higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
NQF LEVEL 4
(First paper)

5 November 2024

This marking guideline consists of 13 pages.

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INSTRUCTIONS AND INFORMATION

1. Examiners are reminded that the subject Mathematics lends itself to various methods and therefore several alternative answers. Mark all mathematical correct answers.

2. The answers provided in this marking guideline are therefore not exhaustive. Marks are allocated for specific principles and markers must adhere to the mark allocation.

3. CA: Consistent accuracy. Follow up as indicated. Do not CA if a mathematical error occurred, which leads to simplification of problem.

   E.g. in differentiation if $\sqrt[3]{x}$ is simplified by candidate to $3x$ … Do not CA if $\sqrt[3]{x}$ is simplified by the candidate to $x^{\frac{2}{3}}$ as the problem is still on the same level.

4. Marks are allocated per step, but if a student omitted a step and there is evidence or reason that he/she can derive at the next step without calculation, then the mark must still be given to the student.

5. Take note that marks have been allocated for simplification. This is to differentiate between an 80 percenter and a 90 percenter.

6. Steps must be marked and not only the final answer. Ticks must correspond with the total. Put ticks and crosses neatly and legibly in red pen only.

7. If the answer does not look exactly as on the marking guideline, this could be an alternative method or a method that is not known to you as marker. Please take a moment before to verify the student's response.

8. Mark all the questions.

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QUESTION 1

Full mark✓ Half mark√ (A): Accuracy (CA): Consistent accuracy or follow up

1.1
1.1.1 A
1.1.2 C
1.1.3 D
1.1.4 B
1.1.5 A

(5 × 1) (5)

1.2 1.2.1 $z_1 = 10 + 6i$ ✓ 1 mark conversion (A) (1)

1.2.2 $z_1 + z_2 = 10 + 6i + 5 - 10i$√
$= 15 - 4i$✓
Half mark conversion z_2(A)
One-mark answer in rectangular (CA)

Polar form:
$= 15{,}52\text{cis}(-14{,}93°)$√
$= 15{,}52\text{cis}(345{,}07°)$✓
Half mark polar answer with negative argument (A)
One mark final answer with positive argument within given restriction (A) (3)

1.2.3 $5 \times 1\,586{,}06 \text{ cis } 92{,}89°$ step 1
√ √
$= 7\,930{,}3\text{cis } 92{,}89°$ step 2✓
Correct application of De Moivre: Step 1
Half mark modulus
Half mark argument
Final answer no half marks
One mark correct answer step 2 (A) (2)

1.3 $(a + bi) = \frac{(-6 - 17i)}{(3 - 4i)} \times \frac{3 + 4i}{3 + 4i}$ √
Half mark multiplies with conjugate/conjugate (A)

$\therefore a + bi = \frac{-18 - 24i - 51i - 68i^2}{9 - 16i^2}$√√
Half mark multiplies denominator (A)
Half mark multiplies numerator (A)

$\therefore a + bi = \frac{50 - 75i}{25}$√√
Half mark simplified denominator (A)
Half mark simplified numerator (A)

$\therefore a + bi = 2 - 3i$√
Half mark *a* + *bi*

ALTERNATIVE:

$(a + bi)(3 - 4i) = -6 - 17i$
$\therefore 3a + 4b + (-4a + 3b)i = -6 - 17i$ √√
$\therefore \quad 3a + 4b = -6 \; eq \; 1 \times 4$√
$-4a + 3b = -17 \; eq \; 2 \times 3$
$\therefore \quad 12a + 16b = -24$
and $-12 + 9b = -51 \therefore 25b = 75$ √
$\therefore \quad b = -3$
$\therefore$ and substitute b into $3a + 4b = -6$√
$\therefore \quad a = 2$
$\therefore \quad a + bi = 2 - 3i$√
Half mark arriving at real term 4b (A)
Half mark equation 1 (A)
Half mark equation 2 (A)
Half mark solution *b*
Half mark solution *a*
Half mark answer $a + bi$ (3)

1.4 $5x^2 + xy + y = 0 \ \ eq\ 1\surd$ Half mark $eq1$ (A)
and $y^2 - 3y - 4 = 0 \ \ eq\ 2\surd$ Half mark $eq2$ (A)
$\therefore (y-4)(y+1) = 0$
$\therefore y = 4\surd \qquad y \neq -1\surd$ Half mark each y

Substitute $y = 4$ in $eq\ 1$
$\therefore 5x^2 + 4x + 4 = 0$

$$\therefore x = \frac{-4 \pm \sqrt{4^2 - 4(5)(4)}}{2(5)}$$ Half mark applying quadratic

$$x = \frac{-4 \pm \sqrt{-64}\surd}{10\surd}$$ $$x = \frac{-4 \pm \sqrt{-64}}{10}$$

$$\therefore x = \frac{-4}{10} \pm \frac{8i}{10}$$ Half mark solving negative root

$$\therefore x = -0{,}4 \pm 0{,}8i \ \text{ or } \ x = \frac{-2}{5}\surd \pm \frac{4i}{5}\surd$$ $$x = \frac{-4 \pm 8i}{10}$$

Half mark imaginary (A)
Half mark real (A) (4)
[18]

QUESTION 2

2.1 Let $(x+2) = 0 \therefore x = -2$

$(-2)^3 + 9(-2)^2 + 26(-2) + 24 = 0\surd$ Half mark substitution (A)

$x + 2$ is a factor$\surd$ Half mark answer (A) (1)

2.2 Let $(x+1) = 0$; $(x-3) = 0$ and $-3c = b$ Half mark substitute $x = 3$
$\therefore x = -1$ or $x = 3\surd\ \surd$ Half mark substitute $x = -1$

For $x = 3$:
$\therefore (3)^3 - a(3)^2 + a(3) + b = 0$ Half mark $eq\ 1$ (CA)
$-6a + b = -27 \quad eq\ 1\surd$

For $x = -1$:
$\therefore (-1)^3 - a(-1)^2 + a(-1) + b = 0$
$-2a + b = 1 \ eq\ 2\surd$ Half mark $eq\ 2$ (CA)
$eq2 - eq1 \therefore 4a = 28$
$\therefore a = 7\checkmark$ One mark correct a (A)

Substitute in *eq* 2:
$\therefore 2(7) + b = 1$
$\therefore b = 15\checkmark$ One mark correct b (A)

$-3c = b \quad eq\ 3 \ \therefore c = \frac{15}{-3} = -5\checkmark$ One mark correct c (A)

ALTERNATIVE:

From eq 2:

$b = 1 + 2a$

Substitute eq 1:

$\therefore -6a + (1 + 2a) = -27 \quad eq\ 1$
$\therefore a = 7$

Substitute in *eq* 2:

$\therefore 2(7) + b = 1$
$\therefore \quad b = 15$

$-3c = b \quad eq\ 3$

Last terms multiplied:

$\therefore c = \dfrac{15}{-3} = -5$ (5)

2.3 2.3.1 $g(\sqrt{3}) = (\sqrt{3})^2 = 3$✓ One mark answer (1)

2.3.2 $f^{-1}(x): x = 2y + 3$

$\therefore f^{-1}(x) \rightarrow y = \frac{x-3}{2}$✓ One mark answer $y =$ … (1)

2.3.3 $(a + 2)^2 - [2(a + 2) + 3] = 0$
$\therefore a^2 + 4a + 4 - 2a - 4 - 3 = 0$✓ One mark simplifying
$\therefore \quad a^2 + 2a - 3 = 0$✓ One mark arriving at quadratic
$\therefore a = -3$√ or $a = 1$√ Half mark each value of a (3)

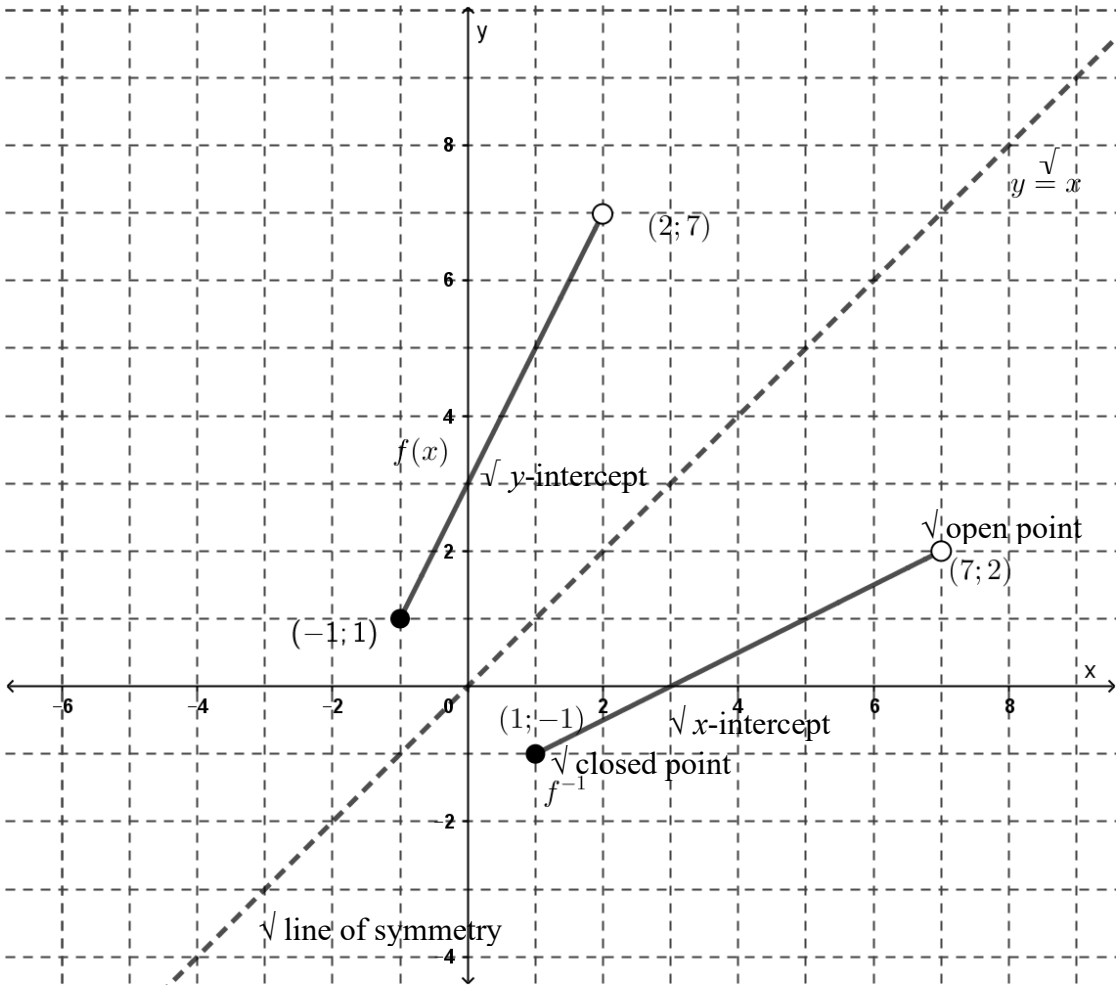
2.3.4 $f(-1) = 2(-1) + 3 = 1$√ Half mark start value (A)
$f(2) = 2(2) + 3 = 7$√ Half mark end value (A)
$\therefore f^{-1}(x)$ domain $\{x: 1 \leq x < 7; x \in \mathrm{R}\}$ Half mark each correct
or $x \in [1; 7)$ for $x \in R$ inequality sign (A) (2)

2.3.5 LHS: $f(f^{-1}(x)) = 2\left(\dfrac{x-3}{2}\right) + 3 = x$√ Half mark substitute $f^{-1}(x)\ in\ f(x)$
Half mark substitute $f(x)\ in\ f^{-1}(x)$

RHS: $f^{-1}(f(x)) = \dfrac{(2x+3)-3}{2} = x$√ Half mark solve LHS
Half mark solve RHS

$\therefore$ LHS = RHS✓ One mark LHS = RHS (3)

2.3.6



Half mark open point at $(7; 2)$
Half mark closed point at $(1; -1)$
Half mark x-intercept at $x = 3$
Half mark line $y = x$
Half mark label $y = x$
Half mark y-intercept of $y = 3$ (3)

[19]

QUESTION 3

3.1

$$f(x) = -\frac{2}{x}$$

$$\therefore\ f(x+h) = -\frac{2}{x+h}$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-(f(x))}{h}\ \surd$$

$$= \lim_{h\to 0}\frac{-\frac{2}{x+h}+\frac{2}{x}}{h}\ \surd$$

$$= \lim_{h\to 0}\frac{-2x+2x+2h}{(x+h)(x)}\times\frac{1}{h}\surd$$

$$= \lim_{h\to 0}\frac{2h}{(x+h)(x)h}\surd$$

$$= \lim_{h\to 0}\frac{2}{(x+h)(x)}\surd$$

$$= \frac{2}{x^2}\surd$$

No marks for final step if there is no evidence of first principles.

Half mark $f(x+h)$

Half mark correct substitution in first principles

Half mark adding fractions

Half mark simplified $f(x+h)-(f(x))$

Half mark cancels h as common factor

Half mark answer

(3)

3.2 3.2.1

$$\frac{dy}{dx} = -15\sin 5x\surd + 8e^{4x}\surd + 36x^2\surd + 0\surd$$

Half mark each term differentiated (A) (2)

3.2.2

$$\frac{dy}{dx} = \frac{2(3)}{3x}\surd\ -\frac{2}{3}x^{-\frac{1}{3}}\surd + 10x^{-6}\surd - \frac{5}{3}x^{-2}\surd$$

$$\therefore\frac{dy}{dx} = \frac{2}{x} - \frac{2}{3x^{\frac{1}{3}}}\surd + \frac{10}{x^6}\surd - \frac{5}{3x^2}\surd$$

$$\therefore\frac{dy}{dx} = \frac{2}{x} - \frac{2}{3\sqrt[3]{x}}\surd + \frac{10}{x^6} - \frac{5}{3x^2}\surd$$

Half mark each term differentiated step 2 (A)
Half mark each term positive exp. where applicable step 3 (CA)
Half mark each term correctly in SURD form final answer (A)
Half mark $\frac{dy}{dx}$ (5)

3.2.3

$$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$$

$$\therefore\frac{dy}{dx} = (3e^{4x})\surd(3cos3x) + 12(e^{4x})\surd(sin3x)$$

$$\therefore\frac{dy}{dx} = (9e^{4x})\surd(cos3x) + 12(e^{4x})\surd(sin3x)$$

Half mark each correct substitution in product rule (A)

(2)
[12]

QUESTION 4

Leave the answer with positive exponents and in surd form where applicable:

4.1 4.1.1 $2\sin 3x + 2x^{5} + 6\ln x + \frac{5}{2}x^{2} + c$ √ √ √ √

Half mark each term (2)

4.1.2 $= -\frac{10e^{5x}}{5} + \frac{\frac{3}{2}x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{\frac{3}{2}x^{\left(\frac{2}{3}\right)}}{-\frac{2}{3}} + \frac{2x^{-1}}{-1} + c$ √

$= -2e^{5x} + 3\sqrt{x} + \frac{9}{4\sqrt[3]{x^{2}}} - \frac{2}{x} + c$

Half mark each term integrated

Half mark 2nd and 3rd term in surd

Half mark 1st and 4th simplified (4)

4.1.3 $\int \left(\frac{4x^{2}-1}{2x-1}\right) dx = \int \left(\frac{(2x-1)(2x+1)}{(2x-1)}\right) dx$ √

$= \int 2x + 1\, dx$ √ √

$= \frac{2x^{2}}{2} + x + c = x^{2} + x + c$ √

Half mark factorise diff in squares (A)

Half mark each integrated and simplified term (CA)

Half mark c (A) (2)

4.2 4.2.1 $s = \int 2t + 3\ dt$✓

$s = t^{2} + 3t\ + c$✓

One mark each term integrated (A) (2)

4.2.2 (a) $s = t^{2} + 3t\ + c$
$2 = 0^{2} + 3(0) + c \therefore c = 2$
$\therefore\ s = t^{2} + 3t\ + 2$✓

One mark solving c (1)

(b) At $t = 5$
$s = (5)^{2} + 3(5) + 2 = 42$ m✓

One mark finding s (1)

4.3 $\int_{4}^{10} \frac{50}{x} dx = 50\ln x \Big|_{4}^{10}$ √ √ √

$= [50\ln 10\ ] - [50\ln 4]$

$= 115{,}13 - 69{,}31$

$= 45{,}82\ u^{2}$✓

Half mark $\int_{4}^{10} y\ dx$
Half mark integration
Half mark subst. each boundary

One mark answer (3)

[15]

QUESTION 5

5.1 5.1.1 $\boldsymbol{s(2) = 10(2)^3 + 5(2) + 10 = 100\ m}$✓ One mark answer (A) (1)

5.1.2 $s'(5) = 30(5)^2 + 5$✓
$= 755$ m/s✓
One mark in substitute s' (A)
One mark answer (CA) (2)

5.1.3 $s''(t) = 60t$√
Acceleration is $60(3) = 180\ ms^{-2}$ √
Half mark finding s''(A)
Half mark answer(A) (1)

5.1.4 $s'(t) = 30t^2 + 5 = 500$✓
$\therefore \quad 30t^2 - 495 = 0$
$\therefore \quad 2t^2 - 33 = 0$

$\therefore t = \dfrac{\sqrt{33}}{\sqrt{2}}$

$= 4{,}06$ s✓
One mark equates $s'\ to\ 500$
One mark solves t (A) (2)

5.2 5.2.1 $x = 3$ is the root, $\rightarrow (x - 3)$ is a factor (1)

5.2.2
$$3\ \Big|\ \begin{array}{l} 1 - 7 + 17 - 15 \rightarrow \\ \downarrow \quad 3 - \ 12 + 15 \ \surd\surd \\ \hline 1 \ -4 + 5 \ \lfloor \ 0 \end{array}$$
In $ax^2 + bx + c$
Half mark a (A)
Half mark b (A)
Half mark c (A)

$\therefore$ Factors are $(x - 3)(x^2 - 4x + 5)$ √

$\therefore x = 3$ (given)No marks √

or $x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2}$ √
Half mark applying quadratic formula (A)

$x = \dfrac{4 \pm \sqrt{-4}}{2}$ √

$\therefore x = 2 + i$√ or $x = 2 - i$√
Half mark each real value (A)
Half mark each imaginary value (A)

ALTERNATIVE:

In $ax^2 + bx + c$
Half mark a (A)
Half mark b (A)
Half mark c (A)
Half mark applying quadratic formula (A)
Half mark each real value (A)
Half mark each imaginary value (A)

$$
\begin{array}{r|llll}
 & & x^2 - 4x + 5 & & \\
\hline
x - 3 & x^3 & -7x^2 & +17x & -15 \\
 & x^3 & -3x^2 & & \\
\hline
 & 0 & -4x^2 & +17x & \\
 & & -4x^2 & +12x & \\
\hline
 & & 0 & 5x & -15 \\
 & & & 5x & -15 \\
\hline
 & & & 0 & +\ 0
\end{array}
$$

Factors are $(x-3)(x^2-4x+5)$

$\therefore x = 3$ (given) No marks

or $x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2}$

$x = \dfrac{4 \pm \sqrt{-4}}{2}$

$\therefore\ x = 2 + i$ or $x = 2 - i$

ALTERNATIVE:

$x^3 - 7x^2 + 17x - 15 = (x-3)(ax^2 + bx + c)$

$\therefore x^3$: $x^3 = ax^3 \therefore a = 1$

Constant $-15 = -3 \times c \ \therefore c = 5$

x^2: $-3ax^2 + bx^2 = -7x^2$

$\therefore \quad -3x^2 + bx^2 = -7x^2$

$\therefore\ b = -4$

$\therefore (x-3)(x^2-4x+5)$ are factors

$\therefore x = 3$ (given)

$x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2}$

$x = \dfrac{4 \pm \sqrt{-4}}{2}$

$\therefore x = 2 + i$ or $x = 2 - i$ (4)

5.2.3 $g(x) = x^3 - 7x^2 + 17x - 15$

$\therefore g'(x) = 3x^2 - 14x + 17$√ Half mark g' (A)

$g''(x) = 6x - 14$√ Half mark g" (A)

At POI $g"(x) = 0 \rightarrow 6x - 14 = 0$

$\therefore x = \frac{7}{3} \cong 2{,}33$✓ One mark x (A)

$y = \left(\frac{7}{3}\right)^3 - 7\left(\frac{7}{3}\right)^2 + 17\left(\frac{7}{3}\right) - 15 \approx -0{,}74$ or $\frac{-20}{27}$✓ One mark y (CA)

POI $= \left(\frac{7}{3}; -0{,}74\right)$ (3)

5.2.4 $g'(x) = 3x^2 - 14x + 17$ Half mark substituting in g' (A)

Gradient $= 3(2)^2 - 14(2) + 17 = 1$√ Half mark answer (CA) (1)

5.3 $y = \frac{k}{x} \therefore y = kx^{-1} \therefore \frac{dy}{dx} = -kx^{-2}$

$\rightarrow \frac{dy}{dx} = -\frac{k}{x^2}$✓ One mark $\frac{dy}{dx}$

Substitute a:

$\therefore \frac{dy}{dx} = -\frac{k}{a^2} =$ gradient Substitute point in $y - y_1 = m(x - x_1)$

$\therefore y - \frac{k}{a} = -\frac{k}{a^2}(x - a)$ √

$\therefore y = -\frac{k}{a^2}x + 2\frac{k}{a}$ √

or $a^2y + kx - 2ka = 0$✓ One mark equation (3)

[18]

QUESTION 6

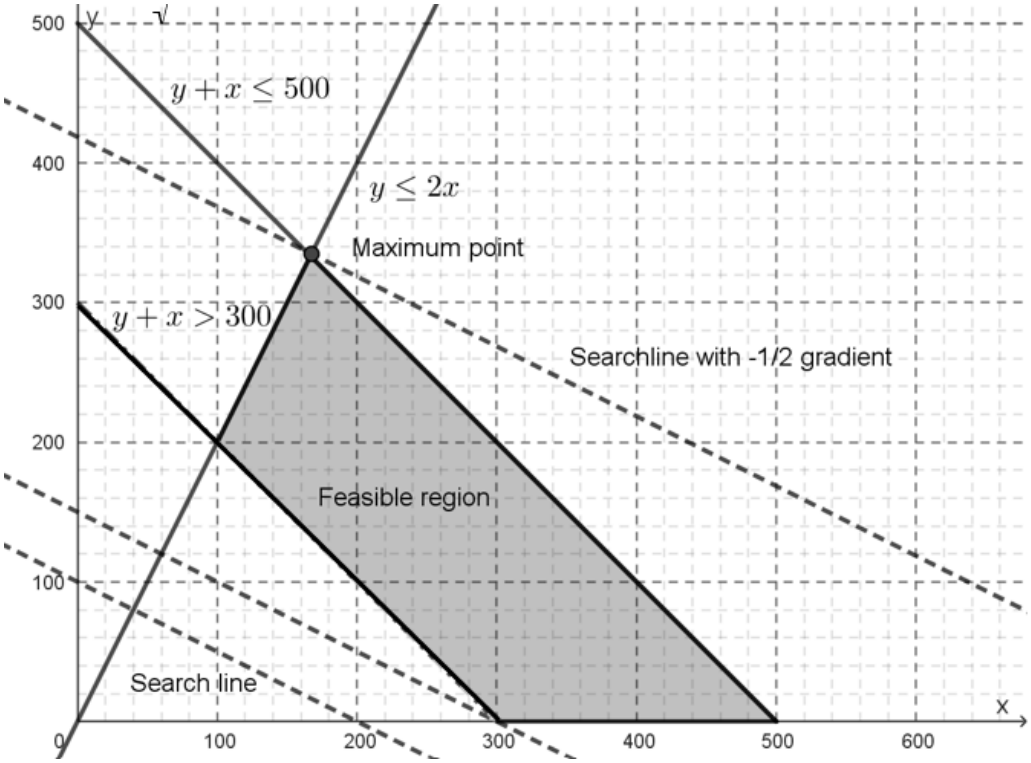
6.1 6.1.1 $y \overset{\surd}{\leq} 2x$ Half mark each equation

$500 - (x + y) \overset{\surd}{<} 200$

$\therefore x + y \overset{\surd}{>} 300$ Half mark each inequality sign

$x + y \overset{\surd}{\leq} 500$ (3)

6.1.2
(a)
(b)
(c)



One mark maximum point✓
One mark feasible region✓
One mark $x + y \leq 500$✓
One mark$x + y > 300$ ✓
One mark $y \leq 2x$✓
One mark search line with gradient $-\frac{1}{2}$✓ (6)

(d) $x + y = 500 \quad eq\ 1$
$y = 2x \quad eq\ 2$

Substitute $eq\ 2$ in $eq\ 1$:
$\therefore 3x = 500$

$\therefore x = 166{,}67$✓
$\rightarrow y = 333{,}33$✓

The question states that analytical method must be followed. No marks if instruction was not followed.
One-mark x
One mark y (2)

	(e)	Maximum cost $c = 20x + 30y + 10(500 - (x + y)$ $\therefore c = 20(166{,}67) + 30(333{,}33) + 10(500 - (500))$ $\therefore$ cost $=$ R133,33✓ To get R20 profit the price must be R133,33 + R20 = R153,33✓	Substitute point in cost equation (CA 6.1.2 d) One-mark cost before profit One-mark cost with profit Answer must be in rand	(2)
6.2	6.2.1	$14x + 35y \overset{\surd}{\leq} 1\,400$ or $2x + 5y \overset{\surd}{\leq} 200$ $\boldsymbol{x + y \geq 50}\surd$ $y \geq 20\surd$	Half mark each inequality sign Half mark each equation	(2)
	6.2.2	D		(1)
	6.2.3	No.✓ $14x + 35y \leq 1\,400$ is constraint $14(62) + 35(24) = 1\,708 > 1\,400$✓		(2) **[18]**

TOTAL: 100

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