higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

22 November 2023

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QUESTION 1

1.1 If $z = e^{xy} \ln y$

$$\frac{\partial z}{\partial x} = ye^{xy} \ln y \quad \checkmark$$

$$\frac{\partial^2 z}{\partial x \partial y} = e^{xy}\ln y \checkmark + y\left(xe^{xy}\ln y + \frac{1}{y}e^{xy}\right) \checkmark\checkmark$$

$$= e^{xy}\ln y + yxe^{xy}\ln y + e^{xy}$$

$$\frac{\partial z}{\partial y} = xe^{xy} \ln y + \frac{1}{y}e^{xy} \checkmark\checkmark$$

$$\frac{\partial^2 z}{\partial y \partial x} = (e^{xy} + xye^{xy})\ln y \checkmark + y\left(\frac{1}{y}e^{xy}\right) \checkmark$$

$$= e^{xy}\ln y + yxe^{xy}\ln y + e^{xy}$$

Therefore, $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$ (8)

1.2 $\frac{dy}{dt} = \frac{\sqrt{3}}{2}t^{\frac{-1}{2}}$ ✓ $\frac{dx}{dt} = \frac{-1}{2}(t+1)^{\frac{-1}{2}}$ ✓

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{\sqrt{3}t^{\frac{-1}{2}}}{(t+1)^{\frac{-1}{2}}} = -\sqrt{3}t^{\frac{-1}{2}}(t+1)^{\frac{1}{2}} \checkmark = \frac{-\sqrt{3}\sqrt{t+1}}{\sqrt{t}} \quad \checkmark$$

ALTERNATIVE

$y = \sqrt{3t} \Rightarrow \frac{y^2}{3} = t$ ✓

$$\therefore x = -\sqrt{t+1} = -\sqrt{\frac{y^2}{3}+1} = -\left(\frac{y^2}{3}+1\right)^{\frac{1}{2}} \quad \checkmark$$

$$\Rightarrow \frac{dx}{dy} = -\frac{1}{2}\left(\frac{y^2}{3}+1\right)^{-\frac{1}{2}}.\frac{2y}{3} = \frac{-y}{3\sqrt{\frac{y^2}{3}+1}} \quad \checkmark$$

$$\therefore \frac{dy}{dx} = \frac{-3\sqrt{\frac{y^2}{3}+1}}{y} = \frac{-3\sqrt{t+1}}{\sqrt{3t}} = \frac{-\sqrt{3}\sqrt{t+1}}{\sqrt{t}} \quad \checkmark$$

(4)
[12]

QUESTION 2

2.1 $8-2x-x^2=-x^2-2x+8$

$=-\left[x^2+2x-8\right]$ ✓

$=-\left[x^2+2x+1-1-8\right]$ ✓

$=-\left[(x+1)^2-9\right]$ ✓

$=9-(x+1)^2$ ✓

ALTERNATIVE

$\frac{4ac-b^2}{4a}+a\left(x+\frac{b}{2a}\right)^2$ ✓

$=\frac{4(-1)(8)-(-2)^2}{4(-1)}-1\left(x+\frac{-2}{-2}\right)^2$ ✓✓

$=9+(x+1)^2$ ✓

$\therefore \int \frac{dx}{8-2x-x^2}=\int \frac{dx}{9-(x+1)^2}$ ✓

$=\frac{1}{3}\tan^{-1}\frac{x+1}{3}+C$ ✓✓✓ (8)

2.2 $f(x)=e^{-x}\Rightarrow f'(x)=-e^{-x}$ and $g'(x)=\cos x\Rightarrow g(x)=\sin x$

$\int e^{-x}\cos x dx=e^{-x}\sin x+\int e^{-x}\sin x dx$ ✓✓

$f(x)=e^{-x}\Rightarrow f'(x)=-e^{-x}$ and $g'(x)=\sin x\Rightarrow g(x)=-\cos x$

$\int e^{-x}\cos x dx=e^{-x}\sin x+\left[-e^{-x}\cos x-\int e^{-x}\cos x dx\right]$ ✓✓

$\int e^{-x}\cos x\,dx=-e^{-x}\cos x+e^{-x}\sin x-\int e^{-x}\cos x\,dx$ ✓

$2\int e^{-x}\cos x dx=e^{-x}\sin x-e^{-x}\cos x+C$ ✓

$\int e^{-x}\cos x dx=\frac{1}{2}e^{-x}\sin x-\frac{1}{2}e^{-x}\cos x+C$ ✓✓ (8)

ALTERNATIVE

$\boldsymbol{f(x) = \cos x \Rightarrow f'(x) = -\sin x \quad \text{and} \quad g'(x) = e^{-x} \Rightarrow g(x) = -e^{-x}}$

$\boldsymbol{\int e^{-x}\cos x\,dx = -e^{-x}\cos x - \int e^{-x}\sin x dx}$ ✓✓

$\boldsymbol{f(x) = \sin x \Rightarrow f'(x) = \cos x \quad \text{and} \quad g'(x) = e^{-x} \Rightarrow g(x) = -e^{-x}}$

$\boldsymbol{\int e^{-x}\cos x\,dx = -e^{-x}\cos x - \left[-e^{-x}\sin x + \int e^{-x}\cos x\,dx\right]}$ ✓✓

$\boldsymbol{\int e^{-x}\cos x\,dx = -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x\,dx}$ ✓

$\boldsymbol{2\int e^{-x}\cos x\,dx = -e^{-x}\cos x + e^{-x}\sin x}$ ✓

$\int e^{-x}\cos x dx = \frac{1}{2}e^{-x}\sin x - \frac{1}{2}e^{-x}\cos x + C$ ✓✓

2.3 $\int \tan^5 3x dx = \int \tan^3 3x \tan^2 3x dx$ ✓

$= \int \tan^3 3x\left(\sec^2 3x - 1\right)dx$ ✓

$= \int \tan^3 3x \sec^2 3x dx - \int \tan^3 3x dx$ ✓

$= \int \tan^3 3x \sec^2 3x dx - \int \tan 3x \tan^2 3x dx$ ✓

$= \int \tan^3 3x \sec^2 3x dx - \int \tan 3x\left(\sec^2 3x - 1\right)dx$ ✓

$= \int \tan^3 3x \sec^2 3x dx - \int \tan 3x \sec^2 3x dx + \int \tan 3x dx$ ✓

$= \frac{1}{3}\int u^3 du - \frac{1}{3}\int u du + \int \tan 3x dx$ ✓

$= \frac{u^4}{12} - \frac{u^2}{6} + \frac{1}{3}\ln sec3x + C$ ✓✓

$= \frac{\left(\tan 3x\right)^4}{12} - \frac{\left(\tan 3x\right)^2}{6} + \frac{1}{3}\ln\left(\sec 3x\right) + C$ ✓

Let $u = \tan 3x$

$du = 3\sec^2 3x dx$

(10)

ALTERNATIVE 1

$$\int \tan^5 3x dx = \int \tan^3 3x \tan^2 3x dx \checkmark$$

$$= \int \tan^3 3x \left(\sec^2 3x - 1\right) dx \checkmark$$

$$= \int \tan^3 3x \sec^2 3x dx - \int \tan^3 3x dx \quad \checkmark$$

$$= \int \tan^3 3x \sec^2 3x dx - \int \tan 3x \tan^2 3x dx \checkmark$$

$$= \int \tan^3 3x \sec^2 3x dx - \int \tan 3x \left(\sec^2 3x - 1\right) dx \checkmark$$

$$= \int \tan^3 3x \sec^2 3x dx - \int \tan 3x \sec^2 3x dx + \int \tan 3x dx \checkmark$$

$$= \frac{1}{3}\int [\boldsymbol{\tan 3x}]^3 \, (\boldsymbol{3\sec^2 3x})\boldsymbol{dx} - \frac{1}{3}\int \boldsymbol{\tan 3x} \, (\boldsymbol{3\sec^2 3x})\boldsymbol{dx} + \int \boldsymbol{\tan 3x dx} \checkmark$$

$$= \frac{(\tan 3x)^4}{12} - \frac{(\tan 3x)^2}{6} + \frac{1}{3}\ln(\sec 3x) + C \checkmark\checkmark\checkmark$$

ALTERNATIVE 2

$$\int \tan^5 3x dx = \int \left(\tan^2 3x\right)^2 \tan 3x dx \checkmark$$

$$= \int \left(\sec^2 3x - 1\right)^2 \tan 3x dx \checkmark$$

$$= \int \left(\sec^4 3x - 2\sec^2 3x + 1\right) \tan 3x dx \checkmark$$

$$= \int \sec^4 3x \tan 3x dx - 2\int \sec^2 3x \tan 3x dx + \int \tan 3x dx \checkmark$$

$$= \frac{1}{3}\int u^3 du - \frac{2}{3}\int u du + \int \tan 3x dx \checkmark\checkmark$$

Let $u = \sec 3x$

$du = 3\sec 3x \tan 3x dx$

$$= \frac{u^4}{12} - \frac{1}{3}u^2 + \frac{1}{3}\ln(\sec 3x) + C \checkmark\checkmark\checkmark$$

$$= \frac{\sec^4 3x}{12} - \frac{1}{3}\sec^2 3x + \frac{1}{3}\ln(\sec 3x) + C \checkmark$$

ALTERNATIVE 3

$$\int \tan^5 3x\,dx = \int \frac{\sin^5 3x}{\cos^5 3x}\,dx \checkmark$$

$$= \int (\sin^2 3x)^2 \sin 3x (\cos 3x)^{-5}\,dx \checkmark$$

$$= \int (1-\cos^2 3x)^2 \sin 3x (\cos 3x)^{-5}\,dx \checkmark$$

$$= \int (1 - 2\cos^2 3x + \cos^4 3x) \sin 3x (\cos 3x)^{-5}\,dx \checkmark$$

$$= \int (\cos 3x)^{-5} \sin 3x\,dx - 2\int (\cos 3x)^{-3} \sin 3x\,dx + \int \tan 3x\,dx \checkmark\checkmark\checkmark$$

$$= -\frac{1}{3}\int (\cos 3x)^{-5}(-3\sin 3x)\,dx + \frac{2}{3}\int (\cos 3x)^{-3}(-3\sin 3x)\,dx + \int \tan 3x\,dx \checkmark$$

$$= \frac{(\cos 3x)^{-4}}{12} - \frac{(\cos 3x)^{-2}}{3} + \frac{1}{3}\ln(\sec 3x) + C \checkmark\checkmark$$

$$= \frac{\sec^4 3x}{12} - \frac{1}{3}\sec^2 3x + \frac{1}{3}\ln(\sec 3x) + C$$

ALTERNATIVE 4

$$\int \tan^5 3x\,dx = \int \frac{\sin^5 3x}{\cos^5 3x}\,dx \checkmark$$

$$= \int \frac{(\sin^2 3x)^2 \sin 3x}{(\cos 3x)^5}\,dx \checkmark$$

$$= \int \frac{(1-\cos^2 3x)^2 \sin 3x}{(\cos 3x)^5}\,dx \checkmark$$

$$= \int \frac{(1-u^2)^2 \sin 3x}{u^5}\frac{du}{-3\sin 3x} \checkmark$$

$u = \cos 3x$

$\Rightarrow \frac{du}{-3\sin 3x} = dx$

$$= -\frac{1}{3}\int \frac{(1-2u^2+u^4)}{u^5}\,du \checkmark$$

$$= -\frac{1}{3}\int \left(u^{-5} - 2u^{-3} + \frac{1}{u}\right)du \checkmark$$

$$= -\frac{1}{3}\left[\frac{u^{-4}}{-4} - \frac{2u^{-2}}{-2} + \ln u\right] + C \checkmark\checkmark\checkmark$$

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$$= \frac{(\cos 3x)^{-4}}{12} - \frac{(\cos 3x)^{-2}}{3} - \frac{1}{3}\ln(\cos 3x) + C \checkmark$$

$$= \frac{(\sec 3x)^{4}}{12} - \frac{(\sec 3x)^{2}}{3} - \frac{1}{3}\ln(\cos 3x) + C$$

2.4 $\int \sin^3 \pi x \cos^5 \pi x dx$

$= \int \sin^2 \pi x \sin \pi x \cos^5 \pi x dx$ ✓

$= \int (1 - \cos^2 \pi x) \sin \pi x \cos^5 \pi x dx$ ✓

$= \int \cos^5 \pi x \sin \pi x dx - \int \cos^7 \pi x \sin \pi x dx$ ✓✓

$= -\frac{1}{\pi} \int u^5 du + \frac{1}{\pi} \int u^7 du$ ✓

$= -\frac{u^6}{6\pi} + \frac{u^8}{8\pi} + C$ ✓✓

$= -\frac{cos^6 \pi x}{6\pi} + \frac{cos^8 \pi x}{8\pi} + C$ ✓ (8)

$u = \cos \pi x$

$\Rightarrow \frac{du}{-\pi \sin \pi x} = dx$

ALTERNATIVE 1

$\int \sin^3 \pi x \cos^5 \pi x dx$

$= \int \sin^2 \pi x \sin \pi x \cos^5 \pi x dx$ ✓

$= \int (1 - \cos^2 \pi x) \sin \pi x \cos^5 \pi x dx$ ✓

$= \int \cos^5 \pi x \sin \pi x dx - \int \cos^7 \pi x \sin \pi x dx$ ✓✓

$= -\frac{\mathbf{1}}{\boldsymbol{\pi}} \int [\cos \boldsymbol{\pi x}]^5 (-\boldsymbol{\pi} \sin \boldsymbol{\pi x}) \boldsymbol{dx} + \frac{\mathbf{1}}{\boldsymbol{\pi}} \int [\cos \boldsymbol{\pi x}]^7 (-\boldsymbol{\pi} \sin \boldsymbol{\pi x}) \boldsymbol{dx}$ ✓✓

$= -\frac{\boldsymbol{cos^6 \pi x}}{\boldsymbol{6\pi}} + \frac{\boldsymbol{cos^8 \pi x}}{\boldsymbol{8\pi}} + \mathbf{C}$ ✓ ✓

ALTERNATIVE 2

$$\int \sin^3 \pi x \cos^5 \pi x dx$$

$$= \int \sin^3 \pi x (\cos^2 \pi x)^2 \cos \pi x dx \checkmark$$

$$= \int \sin^3 \pi x (1 - \sin^2 \pi x)^2 \cos \pi x dx \checkmark$$

$$= \int \sin^3 \pi x (1 - 2\sin^2 \pi x + \sin^4 \pi x) \cos \pi x dx \checkmark$$

$$= \int \sin^3 \pi x \cos \pi x dx - 2\int \sin^5 \pi x \cos \pi x dx + \int \sin^7 \pi x \cos \pi x dx \checkmark$$

$$= \frac{1}{\pi}\int [\sin \pi x]^3 (\pi \cos \pi x) dx - \frac{2}{\pi}\int [\sin \pi x]^5 (\pi \cos \pi x) dx + \frac{1}{\pi}\int [\sin \pi x]^7 (\pi \cos \pi x) dx \checkmark$$

$$= \frac{\sin^4 \pi x}{4\pi} - \frac{\sin^6 \pi x}{3\pi} + \frac{\sin^8 \pi x}{8\pi} + C \checkmark\checkmark\checkmark$$

ALTERNATIVE 3

$$\int \sin^3 \pi x \cos^5 \pi x dx$$

$$= \int \sin^3 \pi x (\cos^2 \pi x)^2 \cos \pi x dx \checkmark$$

$$= \int \sin^3 \pi x (1 - \sin^2 \pi x)^2 \cos \pi x dx \checkmark$$

$$= \int \sin^3 \pi x (1 - 2\sin^2 \pi x + \sin^4 \pi x) \cos \pi x dx \checkmark$$

$$= \int \sin^3 \pi x \cos \pi x dx - 2\int \sin^5 \pi x \cos \pi x dx + \int \sin^7 \pi x \cos \pi x dx \checkmark$$

$$= \frac{1}{\pi}\int u^3 du - \frac{2}{\pi}\int u^5 du + \frac{1}{\pi}\int u^7 du \checkmark$$

$$u = \sin \pi x \Rightarrow \frac{du}{dx} = \pi \cos \pi x \checkmark$$

$$= \frac{u^4}{4\pi} - \frac{u^6}{3\pi} + \frac{u^8}{8\pi} + C \checkmark$$

$$= \frac{\sin^4 \pi x}{4\pi} - \frac{\sin^6 \pi x}{3\pi} + \frac{\sin^8 \pi x}{8\pi} + C \checkmark$$

2.5 $$\int \sin^2\left(\frac{3x}{5}\right)dx = \frac{x}{2} - \frac{\sin\left(\frac{10x}{3}\right)}{\frac{20}{3}} + C \checkmark\checkmark$$ (2)

ALTERNATIVE

$$\int \sin^2\left(\frac{3x}{5}\right)dx = \int \left[\frac{1}{2} - \frac{1}{2}\cos\left(\frac{10x}{3}\right)\right]dx$$

$$= \frac{x}{2} - \frac{\sin\left(\frac{10x}{3}\right)}{\frac{20}{3}} + C \checkmark\checkmark$$

[36]

QUESTION 3

3.1 Let $\frac{2x^2-7x-12}{x(x+2)(x-3)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-3}$ ✓✓

$$2x^2 - 7x - 12 = A(x+2)(x-3) + Bx(x-3) + Cx(x+2) \checkmark$$

$x = 0 \Rightarrow A = 2$✓

$x = 3 \Rightarrow C = -1$✓

$x = -2 \Rightarrow B = 1$✓

$$\int \frac{2x^2-7x-12}{x(x+2)(x-3)} = 2\int \frac{1}{x}dx + \int \frac{1}{x+2}dx - \int \frac{1}{x-3}dx \checkmark$$

$$= 2\ln x + \ln(x+2) - \ln(x-3) + C \checkmark\checkmark\checkmark$$ (10)

3.2

$$x^2 - x - 6 \overline{)\, x^3 + 4x + 9} \quad x + 1 \checkmark$$

$$x^3 - x^2 - 6x$$

$$x^2 + 10x + 9 \checkmark$$

$$x^2 - x - 6$$

$$11x + 15 \checkmark$$

$\therefore \frac{x^3+4x+9}{x^2-x-6} = (x+1) + \frac{11x+15}{x^2-x-6}$ ✓ **where**

$x^2 - x - 6 = (x+2)(x-3)$✓

Let $\frac{11x+15}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$ ✓

$11x+15 = A(x-3) + B(x+2)$ ✓

$x = 3 \Rightarrow \mathrm{B} = \frac{48}{5}$ ✓

$x = -2 \Rightarrow \mathrm{A} = \frac{7}{5}$ ✓

$$\Rightarrow \int \frac{x^3+4x+9}{x^2-x-6}dx = \int (x+1)dx + \frac{7}{5}\int \frac{dx}{x+2} + \frac{48}{5}\int \frac{dx}{x-3}$$ ✓

$$= \frac{x^2}{2} + x + \frac{7}{5}\ln(x+2) + \frac{48}{5}\ln(x-3) + \mathrm{C}$$ ✓✓✓✓ (14)

[24]

QUESTION 4

4.1 $\frac{ds}{dt} + \frac{2}{t+1}s = 3 + \frac{1}{(t+1)^3}$ ✓✓

$\mathrm{IF} = e^{2\int \frac{dt}{t+1}} = e^{2\ln(t+1)} = e^{\ln(t+1)^2} = (t+1)^2$ ✓✓✓

$\frac{d}{dt}\left[s(t+1)^2\right] = \left[3 + \frac{1}{(t+1)^3}\right](t+1)^2$ ✓

$\left[s(t+1)^2\right] = 3\int (t+1)^2\,dt + \int \frac{dt}{t+1}$ ✓✓

$= (t+1)^3 + \ln(t+1) + C$ ✓✓

$2(4+1)^2 = (4+1)^3 + \ln(4+1) + \mathrm{C} \Rightarrow \mathrm{C} = -76.609$ ✓

$\therefore s(t+1)^2 = (t+1)^3 + \ln(t+1) - 76.609$ ✓ (12)

4.2 $r^2 + 6r + 9 = 0$ ✓

$r = -3$ ✓

$y_c = Ae^{-3x} + Bxe^{-3x}$ ✓

$y_p = Cx^2 + Dx + E$ ✓

$\Rightarrow y'_p = 2Cx + D$ ✓ and

$y''_p = 2C$ ✓

$2C + 12Cx + 6D + 9Cx^2 + 9Dx + 9E = x^2$ ✓

$x^2 : 9C = 1 \Rightarrow C = \frac{1}{9}$ ✓

$x : 12C + 9D = 0 \Rightarrow D = \frac{-4}{27}$ ✓

Constant : $2C + 6D + 9E = 0 \Rightarrow E = \frac{2}{27}$ ✓

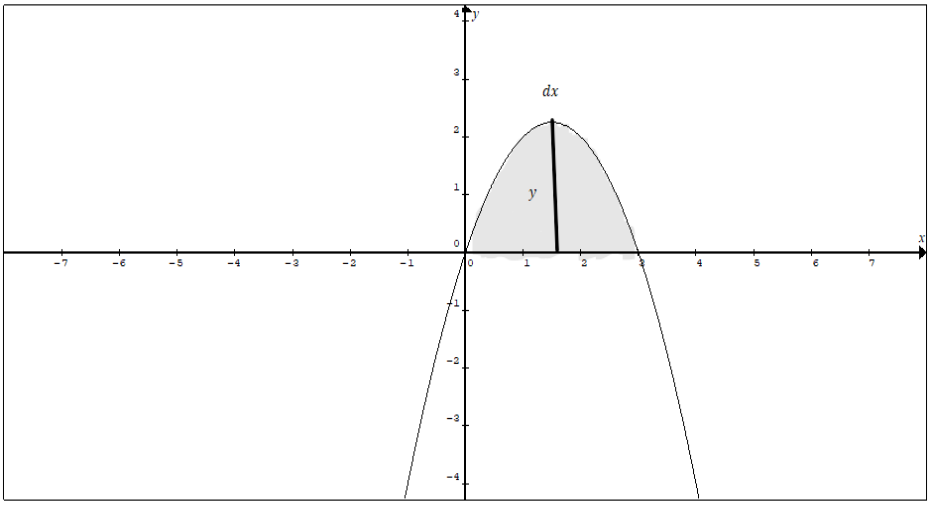
$y_p = \frac{1}{9}x^2 - \frac{4}{27}x + \frac{2}{27}$ ✓

$\therefore y = Ae^{-3x} + Bxe^{-3x} + \frac{1}{9}x^2 - \frac{4}{27}x + \frac{2}{27}$ ✓

(12)
[24]

QUESTION 5

5.1 5.1.1



Graph ✓; Shaded area ✓ ; Strip ✓ ; coordinates (0;0) and (3;0) ✓ (4)

5.1.2 $V_y = 2\pi \int_a^b xydx$ **OR** $V_y = 2\pi \int_a^b x(y_1 - y_2)dx$ ✓

$$V_y = 2\pi\int_0^3 x\left(3x - x^2\right)dx \checkmark\checkmark$$

$$= 2\pi\int_0^3 \left(3x^2 - x^3\right)dx \checkmark$$

$$= 2\pi\left[x^3 - \frac{x^4}{4}\right]_0^3 \checkmark\checkmark$$

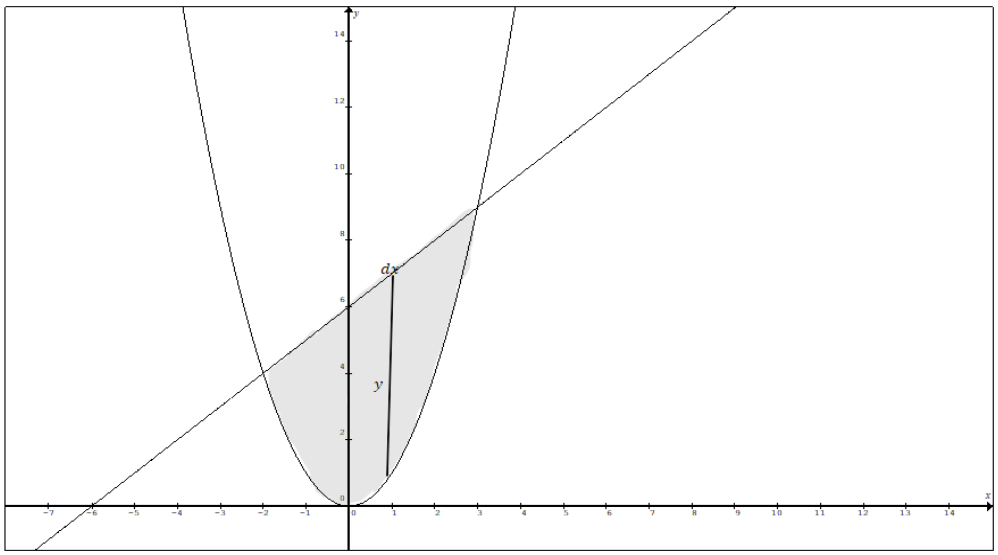
$$= 2\pi\left[3^3 - \frac{3^4}{4}\right] \checkmark$$

$$= 42.412\,(\text{units})^3 \checkmark$$

$$= \frac{27}{2}\pi(\text{units})^3$$

(8)

5.2.1



$\boldsymbol{x^2 = x + 6 \Rightarrow x = -2}$ **or** $\boldsymbol{x = 3}$

and $\boldsymbol{y = 4}$ **or** $\boldsymbol{y = 9}$

$\therefore \boldsymbol{(-2;4)}\checkmark$ ***and*** $\boldsymbol{(3;9)}\checkmark$

Graphs;✓✓ Shaded area;✓ Strip✓ (6)

5.2.2 $$A_x = \int_a^b (y_1 - y_2)dx \checkmark$$

$$= \int_0^3 \left(x + 6 - x^2\right)dx \checkmark\checkmark$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]_0^3 \checkmark$$

$$= \left[\frac{(3)^2}{2} + 6(3) - \frac{(3)^3}{3}\right] - \left[\frac{(0)^2}{2} + 6(0) - \frac{(0)^3}{3}\right] \checkmark$$

$$= 13.5\,(\text{units})^2 \checkmark$$

$$= \frac{27}{2}(\text{units})^2 \quad (6)$$

5.2.3 $$I_y = \int_a^b r^2 dA \checkmark$$

$$I_y = \int_0^3 x^2\left(x + 6 - x^2\right)dx \quad \checkmark\checkmark\checkmark$$

$$= \int_0^3 \left(x^3 + 6x^2 - x^4\right)dx \quad \checkmark$$

$$= \left[\frac{1}{4}x^4 + 2x^3 - \frac{1}{5}x^5\right]_0^3 \quad \checkmark$$

$$= \left[\frac{1}{4}(3)^4 + 2(3)^3 - \frac{1}{5}(3)^5 - 0\right] \quad \checkmark$$

$$= 25.65\,(\text{units})^3 \checkmark$$

$$= \frac{513}{20}(\text{units})^3 \quad (8)$$

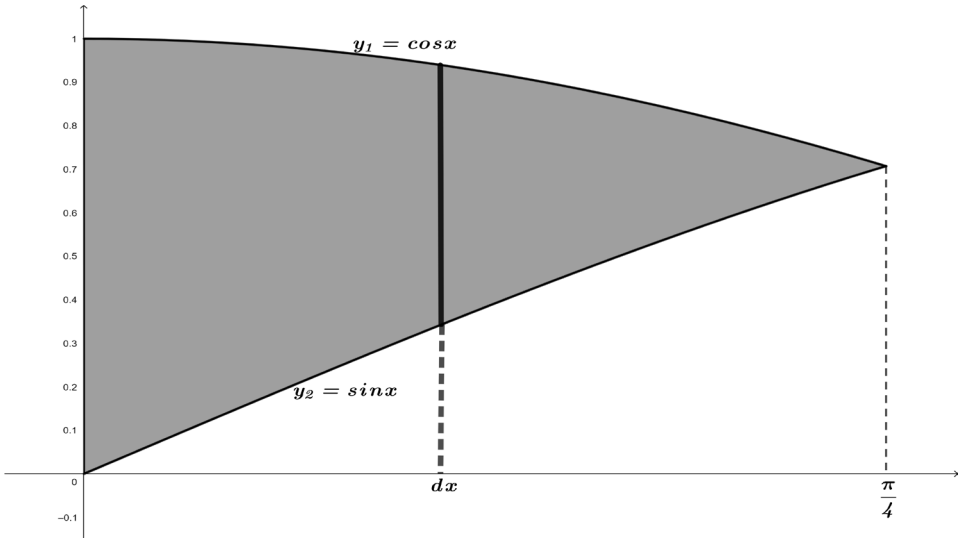
5.2.4 $$I_y = \frac{25.65}{13.5}A \checkmark$$

$$= 1.9A\,(\text{units})^3 \checkmark$$

$$= \frac{19}{10}A\,(\text{units})^3 \quad (2)$$

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5.3 5.3.1



Graphs✓✓
Shaded area✓
Strip✓

$\sin x = \cos x \Rightarrow \tan x = 1$

$\therefore x = \frac{\pi}{4}$ and $y = \frac{\sqrt{2}}{2}$ ✓✓ (6)

5.3.2

$$\mathbf{V_x = \pi \int_a^b (y_1^2 - y_2^2) dx} \checkmark$$

$$\mathbf{= \pi \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x) dx} \checkmark\checkmark\checkmark$$

$$\mathbf{= \pi \int_0^{\frac{\pi}{4}} (\cos 2x) dx} \checkmark$$

$$\mathbf{= \pi \left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}}} \checkmark$$

$$\mathbf{= \pi \left[\frac{\sin \frac{\pi}{2}}{2}\right] - \pi \left[\frac{\sin 0}{2}\right]} \checkmark$$

$$\mathbf{= \frac{\pi}{2}\ (units)^3} \checkmark$$

$$\mathbf{= 1.571\ (units)^3} \quad (8)$$

ALTERNATIVE 1

$$V_x = \pi \int_a^b (y_1^2 - y_2^2)dx \checkmark$$

$$= \pi \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)dx \checkmark\checkmark\checkmark$$

$$= \pi \left[\frac{x}{2} + \frac{\sin 2x}{4} - \frac{x}{2} + \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{4}} \checkmark$$

$$= \pi \left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}} \checkmark$$

$$= \pi \left[\frac{\sin \frac{\pi}{2}}{2}\right] - \pi \left[\frac{\sin 0}{2}\right] \checkmark$$

$$= \frac{\pi}{2} \text{ (units)}^3 \quad \checkmark$$

$$= 1.571 \text{ (units)}^3$$

ALTERNATIVE 2

$$\mathbf{V_x = \pi \int_a^b (y_1^2 - y_2^2)dx} \checkmark$$

$$\mathbf{= \pi \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)dx} \checkmark\checkmark$$

$$\mathbf{= \pi \int_0^{\frac{\pi}{4}} \left(\frac{1}{2} + \frac{1}{2}\cos 2x - \frac{1}{2} + \frac{1}{2}\cos 2x\right)dx} \checkmark$$

$$\mathbf{= \pi \int_0^{\frac{\pi}{4}} (\cos 2x)dx} \checkmark$$

$$\mathbf{= \pi \left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}}} \checkmark$$

$$\mathbf{= \pi \left[\frac{\sin \frac{\pi}{2}}{2}\right] - \pi \left[\frac{\sin 0}{2}\right]} \checkmark$$

$$\mathbf{= \frac{\pi}{2} \text{ (units)}^3} \checkmark$$

$$\mathbf{= 1.571 \text{ (units)}^3}$$

5.3.3 $$I_x = \int_a^b r^2 dm = \rho \int_a^b r^2 dV \checkmark$$

$$= \frac{\rho\pi}{2} \int_a^b (y_1^4 - y_2^4) dx \checkmark$$

$$= \frac{\rho\pi}{2} \int_0^{\frac{\pi}{4}} (\cos^4 x - \sin^4 x) dx \checkmark\checkmark\checkmark$$

$$= \frac{\rho\pi}{2} \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) dx \checkmark$$

$$= \frac{\rho\pi}{2} \int_0^{\frac{\pi}{4}} (\cos 2x) dx \checkmark$$

$$= \frac{\rho\pi}{2} \left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}} \checkmark$$

$$= \frac{\rho\pi}{2} \left[\frac{\sin \frac{\pi}{2}}{2}\right] - \frac{\rho\pi}{2} \left[\frac{\sin 0}{2}\right] \checkmark$$

$$= \frac{\rho\pi}{4} \text{ (units)}^5 \checkmark$$

$$= 0.785\rho \text{ (units)}^5 \quad (10)$$

ALTERNATIVE

$$\mathbf{I_x = \int_a^b r^2 dm = \rho \int_a^b r^2 dV} \ \checkmark$$

$$= \frac{\rho\pi}{2}\int_a^b (y_1^4 - y_2^4)dx \ \checkmark$$

$$= \frac{\rho\pi}{2}\int_0^{\frac{\pi}{4}} (\cos^4 x - \sin^4 x)dx \ \checkmark\checkmark$$

$$= \frac{\rho\pi}{2}\int_0^{\frac{\pi}{4}} [(\cos^2 x)^2 - (\sin^2 x)^2]dx \checkmark$$

$$= \frac{\rho\pi}{2}\int_0^{\frac{\pi}{4}} \left[\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)^2 - \left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)^2\right]dx \checkmark$$

$$= \frac{\rho\pi}{2}\int_0^{\frac{\pi}{4}} \left[\frac{1}{4}+\frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x - \frac{1}{4} + \frac{1}{2}\cos 2x - \frac{1}{4}\cos^2 2x\right]dx$$

$$= \frac{\rho\pi}{2}\int_0^{\frac{\pi}{4}} (\cos 2x)dx \ \checkmark$$

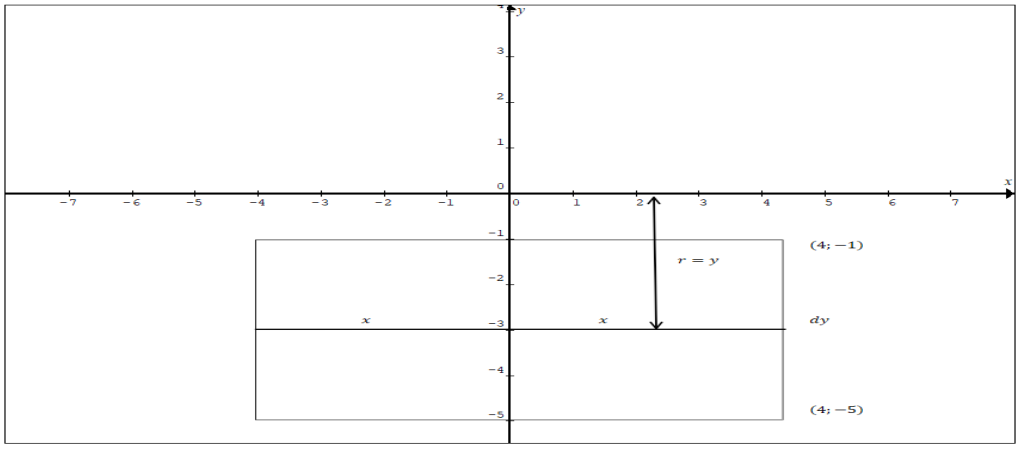
$$= \frac{\rho\pi}{2}\left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}} \ \checkmark$$

$$= \frac{\rho\pi}{2}\left[\frac{\sin\frac{\pi}{2}}{2}\right] - \frac{\rho\pi}{2}\left[\frac{\sin 0}{2}\right] \ \checkmark$$

$$= \frac{\rho\pi}{4}\ (\text{units})^5 \ \checkmark$$

$$= 0.785\rho\,(\text{units})^5$$

5.4 5.4.1



Bottom at $(0;-5)$ ✓; Top at $(0;-1)$ ✓; Coordinates $(4;-1)$ and $4;-5)$ ✓$x = 4$ or strip ✓ (4)

5.4.2 First moment $= \int rdA$ ✓

$$= 8\int_{-5}^{-1} ydy \checkmark\checkmark\checkmark$$

$$= 8\left[\frac{y^2}{2}\right]_{-5}^{-1} \checkmark$$

$$= 8\left[\frac{(-1)^2}{2} - \frac{(-5)^2}{2}\right] \checkmark\checkmark$$

$$= -96(\text{units})^3 \checkmark$$ (8)

5.4.3 Second moment $= \int r^2 dA$ ✓

$$= 8\int_{-5}^{-1} y^2 dy \checkmark\checkmark\checkmark$$

$$= 8\left[\frac{y^3}{3}\right]_{-5}^{-1} \checkmark$$

$$= 8\left[\frac{(-1)^3}{3} - \frac{(-5)^3}{3}\right] \checkmark\checkmark$$

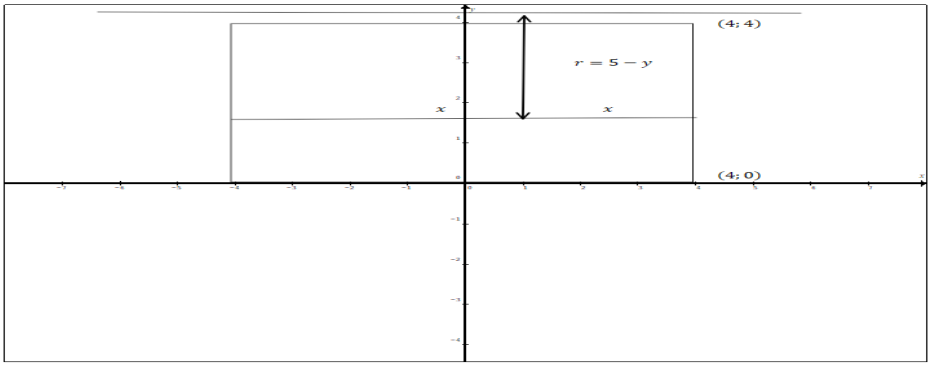
$$= 330.667 \checkmark$$

$$= \frac{992}{3} (\text{units})^4$$

$$\therefore \bar{\bar{y}} = \frac{330.667}{-96} = -3.444 = -\frac{31}{9} \checkmark\checkmark$$ (10)

ALTERNATIVE

5.4 5.4.1



Water level at $(0;5)$ ✓ ; Top at $(0;4)$ ✓ ; coordinates $(4;4)$ and $4;0)$ ✓
$x = 4$ or strip ✓ (4)

5.4.2 First moment $= \int rdA$ ✓

$$= 8\int_0^4 (5-y)\,dy \quad ✓\ ✓\ ✓$$

$$= 8\left[5y - \frac{y^2}{2}\right]_0^4 \ ✓✓$$

$$= 8\left[5(4) - \frac{(4)^2}{2}\right] ✓$$

$$= 96(\text{units})^3 \ ✓$$ (8)

5.4.3 Second moment $= \int r^2 dA$ ✓

$$= 8\int_0^4 (5-y)^2\,dy \quad ✓✓✓$$

$$= 8\int_0^4 (25 - 10y + y^2)\,dy \ ✓$$

$$= 8\left[25y - 5y^2 + \frac{y^3}{3}\right]_0^4 ✓$$

$$= 8\left[25(4) - 5(4)^2 + \frac{(4)^3}{3}\right] ✓$$

$$= 330.667 \ ✓$$

$$= \frac{992}{3}\ (\text{units})^4$$

$$\therefore \bar{\bar{y}} = \frac{330.667}{-96} = -3.444 = -\frac{31}{9} \ ✓✓$$ (10)

[80]

QUESTION 6

6.1 $y = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\frac{dy}{dx} = \frac{1}{2}(e^{x} - e^{-x})$ ✓

$\left(\frac{dy}{dx}\right)^{2} = \left[\frac{1}{2}(e^{x} - e^{-x})\right]^{2} = \frac{1}{4}(e^{2x} - 2 + e^{-2x})$ ✓

$1 + \left(\frac{dy}{dx}\right)^{2} = 1 + \frac{1}{4}(e^{2x} - 2 + e^{-2x})$

$= \frac{1}{4}e^{2x} - \frac{1}{2} + \frac{1}{4}e^{-2x} + 1$ ✓

$= \frac{1}{4}e^{2x} + \frac{1}{2} + \frac{1}{4}e^{-2x}$ ✓

$= \frac{1}{4}\left(e^{2x} + 2 + e^{-2x}\right)$

$= \frac{1}{4}\left(e^{x} + e^{-x}\right)^{2}$ ✓

$S = \int_{a}^{b} \sqrt{\left(\frac{dy}{dx}\right)^{2} + 1}\,dx$ ✓

$= \int_{0}^{2} \sqrt{\frac{1}{4}\left(e^{x} + e^{-x}\right)^{2}}\,dt$ ✓✓

$= \frac{1}{2}\int_{0}^{2}\left(e^{x} + e^{-x}\right)dx$ ✓

$= \frac{1}{2}\left[e^{x} - e^{-x}\right]_{0}^{2}$ ✓

$= \frac{1}{2}\left[e^{2} - e^{-2} - e^{0} + e^{0}\right]$ ✓

$= 3.627$ ✓ (12)

6.2 $x = \ln(\sec t + \tan t) - \sin t$ and $y = \cos t$

$$\frac{dx}{dt} = \frac{1}{\sec t + \tan t} \times (\sec t \tan t + \sec^2 t) - \cos t \checkmark \quad \text{and} \quad \frac{dy}{dt} = -\sin t \checkmark$$

$$= \frac{\sec t(\sec t + \tan t)}{\sec t + \tan t} - \cos t$$

$$= \sec t - \cos t \checkmark$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \sin^2 t + \sec^2 t - 2\sec t \cos t + \cos^2 t \checkmark$$

$$= \sin^2 t + \cos^2 t + \sec^2 t - 2$$

$$= \sec^2 t - 1$$

$$= \tan^2 t \checkmark$$

$$A_x = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= 2\pi \int_0^{\frac{\pi}{3}} \cos t \sqrt{\tan^2 t}\, dt \checkmark\checkmark$$

$$= 2\pi \int_0^{\frac{\pi}{3}} \cos t \times \frac{\sin t}{\cos t}\, dt \checkmark$$

$$= 2\pi \int_0^{\frac{\pi}{3}} \sin t\, dt \checkmark$$

$$= 2\pi \left[-\cos t\right]_0^{\frac{\pi}{3}} \checkmark$$

$$= 2\pi \left[-\cos\left(\frac{\pi}{3}\right) + \cos 0\right] \checkmark$$

$$= 3.142 \checkmark$$

$$= \pi(units)^2$$

(12)
[24]

TOTAL: 200

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