higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

22 November 2023 (X-paper)
09:00–12:00

Scientific calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

050Q1E2322

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Start each section on a new page.
5. Use only a black or blue pen.
6. Write neatly and legibly.

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QUESTION 1

1.1 If $z = e^{xy} \ln y$

Prove that $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$ (4)

1.2 Given: $y = \sqrt{3t}$ and $x = -\sqrt{t+1}$

Determine $\frac{dy}{dx}$. (2)

[6]

QUESTION 2

Determine $\int y dx$ if:

2.1 $y = \frac{1}{8 - 2x - x^2}$ (4)

2.2 $y = e^{-x} \cos x$ (4)

2.3 $y = \tan^5 3x$ (5)

2.4 $y = \cos^5 \pi x \sin^3 \pi x$ (4)

2.5 $y = \sin^2\left(\frac{3x}{5}\right)$ (1)

[18]

QUESTION 3

Use partial fractions to integrate the following:

3.1 $\int \frac{2x^2 - 7x - 12}{x(x+2)(x-3)} dx$ (5)

3.2 $\int \frac{x^3 + 4x + 9}{x^2 - x - 6} dx$ (7)

[12]

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QUESTION 4

4.1 Determine the particular solution of:

$$(t+1)\frac{ds}{dt}+2s=3(t+1)+\frac{1}{(t+1)^2} \text{ where } t=4 \text{ when } s=2$$

(6)

4.2 Determine the general solution of:

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=x^2$$

(6)
[12]

QUESTION 5

5.1 5.1.1 Make a neat sketch of the graph of $y=3x-x^2$ and show the representative strip/element that one will use to calculate the volume of the solid generated when the area bounded by the curve and the line $y=0$ rotates about the y-axis. (2)

5.1.2 Calculate the volume generated if the area described in QUESTION 5.1.1 rotates about the y-axis. (4)

5.2 5.2.1 Calculate the points of intersection of the two curves $y=x+6$ and $y=x^2$ Make a neat sketch of the two curves and show the area bounded by the curves. Show the representative strip/element PERPENDICULAR to the $x-axis$ that one will use to calculate the area bounded by the curves and the $y-axis$ in the first quadrant. (3)

5.2.2 Calculate the area bounded by the curves described in QUESTION 5.2.1. (3)

5.2.3 Calculate the second moment of area when the area bounded by the curves described in QUESTION 5.2.1 is rotated about the y-axis. (4)

5.2.4 Express the answer in QUESTION 5.2.3 in terms of the area. (1)

5.3 5.3.1 Calculate the point of intersection of $y=\sin x$ and $y=\cos x$.

Make a neat sketch of the graphs and clearly indicate the area bounded by the graphs for $0\leq x\leq\frac{\pi}{4}$. Show the representative strip/element that one will use to calculate the volume of the solid generated when the area bounded by the graphs for $0\leq x\leq\frac{\pi}{4}$ rotates about the x-axis. (3)

5.3.2 Calculate the volume described in QUESTION 5.3.1. (4)

5.3.3 Calculate the moment of inertia of the solid obtained when the area described in QUESTION 5.3.1 rotates about the x-axis. (5)

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5.4 5.4.1 A vertical weir in a rectangular canal is 8 m wide and 4 m high. The top of the weir is 1 m below the water surface.

Make a neat sketch of the weir and show the representative strip/element that one will use for calculations. (2)

5.4.2 Calculate, by using integration, the area moment of the weir about the water level in QUESTION 5.4.1. (4)

5.4.3 Calculate, by using integration, the second moment of area of the weir described in QUESTION 5.4.1 about the water level as well as the depth of the centre of pressure on the weir. (5)

[40]

QUESTION 6

6.1 Calculate the length of the curve $y=\frac{1}{2}\left(e^{x}+e^{-x}\right)$ from $x=0$ to $x=2$. (6)

6.2 Calculate the surface area generated when the curves $x=\ln\left(\sec t+\tan t\right)-\sin t$ and $y=\cos t$ from $t=0$ to $t=\frac{\pi}{3}$ are rotated about x-axis. (6)

[12]

TOTAL: 100

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MATHEMATICS N6

FORMULA SHEET

Any other applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x}; \quad \sin x = \frac{1}{\text{cosec}\, x}; \quad \cos x = \frac{1}{\sec x}$$

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$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n \, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx} (ax + b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx} (dx + e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} . \frac{d}{dx} ax$	$x \ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} . \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln \left[\tan \left(\frac{ax}{2} \right) \right] + C$

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$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\text{cosec}\, f(x)$	$-\text{cosec}\, f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

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$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$

$\int [f(x)]^n\, f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$

$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C$

$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$

$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$

$\int \sqrt{a^2 - b^2x^2}\,dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$

$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$

$\int \sqrt{x^2 \pm b^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$

$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$

APPLICATIONS OF INTEGRATION

AREAS

$A_x = \int_a^b y\,dx;\ A_x = \int_a^b (y_1 - y_2)\,dx$

$A_y = \int_a^b x\,dy;\ A_y = \int_a^b (x_1 - x_2)\,dy$

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VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2\pi \int_a^b xy dy$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2\pi \int_a^b xy dx$$

AREA MOMENTS

$$A_{m-x} = \int_a^b r dA \qquad A_{m-y} = \int_a^b r dA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b r dA}{A} \; ; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b r dA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b r dV \quad ; \quad V_{m-y} = \int_a^b r dV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b r dV}{V} \; ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b r dV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

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GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho \pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho \pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3 (cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

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$$A_x = \int_{1\,u}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_{c}^{d} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore \quad ye^{\int Pdx} = \int Qe^{\int Pdx} dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

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