higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
NQF LEVEL 3
(First paper)

8 November 2023

This marking guideline consists of 10 pages.

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QUESTION 1

1.1 1.1.1

$-\left(2+\sqrt{-9}\right)-(-3+\sqrt{-16})$
$=-2-3i+3-4i$
$=1-7i$ ✓real value ✓imaginary value (2)

1.1.2

$\frac{1}{3}(2-i)(6+3i)$
$=\frac{1}{3}(12-3i^2)$ ✓multiplication
$=\frac{1}{3}(15)$ ✓simplify $i^2=-1$
$=5$ ✓answer

OR

$=(\frac{2}{3}-\frac{1}{3}i)(6+3i)$
$=4+2i-2i-i^2$ ✓multiplication of brackets
$=4-(-1)$ ✓simplify $i^2=-1$
$=5$ ✓answer (3)

1.1.3

$2i^3(5i-6i^2-\sqrt{-1})$
$=10i^4-12i^5-2i^3.i$
$=10(-1)^2-12(-1)^2i-2(-1)^2$ ✓✓✓one mark per term
$=10-12i-2$ simplifying powers
$=8-12i$ ✓answer – both values

OR

$=2(-1)i[(5i-6(-1)-i)]$ ✓one mark for $2i^3$
$=-2i(4i+6)$ ✓✓ inside of bracket
$=-8i^2-12i$
$=8-12i$ ✓answer – both values (4)

1.2 1.2.1

$\bar{z}=-\frac{1}{2}-\frac{4,5}{2}i$ **OR** $\bar{z}=-0,5-2,25i$ ✓answer – both values (1)

1.2.2 $r^2 = (-0,5)^2 + (-2,25)^2$ Follow up from 1.2.1

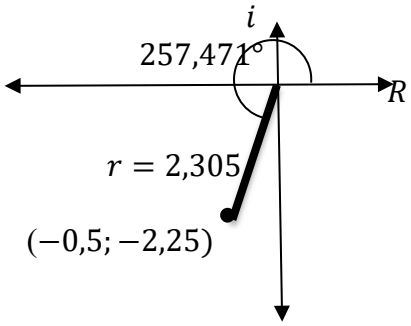
$r^2 = 5,3125$

$r = 2,305$ ✓modulus

$\theta = 180^\circ + tan^{-1}\left(\frac{2,25}{0,5}\right)$

$\theta = 180° + 77,471°$ ✓reference angle

$\theta = 227,471°$ ✓answer



✓standard form values

✓indicate r & θ

(Do not penalise for axes not labelled or incorrect labelling) (5)

[15]

QUESTION 2

2.1 2.1.1 $(z_1)(z_2) = (3,5cis67,8^\circ)(2,4cis12°)$

$= 8,4cis79,8°$ ✓✓one mark per value (2)

2.1.2 $z_1 + z_2$

$= (1,322 + 3,241i) + (2,348 + 0,499i)$ ✓change to polar form

$= 3,67 + 3,74i$ ✓✓one mark per value (3)

2.1.3 $z = \frac{8,4cis79,8°}{3,67 + 3,74i}.$ ✓substitution

$z = \frac{8,4cis79,8°}{5,24\ cis\ 45,54°}$ ✓change to polar form

$z = 1,60\ cis\ 34,3°$ ✓one mark per value

$z = 1,321 + 0,902i$ ✓✓one mark per value

OR

$z = \frac{1,488 + 8,267i}{3,67 + 3,74i}$ ✓substitution

$z = \frac{1,488 + 8,267i}{3,67 + 3,74i} \cdot \frac{3,67 - 3,74i}{3,67 - 3,74i}$ ✓change to rectangular form

$z = \frac{36,489 + 24,775i}{13,469 - 13,988i^2}$ ✓conjugate

$z = \frac{36,489 + 24,775i}{27,456}$ ✓simplification

$z = 1,32 + 0,902i$ ✓both values (5)

2.2 $y = \frac{10-x}{2i} \cdot \frac{2i}{2i} - 3$ ✓conjugate

$y = \frac{20i - 2ix}{4i^2} - 3$

$y = \frac{20i - 2ix}{-4} - 3$

$y = -5i + 0{,}5ix - 3$

$y + 3 = -5i + 0{,}5ix$

$y + 3 = 0$ and $0 = -5i + 0{,}5ix$ ✓✓equate real and imaginary

$y = -3$ $\quad 5 = 0{,}5x$

$10 = x$ ✓✓one mark per value

OR

$2iy = 10 - x - 6i$ ✓multiply by LCM

$2y = -6$ and $0 = 10 - x$ ✓✓equate real and imaginary

$y = -3$ $\quad x = 10$ ✓✓one mark per value (5)

[15]

QUESTION 3

3.1 3.1.1 $q = -1$ ✓value of q

$y = a.2^x - 1$

$1 = a.2^2 - 1$ ✓substitute

$\frac{1}{2} = a$ ✓ value of a (3)

3.1.2 $y > 2; y\epsilon R$ **OR** $y\epsilon(2;\infty)$ ✓answer only

(Do not penalise if yϵR is not included.) (1)

3.1.3 $4 = -\frac{1}{2}x + 3$ ✓value of x

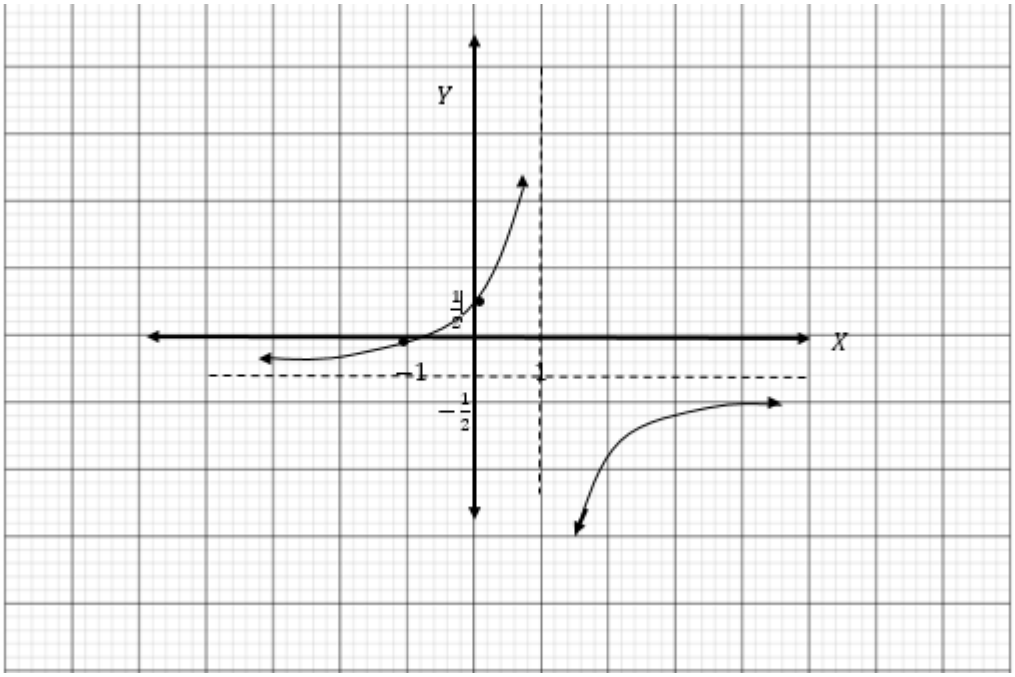
$-2 = x$

Domain: $x\epsilon[-2;\infty)$ $\quad or \quad -2 \leq x < \infty$ ✓Domain

(Follow up on the domain with the calculated value of x.) (2)

3.1.4 $g(x) = -\frac{1}{2}x - 3$ ✓Equation (1)

3.2 3.2.1 ✓shape of the graph
✓showing both correct asymptotes
✓$x-$ intercept
✓$y-$ intercept (4)



3.2.2 $x = 1$ ✓Equation (1)

3.2.3 Function ✓Answer (1)

3.3 3.3.1 $y = a(x + 3{,}5)(x - 1)$ ✓both $x-$ intercepts
Substitute $(-2; 9)$: $9 = a(-2 + 3{,}5)(-2 - 1)$ ✓coordinate
$9 = a\left(\frac{3}{2}\right)(-3)$
$a = -2$ ✓value of a
Equation: $y = -2(x + 3{,}5)(x - 1)$
$y = -2x^2 - 5x + 7$ ✓equation multiplied out
Therefore: $a = -2$; $b = -5$ and $c = 7$ (4)

3.3.2 Turning point: $x = -\frac{5}{4}$ or $x = -1{,}25$
$y = \frac{81}{8}$ or $y = 10{,}125$ ✓value of y
Range: $y \leq 10{,}125$ ✓range (2)

3.3.3 $x > 0$ or $x \epsilon (0; \infty)$ ✓Answer (1)
[20]

QUESTION 4

4.1

$4y + 3x = 50 \therefore y = \frac{50 - 3x}{4}$ ✓y as subject

$x^2 + (\frac{50 - 3x}{4})^2 = 100$ ✓correct substitution

$16x^2 + 2500 - 300x + 9x^2 = 1\,600$

$25x^2 - 300x + 900 = 0$

$x^2 - 12x + 36 = 0$ ✓standard form

$(x - 6)(x - 6) = 0$ ✓factors or formula

$x = 6$ or $x = 6$ ✓value of x

$y = 8$ ✓value of y

OR

$25x^2 - 300x + 900 = 0$

$x = \frac{-(-300) \pm \sqrt{(-300)^2 - 4(25)(900)}}{2(25)}$

$x = \frac{300 \pm \sqrt{0}}{50}$

$x = 6$ or $x = 6$

$y = 8$ (5)

4.2 4.2.1

$\left(\frac{1}{3}x\right)\left(\sqrt[3]{x} - 3\right) = 0$

$x = 0$ or $x = 3^3 = 27$ ✓✓ONE mark per value (2)

4.2.2

$\frac{-1}{2x - 3} - \frac{x - 5}{3} = 0$

$\frac{-3 - (2x - 3)(x - 5)}{3(2x - 3)} = 0$

$\frac{-3 - 2x^2 + 13x - 15}{3(2x - 3)} = 0$

$\frac{-2x^2 + 13x - 18}{3(2x - 3)} = 0$ ✓LCM and numerator

$2x^2 - 13x + 18 = 0$ ✓standard form

$(2x - 9)(x - 2) = 0$ ✓factors

$x = \frac{9}{2}$ or $x = 2$ ✓one mark both values

OR

$-3 - (x - 5)(2x - 3) = 0$ ✓multiply by LCM

$2x^2 - 13x + 18 = 0$ ✓standard form

$(2x - 9)(x - 2) = 18$ ✓factors

$x = \frac{9}{2}$ or $x = 2$ ✓one mark both values (4)

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4.3 $\frac{1}{\frac{b-a}{b}} \div \frac{1}{\frac{a-b}{a}}$ ✓LCM of both denominators

$= \frac{b}{-(a-b)} \div \frac{a}{a-b}$ ✓simplify second fraction

$= \frac{b}{-(a-b)} . \frac{a-b}{a}$ ✓change to multiplication and reciprocal

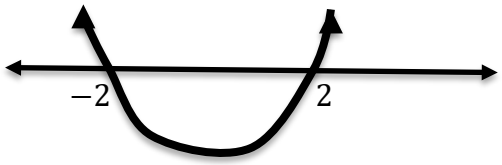
$= -\frac{b}{a}$ ✓answer (4)

4.4 $3x^2 - 4 < 8$ ✓change the sign when x (-2)

$3x^2 - 12 < 0$

$3(x^2 - 4) < 0$

$(x-2)(x+2) < 0$ ✓factors



✓numberline

$x \epsilon (-2; 2)$ ✓correct interval values
✓correct brackets (5)
[20]

QUESTION 5

5.1 5.1.1 $y = 3 - \frac{1}{5\sqrt[3]{x^5}} + 4x$

$y = 3 - \frac{1}{5}x^{-\frac{5}{3}} + 4x$

$\frac{dy}{dx} = \frac{1}{3}\ x^{-\frac{8}{3}} + 4$

$\frac{dy}{dx} = \frac{1}{3\sqrt[3]{x^8}} + 4$ ✓✓✓one mark per derivative

(Penalise with one mark if surd form is not used.) (3)

5.1.2 $f(x) = \dfrac{x^3 - 7x + 9x^2}{7x}$

$f(x) = \frac{1}{7}x^2 - 1 + \frac{9}{7}x$

$f'(x) = \frac{2}{7}x + \frac{9}{7}$ ✓✓✓one mark per term

(Only follow up on terms which were correctly simplified.) (3)

5.2 $f(x) = x^2 - 4$

$f(x + h) = (x + h)^2 - 4$

$f(x + h) = x^2 + 2xh + h^2 - 4$ ✓$f(x + h)$ simplified

$f(x + h) - f(x) = (x^2 + 2xh + h^2 - 4) - (x^2 - 4)$

$= 2xh + h^2$ ✓$f(x + h) - f(x)$

$f'(x) = \lim\limits_{h \to 0} \frac{h(2x + h)}{h}$ ✓common factor

$= 2x$ ✓answer (4)

5.3 Substitute $x = 3$ delivers: $\frac{0}{0}$ no marks allocated

$\lim\limits_{x \to 3} \dfrac{x^2 - 7x + 12}{2x^2 - 4x - 6}$

$\lim\limits_{x \to 3} \dfrac{(x\ -\ 4)(x\ -\ 3)}{2(x^2\ -\ 2x\ -\ 3)}$

$\lim\limits_{x \to 3} \frac{(x - 4)(x - 3)}{2(x + 1)(x - 3)}$ ✓✓factorise fully

$\lim\limits_{x \to 3} \dfrac{x - 4}{2(x + 1)}$

$= \frac{-1}{8}$ ✓Answer

OR

$\lim\limits_{x \to 3} \frac{2x - 7}{4x - 4}$ ✓✓derivatives

$= \frac{-1}{8}$ ✓answer (3)

[13]

QUESTION 6

6.1 $P = 20 - \frac{6}{1} = 14$ thousand or 14 000 ✓answer in thousands
(Answer only: full marks) (1)

6.2 $P = 20 - 6t^{-1}$
$P'(t) = \frac{6}{t^2}$ or $P'(t) = 6t^{-2}$ ✓correct derivative

$t = 3$: $P'(3) = \frac{6}{9} = \frac{2}{3}$ ✓substitution
$= 0{,}667$ thousand or 667 per year ✓answer in thousands (3)

6.3 First year: $P(1) = 14\ 000$
Second year: $P(2) = 17\ 000$ ✓number in 2nd year
Difference: 3 000
Increase in the number of the population from the 1st to the 2nd year.
✓increase (2)

6.4 When $t \to \infty$; $\frac{6}{t} \to 0$
Therefore the population will stabilise at 20 thousand. ✓answer

OR

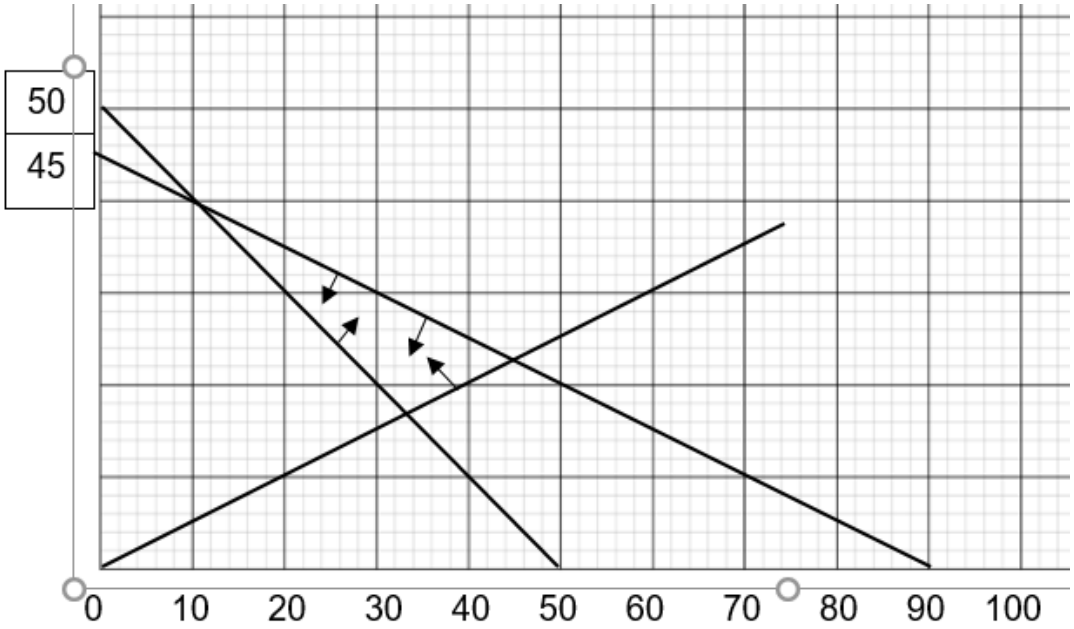
The equation of growth represents the graph of a hyperbola with an asymptote of 20 000. Therefore the population cannot exceed 20 000. (1)
[7]

QUESTION 7

7.1 See below: One mark per correct line: both $x-$ and $y-$ intercepts correct

$1\,000x + 2\,000y \leq 90\,000$: $x = 90$ and $y = 45$✓
$x + y \geq 50$ $x = 50$ and $y = 50$✓
$y \geq \frac{x}{2}$ Coordinate $(0; 0)$✓ and any other coordinate (correct slope)✓ (4)

7.2



Two marks for shading the feasible region✓✓
∗ Incorrect feasible region shaded − do not follow up to QUESTION 7.3 (2)

7.3 Minimum unskilled workers: 10✓✓ and maximum unskilled workers: 45 ✓✓

∗ Only mark QUESTION 7.3 if the feasible region is correct.
∗ Students must indicate minimum and maximum number of unskilled workers in order to obtain full marks.

∗ If all coordinates only are given: No marks
$(10; 40)$; $(45; 20)$; $(35; 15)$

*If a coordinate form is given: $(10; 45)$: One mark only (4)
[10]

TOTAL: 100

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