higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS

(First paper)

NQF LEVEL 4

14 February 2025

This marking guideline consists of 16 pages.

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INSTRUCTIONS TO MARKERS

1. Examiners are reminded that the subject Mathematics lends itself to a variety of methods and therefore several different alternative answers. Mark all mathematical correct answers.

2. The answers provided in this memorandum are therefore not exhaustive. Marks are allocated for specific principle and the markers must adhere to the mark allocation.

3. Please mark the questions and follow up. For example: Error in the first step of differentiation, mistake in converting from surd to exponential, does not imply that the student can't differentiate. Follow up.

4. Marks are allocated per step, but if a student omitted a step and there is evidence or reason that he/she can derive at the next step without calculation, then the mark must still be given to the student.

5. Take note that marks have been allocated to simplification. This is to differentiate between your 80 percenter and 90 percenter.

6. STEPS must be marked and not only the final answer. TICKS must correspond with the total. Put your ticks and crosses neatly and legibly in RED pen only.

7. If the answer does not look exactly the same as on MG, this could be alternative method or a method that is not known to you as marker. Please take a moment before you mark, verify the students responds.

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QUESTION 1

	Full mark ✓ Half mark √ A: Accuracy CA: Consistent Accuracy or follow up				
1.1	1.1.1	A	$19 - 4i$		
	1.1.2	B	$-1 - 5i$		
	1.1.3	D	$(x - 8i)(x + 8i)$		
	1.1.4	B	$8\ \text{cis}(36°)$		
	1.1.5	D	$x = 12$		
			(5 × 1)		(5)

1.2	1.2.1	$z_1 = 13\ \text{cis}\ (-22{,}62°)$ $= \sqrt{13}\ \text{cis}\ 337{,}38°$ √	$0 \leq \theta \leq 360.$ Half mark correct modulus(A) Half mark correct Argument within given interval (A) No marks if argument is negative.	(1)
	1.2.2	$(12 - 5i - 6 + 8i\ )^4 = (6 + \sqrt{3}i)^4$ ✓ $= (6{,}\sqrt{71}\ \text{cis}\ 26{,}\sqrt{57})^4$ √ √ $= 2027{,}17\ \text{cis}\ 106{,}27°$ √ √	One mark adding (A) Converting $R \rightarrow P$ Half mark correct modulus(A) Half mark correct Argument Apply De Moivre Half mark correct modulus(A) Half mark correct Argument (A)	(3)

1.3	Method 1 $a+bi=\frac{-27+21i}{-5+3i}\times\frac{-5-3i}{-5-3i}$ √ $=\frac{(135+81i-105i-63i^2}{25-9i^2}$ √ √ $=\frac{198-24i}{34}$ √ $=\frac{99}{17}-\frac{12}{17}i$ √ $or\ 5{,}82-0{,}71\ i$ Method 2 $(a+bi)(-5+3i)=-27+21i$ $\therefore -5a+3ai-5bi+3i^2b=-27+21i$ $\therefore -5a-3b+3ai-5bi=-27+21i$ Solve simultaneous equations. $\therefore -5a-3b=-27\ \ eq\ 1\ \times 3\rightarrow -15a-9b$ $=-81$ $\therefore 3a-5b=21\quad eq\ 2\ \times 5\rightarrow 15a-25b=105$ $\therefore -34b=24\rightarrow b=-\frac{12}{17}$ Substitute in eq 2 $3a-5\left(-\frac{12}{17}\right)=21$ $\therefore a=\frac{99}{17}$ $\therefore a+bi\ =\frac{99}{17}-\frac{12}{17}i\ \ or\ 5{,}82-0{,}71\ i$	Half mark multiply with conjugate/conjugate (A) Half mark multiply denominator (A) Half mark multiply numerator(A) Half mark simplified denominator (A) Half mark simplified numerator (A) Half mark a+bi	(3)

1.4			
	$-y^2 + 12 = -y$ √ $eq\ 1$ $x^2i + 2xi + yi = 0$ √ $eq2$ ∴ From eq 1 $(y-4)(y+3) = 0$ ∴ y = $\overset{\surd}{-}3$ or y = $4^{\surd}$ $y = -3$ N/A $(\text{not } \in N)$ $y = 4$ in eq 2 $\rightarrow\ x^2 + 2xi + 4i$ $\therefore\ x = \dfrac{-2 \pm \sqrt{2^2 - 4(4)}}{2(1)}$ $= \dfrac{-2 \pm \sqrt{-12}}{2(1)}$ √ $= \dfrac{-2 \pm 2\sqrt{3}i}{2}$ √ √ √ $\therefore x = -1 \pm \sqrt{3}\,i$	Half mark $eq1$ (A) Half mark $eq2$ (A) Half mark each y Half mark applying quadratic $\frac{-2\pm\sqrt{-12}}{2(1)}$ Half mark solving negative root $\frac{-2\pm2\sqrt{3}i}{2(1)}$ Half mark imaginary Half mark real	(4)
			[16]

QUESTION 2

2.1	$Let\ g(x) = 0 \therefore x = 0$ $f(0) = 5(0)^3 + 7(0)^2 - \sqrt{2}(0) - 4 = -4$ $\therefore$ Remainder $= -4$ √	Half mark substitute 0 Half mark answer	(1)
2.2	$(x + 1) = 0 \therefore x = -1$ $(-1)^4 - 15(-1)^2 + p(-1) + 24 = 0$ (√ √) $\therefore p = 10$ ✓	Half mark equate to 0 Half mark substitute -1 One mark answer	(2)
2.3	$(-1; -25)\ given$ Substitute in $g(x)$ $\rightarrow -25 = -2 - m - n$ $\therefore m + n = 23 \quad eq\ 1$ √ and given $m = 40$ $\frac{dy}{dx} = 6x^2 - 2mx + n = 40\ at\ x = -1$ (√ √) $\therefore 34 = 2m + n \quad eq\ 2$ √ $eq\ 2\ -\ eq\ 1$ $\rightarrow m = 11\ and\ n = 23 - m = 12$ $m = 11\ and\ n = 12$ (✓ ✓)	Half mark $eq\ 1$ Half mark differentiation Half mark equates to 40 Half mark $eq\ 2$ One mark answer m One mark answer n	(4)

2.4	2.4.1	g(x) : $y = x^2$ ✓		(1)
	2.4.2	Domain of $g(x)$: $\{x: -3 \leq x \leq 0; x \in R\}$ ✓ or $x \in [-3; 0]\ for\ x \in R$		(1)
	2.4.3	$f^{-1}(x)$: $y = 3x - 8$ ✓		(1)
	2.4.4	$f(f^{-1}(x)) = f(3x - 8)$ $\therefore \frac{f^{-1}(x) + 8}{3}$ $= \frac{3x - 8 + 8}{3} = \frac{3x}{3} = x$ ✓ ✓	One-mark substitute $3x - 8\ in\ f(x)$ (A) One mark answer (A)	(2)
	2.4.5	$f(a + 2) + g(a) = 8$ $\therefore \frac{a + 2 + 8}{3} + a^2 = 8$ √ √ $\therefore\ 3a^2 + a + 10 - 24 = 0$ $\therefore 3a^2 + a - 14 = 0$ √ $\therefore a = -\frac{7}{3} or\ a = 2$ √ √ Test : $f\left(-\frac{7}{3} + 2\right)\ is$ valid for $\{x: -2 \leq x < 7; x \in R\}$ $f(2 + 2) = f(4)$ is valid $g\left(-\frac{7}{3}\right)$ valid for $\{x: -3 \leq x \leq 0; x \in R\}$ $g(2)$ not valid for interval $\therefore a = -\frac{7}{3}$ is only valid answer √	Half mark $f(a + 2)$ Half mark $g(a)$ Half mark quadratic equation Half mark each solution of a Half mark identifying valid answer.	(3)
	2.4.6	√ √ √ √ Range $\{y: 2 \leq y < 5; y \in R\}$ or $y \in [2; 5)\ ; y \in R$	Half mark start value(A) Half mark end value(A) Half mark each correct inequality sign(A)	(2)
	2.4.7	SEE **ADDENDUM A.**		(3)
				[20]

QUESTION 3

3.1	$f(x) = -x^3 \therefore f(x+h) = -(x+h)^3$ $= -x^3 - 3x^2h - 3xh^2 - h^3$ √ $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h\to 0} \frac{-x^3 - 3x^2h - 3xh^2 - h^3 + x^3}{h}$ $= \lim_{h\to 0} \frac{-3x^2h - 3xh^2 - h^3}{h}$ $= \lim_{h\to 0} \frac{h(-3x^2 - 3xh - h^2)}{h}$ √ $= \lim_{h\to 0} -3x^2 - 3xh - h^2$ $= -3x^2$ √		No marks for final answer if it cannot be derived from previous steps. Half mark $f(x+h)$ Half mark correct substitution in first principles Half mark taking h as common factor Half mark answer	(2)
3.2	3.2.1	$\frac{dy}{dx} = 6\cos 3x + 10x^4 + \frac{6}{x} + 0$ √ √ √ √	Half mark each term differentiated (A)	(2)
	3.2.2	$\frac{dy}{dx} = -10e^{5x} + \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{5}{2}} + 2x^{-2}$ √ √ √ √ $\therefore \frac{dy}{dx} = -10e^{5x} + \frac{3}{2\sqrt{x}} - \frac{3}{2\sqrt{x^5}} + \frac{2}{x^2}$ √ √	Half mark each term differentiated step 1 Half mark each term in surd and positive exponent where applicable. Half mark to $\frac{dy}{dx}$ in final answer	(4)

	3.2.3	$\frac{dy}{dx} = (9x^2 + 6)^{\frac{1}{3}}$ $\frac{dy}{dx} = \frac{1}{3}(9x^2 + 6)^{-\frac{2}{3}}(18x)$ ✓ ✓ $\frac{dy}{dx} = \frac{6x}{(9x^2 + 6)^{\frac{2}{3}}}$ ✓ $= \frac{6x}{\sqrt[3]{(9x^2 + 6)^2}}$ ✓ ALTERNATIVE $Let\ u = 9x^2 + 6\ \therefore \frac{du}{dx} = 18x$ √ $and\ \ y = u^{\frac{1}{3}}\ \therefore \frac{dy}{du} = \frac{1}{3}\ u^{-\frac{2}{3}}$ √ $\therefore \frac{dy}{dx} = -4(2x^3 - 4)^{-5}(6x^2)$ ✓ ✓ $\therefore \frac{dy}{dx} = \frac{-24}{(2x^3 - 4)^5}$ $= \frac{6x}{\sqrt[3]{(9x^2 + 6)^2}}$ ✓	One-mark-apply $y = f(x)^n\ rule$ (A) One-mark- d/dx base (A) One-mark- Simplified correctly with positive exponents (CA) One-mark surd form answer Or Half mark -du/dx (A) Half mark- dy/du base (A) One mark -apply $chain\ rule$ One mark- Simplified correctly with positive exponents (CA) One-mark surd form answer	(4)
	3.2.4	$f'(x) = (4e^{3x})(3\sec^2 3x) + (12e^{3x})(\tan 3x)$ √ √ √ √	Half mark for each term correctly applied in product rule	(2)
				[14]

QUESTION 4

4.1	4.1.1	$\int ydx = +3\cos 5x + 2e^{4x} + 12x^3 + 5x + c$	Half mark each term integrated. Half mark positive 3cos 5x and half mark c	(3)
	4.1.2	$\int ydx = \int \left(\frac{2}{x} + \frac{2}{\sqrt[3]{x}} + \frac{10}{x^6} - \frac{5}{3x^2} + 23\right) dx$ $= 2lnx + 3x^{\frac{2}{3}} - 2x^{-5} + \frac{5}{3}x^{-1} + 23x + c$ $= 2lnx + 3\sqrt[3]{x^2} - \frac{2}{x^5} + \frac{5}{3x} + 23x + c$	Half mark each term integrated. Half mark for 2nd,3rd and 4th term simplified to surd and positive exp . No marks for c in 4.1.2	(4)
	4.1.3	$\int (x^2 + 2x + 4)\, dx$ $= \frac{x^3}{3} + \frac{2x^2}{2}x^2 + 4x + c = \frac{x^3}{3} + x^2 + 4x + c$	Half mark each term in simplified fraction Half mark each integrated term No mark for c	(3)
4.2	$x-$ intercept $x^2 - 9 = 0\ x = -3\ or\ x = 3$ $A_1 = \int_0^3 -x^2 + 9\, dx = -\frac{x^3}{3} + 9x \Big|_0^3$ $= \left[-\frac{3^3}{3} + 9(3)\right] - \left[-\frac{0^3}{3} + 9(0)\right]$ $= 18$ (note…three and half mark for A_1) $A_2 = \int_3^6 -x^2 + 9\, dx = -\frac{x^3}{3} + 9x \Big|_3^6$ $= \left[-\frac{6^3}{3} + 9(6)\right] - \left[-\frac{3^3}{3} + 9(3)\right] = -36 = 36$ *area must be positive.* Area total $= 54$ square units	Half mark x intercept Half mark $A_1 = \int_0^3 ydx$ correct boundaries (A) Half mark each integration in A_1 Half mark substituting in 3 and 0 Half mark $+A_1$ (A) Half mark $A_2 = \int_3^6 ydx$ Half mark$-A_2$ (CA) Half mark A_t (CA)		(5)
				[15]

QUESTION 5

5.1	5.1.1	$s(3) = -(3)^2 + 8(3) + 2$ √ $\therefore$ s = 17m √	Half mark substitute s (A) Half mark answer (A)	(1)
	5.1.2	$s'(t) = \overset{\surd}{-2}t + \overset{\surd}{8} = -2(3)_{\surd} + 8 = \overset{\surd}{2}\,ms^{-1}$ Do not penalise if units are omitted.	Half mark differentiation each term Half mark substitute s'(A) Half mark answer (A)	(2)
	5.1.3	$s'' = \overset{\checkmark}{-2}\,ms^{-2}$		(1)
5.2	5.2.1	$f'(x) = 3x^2 - 8x + 1 = 3x^2 - 2qx + 1$✓ $\therefore -2q = -8$ $\therefore\ q = 4$✓	One-mark $f'(x)$ One-mark solution $f(x) = x^3 - 4x^2 + x + 26$	(2)
	5.2.2	$-2\left\|\begin{array}{l}\overrightarrow{1\ -4+1+26}\\ \downarrow\ \ -2+1\ 2-26\\ \overline{1-6+13\ \lfloor 0}\end{array}\right.$. $\therefore x^3 - 4x^2 + x + 26 = (x+2)(x^2 - 6x + 13)$ √ √ √ $x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2} =$ $= \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2}$ ✓ $\therefore x = 3 + 2i$ ✓ and $x = 3 - 2i$ ✓ and $x = \overset{\surd}{-2}$	In $ax^2 + bx + c$ Half mark a Half mark b Half mark c One mark finding answer of negative root…$4i$ One mark each complex root Half mark for $x = -2$ -- given	

		Alternative: $\begin{array}{r\|l} & \overset{\surd}{x^2} \quad \overset{\surd}{-6x} \quad \overset{\surd}{+13} \\ \hline x+2 & x^3 \quad -4x^2 \quad +x \quad +26 \\ & x^3 \quad 2x^2 \\ \hline & 0 \quad -6x^2 \quad +x \\ & \quad -6x^2 \quad -12x \\ \hline & \quad 0 \quad 13x \quad +26 \\ & \quad\quad 13x \quad +26 \\ \hline & \quad\quad 0 \quad + \quad 0 \end{array}$ $\therefore x^3 - 4x^2 + x + 26 = (x+2)(x^2 - 6x + 13)$ $x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2} =$ $\therefore \dfrac{6 \pm \sqrt{-16}}{2} = \dfrac{6 \pm 4i}{2}$ ✓ $\therefore x = \overset{\checkmark}{3 + 2i}$ and $x = \overset{\checkmark}{3 - 2i}$ and $x = \overset{\surd}{-2}$		

		Alternative: $x^3 - 4x^2 + x + 26 = (x+2)(ax^2 + bx + c)$ $\therefore x^3$: $x^3 = ax^3 \therefore a = 1$ √ Constant$26 = 2 \times c \ \therefore c = 13$ √ x^2: $+ 2ax^2 + bx^2 = -4x^2$ $\therefore 2x^2 + bx^2 = -4x^2$ $\therefore \ b = -6$ √ $\therefore x^3 - 4x^2 + x + 26 = (x+2)(x^2 - 6x + 13)$ $x = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2}$ ✓ ✓ ✓ $\therefore x = 3 + 2i$ and $x = 3 - 2i$ and $x = -2$ √		(5)

5.3	5.3.1	The zeros of f $is -3, 2$ and 2 $\therefore f(x) = -(x+3)(x-2)(x-2)$ √ √ √ $= (x^2 - 4x + 4)(-x - 3)$ $= -x^3 - 3x^2 + 4x^2 + 12x - 4x - 12$ $= -x^3 + x^2 + 8x - 12$ √ √ $\therefore a = 1, b = 8$ and $c = -12$. √ √ √	Half mark each factor One mark arriving at cubic Half mark each a,b,c	(4)
	5.3.2	$y = -x^3 + x^2 + 8x - 12$ $\therefore \frac{dy}{dx} = -3x^2 + 2x + 8 = 0$ √ √ √ √ $\therefore x \frac{-2 \pm \sqrt{2^2 - 4(-3)(8)}}{2(-3)} \quad \therefore x = \frac{-4}{3}$ $\therefore \ y = -\left(\frac{-4}{3}\right)^3 + \left(\frac{-4}{3}\right)^2 + 8\left(\frac{-4}{3}\right) - 12$ $\therefore y = -18{,}52 \quad \therefore \mathrm{D} = (1.333; -18{,}52)$ √ √	Half mark each term d/dx (CA) Half mark equate to zero Half mark x Half mark y	(3)
				[18]

QUESTION 6

6.1	6.1.1	a)	$\mathbf{12500x + 10000y \leq 50\,000}$ $\therefore \mathbf{10000y \leq 50\,000 - 12500x}$ √ $\therefore \mathbf{y \leq 5 - \frac{12500}{10000}x}$ $\therefore \mathbf{y \leq 5 - \frac{5}{4}x}$ √	Half mark constraint equation Half mark simplified equation	(1)
		b)	$(\mathbf{1;3}), (\mathbf{1;2})(\mathbf{1;1}), (\mathbf{2;2}), (\mathbf{2;1}), (\mathbf{3;1})$ √ √ √ √	Half mark each for any four correct coordinates	(2)

6.2	6.2.1	$c = 80x + 160y$ ✓			(1)
	6.2.2	$8x + 2y \geq 16$ √ √ $x + y \geq 5$ √ √ $2x + 7y \geq 20$ √ √		Half mark correct inequality sign Half mark each equation	(3)
	6.2.3	a)	See ADDENDUM B		(3)
		b)	See ADDENDUM B		(1)
		c)	See ADDENDUM B		(1)
		d)	See ADDENDUM B		(1)
		e)	$2x + 7y = 20 \quad eq\ 1$ $x + y = 5 \quad eq\ 2$ from $eq\ 2 \rightarrow \quad x = 5 - y$ √ Substitute in $eq\ 1 \rightarrow 2(5 - y) + 7y = 20$ √ $\therefore\ y = 2 \ \therefore x = 3$ √ The point is $(2; 3)$ No mark for answer only.	Half mark manipulates one equation Half mark substitute in second equation Half mark solve x Half mark solve y Analytical solution required.	(2)
		f)	$c = 80(3) + 160(2) = R560$ ✓		(1)
	6.2.4	$K = 8x + 2y = 8 \times 2 + 2 \times 5 = 26$ ✓			(1)
					[17]

			TOTAL	**100**

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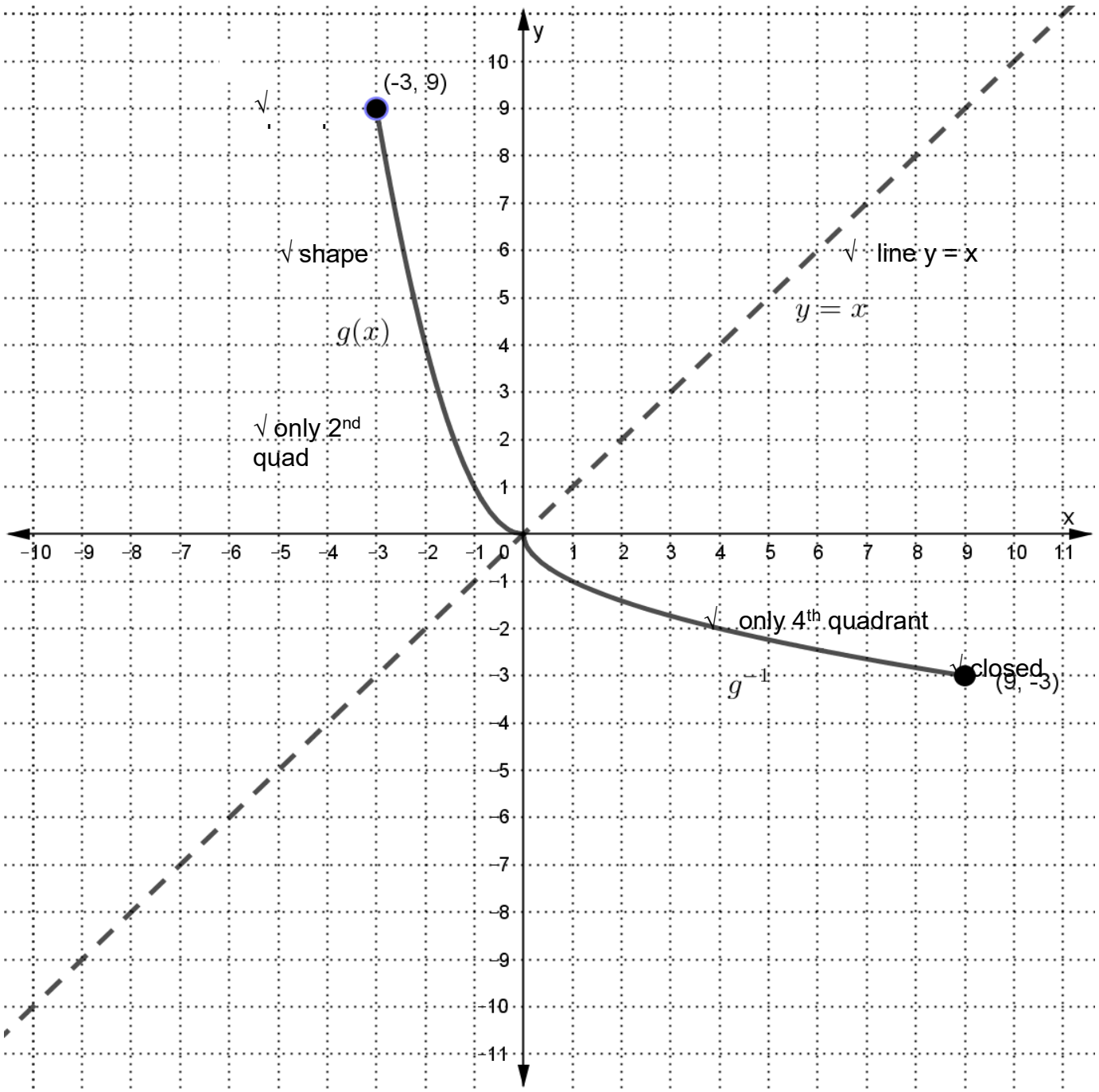
ADDENDUM A

2.4.7



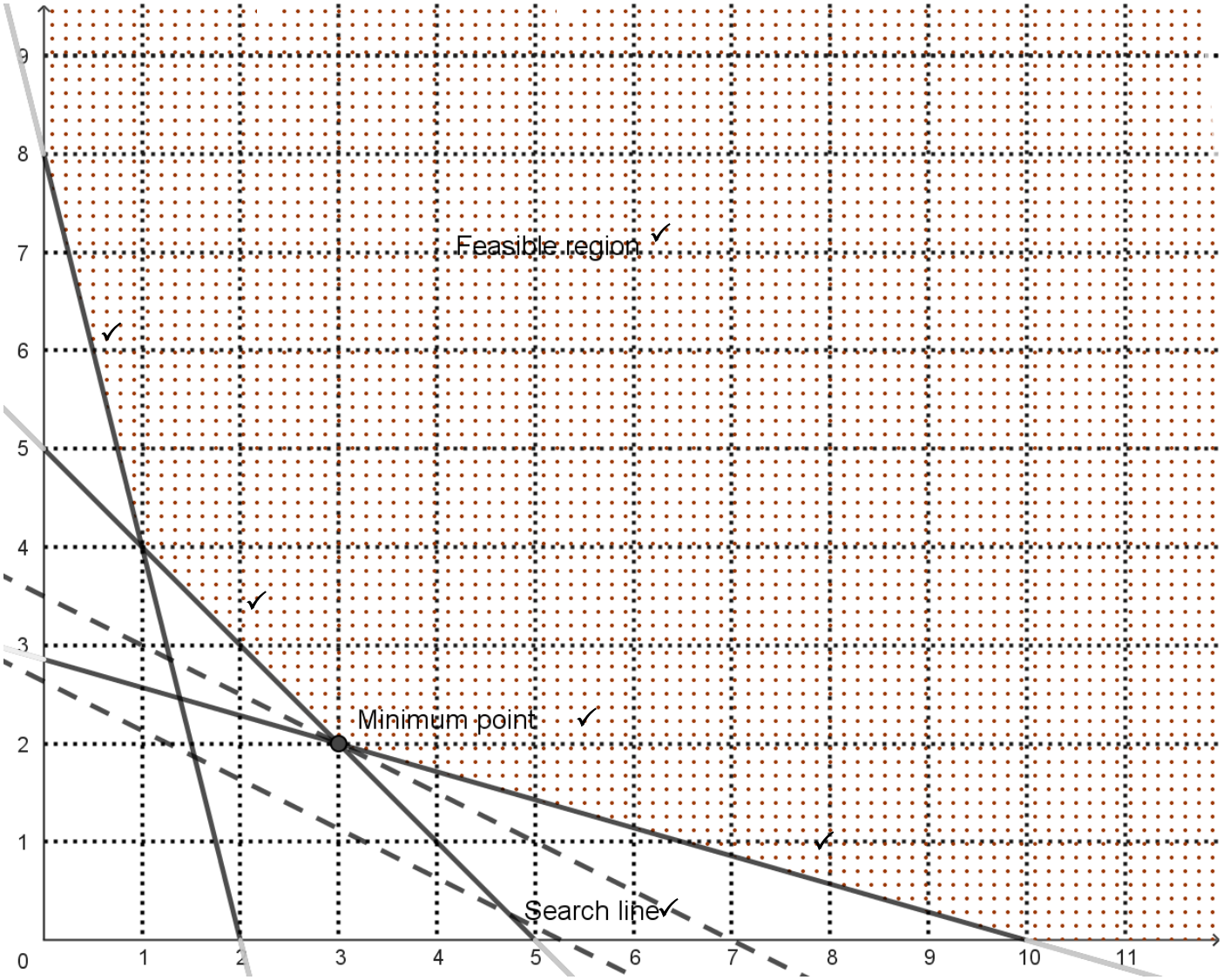
Half mark each closed point
Half mark each for using only 2rd and 4th quadrants
Half mark for curved shape of $g(x)$
Half mark line $y = x$

EXAMINATION NUMBER: | | | | | | | | | | | | | |

CENTRE NUMBER: | | | | | | | | | | | | |

ADDENDUM B

6.2.3



One mark any search line with $m = -2$
One-mark feasible region
One mark $x + y = 5$
One mark $8x + 2y \geq 16$
One mark $2x + 7y \geq 20$
One mark showing correct minimum point

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