higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
NQF LEVEL 3
(First paper)

(10501053)

8 November 2023 (X-paper)
09:00–12:00

Calculators may be used.

This question paper consists of 7 pages, 2 addenda and 1 formula sheet.

009Q1N2308

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)
MATHEMATICS
NQF LEVEL 3
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Clearly show all calculations, diagrams, graphs, etcetera that you have used to determine the answers.
5. Round off answers to THREE decimal places if required unless stated otherwise.
6. Diagrams are not drawn to scale.
7. Complete the EXAMINATION NUMBER and CENTRE NUMBER on both ADDENDUM A and ADDENDUM B (attached). Detach and submit the addenda with the ANSWER BOOK.
8. Use only a black or blue pen. Answers in pencil will not be marked.
9. Write neatly and legibly.

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QUESTION 1

1.1 Calculate each of the following expressions and give the answer in standard form $z = a + bi$:

1.1.1 $-\left(2 + \sqrt{-9}\right) - (-3 + \sqrt{-16})$ (2)

1.1.2 $\frac{1}{3}(2 - i)(6 + 3i)$ (3)

1.1.3 $2i^3(5i - 6i^2 - \sqrt{-1})$ (4)

1.2 Given: $z = \frac{-1 + 4{,}5i}{2}$

1.2.1 Give $\bar{z}$, the conjugate of z, in its simplest form. (1)

1.2.2 Calculate the modulus and positive argument of $\bar{z}$. Show $\bar{z}$ and all calculated values on an Argand diagram. (5)

[15]

QUESTION 2

2.1 The impedance z of a circuit is the resistance of an electrical component against the flow of an alternating or direct electrical current. The impedance is calculated with the formula $z = \frac{(z_1)(z_2)}{z_2 + z_1}$.

2.1.1 If $z_1 = 3{,}5cis67{,}8°$ and $z_2 = 2{,}4(cos12° + isin12°)$, calculate $(z_1)(z_2)$. (2)

2.1.2 Calculate $z_1 + z_2$. (3)

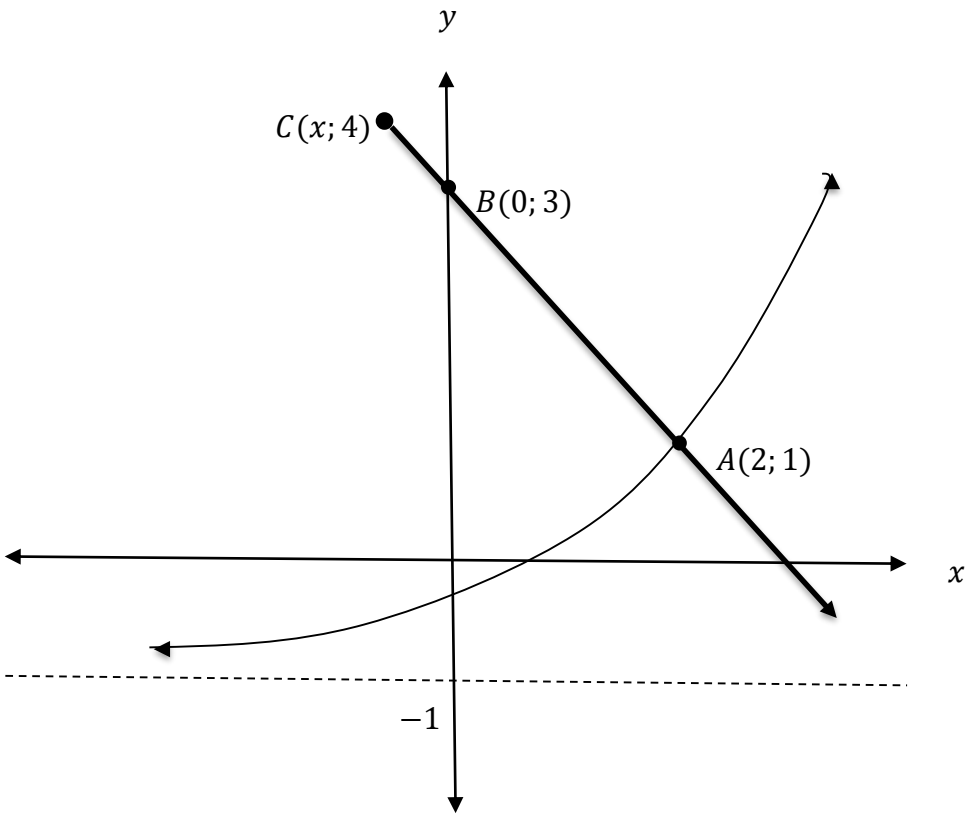
2.1.3 Hence, calculate the value of z, the impedance of the electrical component, using the formula $z = \frac{(z_1)(z_2)}{z_2 + z_1}$. Give the answer in rectangular form. (5)

2.2 Determine the value(s) of x and y if $y = \frac{10 - x}{2i} - 3$. (5)

[15]

QUESTION 3

3.1 The graphs of $f(x) = a.2^x + q$ and $g(x) = \frac{-1}{2}x + 3$ are sketched below. A$(2; 1)$; $B(0; 3)$ and $C(x; 4)$ are points on the graphs of $f(x)$ and $g(x)$.



3.1.1 Determine the values of a and q of the function $f(x)$. (3)

3.1.2 If $f(x) = \frac{1}{2}.2^x - 1$, give the range of the graph of $f(x) + 3$. (1)

3.1.3 Give the domain of the graph of $g(x)$ in interval notation. (2)

3.1.4 Give the equation of $h(x)$ if $g(x)$ is translated six units down. (1)

3.2 Given the equation of $k(x) = \frac{-1}{x-1} - \frac{1}{2}$.

3.2.1 Sketch the graph of $k(x)$. Clearly indicate the x- and y-intercepts. Show the asymptotes. Use ADDENDUM A (attached) to sketch the graph. (4)

3.2.2 Give the equation of the vertical asymptote of $k(x)$. (1)

3.2.3 Is the graph of $k(x)$ a function or nonfunction? (1)

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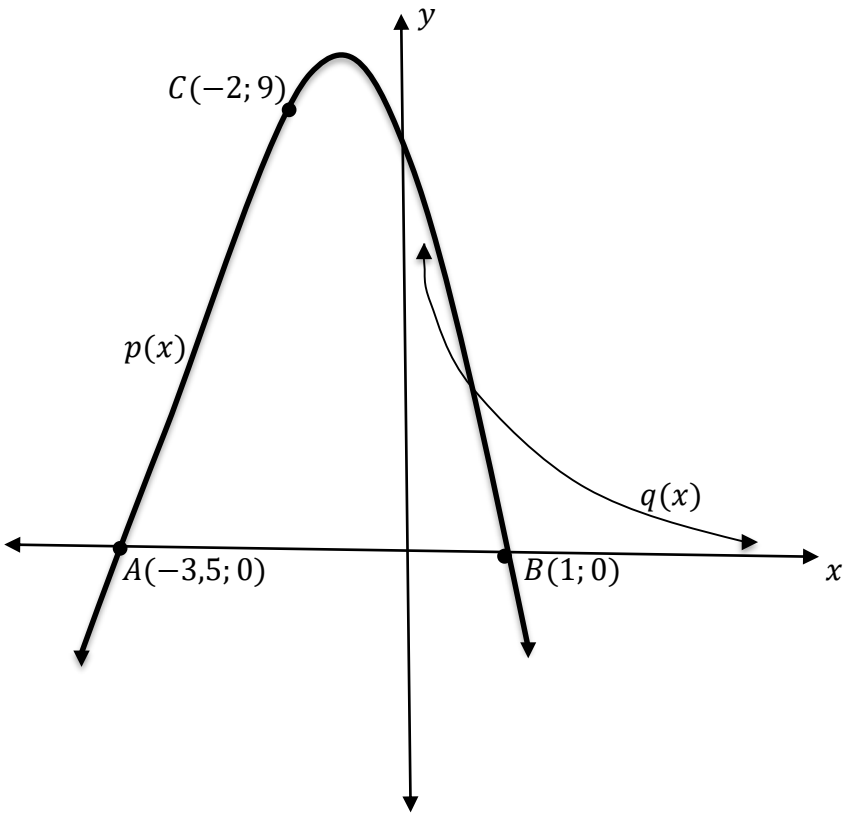
3.3 Given below are the graphs of $p(x) = ax^2 + bx + c$ and $q(x) = \frac{2}{x}; x > 0$. The graph of $f(x)$ intersects the x-axis at points $A(-3{,}5; 0)$ and $B(1; 0)$. $C(-2; 9)$ is a point on the graph of $p(x)$. Functions are not necessarily drawn to scale.



3.3.1 Show, by using calculations, that the value of $a = -2$; $b = -5$ and $c = 7$ of the equation of $p(x)$. (4)

3.3.2 Give the range of $p(x)$. (2)

3.3.3 For which values of x is $q(x)$ decreasing? (1)

[20]

QUESTION 4

4.1 Solve for x and y if $2^{4y+3x} = 2^{50}$ and $x^2 + y^2 = 100$. (5)

4.2 Solve x if:

4.2.1 $\left(\frac{1}{3}x\right)\left(\sqrt[3]{x} - 3\right) = 0$ (2)

4.2.2 $\frac{-1}{2x-3} - \frac{x-5}{3} = 0$ (4)

4.3 Show that $\frac{1}{1-\frac{a}{b}} \div \frac{1}{1-\frac{b}{a}} = -\frac{b}{a}$. (4)

4.4 Solve x if $\frac{3x^2-4}{-2} > -4$. Show all calculations and methods used to solve x. Give the answer in interval notation. (5)

[20]

QUESTION 5

5.1 Determine $\frac{dy}{dx}$ if:

5.1.1 $y = 3 - \frac{1}{5\sqrt[3]{x^5}} + 4x$ (3)

5.1.2 $f(x) = \frac{x^3 - 7x + 9x^2}{7x}$ (3)

5.2 Calculate the derivative of $f'(x)$ using first principles if $f(x) = x^2 - 4$. (4)

5.3 Calculate $\lim_{x \to 3} \frac{x^2 - 7x + 12}{2x^2 - 4x - 6}$. (3)

[13]

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QUESTION 6

It is estimated that t years from now the population of a certain species of insects will be $P(t) = 20 - \frac{6}{t}$ thousand.

6.1 Determine the population in the first year. (1)

6.2 Determine the rate of change in the population in three years' time. (3)

6.3 Does the population increase or decrease yearly? Prove the answer with calculations. (2)

6.4 What will happen to the population in due course? (1)

[7]

QUESTION 7

A factory employs unskilled workers at R1 000 per week and skilled workers at R2 000 per week. The payroll should not exceed R90 000 per week. To keep going, at least 50 operators are necessary. Furthermore, regulations determine that the number of skilled workers should be at least half that of the unskilled workers.

Let x be the number of unskilled workers and y the number of skilled workers.

The following inequalities are obtained:

$1\,000x + 2\,000y \leq 90\,000$

$x + y \geq 50$

$y \geq \frac{x}{2}$

$x; y \geq 0$

$x; y \epsilon N_0$

7.1 Sketch these inequalities on ADDENDUM B (attached). (4)

7.2 Shade and indicate the feasible region on the sketch made in QUESTION 7.1. (2)

7.3 Determine from the graph the minimum and maximum number of unskilled workers that should be employed. (4)

[10]

TOTAL: 100

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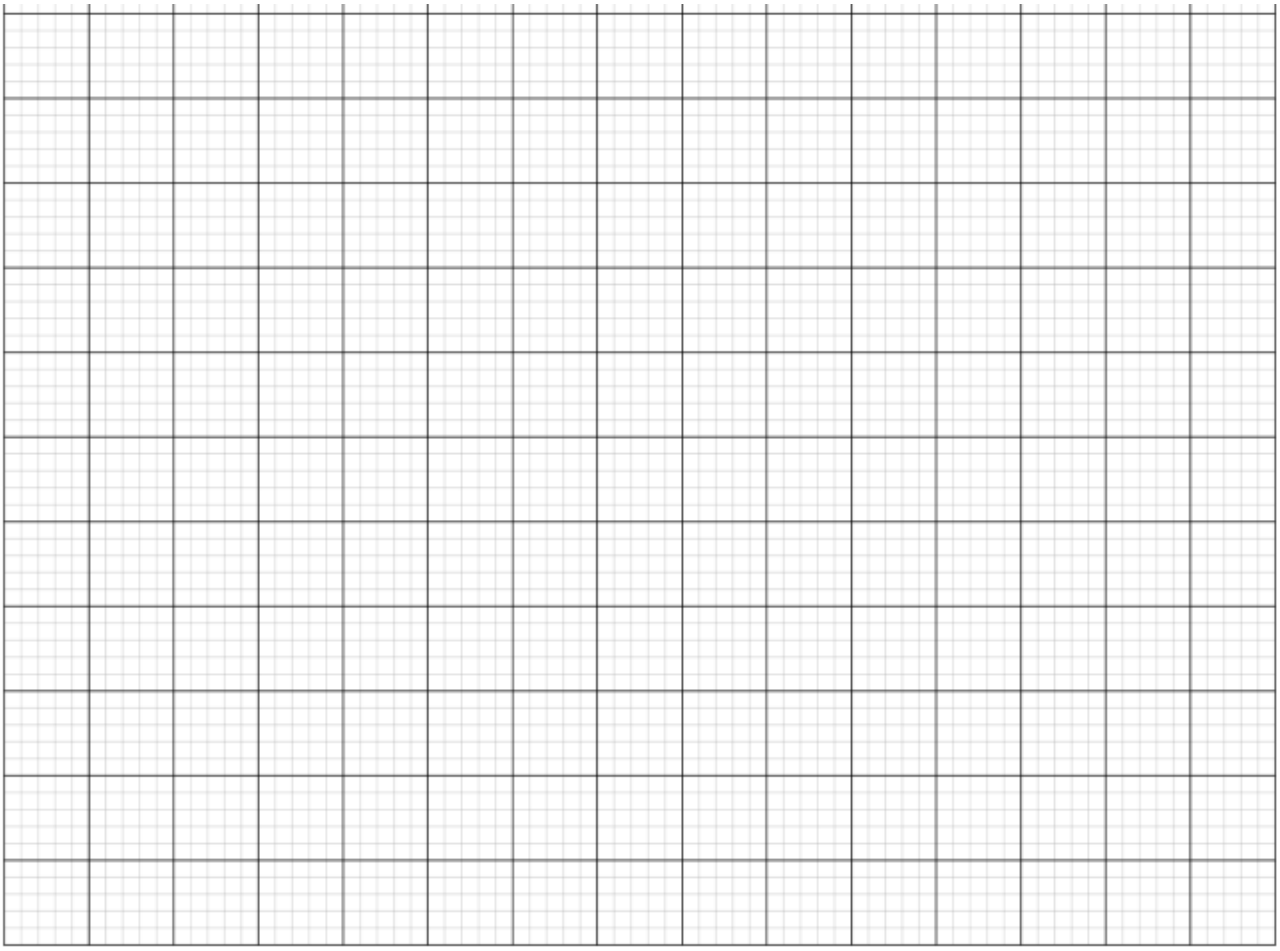
ADDENDUM A **EXAMINATION NUMBER:**

CENTRE NUMBER:

QUESTION 3.2.1



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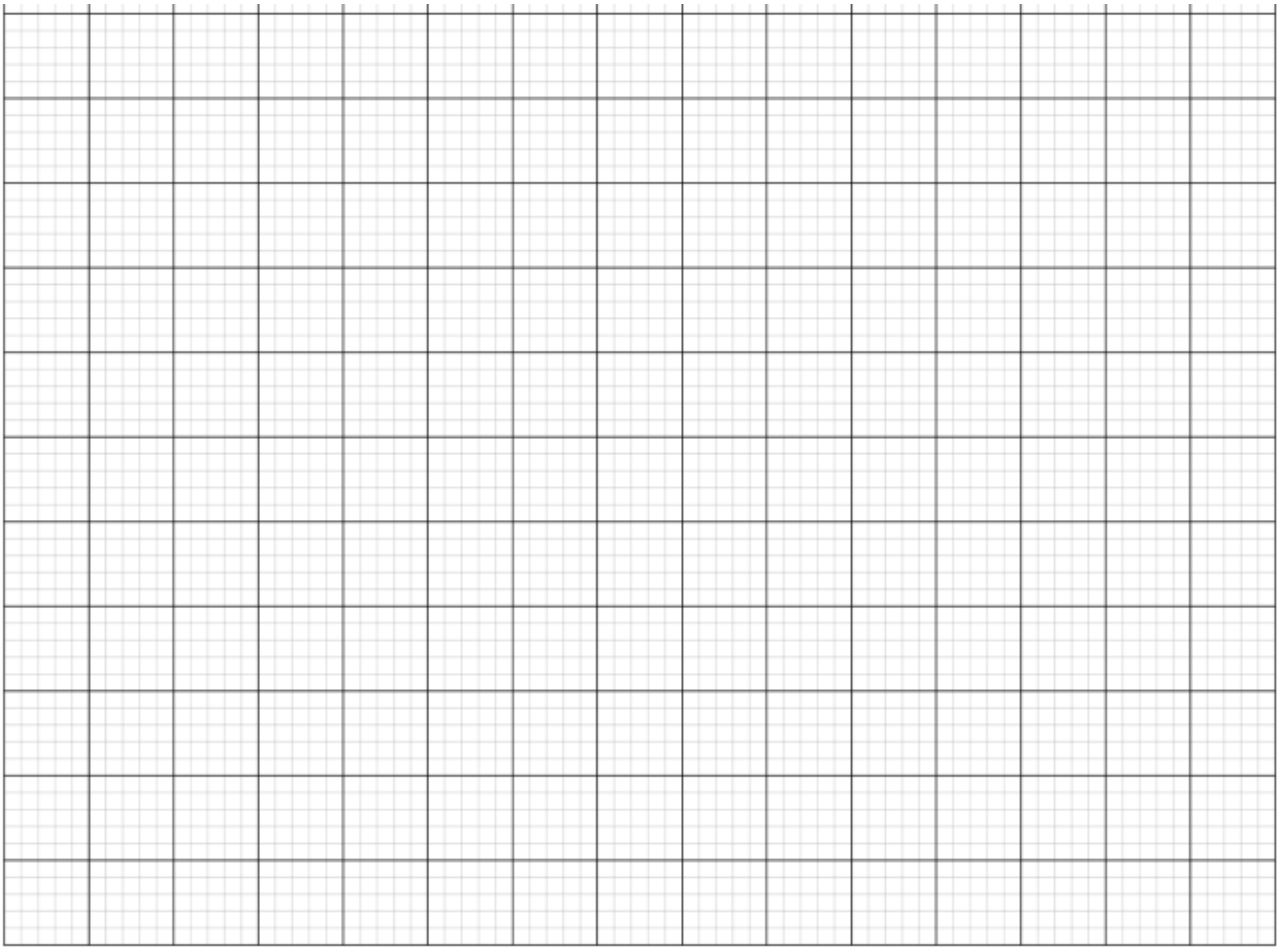
ADDENDUM B **EXAMINATION NUMBER:**

CENTRE NUMBER:

QUESTION 7.1 and 7.2



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(10501053)

FORMULA SHEET

1. $z = r.\cos\theta + r.j\sin\theta$

2. $z = a \pm bj$ or $z = a \pm bi$ where $i = j = \sqrt{-1}$

3. $r = \sqrt{a^2 + b^2}$ or $r = \sqrt{z\ x\ \bar{z}}$

4. $\theta = tan^{-1}\left(\frac{b}{a}\right)$

5. $r\ cis\ \theta$

6. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

7. $y = ax^2 + bx + c$ or $y = a\ (x - p)^2 + q$

8. $y = a(x - x_1)(x - x_2)$

9. $y = \frac{a}{x - p} + q$

10. $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$

11. $\frac{d}{dx} ax^n = n.a\ x^{n-1}$

12. $\frac{d}{dx}\ k = 0$

13. $D_x[k.f(x)] = k.D_x\ [f(x)]$

14. $D_x[f(x) + g(x)] = D_x[f(x)] + D_x[g(x)]$

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