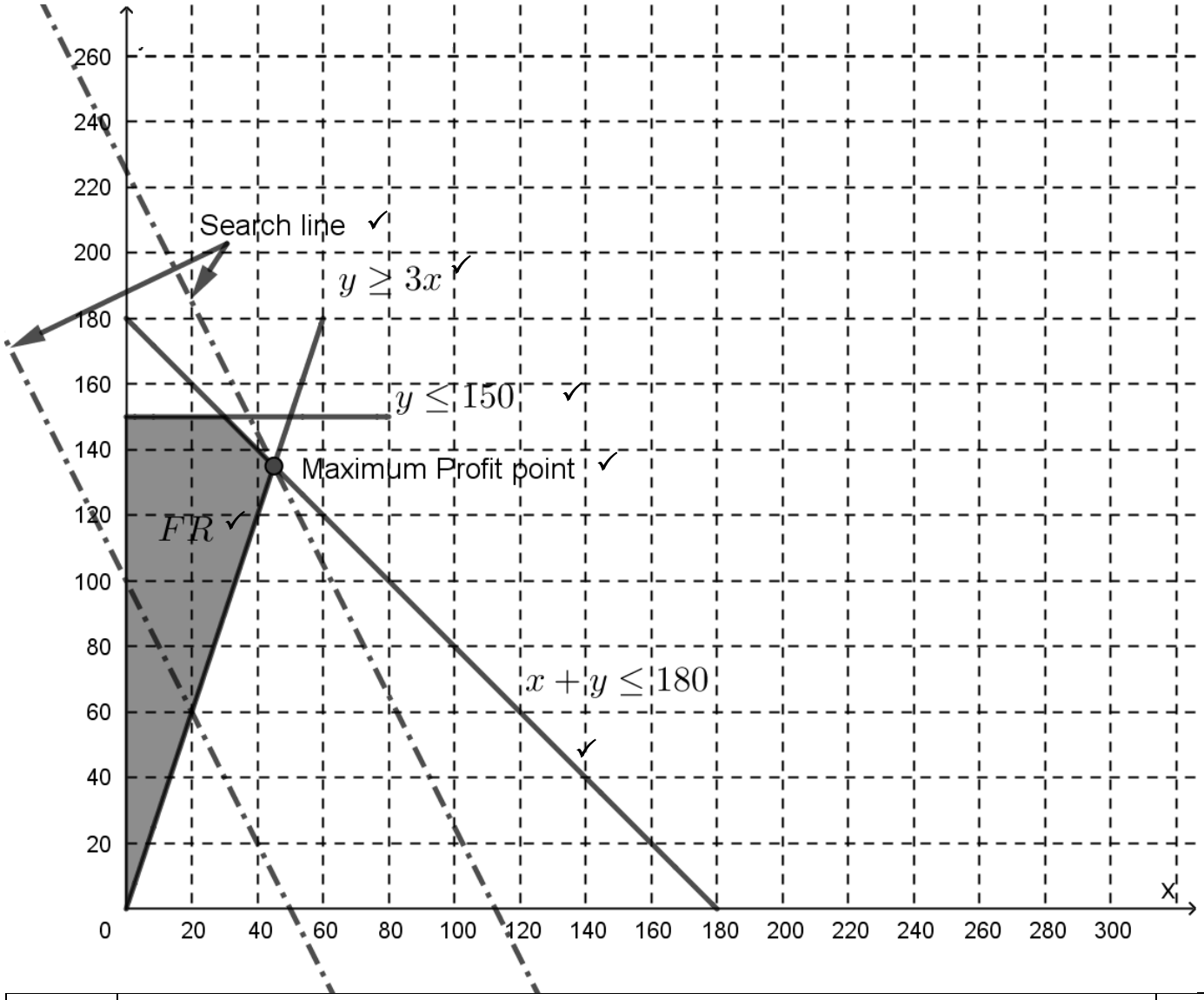
| | | | | |
|---|---|---|---|---|
| | 5.3.3 | $g''(x) = 12x - 14 = 0$<br>$\therefore x = \frac{14}{12} = 1\frac{1}{6} = 1{,}167$<br>$\therefore y = \frac{143}{3} = 17{,}33 \therefore POI = (1{,}167; 17{,}33)$ ✓ | √ mark g" (A)<br>√ mark equates to zero (A)<br>√ mark solve $x$ (A)<br>√ mark solve $y$ (CA) | (2) |

**[19]**

**QUESTION 6**

| | | | | | |
|---|---|---|---|---|---|
| 6.1 | 6.1.1 | a) | (10; 2)✓ | | (1) |
| | | b) | **Test in $3x + 2y \leq 30$**<br>**$\therefore 3(10) + 2(2) = 34$ ✓ $\rightarrow 34 > 30$**<br>It does not meet this restriction because answer is bigger than 30. | | (1) |
| | 6.1.2 | | $\boldsymbol{P = Q(x) + T(y)}$<br>**Given $43 = 4Q + 9T$ *eq* 1** √<br>**$35 = 5Q + 5T$ *eq* 2** √<br>$eq1 \times 5 \rightarrow 215 = 20Q + 45\,T \rightarrow eq3$<br>$eq2 \times 4 \rightarrow 140 = 20Q + 20\,T \rightarrow eq4$<br>$\therefore eq3 - eq4 \rightarrow 75 = 25\,T$<br>$\therefore T = 3$✓<br>Substitute in *eq* 2<br>$\therefore 5Q = 20 \therefore Q = 4$✓ | √mark *eq* 1 (A)<br>√ mark *eq* 2 (A)<br>1 mark solve $T$ (A)<br>1 mark solve Q (CA) | (3) |

| | | | | |
|---|---|---|---|---|
| 6.2 | 6.2.1 | $P = 1\,600x + 800y$ | (A) | (1) |
| | 6.2.2 | $y \leq 150$ ✓<br>$x \leq \frac{1}{3}y \quad \therefore y \geq 3x$ (√ √)<br>$x + y \leq 180$ (√ √) | √ mark each correct inequality sign (A)<br>√ mark each correct equation (A) | (3) |
| | 6.2.3 | | | |




| | | |
|---|---|---|
| (a) | 1 mark $x + y = 180$ | |
| (b) | 1 mark $y = 3x$<br>1 mark $y = 150$<br>1 mark feasible region | |
| (c) | 1 mark showing correct maximum point and 1 Mark showing the search line at the maximum point. | (6) |

**[15]**

**TOTAL: 100**

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