higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

STRENGTH OF MATERIALS AND STRUCTURES N6

6 December 2023

This marking guideline consists of 9 pages.

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QUESTION 1

1.1

$$at\ 100\ mm: a + \frac{b}{0{,}1^2} = 10 \times 10^6 \ldots\ldots\ldots (1)\ \checkmark$$

$$at\ 100\ mm: a - \frac{b}{0{,}1^2} = -30 \times 10^6 \ldots (2)\ \checkmark$$

$$(1) + (2): 2a = -20 \times 10^6$$

$$a = -10 \times 10^6\ \checkmark$$
$$b = 0{,}2 \times 10^6\ \checkmark$$

$$at\ 250\ mm:\ \sigma_H = a + \frac{b}{0{,}25^2}$$

$$= -10 \times 10^6 + \frac{0{,}2 \times 10^6}{0{,}25^2}$$

$$\sigma_H = 6{,}8\ MPa\ \checkmark (compressive)\ \checkmark$$

$$at\ 250\ mm:\ \sigma_R = a - \frac{b}{0{,}2^2}$$

$$= -10 \times 10^6 - \frac{0{,}2 \times 10^6}{0{,}25^2}$$

$$\sigma_R = 13{,}2\ MPa\ \checkmark (compressive)\ \checkmark \quad (8)$$

1.2

$$at\ 250\ mm:\ a - \frac{b}{0{,}25^2} = -13{,}2 \times 10^6 \ldots\ldots\ldots (3)\checkmark$$

$$at\ 400\ mm: a - \frac{b}{0{,}4^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (4)\checkmark$$

$$(3) - (4): -16b + 6{,}25b = -13{,}2 \times 10^6$$

$$b = 1{,}354 \times 10^6\ \checkmark$$
$$a = 8{,}462 \times 10^6\ \checkmark$$

$$at\ 250\ mm:\ \sigma_H = a + \frac{b}{0{,}25^2}$$

$$= 8{,}462 \times 10^6 + \frac{1{,}354 \times 10^6}{0{,}25^2}$$

$$\sigma_H = 30{,}123\ MPa\ \checkmark (tensile)\ \checkmark$$

$$at\ 400\ mm:\ \sigma_H = a + \frac{b}{0{,}4^2}$$

$$= 8{,}462 \times 10^6 + \frac{1{,}354 \times 10^6}{0{,}4^2}$$

$$\sigma_H = 16{,}923\ MPa\checkmark\ (tensile)\ \checkmark \quad (8)$$

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QUESTION 2

2.1 $F_H = \frac{wx_1^2}{2d} = \frac{1 \times 490^2}{2 \times 120} = 1000{,}417\ kN$✓ (1)

2.2 $F_V = wx_2 = 1 \times 510 = 510\ kN$✓

$F_T = \sqrt{F_H^2 + F_V^2} = \sqrt{1000{,}417^2 + 510^2} = 1122{,}913\ kN$✓ (2)

2.3 $A = \frac{F_T}{\sigma} = \frac{1122{,}913 \times 10^3}{180 \times 10^6} = 6{,}238 \times 10^{-3}\ m^2$✓

$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 6{,}238 \times 10^{-3}}{\pi}} = 89{,}123\ mm$✓ (2)

2.4 $for\ frictionless\ pulley\ F_{Tc} = F_{Ta} = 1122{,}913\ kN$✓

$F_{va} = F_{Ta} \sin\theta = 1122{,}913 \sin 60 = 972{,}471\ kN$✓

$R_V = F_{vc} + F_{va} = 510 + 972{,}471 = 1482{,}471\ kN$✓ (3)

2.5 $F_{Ha} = F_{Ta} \cos\theta = 1122{,}913 \cos 60 = 561{,}456\ kN$✓

$R_H = F_{Hc} - F_{Ha} = 1000{,}417 - 561{,}456 = 438{,}96\ kN$✓

$M = (F_{Hc} = F_{Ha})H = (438{,}96) \times 150 = 65844{,}028\ kNm$ ✓ (3)
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QUESTION 3

3.1 $$T_e = \frac{\pi d^3 \tau}{16} = \frac{\pi \times 0{,}1^3 \times 45 \times 10^6}{16} = 8{,}836\ kNm$$✓

$$M^2 + T^2 = T_e^2$$

$$M^2 + 6^2 = 8{,}836^2$$✓

$$M = 6{,}486\ kNm$$✓ (3)

3.2 $$M_e = \frac{\pi d^3 \sigma}{32} = \frac{\pi \times 0{,}1^3 \times 80 \times 10^6}{32} = 7{,}854\ kNm$$✓

$$M_e = 0{,}5\left[M + \sqrt{M^2 + T^2}\right]$$

$$7{,}854 = 0{,}5\left[M + \sqrt{M^2 + 6^2}\right]$$ ✓

$$15{,}708 - M = \sqrt{M^2 + 6^2}$$

$$(15{,}708 - M)^2 = M^2 + 6^2$$✓

$$15{,}708^2 - 31{,}416M + M^2 = M^2 + 6^2$$✓

$$M = 6{,}708\ kNm$$✓ (5)

3.3 Maximum allowed BM is $6{,}486\ kNm$ ✓
This BM will satisfy both given stress limits ✓ (2)

3.4 $$\tau = 45\ MPa$$✓

$$M_e = 0{,}5\left[M + \sqrt{M^2 + T^2}\right] = 0{,}5[6{,}486 + 8{,}836] = 7{,}661\ kNm$$✓

$$\sigma = \frac{32M_e}{\pi d^3} = \frac{32 \times 7{,}661 \times 10^3}{\pi \times 0{,}1^3} = 78{,}034\ MPa$$ ✓ (3)

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QUESTION 4

4.1 *Consider stress limit*:

$M = \sigma \times Z = 150 \times 10^6 \times 1329 \times 10^{-6}$✓ $= 199{,}35\ kNm$✓

$$w = \frac{8M}{L^2} = \frac{8 \times 199{,}35 \times 10^3}{3^2} = 177{,}2\ kN/m$$ ✓

Consider deflection limit:

$$w = \frac{384EI\Delta}{5L^4} = \frac{384 \times 200 \times 10^9 \times 274{,}2 \times 10^{-6} \times 0{,}008}{5 \times 3^4} = 415{,}972\ kN/m$$✓

The maximum allowed weight = total weight − own weight

$w = 177{,}2 \times 10^3 - 74{,}8 \times 9{,}81$ ✓ $= 176{,}466\ kN/m$ ✓ (6)

4.2 $actual\ stress = 150\ MPa$ ✓

$$\Delta = \frac{5wL^4}{384EI} = \frac{5 \times 177{,}2 \times 10^3 \times 3^4}{384 \times 200 \times 10^9 \times 274{,}2 \times 10^{-6}} = 3{,}408 \times mm$$✓ (2)

4.3 $$\Delta = \frac{FL^3}{48EI}$$

$$(3{,}408 - 2) \times 10^{-3}$$ ✓ $$= \frac{F \times 3^3}{48 \times 200 \times 10^9 \times 274{,}2 \times 10^{-6}}$$

$F = 137{,}263\ kN$ ✓ (2)
[10]

QUESTION 5

5.1 $$A = \frac{\pi D^2}{4} = \frac{\pi \times 0{,}15^2}{4} = 17{,}671 \times 10^{-3} m^2 ✓$$

$$\mathrm{I} = \frac{\pi D^4}{64} = \frac{\pi \times 0{,}15^4}{64} = 24{,}85 \times 10^{-6} m^4 ✓$$

$$\sigma_d = \frac{F_1 + F_2}{A} = \frac{1200 + F_2}{17{,}671 \times 10^{-3}} = 67{,}906 \times 10^3 + 56{,}588F_2 ✓$$

$$\sigma_b = \sigma_R - \sigma_d = 215 \times 10^3 - (67{,}906 \times 10^3 + 56{,}588F_2) ✓$$

$$\sigma_b = 147{,}094 \times 10^3 - 56{,}588F_2 ✓$$

$$\sigma_b = \frac{F_2 \times e \times Y}{\mathrm{I}}$$

$$147{,}094 \times 10^3 - 56{,}588F_2 = \frac{F_2 \times 0{,}03 \times 0{,}075}{24{,}85 \times 10^{-6}} ✓$$

$$F_2 = 999{,}755N ✓ \quad (7)$$

5.2 $$\sigma_R = \sigma_d - \sigma_b$$

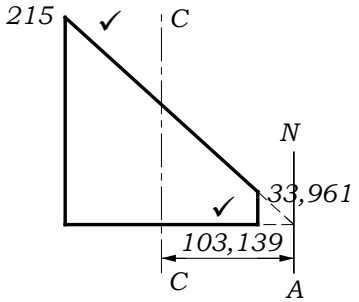
$$= \frac{F_1 + F_2}{A} - \frac{F_2 \times e \times Y}{\mathrm{I}}$$

$$= \frac{1200 + 999{,}755}{17{,}671 \times 10^{-3}} - \frac{999{,}755 \times 0{,}03 \times 0{,}075}{24{,}85 \times 10^{-6}} ✓$$

$$= 124{,}481 \times 10^3 - 90{,}519 \times 10^3 ✓$$

$$\sigma_R = 33{,}961\ kPa ✓ (compressive) ✓ \quad (4)$$

5.3 $$x = \frac{\sigma_d \times I}{F_2 \times e} = \frac{124{,}481 \times 10^3 \times 24{,}85 \times 10^{-6}}{999{,}755 \times 0{,}03} = 103{,}139\ mm \text{ (from centroid)} ✓$$



(3)
[14]

QUESTION 6

6.1 $A = BD - 2bd = 0{,}1 \times 0{,}15 - 2 \times 0{,}045 \times 0{,}013 = 3{,}3 \times 10^{-3}\ \text{m}^2$✓

$$\tau_{ave} = \frac{V}{A} = \frac{15 \times 10^3}{3{,}3 \times 10^{-3}} = 4{,}545\ \text{MPa}$$✓ (2)

6.2 $$I = \frac{BD^3}{12} - 2\left(\frac{bd^3}{12}\right) = \frac{0{,}1 \times 0{,}15^3}{12} - 2\left(\frac{0{,}045 \times 0{,}13^3}{12}\right)$$✓ $= 11{,}648 \times 10^{-6}\ \text{m}^4$✓ (2)

6.3 $Q = a_1\bar{y}_1 = 0{,}1 \times 0{,}01 \times 0{,}07 = 70 \times 10^{-6}\ \text{m}^3$ ✓

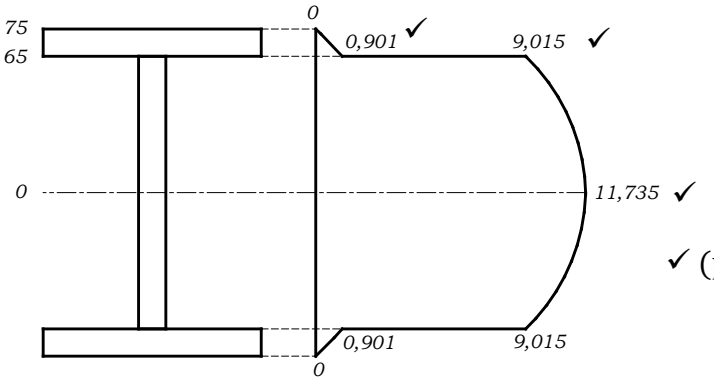
$$\tau = \frac{VQ}{Ib} = \frac{15 \times 10^3 \times 70 \times 10^{-6}}{11{,}648 \times 10^{-6} \times 0{,}1} = 901{,}481\ \text{kPa}$$✓

$$\tau = \frac{VQ}{Ib} = \frac{15 \times 10^3 \times 70 \times 10^{-6}}{11{,}648 \times 10^{-6} \times 0{,}01} = 9{,}015\ \text{MPa}$$✓ (3)

6.4 $Q = a_1\bar{y}_1 + a_2\bar{y}_2 = 70 \times 10^{-6} + 0{,}01 \times 0{,}065 \times 0{,}0325 = 91{,}125 \times 10^{-6}\ \text{m}^3$ ✓

$$\tau = \frac{VQ}{Ib} = \frac{15 \times 10^3 \times 91{,}125 \times 10^{-6}}{11{,}648 \times 10^{-6} \times 0{,}01} = 11{,}735\ \text{MPa}$$✓ (2)

6.5



✓ ✓ ✓ ✓ (parabolic shape) (4)

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QUESTION 7

7.1 $k_e = k_1 + k_2 + \cdots k_{15}$

$k_e = 15 \times 2 = 30 \text{ kN/m}$✓

$x = \sqrt{\frac{2U}{k_e}} = \sqrt{\frac{2 \times 1 \times 10^3}{30 \times 10^3}} = 258{,}199 \text{ mm}$✓ (2)

7.2 $K_W = \frac{4C-1}{4C-4} + \frac{0{,}615}{C} = \frac{4 \times 10 - 1}{4 \times 10 - 4} + \frac{0{,}615}{10} = 1{,}145$✓

$\text{W} = \frac{2U}{x} = \frac{2 \times 1 \times 10^3}{0{,}258} = 7745{,}967 \text{ N}$✓

$W_1 = \frac{W}{n} = \frac{7745{,}967}{15} = 516{,}398 \text{ N (per spring)}$✓

$\tau = K_W \left(\frac{8WD}{\pi d^3}\right)$

$80 \times 10^6 = \frac{1{,}145 \times 8 \times 516{,}398 \times (10d)}{\pi \times d^3}$✓

$d = 13{,}718 \text{ mm}$ ✓ (5)

7.3 $D = 10d = 10 \times 13{,}718 = 137{,}179 \text{ mm}$ ✓

$n = \frac{xGd^4}{8WD^3} = \frac{0{,}258 \times 75 \times 10^9 \times 0{,}0137^4}{8 \times 516{,}398 \times 0{,}137^3} = 64{,}303 \text{ coils}$✓ (2)

7.4 $L_s = (n+2)d = (64{,}303 + 2)13{,}718 = 909{,}536 \text{ mm}$✓ (1)

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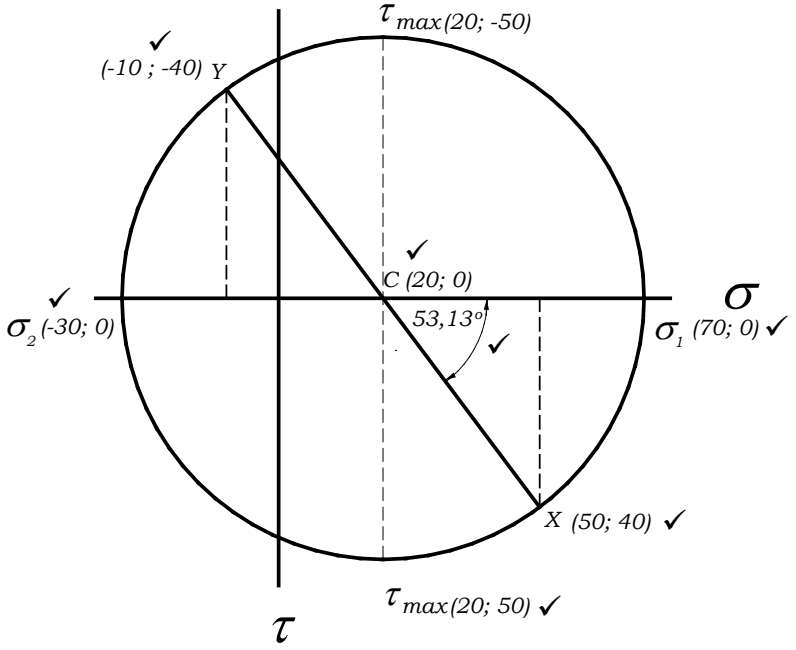
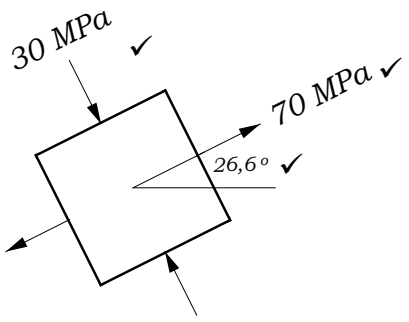
QUESTION 8

8.1



Maximum principal stress = 70 MPa (tensile) ✓
Minimum principal stress = 30 MPa (compressive)✓
Maximum shear stress = 50 MPa ✓ (10)

8.2



(3)
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TOTAL: 100

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