higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT (BEROEPSGERIG)

WISKUNDE
(Tweede vraestel)
NKR-VLAK 4

17 Februarie 2025

Hierdie nasienriglyn bestaan uit 12 bladsye.

Kopiereg voorbehou

VRAAG 1

1.1	**Bewerking**	**Verduideliking**	**Punt**
1.1.1	$r^2 = (2-4)^2 + (3-5)^2 = 8$ ✓ **OF** $r = \sqrt{(2-4)^2 + (3-5)^2} = \sqrt{8}$ $\therefore (x-2)^2 + (y-3)^2 = 8$ ✓	1SF 1CA	(2)
1.1.2	$m_{AB} = \frac{5-3}{4-2} = 1$ √ $\therefore m_{BC} = -1$ √ $\Rightarrow y - 5 = -1(x-4)$√ **OF** $\Rightarrow y = -x + 9$ √ $m_{AB} = \frac{5-3}{4-2} = 1$ √ $\therefore m_{BC} = -1$ √ $5 = (-1 \times 4) + C$ $\Rightarrow C = 9$ √ $\therefore y = -x + 9$ √	1/2A 1/2S 1/2SF 1/2A	(2)
1.1.3	Vals want ✓ $(2-2)^2 + \left(\frac{1}{8} - 3\right)^2 \neq 8$✓	1A; 1R	(2)

1.2			
	$P\hat{O}N = 53°$ (Oorstaande hoeke)✓ $P\hat{N}O = 90°$(Reguitlyn ⊥ Radius)✓ $\therefore O\hat{P}N = 180° - 90° - 53° = 37°$(som v. int.$<_S$ v. ΔPON)✓	1S 1S 1CA	(3)

1.3	**Bewerking**	**Verduideliking**	**Punt**
1.3.1	$\widehat{E_1} = y = \widehat{A_2}$✓ ($<_S$ in dieselfde segment)✓ $\widehat{C_2} = y = \widehat{A_2}$✓ (AO = CO = Radius)✓ $\widehat{E_2} = y = \widehat{C_2}$✓ ($<_S$ in dieselfde segment)	1S; 1R 1S; 1R 1S	(5)
1.3.2	$A\hat{O}C = 180° - 2y = \widehat{O_1} + \widehat{O_2}$✓ (som v. int.$<_S$ v. Δ)✓ **OF** $A\hat{O}C = 180° - 2y = \widehat{O_1} + \widehat{O_2}$✓ (Teen. $<_S$ van ing vierhoek.)✓	1S; 1R	(2)
1.3.3	$\hat{B} = 90° - y$✓ ($<_S$ by middelpunt = 2 ×$<_S$ by omtrek) ✓	1S; 1R	(2)
1.3.4	$\widehat{D_1} = \hat{B} = 90° - y$ (Buite $<_S$ van ing vierh.)✓ $\widehat{E_1} + \widehat{E_2} + \widehat{D_1} + \widehat{A_4} = 180°$ (som van $<_S$ van Δ)✓ $\Rightarrow \widehat{A_4} = 180° - y - y - (90° - y) = 90° - y$ $\Rightarrow \widehat{A_4} = \widehat{D_1}$ ✓ $\therefore EA = ED$ ($<_S$ van gelykbenige Δ)✓	1S & R 1S 1A 1R	(4)

1.3.5	AOCD is nie in ′n ingeskrewe vierhoek nie✓ $\widehat{D_1} = 90° - y \neq 180° - 2y = A\widehat{O}C$✓ (Buite $<_S$ v ing vierh.)✓ Alternatiewe $\widehat{D_2} = 180° - \widehat{D_1} = 180° - (90° - y) = 90° + y$ ✓ $\Rightarrow \widehat{D_2} + A\widehat{O}C = 90° + y + 180° - 2y = 270° - y \neq 180°$✓ $\therefore$AOCD is nie ′n ingeskrewe vierhoek nie✓	1S 1S; 1R	(3)

[25]

VRAAG 2✓

2.1	$\sqrt{2\cos 2B + 2}$ $= \sqrt{2(\cos 2B + 1)}$ ✓ $= \sqrt{2(2\cos^2 B - 1 + 1)}$ ✓ $= \sqrt{4\cos^2 B}$ $= 2\cos B$ ✓	1F 1F 1 A	(3)

2.2	**Bewerking**	**Verduideliking**	**Punt**
2.2.1	$\cos 87° \cos 23° + \sin 87° \sin 23° = 1 - 2\sin^2 x$ $\Rightarrow \cos(87° - 23°) = \cos 2x$ ✓ $\Rightarrow \cos 64° = \cos 2x$ √ $\Rightarrow 64° = 2x$ √ $\Rightarrow 32° = x$ ✓	 1 B/D 1/2 S 1/2 S 1 A	(3)
2.2.2	$\sqrt{(1 - \cos x)(1 + \cos x)} = \frac{1}{2}$ $\sqrt{1 - cos^2 x} = \frac{1}{2}$ ✓ $\sqrt{sin^2 x} = \frac{1}{2}$✓ $\sin x = \frac{1}{2}$ $x = 45°$✓	 1 S 1 S 1 A	(3)

2.3	**Bewerking**	**Verduideliking**	**Punt**
2.3.1	LHS $= \dfrac{1+\sin 2x+\cos 2x}{\sin x+\cos x}$ $= \dfrac{1+2\sin x\cos x+2\cos^2 x-1\checkmark}{\sin x+\cos x}$ $= \dfrac{2\sin x\cos x+2\cos^2 x\checkmark}{\sin x+\cos x}$ $= \dfrac{2\cos x\,(\sin x+\cos x)\ \checkmark}{\sin x+\cos x}$ $= 2\cos x$ $=$ RHS Alternatiewe LHS $= \dfrac{1+\sin 2x+\cos 2x}{\sin x+\cos x}$ $= \dfrac{\cos^2 x+\sin^2 x+2\sin x\cos x+\cos^2 x-\sin^2 x\ \checkmark}{\sin x+\cos x}$ $= \dfrac{2\sin x\cos x+2\cos^2 x\ \checkmark}{\sin x+\cos x}$ $= \dfrac{2\cos x\,(\sin x+\cos x)\checkmark}{\sin x+\cos x}$ $= 2\cos x$ $=$ RHS	1 B/D 1 Teller 1 Faktorisering 1 B/D 1 Teller 1 Faktorisering	(3)

2.3.2	LHS $= \dfrac{\sin 3x}{\sin x}-\dfrac{\cos 3x}{\cos x}$ $= \dfrac{\sin 3x\cos x-\cos 3x\sin x\ \checkmark}{\sin x\cos x\ \checkmark}$ $= \dfrac{\sin(3x-x)\checkmark}{\sin x\cos x}$ $= \dfrac{\sin 2x\ \surd}{\sin x\cos x}$ $= \dfrac{2\sin x\cos x\ \surd}{\sin x\cos x}$ $=$ RHS	1 Teller 1 Noemer 1 B/D ½ $\sin 2x$ ½ F	

	Alternatiewe $\text{LHS} = \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$ $= \frac{\sin(2x+x)}{\sin x} - \frac{\cos(2x+x)}{\cos x}$ √ $= \frac{\sin 2x \cos x + \sin x \cos 2x}{\sin x} - \frac{(\cos 2x \cos x - \sin 2x \sin x)}{\cos x}$ ✓ $= \frac{\sin 2x \ \cos x}{\sin x} + \cos 2x - \cos 2x + \frac{\sin 2x \ \sin x}{\cos x}$ ✓ $= \frac{2\sin x \cos^2 x}{\sin x} + \frac{2\sin^2 x \cos x}{\cos x}$ √ $= 2\cos^2 x + 2\sin^2 x$ √ $= 2(\cos^2 x + \sin^2 x)$ √ $= \text{RHS}$	1 Teller 1 Noemer 1 B/D ½ $\sin 2x$ ½ F	(4)

2.4	**Bewerking**	**Verduideliking**	**Punt**
2.4.1	$\tan 20° = \frac{15}{\text{BC}}$ ✓ $\Rightarrow \text{BC} = \frac{15}{\tan 20°} = 41.21\text{m}$✓ Alternatief 1 In ΔBDC; $\hat{\text{D}} = 180° - 90° - 20° = 70°$√ $\Rightarrow \frac{\text{BC}}{\sin 70°} = \frac{15}{\sin 20°}$√ $\Rightarrow \text{BC} = \frac{15\sin 70°}{\sin 20°} = 41.21\text{m}$✓ Alternatief 2 $\tan 70° = \frac{\text{BC}}{15}$ ✓ $\Rightarrow \text{BC} = 15\tan 70° = 41.21\text{m}$✓	1 S 1 CA ½ CA ½ SF 1 CA	(2)

2.4.2	$AC^2 = (41.21)^2 + (170)^2 - 2(41.21)(170)\cos 35°$✓ $\Rightarrow AC = 138.28$m✓ $\therefore \tan\alpha = \frac{15}{138.28}$ ✓ $\Rightarrow \alpha = \tan^{-1}\left(\frac{15}{138.28}\right) = 6.19°$ ✓	1SF 1 CA 1S 1CA	(4)
2.4.3	$(170)^2 = (41.21)^2 + (138.28)^2 - 2(41.21)(138.28)\cos\hat{C}$✓ $\Rightarrow \cos\hat{C} = -0.709$✓ $\Rightarrow \hat{C} = \cos^{-1}(-0.709) = 135.15°$ ✓ Alternatiewe $\frac{170}{\sin\hat{C}} = \frac{138.28}{\sin 35°}$ ✓ $\Rightarrow \sin\hat{C} = \frac{170\sin 35°}{138.28} = 0.7051$ ✓ $\Rightarrow \hat{C} = 180° - \sin^{-1}(0.7051) = 135.16°$ ✓	1SF 1S 1CA 1SF 1S 1CA	(3)
			[25]

VRAAG 3

3	**Bewerking**	**Verduideliking**	**Punt**
3.1	Sien ADDENDUM A hier onder✓✓✓		(3)
3.2	Sien ADDENDUM A hier onder✓		(1)
3.3	Negatiewe verhouding✓	1S	(1)
3.4	OPSIE 1 Sien ADDENDUM B hier onder √√√√√√ $b = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - \left(\sum x\right)^2}$ $= \frac{(8\times 469.5) - (140\times 32)}{(8\times 2804) - (140)^2}\surd$ $= -0.256\surd$ $a = \bar{y} - b\bar{x}$ $= 4 - (-0.256\times 17.5) = 8.48\surd$ $\therefore y = 8.48 - 0.256x\surd$	1/2 punt vir elke som/gemiddelde in tabel 1/2 punt vir subst. in formule van b (SF) 1/2 punt vir b waarde (CA) 1/2 punt vir a waarde (CA) 1/2 punt vir reg. verg.	

	OPSIE 2 Sien ADDENDUM B hier onder √√√√√√ $b = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$ $= \frac{-90.5}{354}\surd$ $= -0.256\surd$ $a = \bar{y} - b\bar{x}$ $= 4 - (-0.256 \times 17.5) = 8.48\surd$ $\therefore y = 8.48 - 0.256x\surd$		(5)
3.5	$y = 8.48 - 0.256(17)$✓ $\Rightarrow y = \frac{516}{125} = 4.128$✓	1SF 1CA	(2)
3.6	$\bar{x} = 17.5$ (Sien ADDENDUM B hier onder) $s^2 = \frac{(8-17.5)^2 + (13-17.5)^2 + \cdots + (30-17.5)^2}{8}$ ✓ $= 44.25$ ✓ $\Rightarrow s = \sqrt{44.25} = 6.65$ ✓	1SF 1CA 1CA	(3)

ALTERNATIEF VIR 3.6

x	$x - \bar{x}$	$(x - \bar{x})^2$
8	-9.5	90.25
13	-4.5	20.25
14	-3.5	12.25
15	-2.5	6.25
15	-2.5	6.25
20	2.5	6.25
25	7.5	56.25
30	12.5	156.25
TOTAL	0	354✓

$s^2 = \frac{354}{8} = 44.25$ ✓

$\Rightarrow s = \sqrt{44.25} = 6.65$ ✓

1SF
1CA
1CA

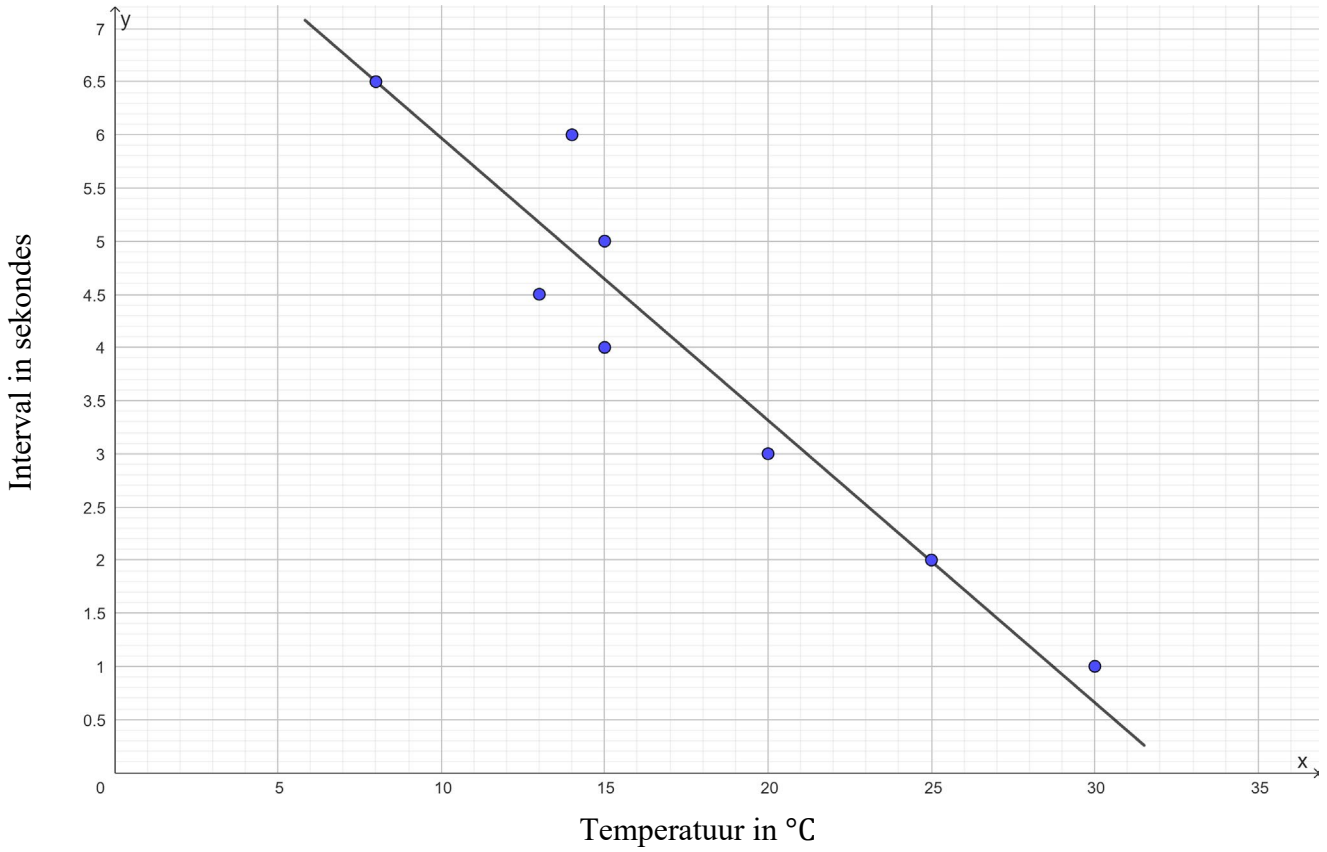
[15]

ADDENDUM A

EKSAMENNOMMER:

SENTRUMNOMMER:

3.1
3.2



Lyn van die beste passing ✓

Let wel: 'n Lyn van die beste passing (‘eyeball method’) is 'n reguit lyn wat deur die maksimum aantal punte op 'n verspreidingsdiagram getrek word; balanseer ongeveer 'n gelyke aantal punte onder en bo die lyn.

Verspreidingsdiagram ✓✓✓

Let wel: Trek 1 punt af vir enige 3 punte wat verkeerd geplot is.

ADDENDUM B

EKSAMENNOMMER: | | | | | | | | | | | | |

SENTRUMNOMMER: | | | | | | | | | | | |

3.4

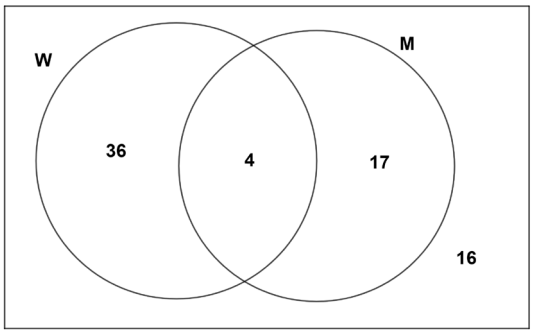
OPSIE 1

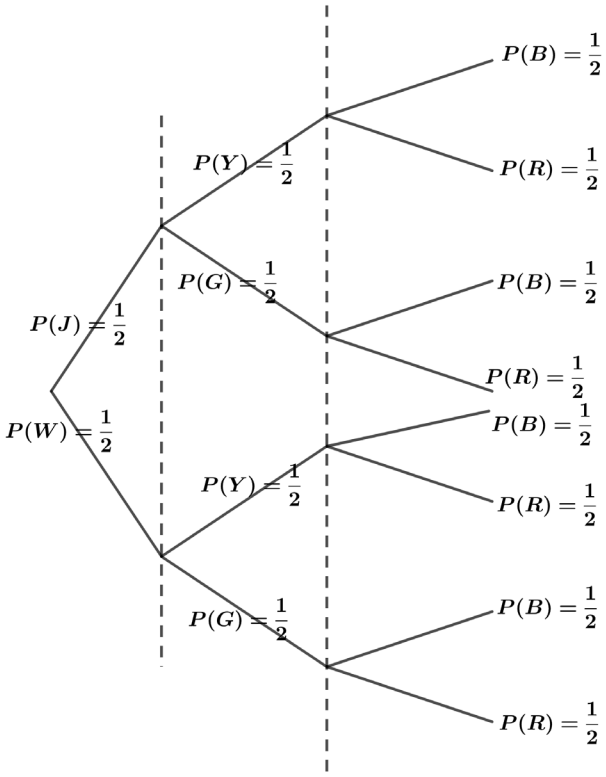
Temperatuur in °C (x)	**Interval in sekondes (y)**	xy	x^2
8	6.5	52	64
13	4.5	58.5	169
14	6	84	196
15	5	75	225
15	4	60	225
20	3	60	400
25	2	50	625
30	1	30	900
$\sum x = 140$ √	$\sum y = 32$ √	$\sum xy = 469.5$ √	$\sum x^2 = 2804$√
$\bar{x} = 17.5$√	$\bar{y} = 4$√		

OPSIE 2

Temperatuur in °C (x)	**Interval in sekondes (y)**	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$
8	6.5	-23.75	90.25
13	4.5	-2.25	20.25
14	6	-7	12.25
15	5	-2.5	6.25
15	4	0	6.25
20	3	-2.5	6.25
25	2	-15	56.25
30	1	-37.5	156.25
$\sum x = 140$ √	$\sum y = 32$ √	$\sum (x - \bar{x})(y - \bar{y}) = -90.5$ √	$\sum (x - \bar{x})^2 = 354$√
$\bar{x} = 17.5$√	$\bar{y} = 4$√		

VRAAG 4

4.1	**Bewerking**	**Verduideliking**	**Punt**
4.1.1	$S = 6;\ U = 5;\ T = 31;\ V = 73$✓✓	1/2 punt elk	(2)
4.1.2	$P(\overline{W \text{ en } M}) = \frac{6 + 10 + 17 + 5 + 31}{73} = \frac{69}{73} = 0.945$✓✓ **OF** $P(\overline{W \text{ en } M}) = \frac{73 - 4}{73} = \frac{69}{73} = 0.945$✓✓ **OF** $P(\overline{W \text{ en } M}) = 1 - P(W \text{ en } M)$ $= 1 - \frac{4}{73}$ ✓ $= 0.945$ ✓	1S; 1CA 1S; 1CA 1S 1CA	(2)
4.1.3	$P(G \text{ en } H) = \frac{10}{73} = 0.137$ ✓ $P(G) \times P(H) = \frac{33}{73} \times \frac{41}{73} = \frac{1353}{5329} = 0.254$ ✓ $\therefore$ Die gebeure is sedertdien nie onafhanklik nie $0.137 \neq 0.254$ ✓	1 CA 1 CA 1S	(3)
4.1.4	 √√√√	1/2 punt vir elke nommer	(2)

4.2	**Bewerking**	**Verduideliking**	**Punt**
4.2.1	 ✓✓✓✓	1/2 punt vir elk van die 8 takke wat die korrekte waarskynlikhede bevat	(4)
4.2.2	$\text{P(W en Y en R)} = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8} = 0.125$ ✓✓	1S; 1A	(2)

[15]

VRAAG 5

5.1	Reguitlyn-depresiasie	1 S	(1)

5.2			
	<u>Eerste 7 jaar</u> $\text{A} = \text{P}(1-\text{i})^{\text{n}}$		
	$\text{A} = 400\,000\left(1-\frac{8.5}{100}\right)^{7}$ ✓	1SF	
	$= \text{R}214\,786.83$ ✓	1CA	
	<u>Volgende 3 jaar</u> $\text{A} = \text{P}(1-\text{in})$		
	$= 214\,786.83\left(1-\frac{4.5}{100}\times 3\right)$ ✓	1SF	
	$= \text{R}185\,790.61$ ✓	1CA	(4)

5.3	**Bewerking**	**Verduideliking**	**Punt**
5.3.1	Deposito $= 20\%$ of $\text{R}15000 = 15000 \times \frac{20}{100} = R3000$ ✓✓	1SF, 1A	(2)
5.3.2	Uitstaande balans $= R15000 - R3000 = R12000$ ✓	1 A	(1)
5.3.3	$A = P\,(1 + \frac{r}{100} \times n)$ $A = 12000\,(1 + \frac{18}{100} \times 3$✓$)$ $A = R\,18480$ ✓ Maandelikse paaiement $\frac{18480}{36} + R50$✓ $= R\,563{,}33$✓	1 SF 1A 1 SF , 1A	(4)

5.4	**Bewerking**	**Verduideliking**	**Punt**
5.4.1	'n Belastingdrempel is die vlak waarop inkomste belasbaar is.✓	1SF	(1)
5.4.2	'n Belastingkrediet is 'n bedrag/item wat die jaarlikse belasting verminder✓	1 SF	(1)

5.5	**Bewerking**	**Verduideliking**	**Punt**
5.5.1	Bruto jaarlikse inkomste $= (12 \times 20\,000) = \text{R}240\,000$ ✓ Aftrekkings $= (5\,000 \times 12) + (1\% \times 240\,000) = \text{R}62\,400$✓ Belasbare inkomste $= \text{R}240\,000 - \text{R}62\,400 = \text{R}177\,600$ ✓ Belasting betaalbaar $= 18\% \times 177\,600 = \text{R}31\,968$ ✓ Jaarlikse belastingplig $= \text{R}31\,968 - \text{R}17\,235 = \text{R}14\,733$ ✓	1A 1A 1A 1A 1A	(5)
5.5.2	Maandelikse bedrag uit $= \text{R}20\,000 - 5\,000 - 200 - 5000$ $= \text{R}9\,800$ ✓	1A	(1)

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TOTAAL: 100

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