higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

16 November 2022

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QUESTION 1

1.1 $$Q = (x+y)\ln\left(\frac{x}{y}\right)$$

$$\frac{\partial Q}{\partial x} = \ln\left(\frac{x}{y}\right) + (x+y)\times\frac{1}{\frac{x}{y}}\times\frac{1}{y}$$

$$= \ln\left(\frac{x}{y}\right) + \frac{x+y}{x} \checkmark$$

$$\frac{\partial Q}{\partial y} = \ln\left(\frac{x}{y}\right) + \frac{x+y}{\frac{x}{y}}\times -xy^{-2}$$

$$= \ln\left(\frac{x}{y}\right) - \frac{x+y}{y} \checkmark$$

$$x\left(\ln\left(\frac{x}{y}\right) + \frac{x+y}{x}\right) + y\left(\ln\left(\frac{x}{y}\right) - \frac{x+y}{y}\right) \checkmark$$

$$= xln\left(\frac{x}{y}\right) + x + y + yln\left(\frac{x}{y}\right) - x - y \checkmark$$

$$= x\ln\left(\frac{x}{y}\right) + y\ln\left(\frac{x}{y}\right) \checkmark$$

$$= (x+y)\ln\left(\frac{x}{y}\right) \checkmark$$

$$= Q$$ (3)

1.2 $$z = 3x^2y^5$$

$$x = 3 \qquad y = 2$$
$$\delta x = -5 \qquad \delta y = 2.5$$

$$\therefore \frac{\partial z}{\partial x} = 6xy^5 \checkmark \quad \text{and} \quad \frac{\partial z}{\partial y} = 15x^2y^4 \checkmark$$

$$\delta z = \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y \checkmark$$
$$= \left(6xy^5\right)(-5) + \left(15x^2y^4\right)(2.5) \checkmark$$
$$= 6(3)(2)^5(-5) + 15(3)^2(2)^4(2.5) \checkmark$$
$$= 2520 \checkmark$$ (3)

[6]

QUESTION 2

2.1 $\int -2xcosec^2 2xdx = xcot2x - \int cot2xdx$ ✓✓

$= xcot2x - \frac{1}{2}lnsin2x + C$✓✓

$f(x) = -2x \Rightarrow f'(x) = -2$

$g'(x) = cosec^2 2x \Rightarrow$

$g(x) = -\frac{1}{2}cot2x$

(2)

2.2 $\int x^3 cosxdx = x^3 sinx - 3\int x^2 sinxdx$✓✓

$= x^3 sinx - 3\left[-x^2 cosx + 2\int xcosxdx\right]$✓✓

$= x^3 sinx + 3x^2 cosx - 6\left[xsinx - \int sinxdx\right]$✓✓✓✓

$= x^3 sinx + 3x^2 cosx - 6xsinx + 6(-cosx) + C$✓

$= x^3 sinx + 3x^2 cosx - 6xsinx - 6cosx + C$ ✓

(5)

2.3 $\int \frac{dx}{(x+1)(x+4)}$

$= -\int \frac{dx}{\frac{9}{4} - \left(x+\frac{5}{2}\right)^2}$ ✓

$= -\frac{1}{2\left(\frac{3}{2}\right)} \ln\left[\frac{\frac{3}{2}+\left(x+\frac{5}{2}\right)}{\frac{3}{2}-\left(x+\frac{5}{2}\right)}\right] + C$ ✓✓

$= -\frac{1}{3} \ln\left[\frac{\frac{3}{2}+\left(x+\frac{5}{2}\right)}{\frac{3}{2}-\left(x+\frac{5}{2}\right)}\right] + C$ ✓

$(x+1)(x+4) = x^2 + 5x + 4$ ✓

$= x^2 + 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 4$ ✓

$= x^2 + 5x + \frac{25}{4} - \frac{25}{4} + 4$

$= \left(x+\frac{5}{2}\right)^2 - \frac{9}{4}$ ✓

$= -\left(\frac{9}{4} - \left(x+\frac{5}{2}\right)^2\right)$ ✓

$ax^2 + bx + c = \frac{4ac-b^2}{4a} + a\left(x+\frac{b}{2a}\right)^2$ ✓

$\Rightarrow x^2 + 5x + 4 = \frac{4(4)(1)-(5)^2}{4(1)} + 1\left(x+\frac{5}{2(1)}\right)^2$ ✓

$= -\frac{9}{4} + \left(x+\frac{5}{2}\right)^2$ ✓

$= -\left[\frac{9}{4} - \left(x+\frac{5}{2}\right)^2\right]$ ✓

(4)

2.4 $\int \sin^5 3x dx$

$= \int \sin^4 3x \sin 3x dx$

$= \int \left(\sin^2 3x\right)^2 \sin 3x dx$ ✓

$= \int \left(1-\cos^2 3x\right)^2 \sin 3x dx$ ✓

$= \int \left(1-2\cos^2 3x+\cos^4 3x\right)\sin 3x dx$ ✓

$= \int \sin 3x dx - 2\int \cos^2 3x \sin 3x dx + \int \cos^4 3x \sin 3x dx$ ✓

$= \frac{-\cos 3x}{3} + \frac{2}{3}\int u^2 du - \frac{1}{3}\int u^4 du$ ✓✓✓

$= \frac{-\cos 3x}{3} + \frac{2u^3}{9} + \frac{u^5}{15} + C$ ✓✓

$= \frac{-\cos 3x}{3} + \frac{2\cos^3 3x}{9} - \frac{\cos^5 3x}{15} + C$ ✓ (5)

$u = \cos 3x$
$du = -3\sin 3x dx$
$\frac{-du}{3} = \sin 3x dx$

2.5 $\int 5xe^{-2x}dx = -\frac{5}{2}xe^{-2x} + \frac{5}{2}\int e^{-2x}dx$ ✓✓

$= -\frac{5}{2}xe^{-2x} + \frac{5}{2}\left[\frac{e^{-2x}}{-2}\right] + C$ ✓

$= -\frac{5}{2}xe^{-2x} - \frac{5}{4}e^{-2x} + C$ ✓ (2)

[18]

QUESTION 3

3.1 $$\int \frac{x^3+6x-7}{\left(x^2-2x-3\right)^2}dx$$

Let $\frac{x^3+6x-7}{(x+1)^2(x-3)^2}=\frac{A}{(x+1)^2}+\frac{B}{(x+1)}+\frac{C}{(x-3)^2}+\frac{D}{(x-3)}$ ✓✓

$x^3+6x-7=A(x-3)^2+B(x+1)(x-3)^2+C(x+1)^2+D(x-3)(x+1)^2$ ✓

Let $x=3$ Let $x=-1$

$38=16C$ $-14=16A$

$C=\frac{19}{8}$ ✓ $A=\frac{-7}{8}$ ✓

x^3: $1=B+D$

x^2: $0=A+C-D-5B$

$\frac{-3}{2}=-D-5B$

$\frac{-1}{2}=-4B$

$B=\frac{1}{8}$ ✓

$D=\frac{7}{8}$ ✓

$=\frac{-7}{8}\int\frac{dx}{(x+1)^2}+\frac{1}{8}\int\frac{dx}{x+1}+\frac{19}{8}\int\frac{dx}{(x-3)^2}+\frac{7}{8}\int\frac{dx}{x-3}$ ✓

$=-\frac{7}{8}\int(x+1)^{-2}dx+\frac{1}{8}\int\frac{dx}{x+1}+\frac{19}{8}\int(x-3)^{-2}dx+\frac{7}{8}\int\frac{dx}{x-3}$

$=\frac{7}{8(x+1)}+\frac{1}{8}\ln(x+1)-\frac{19}{8(x-3)}+\frac{7}{8}\ln(x-3)+C$ ✓✓✓✓ (6)

3.2 $\int \frac{x^2}{(1-x)(x^2+1)}dx$

Let $\frac{x^2}{(1-x)(x^2+1)} = \frac{A}{1-x} + \frac{Bx+C}{x^2+1}$ ✓

$$x^2 = A(x^2+1) + (Bx+C)(1-x)$$ ✓

Let $x = 1$:

$1 = 2A$

$A = \frac{1}{2}$ ✓

x^2: $1 = A - B$

$B = \frac{-1}{2}$ ✓

$constant$: $0 = A + C$

$C = \frac{-1}{2}$ ✓

$= \frac{1}{2}\int \frac{dx}{1-x} + \int \frac{\frac{-1}{2}x - \frac{1}{2}}{x^2+1}dx$ ✓

$= \frac{1}{2}\int \frac{dx}{1-x} - \frac{1}{2}\int \frac{xdx}{x^2+1} - \frac{1}{2}\int \frac{dx}{x^2+1}$ ✓

$= -\frac{1}{2}\int \frac{-1}{1-x}dx - \frac{1}{4}\int \frac{2x}{x^2+1}dx - \frac{1}{2}\int \frac{dx}{x^2+1}$ ✓✓

$= \frac{-1}{2}\ln(1-x) - \frac{1}{4}\ln(x^2+1) - \frac{1}{2}\tan^{-1}x + C$ ✓✓✓

(6)
[12]

QUESTION 4

4.1 $$(t-1)^3\frac{ds}{dt}+4(t-1)^2 s=t+1$$

$$\frac{ds}{dt}+\left(\frac{4}{t-1}\right)s=\frac{t+1}{(t-1)^3}\checkmark$$

IF $= e^{4\int\frac{dt}{t-1}}$

$= e^{4\ln(t-1)}\checkmark$

$= e^{\ln(t-1)^4}\checkmark$

$= (t-1)^4\checkmark$

$$\frac{d}{dt}\left[(t-1)^4 s\right]=\frac{t+1}{(t-1)^3}\times(t-1)^4\checkmark$$

$$(t-1)^4 s=\int(t^2-1)dt\checkmark\checkmark$$

$$=\frac{t^3}{3}-t+C\checkmark\checkmark$$

$$\therefore s=\frac{t^3}{3(t-1)^4}-\frac{t}{(t-1)^4}+\frac{C}{(t-1)^4}\checkmark$$ (5)

4.2 $\frac{d^2s}{dt}+4\frac{ds}{dt}+13s=39t^2+11t+54$ when $t=0;\ s=1$ and $\frac{ds}{dt}=8$ when $t=0$

$r^2+4r+13=0$ ✓

$r=-2\pm 3j$ ✓

$s_c=e^{-2t}\left(A\cos 3t+B\sin 3t\right)$ ✓

$s_p=Ct^2+Dt+E$ ✓

$s'_p=2Ct+D$ ✓

$s''_p=2C$ ✓

$2C+8Ct+4D+13Ct^2+13Dt+13E=39t^2+11t+54$

$t^2:\quad 13C=39\ \Rightarrow\ C=3$ ✓

$t:\quad 8C+13D=11\ \Rightarrow\ 13D=-13\ \Rightarrow\ D=-1$ ✓

Constant: $2C+4D+13E=54\ \Rightarrow 13E=52\ \Rightarrow E=4$ ✓

$s_p=3t^2-t+4$

$s=s_c+s_p$

$=e^{-2t}\left(A\cos 3t+B\sin 3t\right)+3t^2-t+4$ ✓

$1=A+4\ \Rightarrow\ A=-3$ ✓

$s'=-2e^{-2t}A\cos 3t-3Ae^{-2t}\sin 3t-2e^{-2t}B\sin 3t+3Be^{-2t}\cos 3t$ ✓

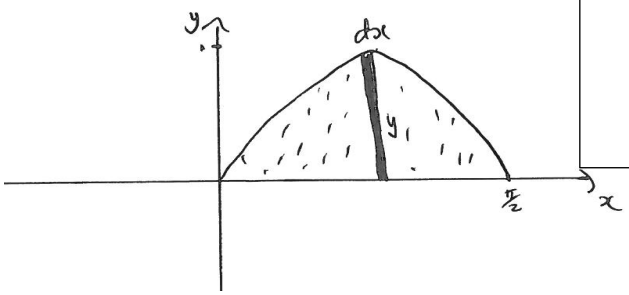
$8=-2A+3B-1\ \Rightarrow\ 3=3B\ \Rightarrow\ B=1$ ✓

$\therefore\ s=e^{-2t}\left(-3\cos 3t+\sin 3t\right)+3t^2-t+4$ ✓ (7)

[12]

QUESTION 5

5.1 5.1.1 $y = \sin 2x$



strip✓

$x - intercepts$✓

area ✓

graph ✓

(2)

5.1.2 $dV_x = \pi y^2 dx$ ✓

$$V_x = \pi \int_0^{\frac{\pi}{2}} \sin^2 2x dx \checkmark$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} (1 - \cos 4x)\, dx \checkmark$$

$$= \frac{\pi}{2} \left[x - \frac{\sin 4x}{4} \right]_0^{\frac{\pi}{2}} \checkmark$$

$$= \frac{\pi}{2} \left[\frac{\pi}{2} - \frac{sin2\pi}{4} \right] - \frac{\pi}{2}[0 - 0] \checkmark$$

$$= \frac{\pi^2}{4} \text{ or } 2.467\,(\text{units})^3 \checkmark$$

$V_x = \pi \int_a^b y^2 dx$ ✓

$V_x = \pi \int_0^{\frac{\pi}{2}} sin^2 2x dx$ ✓✓

$= \pi \left[\frac{x}{2} - \frac{sin4x}{8} \right]_0^{\frac{\pi}{2}}$ ✓

$= \pi \left[\frac{\pi}{4} - \frac{sin2\pi}{8} \right] - \pi[0 - 0]$✓

$= \frac{\pi^2}{4}$ ✓

(3)

5.1.3 $dVmoy = x\pi y^2 dx$

$$Vmoy = \pi\int_0^{\frac{\pi}{2}} x\sin^2 2x dx \checkmark\checkmark$$

$$= \pi\left[x\left(\frac{x}{2}-\frac{\sin 4x}{8}\right)-\int\left(\frac{x}{2}-\frac{\sin 4x}{8}\right)dx\right]_0^{\frac{\pi}{2}} \checkmark\checkmark$$

$$= \pi\left[\frac{x^2}{2}-\frac{xsin4x}{8}-\left(\frac{x^2}{4}+\frac{cos4x}{32}\right)\right]\checkmark\checkmark$$

$$= \pi\left[\frac{x^2}{4}-\frac{x\sin 4x}{8}-\frac{\cos 4x}{32}\right]_0^{\frac{\pi}{2}}$$

$$= \pi\left[\frac{\pi^2}{8}-\frac{\frac{\pi}{2}\sin\pi}{8}-\frac{\cos\pi}{32}+\frac{1}{32}\right]\checkmark$$

$$= 4.0721(\text{units})^4 \checkmark$$

$$\bar{x} = \frac{Vmoy}{V}$$

$$= \frac{4.0721}{2.467}\checkmark$$

$$= 0.912\text{ units}\checkmark$$ (5)

5.2 5.2.1 $y = 2x$ and $y = x^2$

$x^2 - 2x = 0$

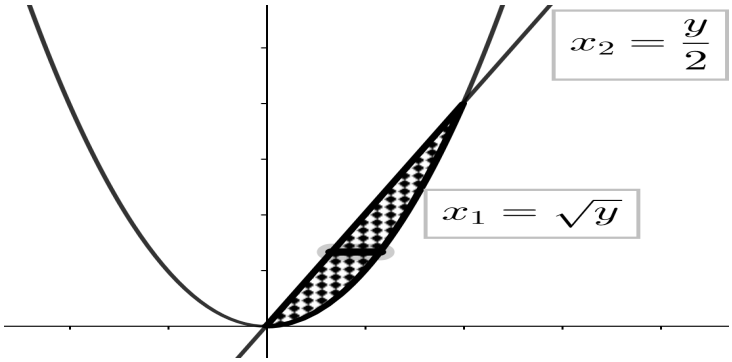
$x(x-2) = 0$

$x = 0$ or $x = 2$

$y = 0 \qquad y = 4$

(0;0) and (2;4) ✓✓

strip✓
area✓
Each graph ✓✓



(3)

5.2.2 $dA = (x_2 - x_1)\,dy$ ✓ **OR** $A_y = \int_c^d (x_1 - x_2)dy$

$$A = \int_0^4 \left(y^{\frac{1}{2}} - \frac{y}{2} \right) dy \text{ ✓✓}$$

$$= \left[\frac{2y^{\frac{3}{2}}}{3} - \frac{y^2}{4} \right]_0^4 \text{ ✓}$$

$$= \left[\frac{2}{3}(4)^{\frac{3}{2}} - \frac{(4)^2}{4} - 0 \right] \text{ ✓}$$

$$= \frac{4}{3} \text{ or } 1.333\,(\text{units})^2 \text{ ✓}$$

(3)

5.2.3 $dI_x = \text{Area} \times (\text{distance})^2$ ✓ **OR** $I_x = \int_a^b r^2 dA$

$$I_x = \int_0^4 y^2 \left(x_2 - x_1\right) dy \checkmark\checkmark$$

$$= \int_0^4 y^2 \left(y^{\frac{1}{2}} - \frac{y}{2} \right) dy \checkmark$$

$$= \int_0^4 \left(y^{\frac{5}{2}} - \frac{y^3}{4} \right) dy \checkmark$$

$$= \left[\frac{2}{7} y^{\frac{7}{2}} - \frac{y^4}{8} \right]_0^4 \checkmark$$

$$= \left[\frac{2}{7}(4)^{\frac{7}{2}} - \frac{(4)^4}{8} - 0 \right] \checkmark$$

$$= \frac{32}{7} \text{ or } 4.571\,(\text{units})^4 \checkmark$$

(4)

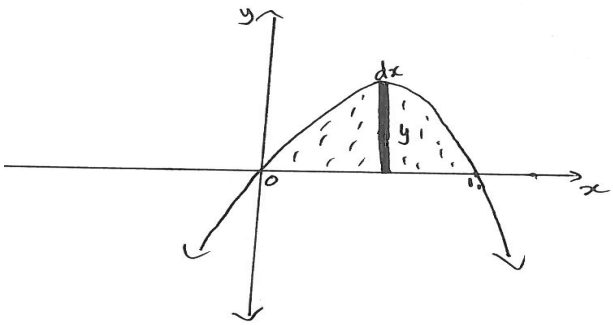
5.2.4 $I_x = 4.571 \frac{A}{A} \checkmark$

$$= \frac{4.571}{1.333} A$$

$$= 3.429\,(\text{units})^4 \checkmark$$

(1)

5.3 5.3.1



strip✓

$x - intercepts$✓

area✓

graph✓

(2)

5.3.2 $dV_x = \pi y^2 dx$

$$V_x = \pi \int_0^1 y^2 dx \checkmark$$

$$= \pi \int_0^1 \left(-x^2 + x\right)^2 dx \checkmark$$

$$= \pi \int_0^1 \left(x^4 - 2x^3 + x^2\right) dx \checkmark$$

$$= \pi \left[\frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3}\right]_0^1 \checkmark$$

$$= \pi \left[\frac{1}{5} - \frac{1}{2} + \frac{1}{3} - 0\right] \checkmark$$

$$= \frac{\pi}{30}$$

$$= 0.105 \text{ (units)}^3 \checkmark \quad (3)$$

5.3.3 $dI_x = \frac{1}{2} r^2 dm$

$$I_x = \frac{\rho\pi}{2} \int_0^1 y^4 dx \checkmark\checkmark$$

$$= \frac{\rho\pi}{2} \int_0^1 \left(x^4 - 2x^3 + x^2\right)^2 dx \checkmark$$

$$= \frac{\rho\pi}{2} \int_0^1 \left(x^8 - 4x^7 + 6x^6 - 4x^5 + x^4\right) dx \checkmark\checkmark$$

$$= \frac{\rho\pi}{2} \left[\frac{x^9}{9} - \frac{x^8}{2} + \frac{6x^7}{7} - \frac{2x^6}{3} + \frac{x^5}{5}\right]_0^1 \checkmark$$

$$= \frac{\rho\pi}{2} \left[\frac{1}{9} - \frac{1}{2} + \frac{6}{7} - \frac{2}{3} + \frac{1}{5}\right] \checkmark$$

$$= \frac{\rho\pi}{1260}$$

$$= 0.002493\rho \text{ (units)}^5 \checkmark \quad (4)$$

5.4 5.4.1 $y = ax^2 - 5$ ✓ through (4;-1)

$-1 = a(4)^2 - 5$

$a = \frac{1}{4}$ ✓

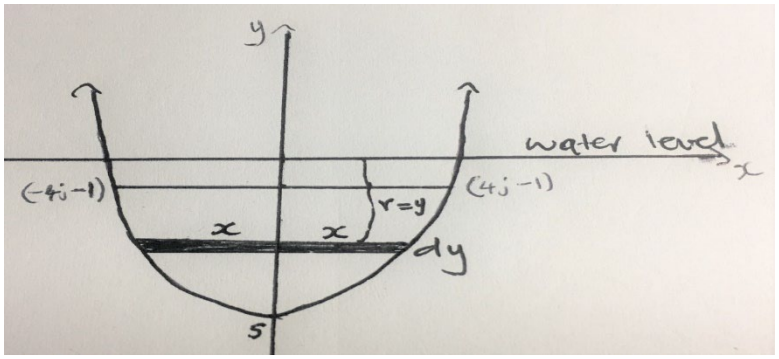
$y = \frac{1}{4}x^2 - 5$ ✓

$x = 2\sqrt{y+5}$

Strip✓

Top of gate at $y = -1$ ✓

Bottom of gate at $y = -5$ ✓



(3)

5.4.2 First moment

$$= \int_a^b r dA$$

$$= 4\int_{-5}^{-1} y\sqrt{y+5}\,dy \quad \checkmark$$

$dA = 2xdy = 4\sqrt{y+5}$

$$= 4\int_{-5}^{-1} (u-5)u^{\frac{1}{2}}du \checkmark$$

$let\ u = y + 5$

$\Rightarrow u - 5 = y$

$\Rightarrow\ du = dy$

$$= 4\int_{-5}^{-1} \left(u^{\frac{3}{2}} - 5u^{\frac{1}{2}}\right)du \checkmark$$

$$= 4\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{10}{3}u^{\frac{3}{2}}\right]_{-5}^{-1} \checkmark$$

$y = -1 \Rightarrow u = 4$

$y = -5 \Rightarrow u = 0$

$$= 4\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{10}{3}u^{\frac{3}{2}}\right]_0^4 \checkmark$$

$$= 4\left[\frac{2}{5}(y+5)^{\frac{5}{2}} - \frac{10}{3}(y+5)^{\frac{3}{2}}\right]_{-5}^{-1}$$

$$= 4\left[\frac{2}{5}(4)^{\frac{5}{2}} - \frac{10}{3}(4)^{\frac{3}{2}} - 0\right] \checkmark$$

$$= 4\left[\frac{2}{5}(4)^{\frac{5}{2}} - \frac{10}{3}(4)^{\frac{3}{2}} - 0\right] \checkmark$$

$$= -55.467\ \text{m}^3 \checkmark$$

$= -55{,}467m^3 \checkmark$ (3)

5.4.3 Second moment

$$= \int_a^b r^2 dA$$

$$= 4\int_{-5}^{-1} y^2\sqrt{y+5}\,dy$$

$dA = 2xdy = 4\sqrt{y+5}$

$$= 4\int_{-5}^{-1}(u-5)^2 u^{\frac{1}{2}}du \quad \checkmark$$

$let\ u = y+5$

$\Rightarrow u-5 = y$

$\Rightarrow\ du = dy$

$$= 4\int_{-5}^{-1}\left(u^2-10u+25\right)u^{\frac{1}{2}}du \checkmark$$

$$= 4\int_{-5}^{-1}\left(u^{\frac{5}{2}}-10u^{\frac{3}{2}}+25u^{\frac{1}{2}}\right)du \checkmark$$

$$= 4\left[\frac{2}{7}u^{\frac{7}{2}}-4u^{\frac{5}{2}}+\frac{50}{3}u^{\frac{3}{2}}\right]_{-5}^{-1} \checkmark$$

$$= 4\left[\frac{2}{7}(y+5)^{\frac{7}{2}}-4(y+5)^{\frac{5}{2}}+\frac{50}{3}(y+5)^{\frac{3}{2}}\right]_{-5}^{-1}$$

$$= 4\left[\frac{2}{7}(4)^{\frac{7}{2}}-4(4)^{\frac{5}{2}}+\frac{50}{3}(4)^{\frac{3}{2}}\right] \checkmark$$

$y = -1 \Rightarrow u = 4$

$y = -5 \Rightarrow u = 0$

$= 4\left[\frac{2}{7}u^{\frac{7}{2}}-4u^{\frac{5}{2}}+\frac{50}{3}u^{\frac{3}{2}}\right]_0^4 \checkmark$

$= 4\left[\frac{2}{7}(4)^{\frac{7}{2}}-4(4)^{\frac{5}{2}}+\frac{50}{3}(4)^{\frac{3}{2}}\right] \checkmark$

$= 167{,}619\ m^4 \checkmark$

$= 167.619\ \text{m}^4 \checkmark$

$$\bar{y} = \frac{167.619}{-55.467} \checkmark$$

$= -3.022\ \text{m} \checkmark$ (4)

(ALTERNATIVE ANSWERS TO QUESTION 5.4)

5.4 5.4.1 $y = ax^2 \; through \; (4;4)$✓

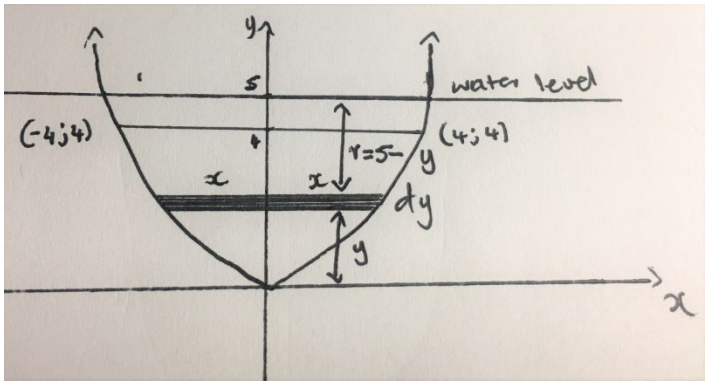
$4 = 16a \;\; \Rightarrow a = \frac{1}{4}$✓

$\therefore y = \frac{1}{4}x^2 \;\; \Rightarrow x = 2\sqrt{y}$ ✓

Strip ✓

Top of gate at $y = 4$ ✓

Water level at $y = 5$ ✓



(3)

5.4.2 First moment

$$= \int_a^b rdA$$

$$= 4\int_0^4 (5-y)\, y^{\frac{1}{2}} dy \;\checkmark\checkmark$$

$dA = 2xdy = 4\sqrt{y}$

$$= 4\int_0^4 \left(5y^{\frac{1}{2}} - y^{\frac{3}{2}}\right) dy \;\checkmark$$

$$= 4\left[\frac{10}{3}y^{\frac{3}{2}} - \frac{2}{5}y^{\frac{5}{2}}\right]_0^4 \;\checkmark$$

$$= 4\left[\frac{10}{3}(4)^{\frac{3}{2}} - \frac{2}{5}(4)^{\frac{5}{2}} - 0\right] \checkmark$$

$$= 55.467 \text{ m}^3 \checkmark$$

(3)

5.4.3 Second moment

$$\int_a^b r^2 dA$$

$$= 4\int_0^4 (5-y)^2 y^{\frac{1}{2}} dy \checkmark$$

$dA = 2xdy = 4\sqrt{y}$

$$= 4\int_0^4 \left(25 - 10y + y^2\right) y^{\frac{1}{2}} dy \checkmark$$

$$= 4\int_0^4 \left(25y^{\frac{1}{2}} - 10y^{\frac{3}{2}} + y^{\frac{5}{2}}\right) dy \checkmark$$

$$= 4\left[\frac{50}{3}y^{\frac{3}{2}} - 4y^{\frac{5}{2}} + \frac{2}{7}y^{\frac{7}{2}}\right]_0^4 \checkmark$$

$$= 4\left[\frac{50}{3}(4)^{\frac{3}{2}} - 4(4)^{\frac{5}{2}} + \frac{2}{7}(4)^{\frac{7}{2}} - 0\right] \checkmark$$

$$= 167.619 \text{ m}^4 \checkmark$$

$$\bar{y} = \frac{167.619}{55.467} \checkmark$$

$$= 3.022 \text{ m} \checkmark$$ (4)

[40]

QUESTION 6

6.1 $x = 3\cos t - \cos 3t$ and $y = 3\sin t - \sin 3t$

$\frac{dx}{dt} = -3\sin t + 3\sin 3t$ ✓ $\frac{dy}{dt} = 3\cos t - 3\cos 3t$ ✓

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left(-3\sin t + 3\sin 3t\right)^2 + \left(3\cos t - 3\cos 3t\right)^2$$

$$= 9\sin^2 t - 18\sin t \sin 3t + 9\sin^2 3t + 9\cos^2 t - 18\cos t \cos 3t + 9\cos^2 3t \checkmark$$

$$= 9\left(\sin^2 t + \cos^2 t\right) + 9\left(\sin^2 3t + \cos^2 3t\right) - 18\left(\cos t \cos 3t + \sin t \sin 3t\right) \checkmark$$

$$= 9(1) + 9(1) - 18\cos(-2t)$$

$$= 18 - 18\cos 2t$$

$$= 18\left(1 - \cos 2t\right) \checkmark$$

$$= 18\left(2\sin^2 t\right)$$

$$= 36\sin^2 t \checkmark$$

$\cos(-2t) = \cos(t - 3t) = \cos t \cos 3t + \sin t \sin 3t$

$\cos 2t = 1 - 2\sin^2 t$

Therefore,

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{\pi} \sqrt{36\sin^2 t}\, dt \checkmark\checkmark$$

$$= 6\int_0^{\pi} \sin t\, dt \checkmark$$

$$= -6\cos t\Big|_0^{\pi} \checkmark$$

$$= -6\cos\pi - \left(-6\cos 0\right) \checkmark$$

$$= -6(-1) - (-6(1))$$

$$= 12 \checkmark$$ (6)

6.2 $y = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\frac{dy}{dx} = \frac{1}{2}\left(e^{x} - e^{-x}\right)$ ✓

$\left(\frac{dy}{dx}\right)^{2} + 1 = \frac{1}{4}(e^{x} - e^{-x})^{2} + 1$

$= \frac{e^{2x} - 2 + e^{-2x} + 4}{4}$ ✓

$= \frac{e^{2x} + 2 + e^{-2x}}{4}$

$= \frac{(e^{x} + e^{-x})^{2}}{4}$ ✓

$A_x = 2\pi \int y\sqrt{\left(\frac{dy}{dx}\right)^{2} + 1}dx$

$= 2\pi \int_0^2 \frac{1}{2}\left(e^{x} + e^{-x}\right)\sqrt{\frac{\left(e^{x} + e^{-x}\right)^{2}}{4}}dx$ ✓✓✓

$= \frac{\pi}{2}\int_0^2 \left(e^{x} + e^{-x}\right)\left(e^{x} + e^{-x}\right)dx$ ✓

$= \frac{\pi}{2}\int_0^2 \left(e^{2x} + 2 + e^{-2x}\right)dx$ ✓

$= \frac{\pi}{2}\left[\frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2}\right]_0^2$ ✓✓

$= \frac{\pi}{2}\left[\frac{e^{4}}{2} + 4 - \frac{e^{-4}}{2} - \frac{1}{2} + \frac{1}{2}\right]$ ✓

$= 49.150$ ✓ (6)

[12]

TOTAL: 100

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